MIDTERM REVIEW

Observational Evidence for mass discrepancies

- Oort discrepancy in solar neighborhood
- flat rotation curves of spiral galaxies
- clusters of galaxies
  - velocity dispersions
  - hydrostatic equilibrium of X-ray gas
  - gravitational lensing of background galaxies
- large scale structure
- $\Omega_m > \Omega_b$

Early indications included

Oort (1932) - factor of ~ 2 discrepancy in solar neighborhood

Zwicky (1933) - factor of ~ 100 discrepancy in clusters

Ostriker & Peebles (1973) - factor of ~ 10 discrepancy in bar instability in disks

WIMPS favored because
- $\Omega_m > \Omega_b$
- LSS formation

Dark Matter Candidates

Baryonic

e.g. Brown dwarfs
black holes
cold H_2
Jupiters
old white dwarfs
etc.

Non - Baryonic

Hot
ν

other
Warm DM
SIDM
pretty much anything you wanna make up.

Cold (CDM)
WIMPs
Black holes
Strange nuggets
Axions
other particle DM candidates

Dark matter is the usual inference; the discrepancy might also indicate a change in dynamical laws

Virial Theorem

Can be derived from stationary moment of inertial tensor

boils down to $2\langle K\rangle + \langle W\rangle = 0$

Kinetic E Potential Energy

for N particles of equal mass m such that $M = Nm$,

$$M = \frac{2\sigma^2 R_{hms}}{G}$$

where the harmonic radius R_{hms} is usually approximated as $R_{hms} \gtrsim 1.25 R_e$

Vertical Force (Oort: restoring force to disk)

$$K_z = -\frac{\partial \Phi}{\partial z} = \frac{1}{\nu}\frac{\partial(\nu\sigma^2)}{\partial z}$$

where $\nu(z)$ is the vertical profile of tracer population

locally this boils down to

$$\sigma_z^2 = 2\pi G \Sigma z_0$$

e.g. $\nu(z) = \nu_0 e^{-z/z_0}$

Disk Stability

LOCAL: Toomre Q: $Q = \frac{\sigma_R K}{3.36 G\Sigma}$ locally stable if $Q \gtrsim 1$

GLOBAL: $X_m = \frac{K^2 R}{2\pi m G \Sigma}$ higher surface densities less stable

Ostriker & Peebles: $t \lesssim 0.14$ where $t = \frac{T}{|W|}$

with $K = T + \frac{1}{2}\Pi$. T = rotational kinetic energy

Π = kinetic energy in random motions

Exponential Disks

2D face-on surface brightness profile

$$\Sigma(R) = \Sigma_0 e^{-R/R_d}$$

Σ_0 = central surface density
R_d = scale length

This integrates to a total luminosity $L = 2\pi \Sigma_0 R_d^2$

The enclosed luminosity is a simple fn of ~~the~~ scale length

$$L(<x) = 2\pi \Sigma_0 R_d^2 \left[1 - (1+x)e^{-x}\right] \quad \text{where} \quad x = \frac{R}{R_d}$$

in 3D one can have a "double exponential" model

$$\rho = \rho_0 e^{-R/R_d} e^{-z/z_0}$$

For general light profiles we can fit the

Sersic profile

$$\Sigma(R) = \Sigma_e e^{-b_n\left[(R/R_e)^{\frac{1}{n}} - 1\right]}$$

which reduces to the exponential form for $n = 1$
and is equivalent to the de Vaucoulers profile for $n = 4$

Potential - Density Pairs

Poisson Eqn $\quad \nabla^2 \Phi = 4\pi G \rho$

has an analytic solution for a handful of Φ-ρ pairs.

$\therefore$ It helps to know ∇^2 in the right coordinate system

Phase space $f(\vec{x}, \vec{v}, t)$: the distribution function

describes the density distribution of particles in both configuration space (x, y, z) and momentum (v_x, v_y, v_z).

For a stationary (stable) systems of widely separated stars, we have the collisionless Boltzmann eqn

$$\frac{\partial f}{\partial t} + \vec{v} \cdot \vec{\nabla} f - \vec{\nabla} \Phi \cdot \frac{\partial f}{\partial \vec{v}} = 0$$

Note that the density in configuration space ν is just the integral over f in momentum: $\nu = \int f d^3 v$

We usually use this to break the collisionless Boltzmann eqn into the

Jean's eqns

$$\frac{\partial \nu}{\partial t} + \frac{\partial (\nu \bar{v}_i)}{\partial x_i} = 0$$

$$\frac{\partial (\nu \bar{v}_j)}{\partial t} + \frac{\partial (\nu \overline{v_i v_j})}{\partial x_i} + \nu \frac{\partial \Phi}{\partial x_j} = 0$$

$$\nu \frac{\partial v_j}{\partial t} + \nu \bar{v}_i \frac{\partial \bar{v}_j}{\partial x_i} = -\nu \frac{\partial \Phi}{\partial x_j} - \frac{\partial (\nu \sigma_{ij}^2)}{\partial x_i}$$

where we have the "moments" of f

$$\nu = \int f d^3 v \qquad \overline{v_i v_j} = \frac{1}{\nu} \int v_i v_j f d^3 v$$

$$\bar{v}_i = \frac{1}{\nu} \int v_i f d^3 v \qquad \sigma_{ij}^2 = \overline{v_i v_j} - \bar{v}_i \bar{v}_j$$

