NFW Density Profile

Halo only

Navarro, Frenk & White (1996, ApJ, **462**, 563) define the spherical density profile

$$\rho(r) = \frac{\rho_s r_s^3}{r(r+r_s)^2}.$$

This integrates to a mass profile

$$M(r) = 4\pi\rho_s r_s^3 \left[\ln(1+x) - \frac{x}{1+x}\right] = M_* \left[\ln(1+x) - \frac{x}{1+x}\right],$$

where $x = r/r_s$ and $M_* = 4\pi\rho_s r_s^3$. For small x,

$$M(x) \simeq M_* \left[\frac{x^2}{2} - \frac{2x^3}{3} + \frac{3x^4}{4}\right] \simeq 2\pi\rho_s r_s r^2.$$

The gravitational potential is the work done

$$\Phi(r) = -\int_r^\infty \frac{GM(r')}{r'^2} dr'.$$

Inserting the above expression for $M(r)$, we obtain

$$\Phi(r) = -\frac{GM_*}{r_s}\frac{\ln(1+x)}{x}.$$

Notice that this does tend to zero as $r \to \infty$ even though the total mass is logarithmically divergent. At small radii

$$\Phi(r) \simeq -\frac{GM_*}{r_s}\left[1 - \frac{x}{2} + \frac{x^2}{3} - \frac{x^3}{4}\right]$$

The radial acceleration is

$$-\frac{d\Phi}{dr} = \frac{GM_*}{r_s^2}\left[\frac{1}{x(1+x)} - \frac{\ln(1+x)}{x^2}\right],$$

and the circular speed is

$$V^2(r) = \frac{GM_*}{r}\left[\ln(1+x) - \frac{x}{1+x}\right].$$

Since

$$\kappa^2 = 2\Omega\left(\Omega + \frac{dV}{dr}\right) \equiv \frac{2V^2}{r^2} + \frac{1}{r}\frac{dV^2}{dr},$$

we find

$$\kappa^2(r) = \frac{GM_*}{r^3}\left[\ln(1+x) - \frac{x}{1+x} + \frac{x^2}{(1+x)^2}\right].$$

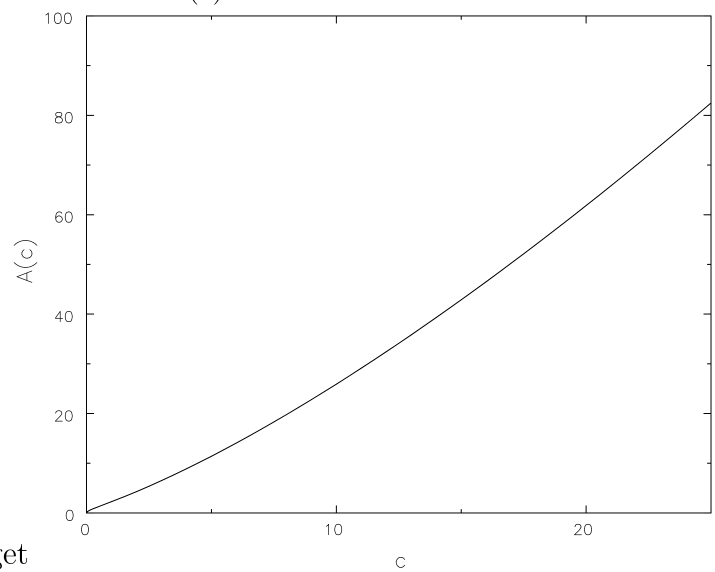
NFW introduce the "virial radius" r_{200}, inside of which the *average* density is $200\rho_{\rm crit}$, with $\rho_{\rm crit} = 3H_0^2/(8\pi G) = 1.88 \times 10^{-29} h^2$ g cm^{-3} $= 2.76 \times 10^{-7} h^2$ M$_\odot$ pc^{-3}. They further define $c = r_{200}/r_s$. Now

$$V_{200}^2 = \frac{GM_{200}}{r_{200}} = \frac{G}{cr_s} 200 \frac{4\pi}{3} c^3 r_s^3 \frac{3H_0^2}{8\pi G}, \qquad \text{or} \qquad V_{200} = 10cr_sH_0,$$

and

$$\frac{\rho_s}{\rho_{\rm crit}} = \frac{200c^3}{3[\ln(1+c) - c/(1+c)]} \equiv \frac{200}{3} [A(c)]^2 .$$

For small values of c we have $[A(c)]^2 \simeq 6c/(3-2c)$. (For those interested in trivia, $\rho_s = \rho_{\rm crit}$ when $c \approx 3/402$.) The function $A(c)$ looks like:



From this, we also get

$$V(x) = V_{200} \left[\left(\frac{c}{x}\right) \frac{\ln(1+x) - x/(1+x)}{\ln(1+c) - c/(1+c)} \right]^{1/2},$$

which has a maximum at $x \simeq 2.162582$ with the value

$$V_{\rm max} \simeq 0.465 V_{200} \left[\frac{c}{\ln(1+c) - c/(1+c)} \right]^{1/2} = \frac{0.465 V_{200}}{c} A(c) = 4.65 r_s H_0 A(c).$$

The average density inside r_s is

$$\bar\rho_1 = \frac{3V^2(1)}{4\pi G r_s^2} = \frac{3V_{200}^2}{4\pi G r_s^2} \frac{A^2(c)}{c^2} [\ln 2 - 1/2] = \frac{3 \times 0.193 A^2(c)}{4\pi G} 100 H_0^2.$$

In terms of the critical density, this is

$$\frac{\bar\rho_1}{\rho_{\rm crit}} = \frac{3 \times 19.3 A^2(c)}{4\pi G} H_0^2 \times \frac{8\pi G}{3H_0^2} = 38.6 A^2(c),$$

which is less than ρ_s because $\rho(r_s) = \rho_s/4$ – in fact, $0.58\rho_s \approx \bar\rho_1 \approx 2.32\rho(r_s)$.