DARK MATTER

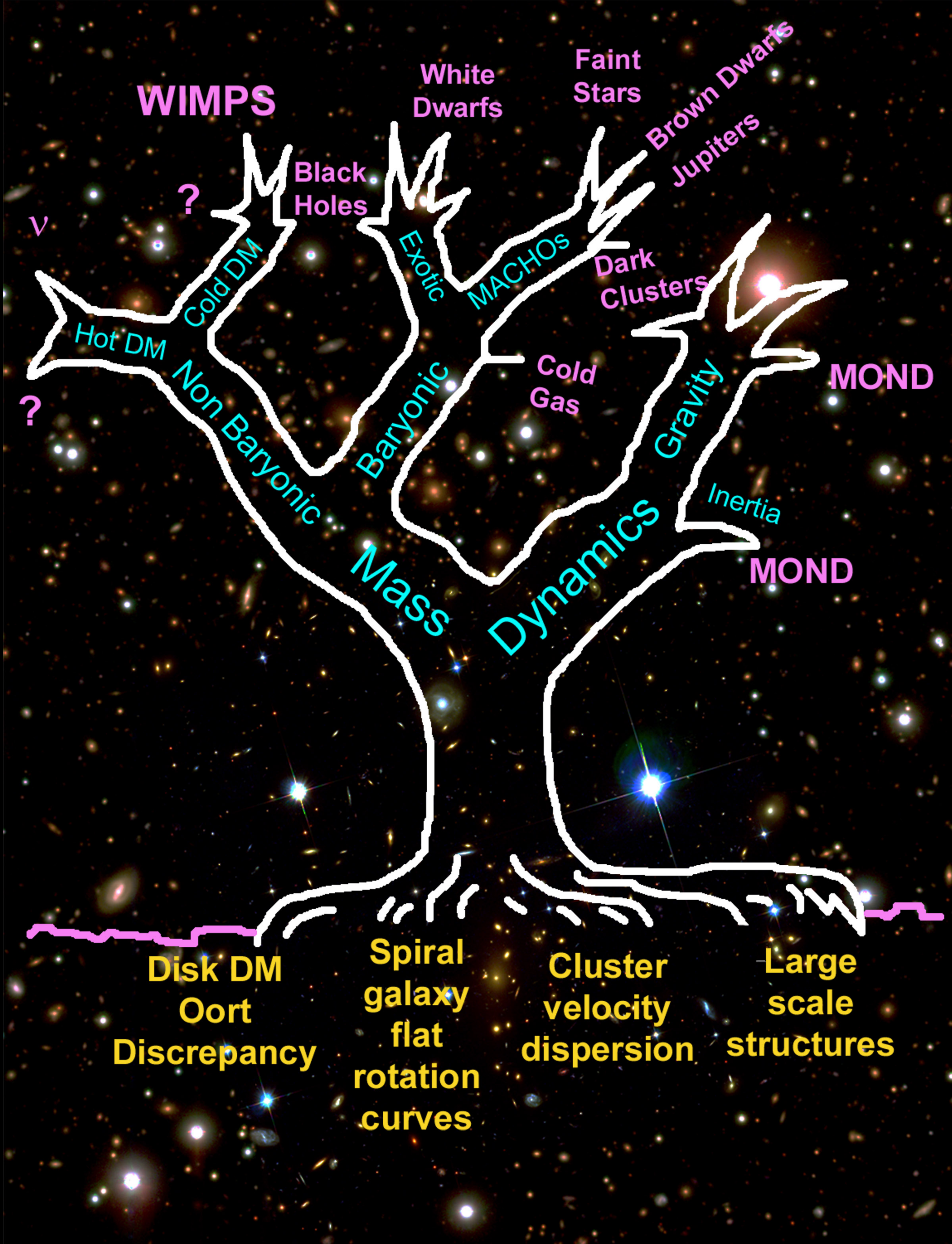
ASTR 333/433

TODAY

COSMOLOGICAL DARK MATTER

POWER SPECTRA

Homework 3
Due Thursday, April 9
(next time)



