

Oort constants $A \neq B$

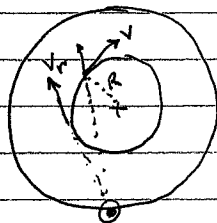
local measures of Galactic kinematics

A is a measure of the local SHEAR (differential rotation)

$$A \equiv -\frac{1}{2} \left[R \frac{dR}{dR} \right]_{R_0} = \frac{1}{2} \left(\frac{V}{R} - \frac{dV}{dR} \right)_{R_0}$$

This arises because the terminal velocity along the line of sight is related to position by

$$V_r = R_0 \sin l \left(\frac{V}{R} - \frac{V_0}{R_0} \right)$$



For stars near to us ($d \ll R_0$),

$R - R_0$ is small. Doing a bit of trigonometry

$$V_r \approx d \sin(2l) \left[-\frac{R}{2} \frac{dV}{dR} \right]_{R_0} = A d \sin(2l)$$

Handy for nifty reason:

If you know A , you can get a kinematic estimate of d

Similarly, you can estimate A if you know d to a bunch of stars

Making a further trigonometric trick

(expanding around $l \approx 90^\circ$)

we get

$$V_r \approx 2AR_0(1 - \sin l) \quad \text{only valid near } l \approx 90^\circ$$

so by measuring V_r in the direction we're headed,

we get AR_0 . This combination is well measured

$$AR_0 \approx 115 \text{ km s}^{-1}$$

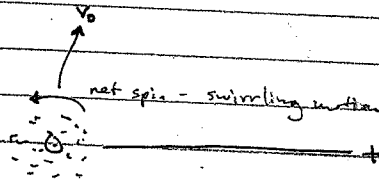
even if $A \neq R_0$ are separately uncertain

line of sight radial velocity

VORTICITY or CIRCULATION

Orbit constant B

measure of vorticity

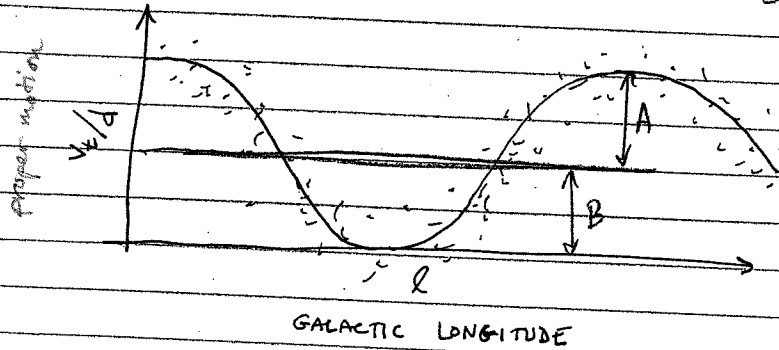


$$B = -\frac{1}{2} \left(\frac{V}{R} + \frac{dV}{dR} \right)_{R_0}$$

whereas A only depends on radial velocities v_r
need tangential velocities v_θ (proper motions) for B

$$B = \frac{v_\theta}{d} - A \cos(2\ell)$$

basically the "excess" circulation
motion not already attributable to
shear



Note that these \neq s are not independent:

$$\Omega_0 = \frac{V_0}{R_0} = A - B \quad \text{angular speed of LSR}$$

$$\frac{dV}{dR} \Big|_{R_0} = -(A + B) \quad \text{local slope of rotation curve}$$

$A \ll |B|$ $v(r)$ rising

$A \gg |B|$ $v(r)$ declining

$A = |B|$ flat $v(r)$

In non-axisymmetric potentials (e.g. barred spirals like the Milky Way) there can be a net radial flow around some arbitrary point as well as net circulation

