Anisotropy parameter

In general, orbits of stars in pressure supported systems need not be isotropic, i.e. ($\sigma_{l.o.s.} \neq \sigma_{max}$)

σ is different in different directions

σ_r, $\sigma_t = \sigma_\theta = \sigma_\phi$ radial & tangential

In general

$$M(r) = \frac{r\sigma_r^2}{G}\left(\gamma_* + \gamma_\sigma - 2\beta\right)$$

where $\gamma_* = -\frac{d\ln n_*}{d\ln r}$ logarithmic slope of stellar density profile (measurable)

$\gamma_\sigma = -\frac{d\ln \sigma_r^2}{d\ln r}$ logarithmic slope of $\sigma_r^2(r)$ [radial, not l.o.s]

$\beta = 1 - \frac{\sigma_t^2}{\sigma_r^2}$ anisotropy parameter

σ_t = " " " tangential (θ, ϕ) direction

σ_r = velocity dispersion in radial direction within body

extreme cases ~~β~~

circular orbits	$\sigma_r = 0$	$\beta = -\infty$
isotropy	$\sigma_r = \sigma_t$	$\beta = 0$ — implicitly assumed in most virial estimates: $\sigma_{los} = \sigma_r = \sigma_t$
radial	$\sigma_t = 0$	$\beta = 1$

more generally, $\beta < 0$ "tangential" bias

$\beta > 0$ "radial" bias

β can vary with r. This is the biggest systematic uncertainty in mass modeling elliptical galaxies

Scaling relations

TF, Faber-Jackson : $L \sim \sigma^4$

Fundamental Plane

Virial Fundamental Plane : $M \sim \sigma^2 R \sim \Sigma R^2$

$\sigma^2 \sim \Sigma R$

observed FP "titled" $R \sim \sigma^2 \Sigma^{-1}$

wrt virial version : $R \sim \sigma^{1.4} I^{-0.8}$

depends on who you ask

Baryon fractions :

universal cosmic baryon fraction $f_b = \frac{\Omega_b}{\Omega_M}$ — baryon density / gravitating mass density

in individual galaxies, we have the "disk" baryon fraction:

$$f_d = \frac{M_b}{f_b M_{tot}} = \frac{M_* + M_g \text{ (observed)}}{f_b \text{ (Mtotal dynamical mass)}}$$

don't know what M_{tot} is; typically associate it with M_{200}

For rich clusters of galaxies (w/ $M_{tot} \approx 10^{15} M_\odot$)

it works out :

$$M_b \approx f_b M_{tot} \quad \text{so} \quad f_d \approx 1$$

If we look at smaller systems, f_d departs systematically from unity ($f_d < 1$) becoming ever smaller for smaller objects

$$f_d \approx \tanh\left(\frac{V_f}{900}\right)$$ works ok — maybe 700 km/s