

Generic flavors of gravitational lensing

- Strong lensing - multiple images
- Weak lensing - mild distortion of single image
- microlensing - unresolved brightening from strong lens

more specifically

weak lensing is a mild distortion that preserves a 1:1 relation between lens plane & image plane.

strong lensing generates multiple ~~images~~ images with $\#$ depending on lens structure $\Sigma(r, \theta)$ and source location.

strong & weak lensing separated by a critical surface density

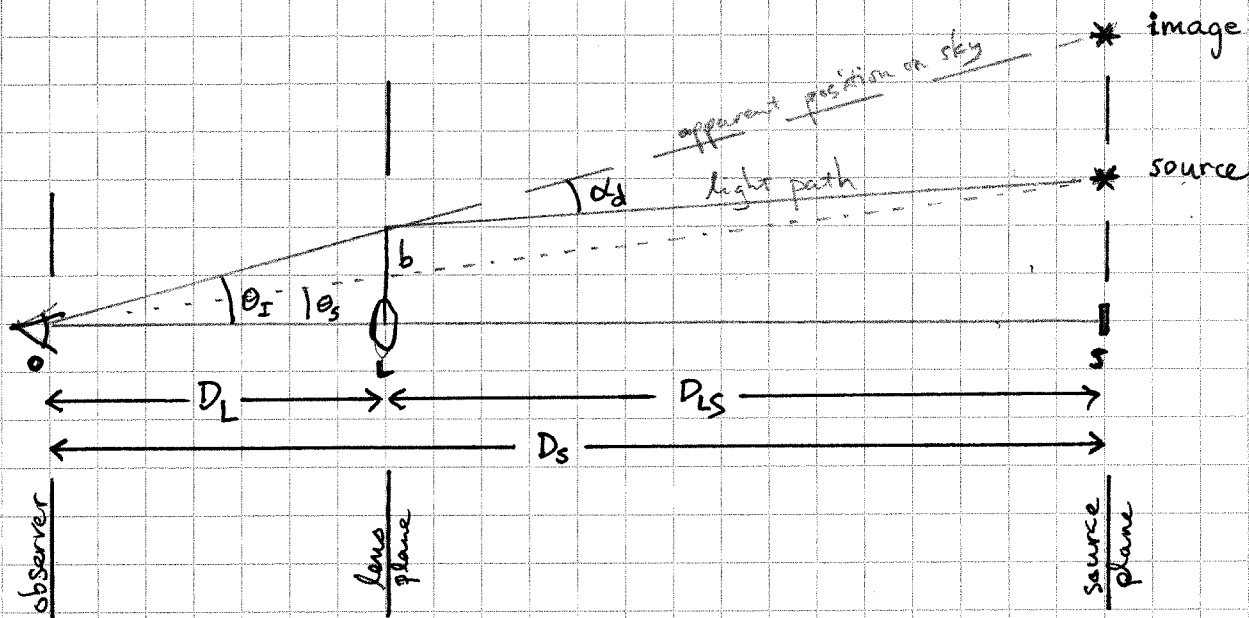
$$\Sigma_{\text{crit}} = \frac{c^2}{4\pi G} \frac{D_S}{D_L D_{LS}}$$

Gravitational lensing conserves surface brightness. Results in a net magnification of source brightness as the lens bends light into our path that otherwise would have missed us.

Gravitational Lensing

a relativistic phenomenon: GR required.

See Mo, vdB, & White § 6.6
Peacock ch. 4



factor of 2 in excess of Newton

Bend angle
$$\alpha_d = \frac{2}{c^2} \int a_{\perp} dl = \frac{4GM}{c^2 b}$$
 for circular $M(<b)$

line integral of gravitational acceleration

impact parameter

Provides good constraint on mass enclosed by lens $M(<b)$

Thin lens approximation: sudden deflection

region over which light rays bent small compared to the distance to the source. Usually a good approx. in astro!
BUT: only holds if there is only a single lens along the l.o.s.!

$$\alpha_d = \frac{D_S}{D_L} (\theta_I - \theta_S)$$

D_S, D_{LS} observed (redshift); θ_I observed; θ_S follows from geometry.

above Σ_{crit} , get multiple images (strong lensing)

For cosmological source & lens at $z_s = 2$ & $z_L = 0.5$

$$\Sigma_{\text{crit}} \approx 0.8 \text{ g cm}^{-2} \approx 3800 M_{\odot} \text{ pc}^{-2}$$

surface densities in excess of $3800 M_{\odot} \text{ pc}^{-2}$ typically only occur near the centers of dense systems like giant elliptical galaxies.

Get multiple images, sometimes Einstein rings, useful for measuring H_0 through quasar variability (geometric path length different for each image; measure time delay, use speed of light to get distance)

SVT
subject to
mass sheet degeneracy
(constant, uniform
→ causes effect
in H_0)

Strong lensing always happens in bright places - no help with dark matter.

Mass inside giant arc:

$$M(<\theta_s) = \pi \Sigma_{\text{crit}} = (1.1 \times 10^{14} M_{\odot}) \left(\frac{\theta_s}{30''}\right)^2 \left(\frac{D_L}{D_S}\right) \left(\frac{D_S}{1 \text{ Gpc}}\right)$$

applied to clusters of galaxies showing arcs,
gives

$$M/L \approx 200 M_{\odot}/L_{\odot}$$

Weak lensing: statistical detection of
net arc-like features around lens

examples: Bullet cluster (far from cluster)

groups of galaxies

L^* galaxies - many individuals stacked

convergence: $K = \frac{\Sigma}{\Sigma_c}$ (< 1 for weak lensing - Enclosed K)

gravitational shear:

$$\gamma = \gamma_1 + i\gamma_2$$

basically just residual shapes
of galaxies

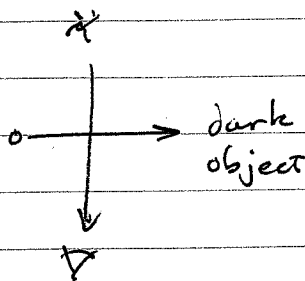
$$\nabla K = \begin{pmatrix} \gamma_{1,1} + \gamma_{2,2} \\ \gamma_{2,1} - \gamma_{1,2} \end{pmatrix}$$

get K from shapes

Micro lensing

Magnification of
background source
caused by unresolved

Einstein ring as lens moves across image



results in a net amplification A (can be large!)

of expected microlensing events

$$N(>A) = \frac{2\tau}{(A^2 - 1) + A\sqrt{A^2 - 1}}$$

where the lensing optical depth is

$$\tau = \frac{4\pi G}{c^2} \int \frac{D_L D_{LS}}{D_S} \rho dl$$

for thin screen of lenses, $\tau \rightarrow K = \frac{z}{z_{crit}}$

for MACHOs in the halo,

$$\tau = 2\pi \frac{\sigma_v^2}{c^2} \frac{D_L D_{LS}}{r D_S}$$

to LMC $\tau = 5 \times 10^{-7}$ if all halo in MACHOs

$\tau < 1 \times 10^{-7}$ observed