V_ϕ = speed in circular direction

V_R = speed in radial direction

Now there can also be

C radial shear

Dark local divergence

C & K probably

a few $\text{km s}^{-1} \text{kpc}^{-1}$

Chandrasekhar 1942). At each position x in the Galaxy, there is a unique streaming velocity $v = (U, V)$ (here we ignore the possibility of orbit crossing). The streaming velocity at some point x in the Galaxy with respect to an observer at the Sun may be expanded in a Taylor series (with local Cartesian coordinates: \hat{e}_x and \hat{e}_y pointing in directions $l = 0^\circ$ and $l = 90^\circ$),

$$v = -v_0 + H \cdot x + \mathcal{O}(x^2), \quad (1)$$

where v_0 is the velocity of the Sun with respect to the local streaming and

$$H = \begin{pmatrix} \partial v_x / \partial x & \partial v_x / \partial y \\ \partial v_y / \partial x & \partial v_y / \partial y \end{pmatrix}_{x=0} \equiv \begin{pmatrix} K + C & A - B \\ A + B & K - C \end{pmatrix}. \quad (2)$$

The parameters A , B , C , and K are the Oort constants; they measure the local divergence (K), vorticity (B), and azimuthal (A) and radial (C) shear of the velocity field generated by closed orbits. The Oort constants may also be expressed in terms of cylindrical coordinates (R , φ) with the Sun at $(R_0, 0)$ (cf. Chandrasekhar 1942),

$$2A = \frac{v_\varphi}{R} - \frac{\partial v_\varphi}{\partial R} - \frac{1}{R} \frac{\partial v_R}{\partial \varphi}, \quad (3a)$$

$$2B = -\frac{v_\varphi}{R} - \frac{\partial v_\varphi}{\partial R} + \frac{1}{R} \frac{\partial v_R}{\partial \varphi}, \quad (3b)$$

$$2C = -\frac{v_R}{R} + \frac{\partial v_R}{\partial R} - \frac{1}{R} \frac{\partial v_\varphi}{\partial \varphi}, \quad (3c)$$

$$2K = \frac{v_R}{R} + \frac{\partial v_R}{\partial R} + \frac{1}{R} \frac{\partial v_\varphi}{\partial \varphi}, \quad (3d)$$

evaluated at the solar position (we use the convention that φ increases clockwise, i.e., in the direction of Galactic rotation). In the axisymmetric limit, $C = K = 0$,⁴ and

$$A_{\text{sym}} = \frac{1}{2} \left(\frac{v_\varphi}{R} - \frac{\partial v_\varphi}{\partial R} \right)_{R=R_0}, \quad (4a)$$

$$B_{\text{sym}} = \frac{1}{2} \left(-\frac{v_\varphi}{R} - \frac{\partial v_\varphi}{\partial R} \right)_{R=R_0}, \quad (4b)$$

equivalent to the equations actually given by Oort, where R_0 is the Sun's distance to the Galactic center. In an axisymmetric galaxy, the circular (closed) orbits have velocity $v_\varphi^2 = R \partial \Phi / \partial R$, and measurements of the Oort constants provide a direct constraint on the Galactic potential Φ . For instance, a harmonic potential results in solid-body rotation, $A = 0$, and B equal to the (constant) rotation fre-

Olling & Dehnen estimate $C \approx 10 \text{ km s}^{-1} \text{kpc}^{-1}$ - pretty big!
 Küster & Tremaine (1994) had estimated $C \approx |K| < 1 \text{ km s}^{-1} \text{kpc}^{-1}$ - very small!

Classical Integrals of Motion

conserved quantities:

Energy
Angular momentum
Third Integral *

* For many cases of interest (e.g., axisymmetric potentials) there is an effective 3rd integral of motion that can not generally be expressed analytically, but does limit the possible orbit families.

In cylindrical coordinates R, ϕ, z

$$\text{Energy: } E = \frac{1}{2}(V_R^2 + V_\phi^2 + V_z^2) + \Phi(R, z) \quad (\text{Hamiltonian representation})$$

$$\text{Angular momentum } L_z = R V_\phi = R^2 \Omega = R^2 \dot{\phi}$$

Since angular momentum is conserved, can write

$$\text{effective potential} \quad \Phi_{\text{eff}}(R, z) = \Phi(R, z) + \frac{L_z^2}{2R^2}$$