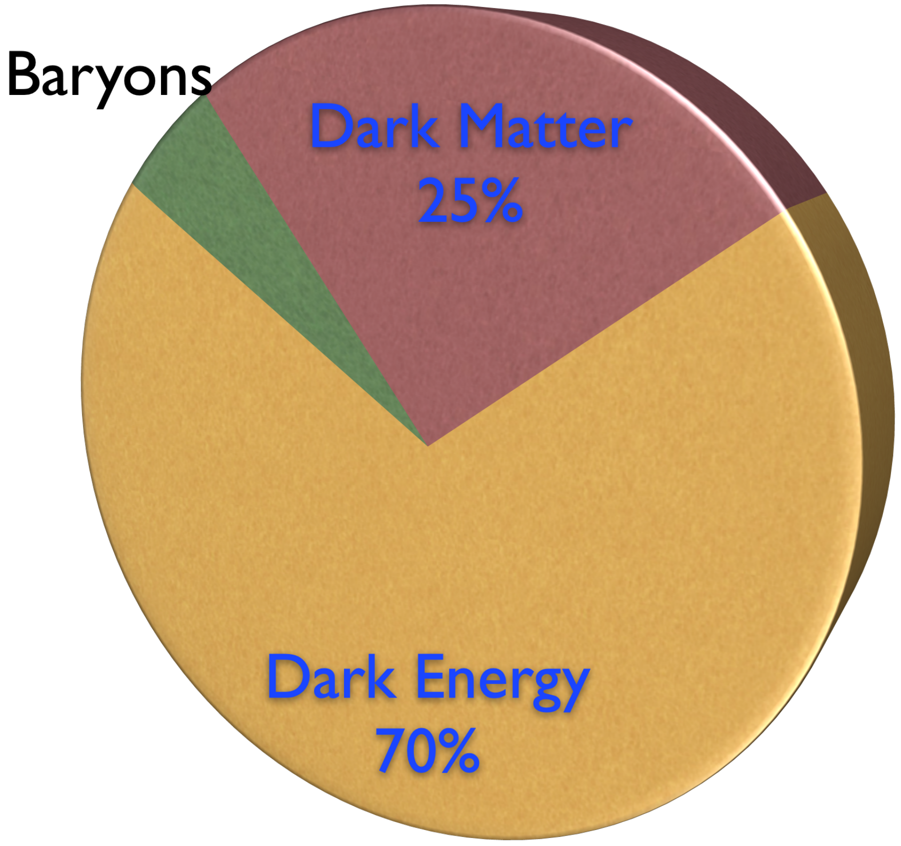
Empirical Pillars of the Hot Big Bang

1. Hubble Expansion
2. Big Bang Nucleosynthesis Ω_b
3. Cosmic Microwave Background

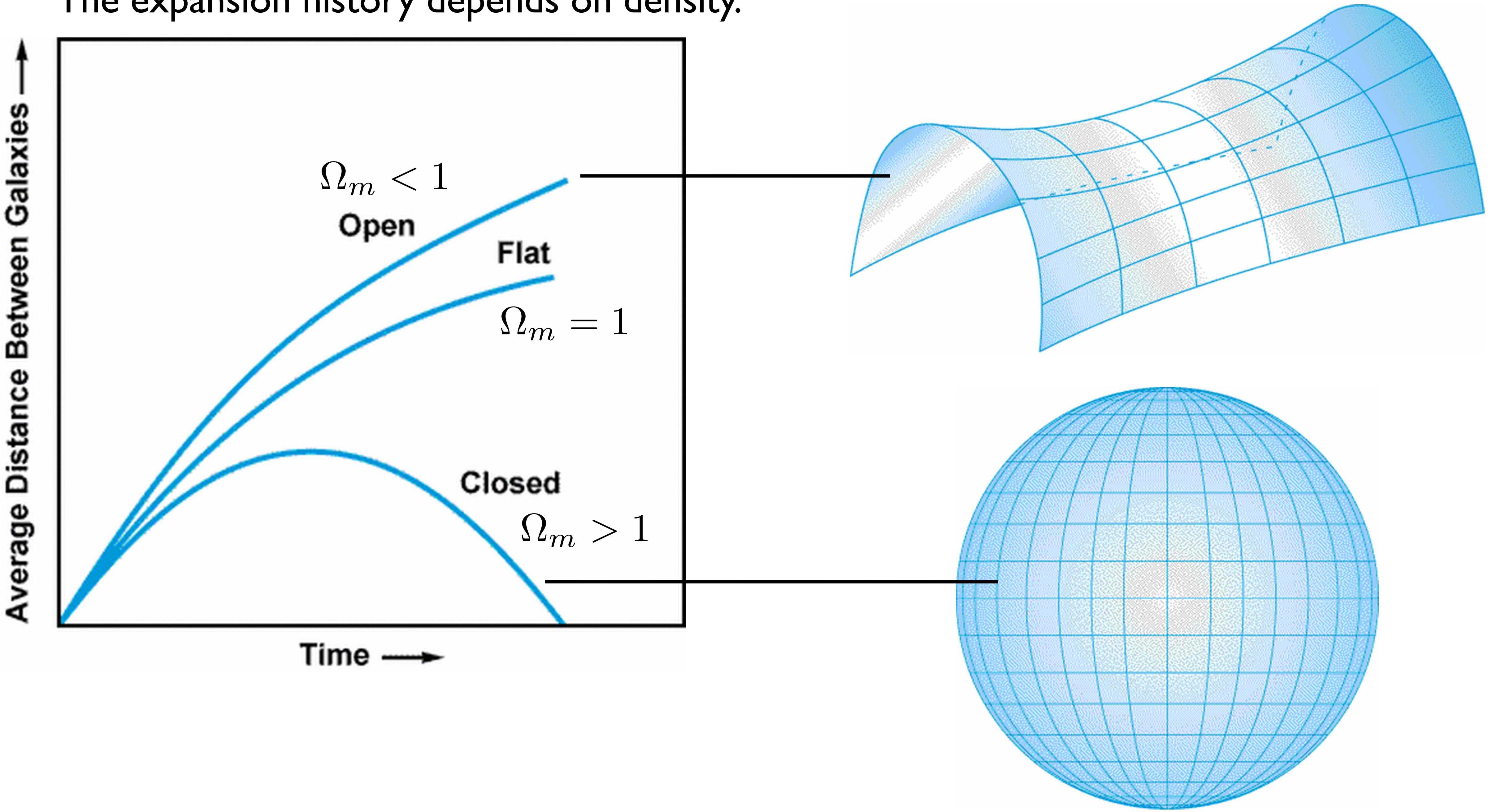
$\Omega_m = \Omega_b + \Omega_{DM}$

Auxiliary Hypotheses

- Dark matter Ω_{DM}
- Dark Energy Ω_Λ



The expansion history depends on density.



The expansion history and the geometry of the universe are both related to the density.

$$\Omega_m + \Omega_\Lambda + \Omega_k = 1$$

mass density

cosmological constant

curvature

Measurements of the gravitating mass density

- Cluster M/L
  - measure M/L of a cluster, combine with measured luminosity density of universe.
- Weak lensing
  - measure shear over large scales
- Peculiar Velocity Field
  - measure deviations from Hubble flow
- Power spectrum of galaxies
- CMB fits

Measurements of the gravitating mass density

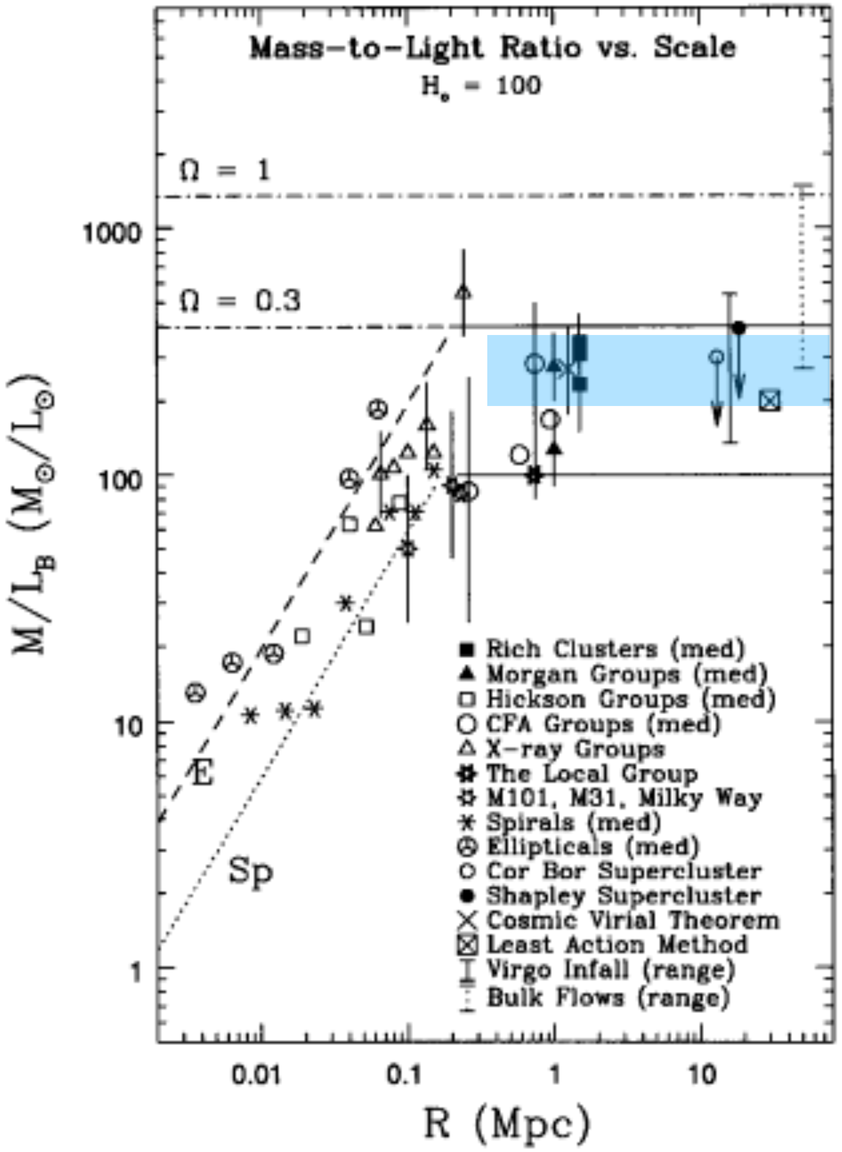
- Cluster M/L
  - measure M/L of a cluster, combine with measured luminosity density of universe.
  - j from integrating the luminosity function of galaxies:

$$\rho_m = \left(\frac{M}{L}\right) j$$

  - Also, cluster baryon fractions:

$$f_b = \frac{M_b}{M_{tot}} \quad \circ\!\!\longrightarrow \quad \Omega_m = \frac{\Omega_b}{f_b}$$

  - both assume clusters are representative of the whole.



$\Omega_m \approx \frac{1}{4}$

FIG. 2.—Composite mass-to-light ratio of different systems—galaxies, groups, clusters, and superclusters—as a function of scale. The best-fit $M/L_B \propto R$ lines for spirals and ellipticals (from Fig. 1) are shown. We present median values at different scales for the large samples of galaxies, groups and clusters, as well as specific values for some individual galaxies, X-ray groups, and superclusters. Typical 1 σ uncertainties and 1 σ scatter around median values are shown. Also presented, for comparison, are the M/L_B (or equivalently Ω) determinations from the cosmic virial theorem, the least action method, and the *range* of various reported results from the Virgocentric infall and large-scale bulk flows (assuming mass traces light). The M/L_B expected for $\Omega = 1$ and $\Omega = 0.3$ are indicated.

