TF, non-self-gravitating disk (DM everywhere dominant)

$$M_\Delta = \frac{4\pi}{3}\Delta \rho_{crit} R_\Delta^3 \qquad V_\Delta = \frac{GM_\Delta}{R_\Delta}$$

"virial" quantities

combining these with $\rho_{crit} = \frac{3H_0^2}{8\pi G}$ gives

$$M_\Delta = \left(\frac{\Delta}{2}\right)^{-1/2} \left(\frac{1}{GH_0}\right) V_\Delta^3$$

so $M_\Delta = C\, V_\Delta^3$

$$M_b = f_d f_b M_\Delta \qquad V_f = f_v V_\Delta$$

$$\frac{M_b}{f_d f_b} = C \left(\frac{V_f}{f_v}\right)^3$$

$f_b = 0.15$ universal baryon fraction $f_b = \frac{\Omega_b}{\Omega_m}$

$f_d = ?$ disk fraction ≤ 1 only constraint

$f_v = f(v) \simeq$ unity $\longrightarrow$ 1–1.3 typically

should increase w/ Σ & therefore with L

if f_d & f_v constant,

$M_b \sim V_f^3$ slope too shallow, $f_v(\Sigma)$: $3 \to 3.3 \pm 0.2$

so infer $f_d \propto V_f$ (or f_v or both)

adiabatic compression

more fine-tuning, feedback

observed baryon content of bound structures a strong function of mass: halo-by-halo missing baryon problem

Scaling relations

Tully-Fisher

self gravitating disks:

$$V^2 = \frac{GM}{r} \qquad M \text{ disk dominated}$$

$$V^4 = \frac{G^2M^2}{r^2} \qquad \Sigma = \frac{M}{r^2} = \text{surface density enclosed by } r$$

$$V^4 = G^2 M \Sigma \qquad M = \Upsilon L$$

or observationally $\qquad I = \Upsilon \Sigma \qquad \Upsilon = \text{dynamical } M/L$

I → surface brightness

$$V^4 = G^2 \Upsilon^2 L I$$

i.e. $L \propto V^4$ provided Υ & I constant.

It was once thought all galaxies had the same surface brightness (Freeman's Law).

This derivation predicts a shift in TF with surface brightness I. Such a shift is not observed. Consequently, there is a fine-tuning:

$$\Upsilon^2 I = \text{constant}$$

so $\quad I \sim \Upsilon^{-1/2}$

lower surface brightness galaxies must be progressively more DM dominated

Example feedback scheme (Dutton 2009 MNRAS, 396, 121)

Supernova drives outflows, which are assumed to move at the local escape velocity to maximize mass removal. ($< V_{esc}$ doesn't escape
$> V_{esc}$ moves less mass for same energy)

Energy driven wind model:

$$\Delta M_{eject}(R) = \frac{2\epsilon_{EFB}\,\eta_{SN}\,E_{SN}}{V_{esc}^2(R)}\Delta M_{*}(R)$$

ejected mass from radius R

mass of stars formed at radius R

$E_{SN} \simeq 10^{51}$ erg $= 5\times10^{7}$ km^2 s^{-2} $M_\odot$

$\eta_{SN} = 8.3\times10^{-3}$ # SN per $M_\odot$ of stars formed (this # for a Chabrier IMF)

ϵ_{EFB} = fraction of kinetic energy injected into wind
usually a large # in simulations (0.25 − 1)
usually a small # observed (0.02 − 0.1)

Momentum driven wind model

$$\Delta M_{eject}(R) = \frac{\epsilon_{MFB}\,p_{SN}\,\eta_{SN}}{V_{esc}(R)}\Delta M_{*}(R)$$

$p_{SN} = 3\times10^{4}\ M_\odot$ km s^{-1} is momentum produced by one SN

ϵ_{MFB} is again the coupling efficiency to the ISM

This formulation maximizes the impact of SN.