The Jean's eqn for obtaining the circular velocity of a test particle from the moments of a population of tracers on non-circular orbits:

$$V_c^2(R) = \langle v_\phi^2 \rangle + \langle v_R^2 \rangle \left(1 + \frac{\partial \ln \nu}{\partial \ln R} + \frac{\partial \ln \langle v_R^2 \rangle}{\partial \ln R} \right)$$

where

V_c is the circular velocity of the gravitational potential

V_ϕ is the [quasi-circular] velocity in the tangential direction

V_R is the velocity in the radial direction

ν is the density of tracers (e.g. stars)

so $\frac{\partial \ln \nu}{\partial \ln R}$ is the slope of the radial variation of $\nu(R)$

$\frac{\partial \ln \langle v_R^2 \rangle}{\partial \ln R}$ is the slope of the radial variation of the radial velocity dispersion

$\langle v_\phi^2 \rangle$ & $\langle v_R^2 \rangle$ are the averages over the population, i.e., the net rotation $\langle v_\phi \rangle$ and the radial velocity dispersion $\langle v_R^2 \rangle^{1/2} = \sigma_r$

example application: Gaia data for Milky Way
Eilers et al 2019
McGaugh 2019

For constant thickness disk, $\frac{\partial \ln \nu}{\partial \ln R} = \frac{\partial \ln \Sigma}{\partial \ln R}$

Energy & Angular momentum

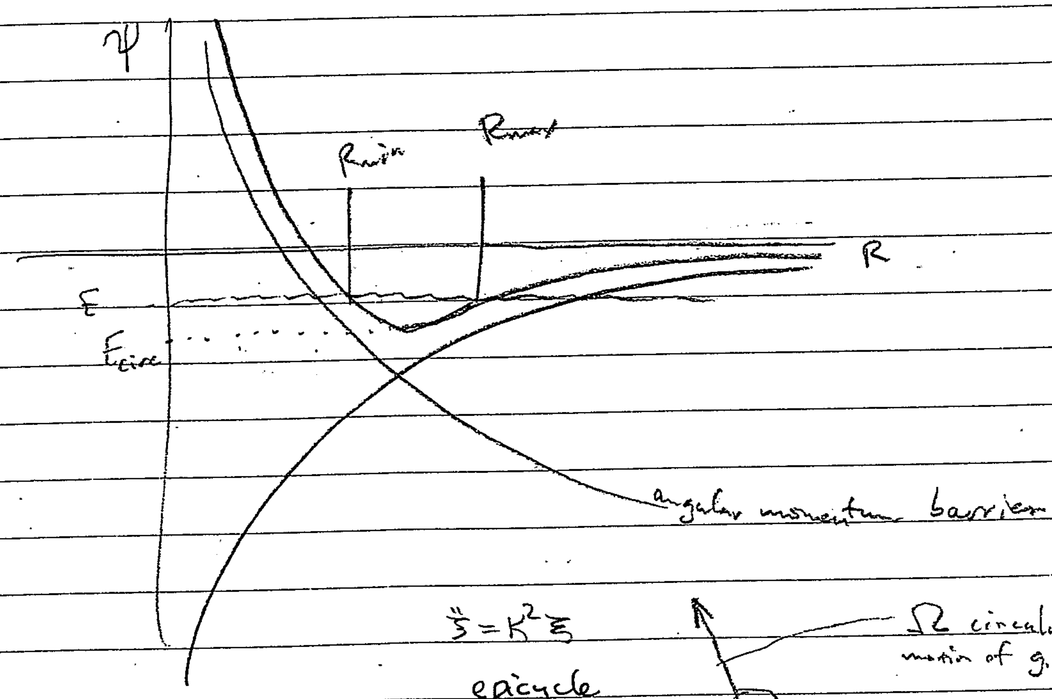
$$E = \frac{1}{2}\left(V_R^2 + V_\phi^2 + V_z^2\right) + \Phi(R,z)$$ energy per unit mass

in cylindrical coordinates

$$J_z = R V_\phi$$ angular momentum per unit mass

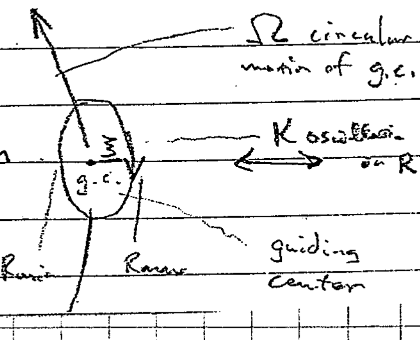
Effective Potential

$$\Psi(R,z) = \Phi(R,z) + \frac{J_z^2}{2R^2}$$



$\ddot{\xi} = K^2 \xi$

epicycle approximation



Galactic constants

R_0 - distance to Galactic Center

V_0 - circular velocity of LSR (sometimes called Θ_0)