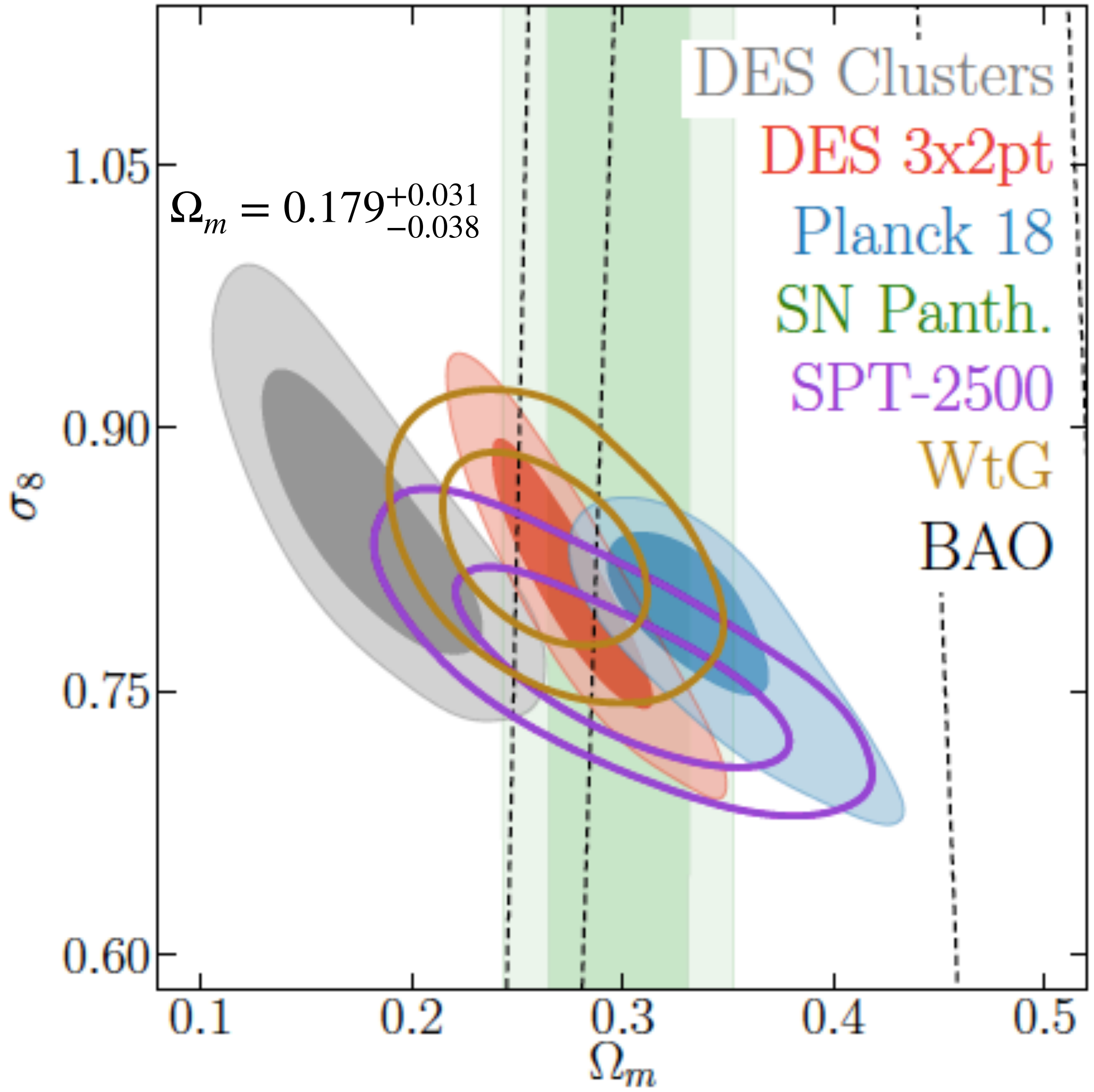
Bahcall, Lubin, & Dorman (1995)

Measurements of the gravitating mass density

- Weak lensing
  - – measure shear over large scales

$\Omega_m \approx 0.18 \pm 0.04$

Dark Energy Survey
arxiv:2002.11124



Measurements of the gravitating mass density

- Peculiar Velocity Field
  - – measure deviations from Hubble flow

in linear regime $\frac{\delta\rho}{\rho} \ll 1$

$$\frac{\delta v}{v} \approx \frac{d \ln H}{d \ln \rho} \frac{\delta \rho}{\rho} \approx -\frac{1}{3} \frac{\Omega_m^{0.6}}{b} \frac{\delta \rho_g}{\rho_g}$$

