Disk Stability

Ostriker & Peebles (1973)

- very early, rather primitive numerical simulations
- showed that thin, cold, rotating disks are unstable to the growth of $m=2$ modes (the bar instability).
- this problem remains unresolved (arXiv: 1601.03406).

O & P solution was to embed spiral disks in a deep potential well – a dark matter halo.

Split kinetic energy into rotational T and pressure Π components

so

$$K = T + \frac{1}{2}\Pi$$ (T all in plane of disk. Π not)

$$K = \frac{1}{2}|W|$$ Virial equilibrium condition
kinetic energy = half the potential energy

O & P found stability was achieved when

$$t \lesssim 0.14 \qquad t \equiv \frac{T}{|W|}$$ the rotational component of the KE normalized to the P.E.

This is crude & almost certainly wrong in detail, but was important early evidence for Dark Matter

e.g.: the Milky Way has $V_c \approx 220$ km s^{-1}, $\sigma \approx 20$ km s^{-1} — $T \sim V_c^2$, $\Pi \sim \sigma^2$

working those numbers, $t \approx 0.45$ [0.5 is the maximum]

Such a cold disk should be wildly unstable, yet spiral galaxies persist over a Hubble time

Dark Matter halos provide stability (maybe) provided $\Pi \gg T$: they must be dynamically hot, quasi-spherical, and dominate the mass

Other stability criteria

<u>Global</u> stability against non-axis-symmetric perturbations

$$X_m = \frac{\kappa^2 R}{2\pi m G\Sigma}$$

Goldreich & Tremaine '78, '79
Toomre 1981

m is mode number of perturbation − $m = 2$ for a bar or grand design spiral

κ is the epicyclic frequency
Σ the surface density

The higher the surface density Σ, the less stable the disk

The precise stability criterion is determined numerically and depends on the potential

$X > 3$ required for stability for a flat rotation curve

$X > 1$ suffices for a rising rotation curve

Lowest modes m suppressed first (Athanasoula et al 1987)

So a very high surface density disk is unstable
a merely high " " " can have $m=2$ modes (bars, grand design spirals)
a low " " " will have $m=2$ suppressed.

It is tempting to conclude the grand design spirals are nearly maximal − high enough surface density to drive $m=2$ modes
Perhaps flocculent spirals are lower Σ so $m=2$ suppressed

Bars & spiral arms are the natural consequence of disk self-gravity. If the halo becomes too dominant, all modes get suppressed.

The amplitudes of modes are usually expressed as Fourier components

$$A_m = \frac{1}{N} \sum_{j=1}^{N} e^{i\left[m\theta_j + p\ln(r_j)\right]}$$

A is the amplitude of mode m for N stars

m = 2 gives a bar for $p = 0$
" " grand design spiral for $p > 0$

p is the "pitch angle" of a "logarithmic spiral"

LOCAL disk stability criterion "Toomre Q"
(Toomre 1964)

$$Q = \frac{\sigma_r \, K}{3.36\, G \Sigma}$$

σ_r = radial velocity dispersion
Σ = local disk surface density
K = epicyclic frequency

3.36 Constant chosen so that stability occurs for $Q \gtrsim 1$

Numerically, $Q \approx 1.4$ in solar neighborhood
Instability $Q \lesssim 1$ sometimes thought to be a criterion for star formation.

EPICYCLIC FREQUENCY

$$K^2 = \frac{\partial^2 \Phi}{\partial R^2} = R\frac{\partial \Omega^2}{\partial R} + 4\Omega^2$$

where $\Omega = \frac{V}{R}$ is the orbital frequency.

K is the frequency with which a star oscillates (in the plane) about the "guiding center" of a closed (circular) orbit.