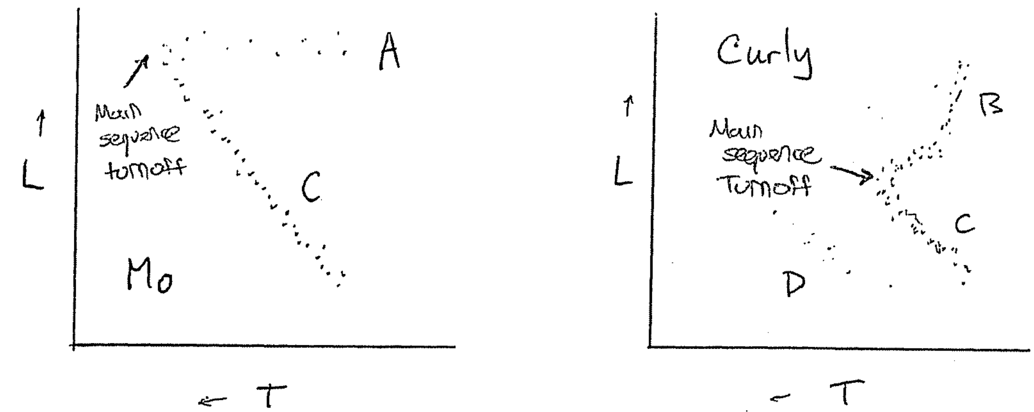
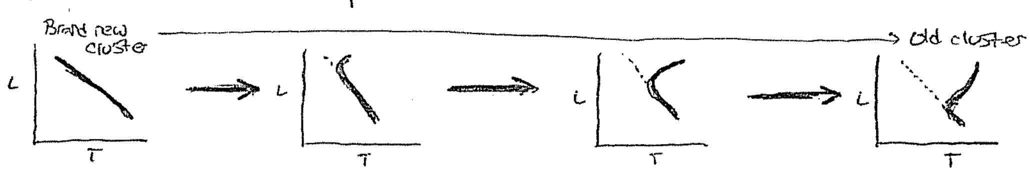
Part II: Short Answer Questions.
There are 4 short answer questions worth 10 points each.
Give a full yet concise answer to the question posed, explicitly showing any mathematical work if required.

32. Below are Hertzsprung-Russel diagrams for two clusters. Use these to answer the questions below.

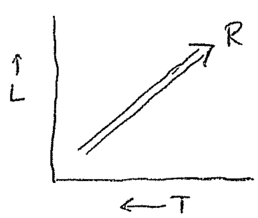


a. (4 points) Which cluster is older, Mo or Curly? How do you make this determination?

Curly is older, as the main-sequence turnoff is further 'down' the main sequence.



b. (2 points) Which are larger - stars around A or C?



Radius increases diagonally (this is from the Stefan-boltzman law: $L = 4\pi R^2 \sigma T^4$

Thus, stars A are larger than stars C

c. (4 points) Identify each of the following branches of the H-R diagram by the associated letter:

Main Sequence → C

Supergiants → A

White Dwarfs → D

Red Giants → B

33. Imagine a star, Hyperion, which has a radius 16 times that of the sun but whose surface temperature is half as hot.

a. (2 points) What is the name of the relation between luminosity, radius, and temperature? Write down the mathematical expression of this relation.

$L = 4\pi R^2 \sigma T^4$ ← Stefon-Boltzmann's Law

- L — Luminosity (W)
- R — Radius ([redacted] m)
- T — Temperature (K)

4, π, and σ are constants

b. (4 points) Use the above relation to express the luminosity of Hyperion in terms of that of the sun. (i.e., How many times brighter or fainter is Hyperion than the sun?)

sun → $L_{sun} = 4\pi R_{sun}^2 \sigma T_{sun}^4$

hyperion → $L_H = 4\pi R_H^2 \sigma T_H^4$

Givens: $R_H = 16 R_{sun}$, $T_H = \frac{1}{2}$ [redacted] T_{sun}

want: $\frac{L_H}{L_s}$ (taking a ratio!) $\Rightarrow \frac{L_H}{L_s} = \frac{4\pi R_H^2 \sigma T_H^4}{4\pi R_s^2 \sigma T_s^4} = \left(\frac{R_H}{R_s}\right)^2\left(\frac{T_H}{T_s}\right)^4 = \frac{L_H}{L_s}$

plug in values $= (16)^2\left(\frac{1}{2}\right)^4 = 16^2\left(\frac{1}{2}\right)^2 = 8^2 = 64$

$\boxed{\frac{L_H}{L_s} = 64}$

or, $L_H = 64 L_s$

Hyperion is 64 times as luminous as the sun.

c. (2 points) The wavelength at which the Sun's radiation peaks is 500 nm. At what wavelength does the radiation of Hyperion peak?