peculiar velocity

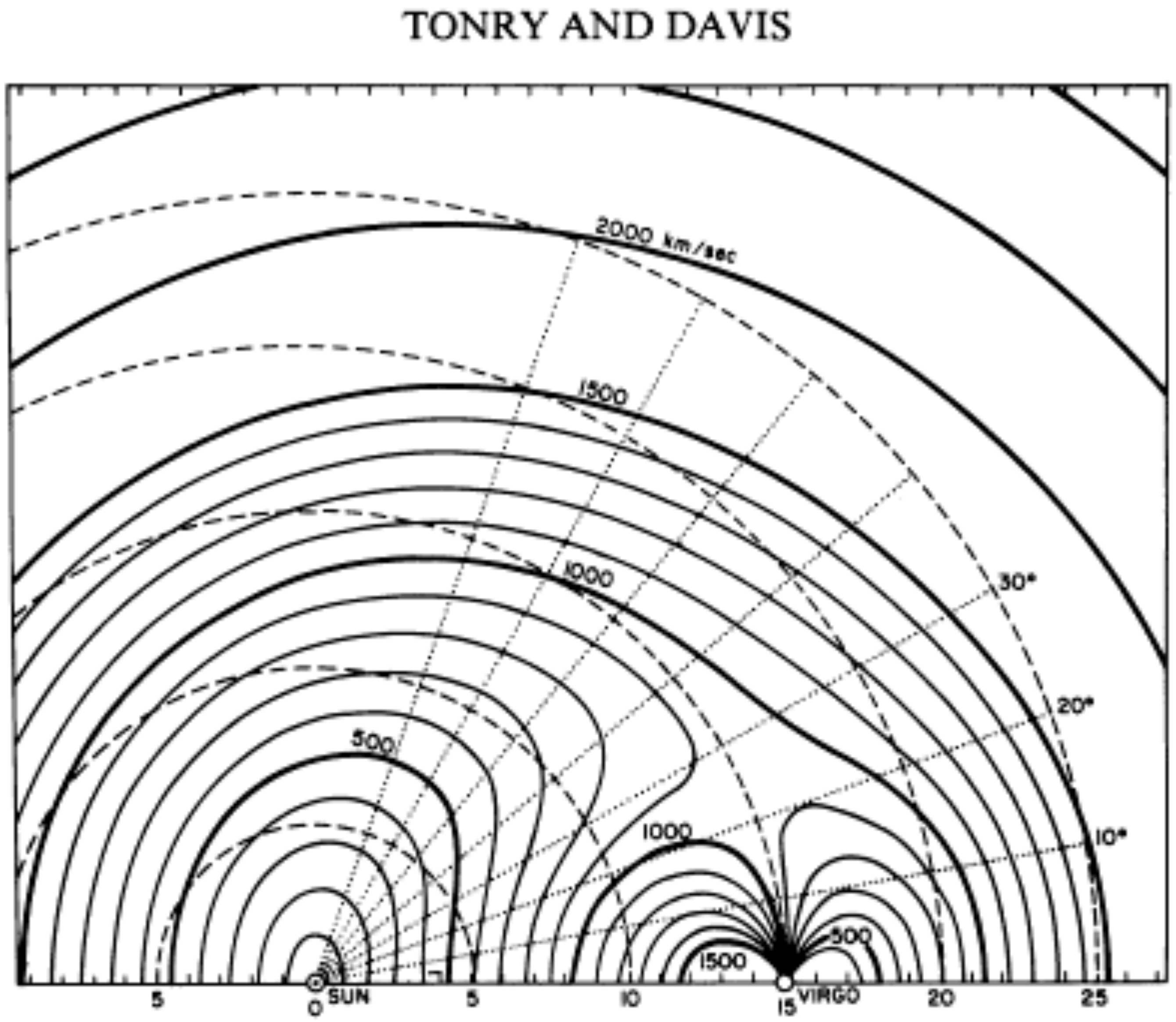
distortion in Hubble flow induced by

mass over-density

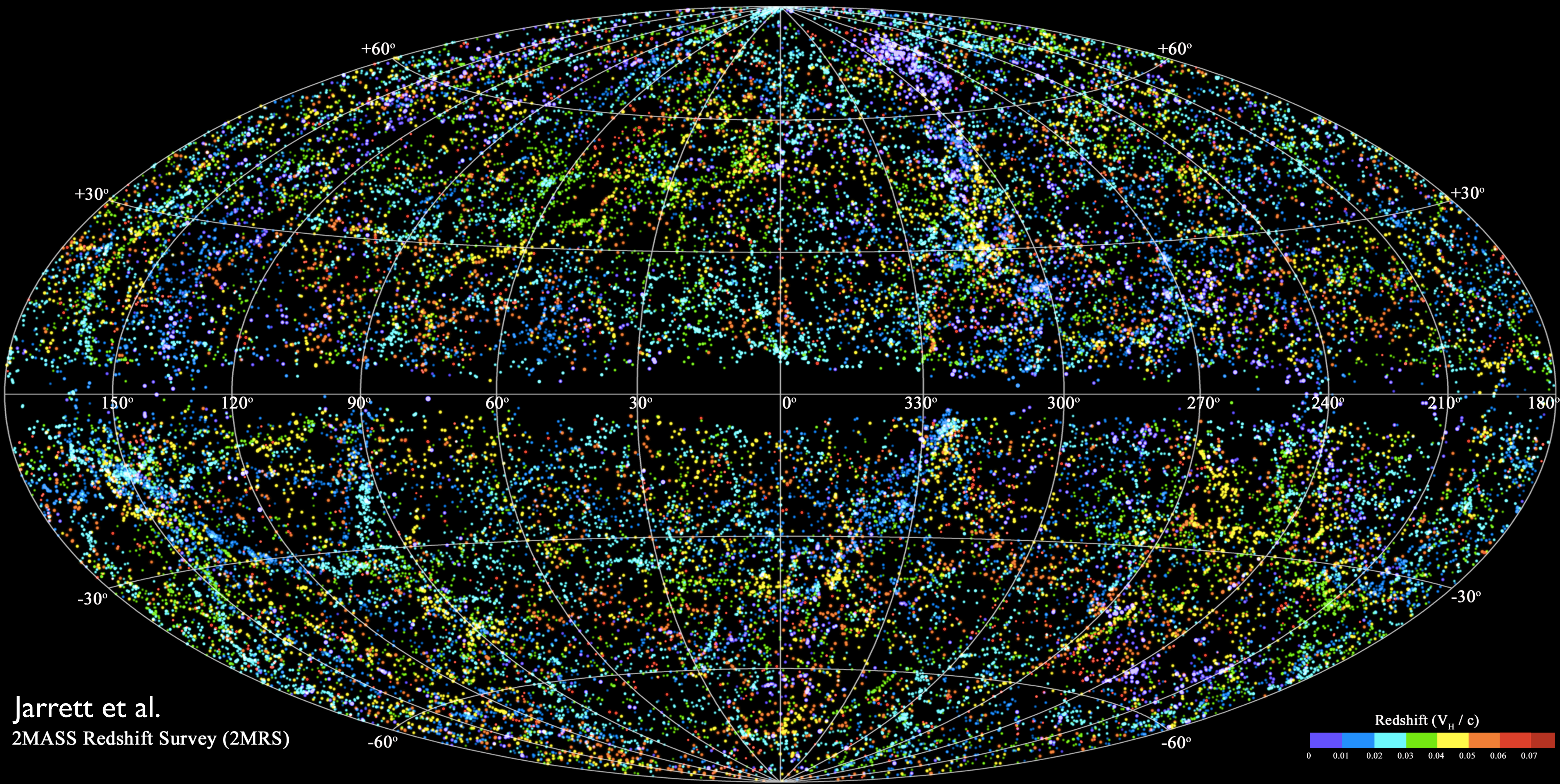
bias

BIAS ***b***
relates
galaxy over-densities
to mass over-densities

$$\Omega_m = 0.25 \pm 0.05$$



The power spectrum provides a statistical description of the clustering of galaxies, which are not randomly distributed in space



Jarrett et al.
2MASS Redshift Survey (2MRS)

- Power spectrum of galaxies

$$\delta \equiv \frac{\delta\rho}{\rho}$$

The power spectrum is commonly used to quantify large scale structure. It is the related to the 2 point correlation function via Fourier transform.

2 point correlation function: $\xi(\vec{r}) = \langle \delta(\vec{x})\delta(\vec{x} + \vec{r}) \rangle$

The 2 point correlation function is the probability of finding one galaxy near another in excess over a random distribution.

Power spectrum: $P(k) = \langle |\delta_k|^2 \rangle$ where $k = \dfrac{2\pi}{\lambda}$

where k is the wavenumber corresponding to the scale λ

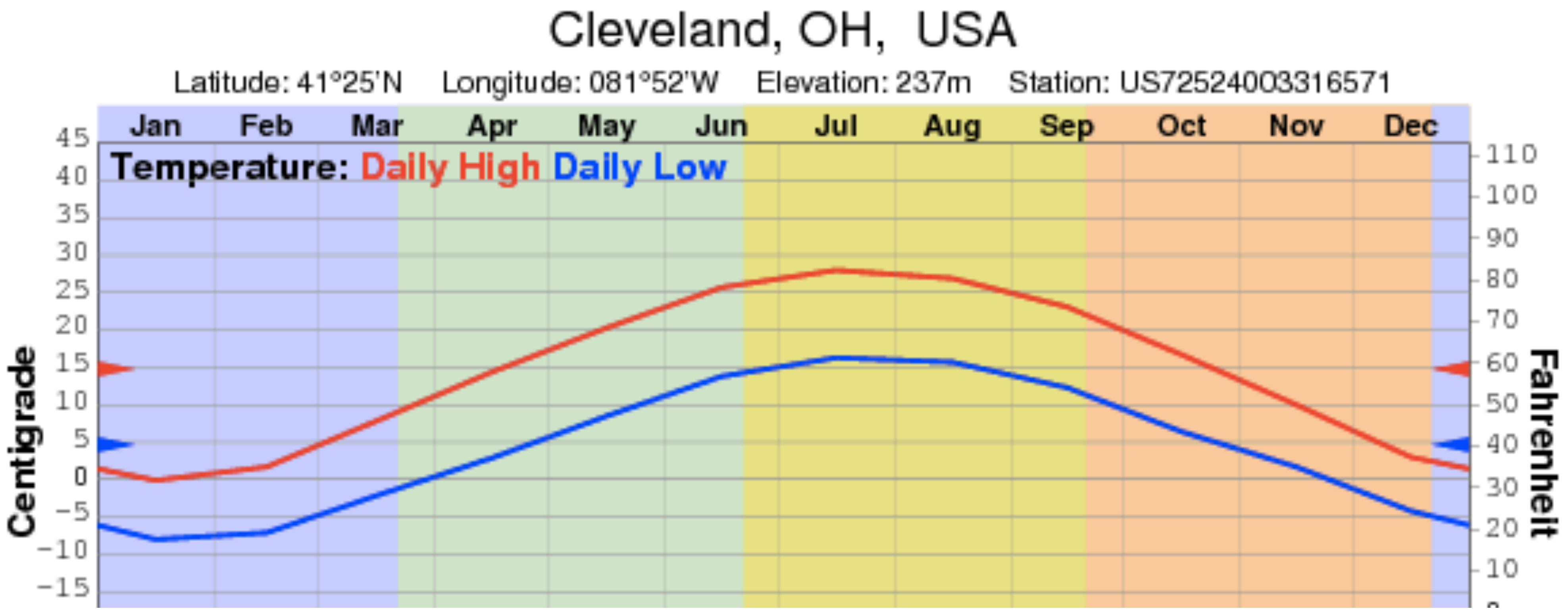
Fourier transform:

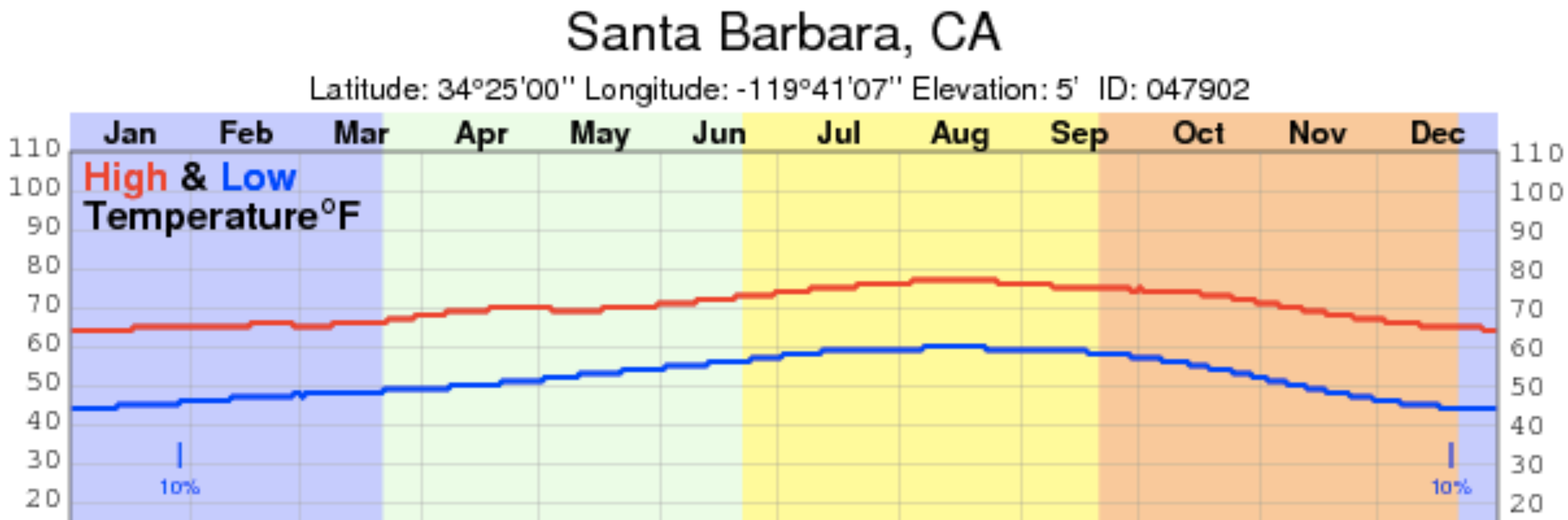
$$\xi(\vec{r}) = \frac{V}{(2\pi)^3} \int \underset{\overset{\uparrow}{P(k)}}{|\delta_k|^2} e^{-i\vec{k}\cdot\vec{r}d^3k}$$

averaged over volume V

Power Spectrum

Example: weather in Cleveland and Santa Barbara
More power on long time scales in Cleveland (seasonal variation)

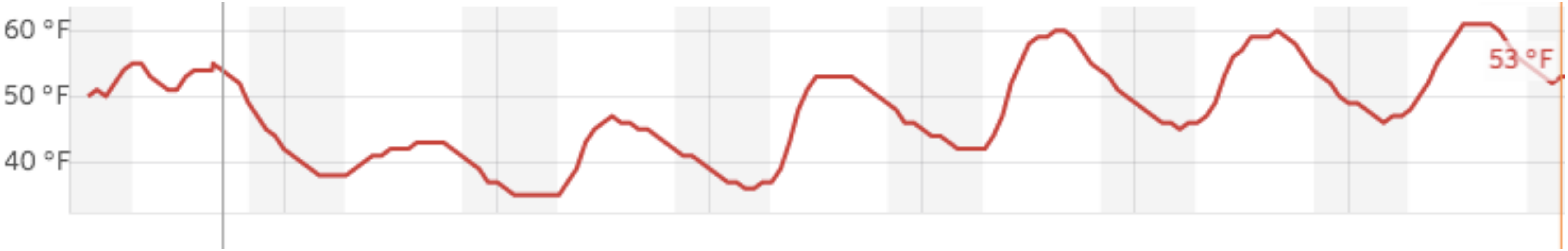




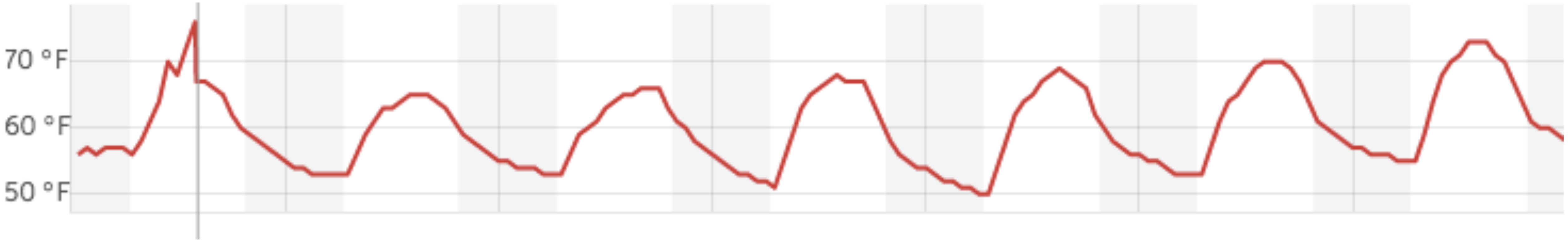
Power Spectrum

Example: weather in Cleveland and Santa Barbara
Similar power on short time scales in Santa Barbara (diurnal variation)

Cleveland forecast



Santa Barbara forecast



A power spectrum is a Fourier transform that
quantifies the relative variability on different scales

- Power spectrum of galaxies $\delta \equiv \frac{\delta\rho}{\rho}$ $k = \frac{2\pi}{\lambda}$

Power law power spectrum: $P(k) = \langle |\delta_k|^2 \rangle \propto k^n$

where n = 1 is scale free, with the same power on all scales.
This is observed to be nearly the case on large scales that have not yet collapsed. It is modulated on small scales by structure formation.

One way to think of it is the rms variation at each scale λ

$$M \sim \lambda^3 \qquad \delta_{\mathrm{rms}} \propto M^{-(n+3)/6}$$

There is more rms variance on small scales, so more power there.
[On very large scales, the universe is homogeneous, so no variance.]

