Basic tenets of MOND

1. New scale with dimensions of acceleration, a_0

2. Standard (Newtonian) dynamics hold when $a \gg a_0$

3. Dynamics scale invariant for $a \ll a_0$

Scale invariance : $(\vec{r}, t) \to \lambda(\vec{r}, t)$

This invariance doesn't hold for Newton, where the constant G obstructs invariance. To obtain invariance, the relevant constant is

$$\mathcal{A}_0 = a_0 G$$

Dynamics are then scale invariant for constant $\mathcal{A}_0$ as $a_0 \to \infty$ and $G \to 0$

Modern data yield

$$a_0 = 1.20 \times 10^{-10}\ \mathrm{m\,s^{-2}} \quad \underset{\text{random}}{\pm 0.02} \quad \underset{\text{systematic}}{\pm 0.24}$$

The systematic uncertainty stems from the distance scale (H_0) and — more importantly — the absolute value of the stellar mass-to-light ratio. Uncertainty in M_*/L is NOT "Gaussian". If M_*/L is way off, it affects the shape of the interpolation fcn — you can't "just" adjust a_0 but have to consider the entire shape of the RAR.

This a_0 for $$g = \frac{g_b}{1 - e^{-\sqrt{g_b/a_0}}}$$

MOND - Modified gravity (or inertia)

All we really know is that there is a discrepancy: ordinary gravity (Newton & Einstein) cannot explain the data with only the known baryons (what we can see).

Obvious hypothesis: rather than invoke dark matter, try tweaking the force law

Many attempts to modify gravity have been made. Tried and failed - things go south quickly if you write down the wrong force law.

e.g., ALL length-scale based modifications can be EXCLUDED.

The mass scale does not appear at a characteristic length scale.

Generically: predict the wrong slope for Tully-Fisher

e.g., suppose force law becomes $\frac{1}{r}$ at $r > R_+$

$$\frac{F}{m} = a = \frac{GM}{r^2} \longrightarrow \frac{GM}{rR_+} \quad \text{for } r > R_+$$

$$\frac{V^2}{r} = \frac{GM}{rR_+}$$

$$V^2 = \left(\frac{G}{R_+}\right) M \quad \text{hence:}$$

Rotation curves flat ✓ $V \sim$ const

Tully-Fisher slope 2 ✗ $M \sim V^2$

MOND: alter force law at <u>acceleration</u> scale, a_0

empirically, $a_0 \approx 10^{-10}\ m\,s^{-2}$

limits:

g_N = normal Newtonian expr.

$a \to g_N \qquad a \gg a_0$

$a \to \sqrt{g_N a_0} \qquad a \ll a_0$

$(a^2 = g_N a_0)$

$g_N = \frac{GM}{r^2}$ for point mass

transition made smoothly by interpolation fcn $\mu(x)$

$x = \frac{a}{a_0} \qquad \mu \to 1$ for $x \gg 1$

$\mu \to x$ for $x \ll 1$

so

$\mu(x)\, a = g_N$

in practice $\quad \mu(x) = \frac{x}{1+x}$

or $\frac{x}{\sqrt{1+x^2}}$

or $1 - e^{-x}$

OR $\mu \leftrightarrow \nu^{-1}$

$a = \nu(y)\, g_N$

Can alternatively write $\quad a = \nu g_N \quad$ i.e. $V_c^2 = \nu(y) \cdot V_b^2$

with $\nu(y) = \left[1 - e^{-\sqrt{y}}\right]^{-1}$

$\nu \to 1 \quad a \gg a_0$

$\nu \to y^{-1/2}$

MOND can be interpereted as either a modification of

GRAVITY $\quad F = \frac{GMm}{r^2} \quad$ (modification of Newton NOT GR)

OR

In deep MOND limit

$$a = \sqrt{g_N a_0}$$

$$\frac{V^2}{r} = \sqrt{\frac{GM}{r^2} a_0}$$

$V^4 = \chi a_0 GM \longrightarrow$ TF. No Σ or R_d residuals

χ geometry term $\chi \approx 0.8$ for disks

For pressure supported systems

$$\sigma^4 = \frac{4 a_0 GM}{81}$$

geometry term for spheres

In EFE regime, $a_{in} < a_{ex} < a_0$

a_{in}: e.g. dSph; a_{ex}: e.g., MW

$$\mu(x) V^2 = \frac{GM}{r}$$

where $x \approx g_{ex}/a_0$

(really, vector sum of all $\vec{g}$)

$\mu(x)$ $= 1$ in Newton; otherwise the same. Looks like G is bigger

$$G_{eff} = \frac{G}{\mu} \approx \frac{G a_0}{g_{ex}}$$

Original form doesn't conserve momentum or energy

Fixed by modified Poisson eqn

$$\nabla \cdot \left[\mu\left(\frac{\nabla\Phi}{a_0}\right) \nabla\Phi \right] = 4\pi G\rho$$

(conformally invariant in 3D. Regular Poisson only in 2D)

In MOND regime, dynamics are invariant under transformations $(t, \vec{x}) \rightarrow \lambda(t, \vec{x})$