$\Omega_0 = \frac{V_0}{R_0}$ - orbital frequency

o subscript denotes solar location

orbital period $P = \frac{2\pi}{\Omega}$

Oort "constants"

$$A = -\frac{1}{2}\left[R\frac{d\Omega}{dR}\right]_{R_0} = \frac{1}{2}\left(\frac{V}{R} - \frac{dV}{dR}\right)_{R_0}$$ SHEAR

$$B = \qquad = -\frac{1}{2}\left(\frac{V}{R} + \frac{dV}{dR}\right)_{R_0}$$ VORTICITY due to angular momentum gradient

NOTE : $\Omega = A - B$

$-\frac{dV}{dR} = A + B$

epicyclic frequency : $K^2 = -4B\Omega$

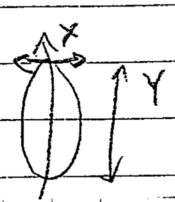
frequencies $\omega_z > K > \Omega$ so orbits not closed

more vertical oscillations than radial than complete orbits

$\omega_z \simeq 48$

$K \simeq 37$

$\Omega \simeq 30$ km s^{-1} kpc^{-1} in solar neighborhood



size of ellipsoid $\frac{Y}{X} = \frac{2\Omega}{K} = \frac{\sigma_x}{\sigma_y}$

timescales

crossing time $\quad t_c = \frac{2R}{V}$

dynamical time $\quad t_d = \sqrt{\frac{3\pi}{16 G\rho}}$

relaxation time $\quad \frac{t_R}{t_c} = \frac{N}{48 f^2} \simeq \frac{N}{6 \ln (N/2)}$

3 LAWS of GALACTIC ROTATION

1. Flat rotation curves

The rotation curves of rotating galaxies tend to approach an approximately constant velocity that persists to indefinitely large radii.

2. Baryonic Tully-Fisher Relation : $M_b = A V_f^4$

The total baryonic mass of a rotating galaxy scales as the fourth power of its flat rotation velocity.

3. Mass Discrepancy - Surface Density relation

The amplitude of the mass discrepancy scales roughly as $\Sigma_b^{1/2}$.
(Holds both globally and locally)

Halo models

pseudo-isothermal empirically motivated

$$\rho_{iso}(r) = \frac{\rho_0}{1+(r/R_c)^2}$$

characterized by

core radius R_c

flat velocity V_∞

$$V(r) = V_\infty \sqrt{1 - \frac{R_c}{R}\tan^{-1}\left(\frac{R}{R_c}\right)}$$

$$V_\infty = \sqrt{4\pi G \rho_0 R_c^2}$$

NFW derived from simulations

$$\rho_{NFW}(r) = \frac{4\rho_s}{(r/r_s)(1+r/r_s)^2}$$

characterized by

c

$R_{200}/V_{200}/M_{200}$

$$V(r) = V_{200}\sqrt{\frac{\ln(1+cx) - \frac{cx}{1+cx}}{x\left[\ln(1+c) - \frac{c}{1+c}\right]}}$$

$$x = \frac{r}{R_{200}} \qquad c = \frac{R_{200}}{r_s}$$

$$V_{200} = h R_{200} \quad \text{where} \quad h = \frac{H_0}{100}$$

in km/s in kpc

$$M_{200} = \frac{4\pi}{3}(200) R_{200}^3$$

mention Einasto

Burkert

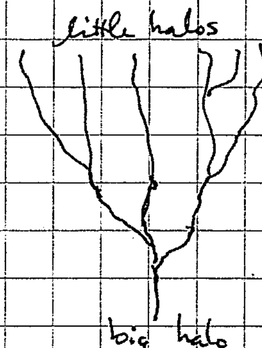
Cosmology essentials

Friedmann eqn: $$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{a^2} + \frac{c}{3}\Lambda$$

$$H = \frac{\dot{a}}{a} \qquad \Omega_m = \frac{\rho}{\rho_{crit}} \qquad \rho_{crit} = \frac{3H^2}{8\pi G}$$

In the early universe, regions with local $\Omega_m > 1$ $\left(\frac{\delta\rho}{\rho} > 1\right)$ can begin to gravitationally collapse
IFF they are composed of a form of dark matter that does not interact with photons

Monolithic galaxy formation
replaced by hierarchical galaxy formation



Overdensities

$$M_\Delta = \frac{4\pi}{3}\Delta\rho_{crit}R_\Delta^3$$

$$V_\Delta = \frac{GM_\Delta}{R_\Delta}$$

$$\therefore \quad M_\Delta = \frac{\Delta}{2G}H_0^2R_\Delta^3 = \sqrt{\frac{2}{\Delta}}\,\frac{V_\Delta^3}{GH_0}$$