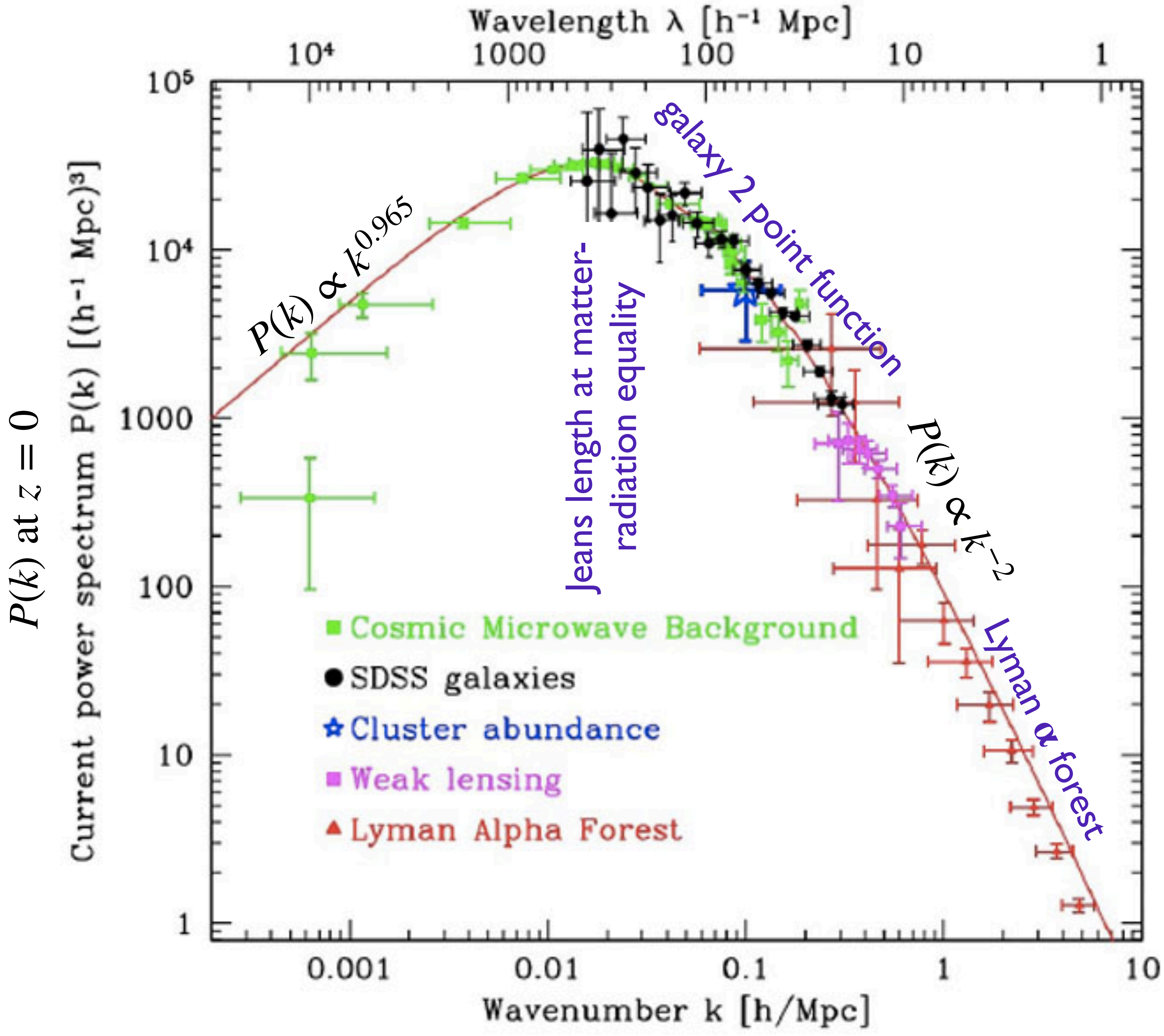
By convention, the normalization is set on a scale of 8 Mpc, where

$\frac{\delta N_{gal}}{N_{gal}} = 1$ with corresponding mass variance σ_8

Planck estimates:

$$n = 0.965 \pm 0.004$$

$$\sigma_8 = 0.811 \pm 0.006$$



Jeans length at matter-radiation equality

$$\lambda_J = c_s \sqrt{\frac{\pi}{G\rho}}$$

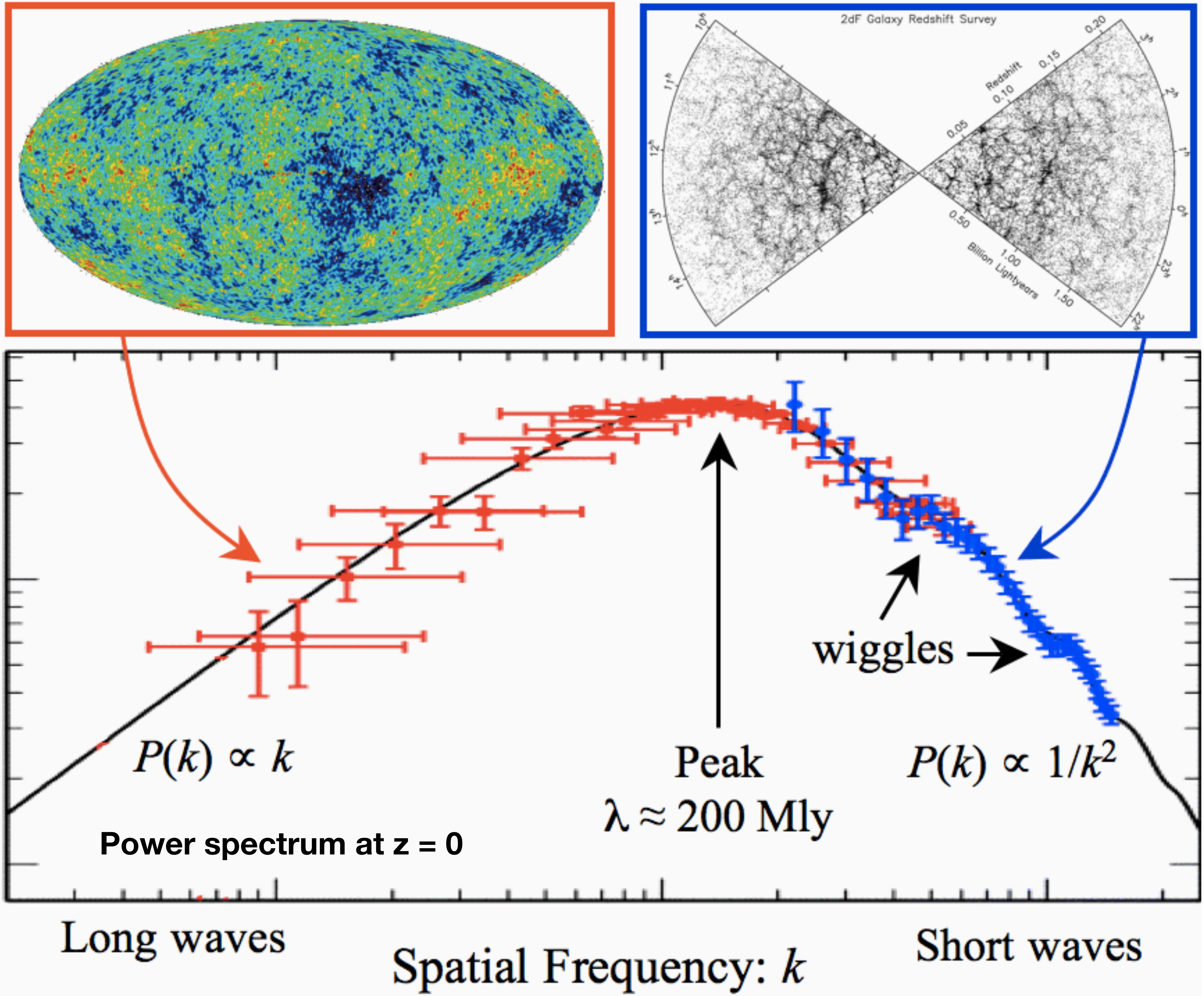
sound speed

$$c_s^2 = \frac{\partial P}{\partial \rho} = \frac{1}{3} c^2$$

at smaller scales, things go non-linear from gravitational collapse, pressure, dissipation, feedback, etc. Described by a Transfer function

$$T(k) \equiv \frac{\delta_k(z = 0)}{D(z)\delta_k(z)}$$

where D(z) is the linear growth factor - what it would have been without all these nasty non-linear effects.



Power spectrum at z = 0

From an accident report in the *Boston Driver's Handbook*:
"The guy was all over the road. I had to swerve several times before I hit him."

The power spectrum of SCDM missed badly:
too much power on small scales;
too little power on large scales.

<u>SCDM</u> ("Standard" CDM)

$$\Omega_m = 1$$
$$H_0 = 50$$

$\Omega_m h = 0.5$ expected

$\Omega_m h \approx 0.2$ observed

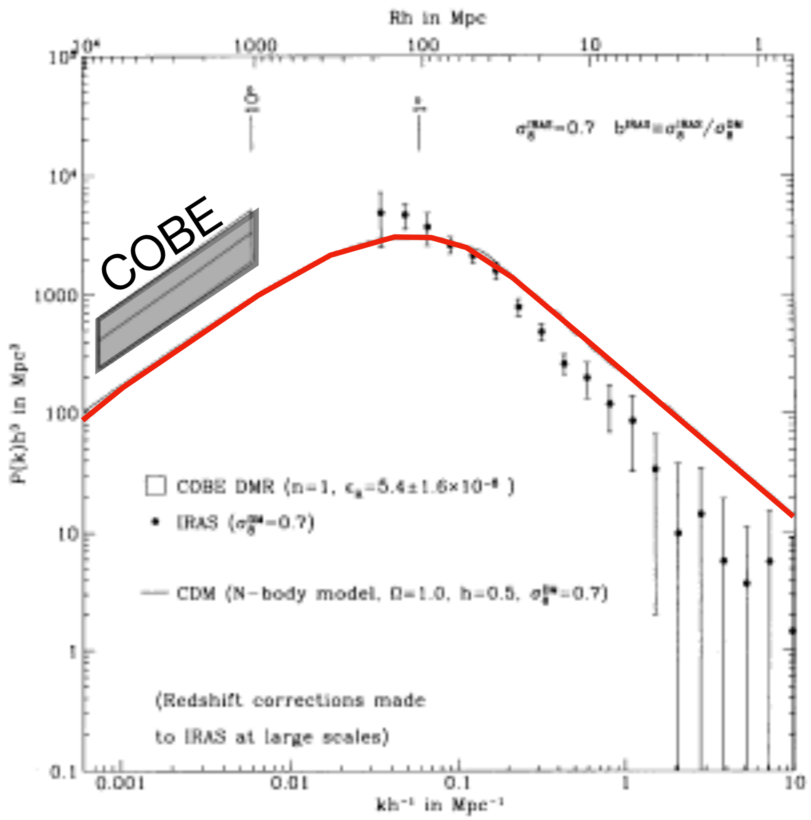


FIG. 10.—Solid curve is the real space power spectrum of the full nonlinear CDM *N*-body simulation (as in Fig. 3) normalized to the real space variance of *IRAS* galaxies ($\sigma_8 = 0.7$). The points are the *IRAS* redshift space $\tilde{P}(k)$ from Fig. 4, rescaled by eq. (17) with $\Omega = 1$ and $b = 1$; this is then, apart from the effects of the convolution in eq. (14), an approximation to the power spectrum of *IRAS* galaxies in *real* space on large scales if the *IRAS* galaxies are unbiased. The box indicates the power spectrum inferred from the *COBE* DMR measurements, assuming a $n = 1$ spectral index and $\epsilon_H = (5.4 \pm 1.6) \times 10^{-6}$ (Smoot et al. 1992; Wright et al. 1992). Note that when the CDM model is normalized to the *IRAS* variance, it produces excessive power on small scales while simultaneously failing to produce sufficient power on large scales to match the *COBE* results.

SCDM
$\Omega_m h = 0.5$
$\sigma_8 = 0.7$

Fisher et al. (1993) ApJ, 402, 42

LCDM

$$\Omega_m = 0.3$$

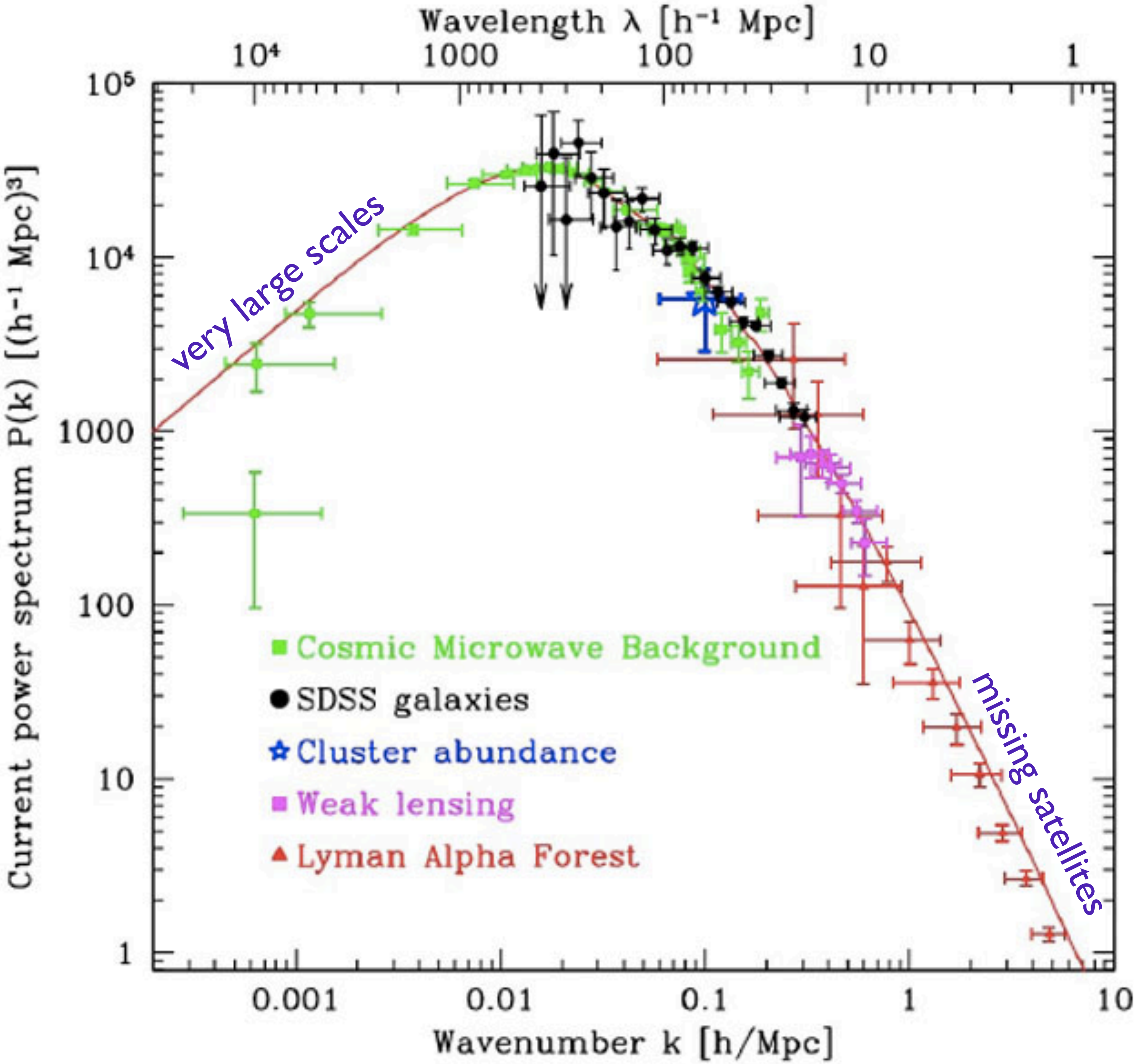
$$H_0 = 70$$

$$\Omega_\Lambda = 0.7$$

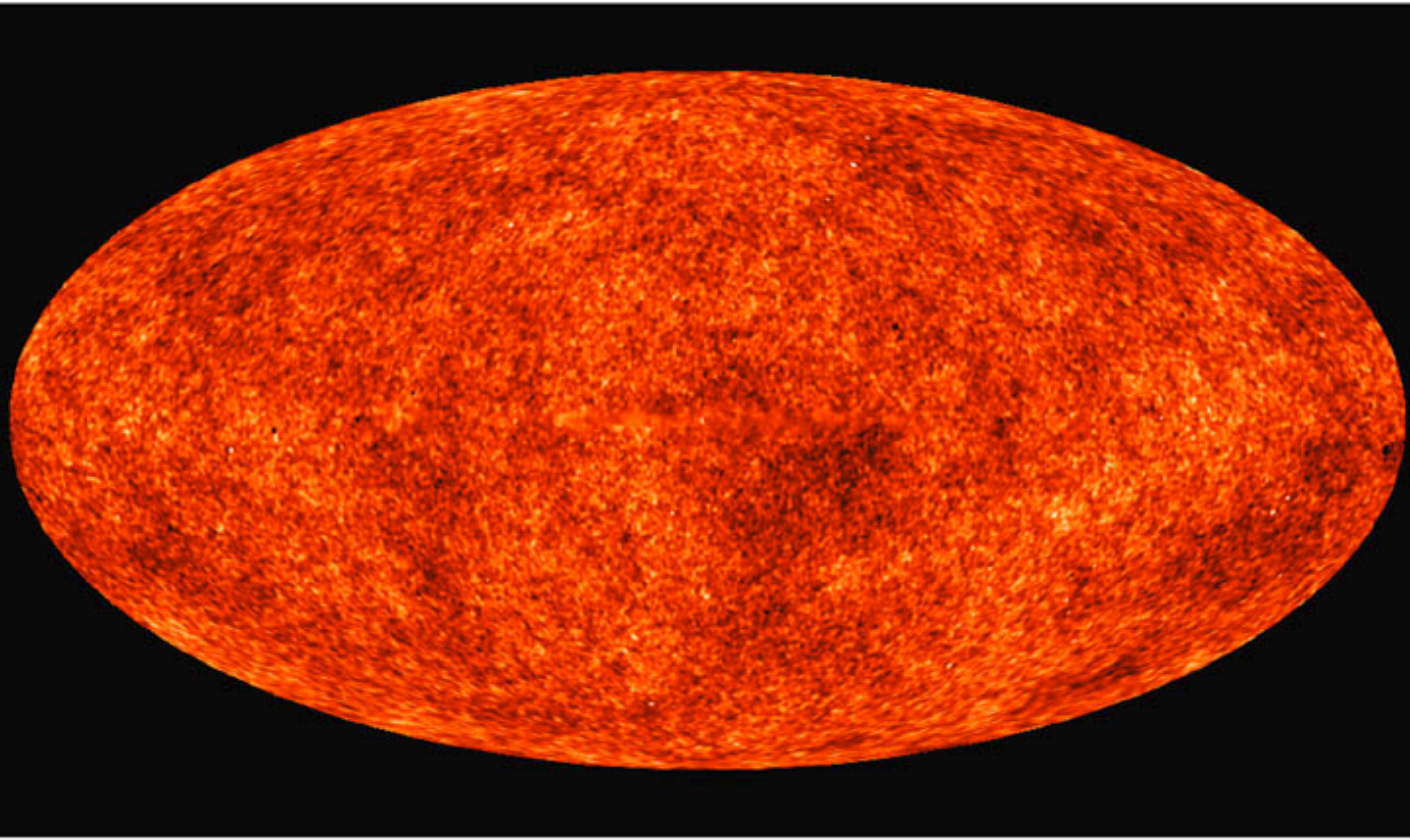
$$\Omega_m h = 0.21$$

$$\sigma_8 = 0.83$$

$$n_s = 0.965$$



CMB: Baby picture of the universe (370,000 years old)



Universe very uniform at $z = 1090$ (370,000 years old)

CMB temperature fluctuations directly related to density fluctuations

$$\frac{\delta T}{T} = \frac{1}{3}\frac{\delta\rho}{\rho} \sim 10^{-5}$$

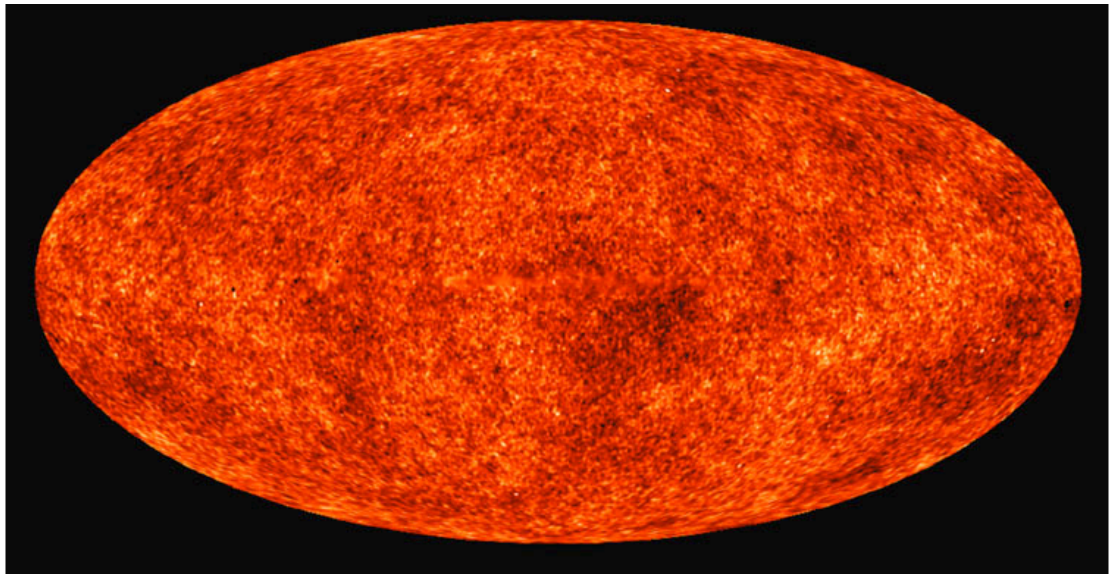
Basic problem:
not enough time for structure to grow.

$$\delta \propto a = 1091 \text{ since } z = 1090$$

Gravity will grow the observed large scale structure, but it works slowly. Can't get here from there in a Hubble time: need a factor of 100,000 but only get 1,000. Cold dark matter speeds up the process while not overproducing the temperature fluctuations.

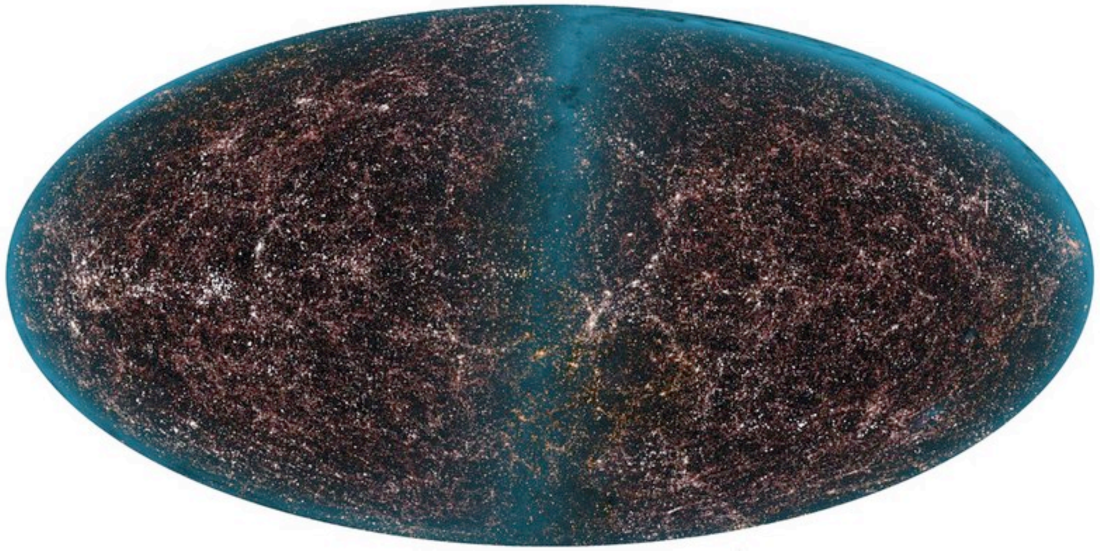
There isn't enough time to form the observed cosmic structures from the smooth initial conditions unless there is a component of mass independent of photons.

$t = 3.8 \times 10^5$ yr



very smooth: $\delta\rho/\rho \sim 10^{-5}$

$t = 1.4 \times 10^{10}$ yr



very lumpy: $\delta\rho/\rho \sim 1$

$$\delta\rho/\rho \propto t^{2/3}$$