Wien's law $\lambda_{max} = \frac{b}{T}$

sun: $500\text{ nm} = \frac{b}{T_{sun}}$

hyperion: $\lambda_H = \frac{b}{(\frac{1}{2}T_s)}$

$\lambda_H = 2\left(\frac{b}{T_s}\right) = 2\lambda_s = 2 \cdot 500\text{ nm} = \boxed{1000\text{ nm} = \lambda_H}$

Hyperion has a peak wavelength of 1000 nm

d. (2 points) What color would Hyperion appear to be - red or blue-white?

Hyperion is cooler than the sun ($T_H = \frac{1}{2}T_s$), and so it is likely red. (The sun is yellow/white by comparison.)

34. Newton's generalization of Kepler's Third Law reads

$$(M1+M2)\, P^2 = a^3$$

total mass ← UNITS!

when mass is measured in solar masses, period is measured in years, and separation in AU.

a. (4 points)
First, consider an exoplanet orbiting a sun-like star, so that M1 >> M2 and M1 = 1 solar mass. If we observe a sinusoidal Doppler pattern that repeats every 8 hours and 46 minutes, what is the separation between the star and the planet?
[Hint: 8 hours, 46 minutes = (1 year)/1,000]

"$>>$" means much much greater than.

when $M_1 >> M_2$, we basically can ignore M_2. So $(M_1+M_2) \approx M_1$

$P = 8$ hours, 46 m $= \frac{1}{1000}$ yr

$$M_1 P^2 = a^3 \Rightarrow (1)\left(\frac{1}{1000}\right)^2 = a^3 \Rightarrow \left(\frac{1}{1000}\right)^{2/3} = a = \left(\frac{1}{10^3}\right)^{2/3} = \frac{1^2}{10^2} = \frac{1}{100}$$

$$\boxed{a = \frac{1}{100}\text{ AU}}$$

b. (4 points)
Now consider a binary star system in which M1 = 2 M2.
If we measure the orbit and find P = 3 years and a = 3 AU, what is the mass of each star?

$M_1 = 2M_2$
$P = 3$ yr
$a = 3$ AU

$$M_{total} = \frac{a^3}{P^2}$$ ↑ in M_{sun}

$$\frac{3^3}{3^2} = \boxed{3 = M_{total}}$$

$$M_{total} = M_2 + 2M_2 = 3M_2$$

$$\boxed{M_2 = 1\, M_{sun} \quad M_1 = 2\, M_{sun}}$$

c. ~~(2~~ points) 4 points (typo)
Finally, consider a binary star system where both stars are type K9 V. Their orbit is measured to have a = 100 AU and P = 1000 years. What is the mass of one of these stars?

→ This means both are likely the same mass : $M_1 \sim M_2$

$$M_{total} = \frac{a^3}{P^2} = \frac{100^3}{1000^2} = \frac{(10^2)^3}{(10^3)^2} = \frac{10^6}{10^6} = \boxed{1\, M_{sun} = M_{total}}$$

since this is composed of two equal mass stars, then the mass of one by itself is just $1\, M_{sun}/2 = \boxed{0.5\, M_{sun}}$

35. Kirchhoff's Laws:

In the questions below, use your knowledge of Kirchhoff's Laws to draw a line connecting each source with the type of spectrum it produces. Parts (a) and (b) are identical except that concrete examples of sources are given in part (b).

a. (6 points) Match the source with the type of spectrum it produces

SOURCE	SPECTRUM
A hot, dense object produces	an emission line spectrum
A hot, diffuse gas produces	a continuum spectrum (rainbow)
A cool gas obscuring a light source produces	an absorption line spectrum

b. (6 points) Match the source with the type of spectrum it produces

SOURCE	SPECTRUM
A star produces	an emission line spectrum
A light bulb produces	a continuum spectrum (rainbow)
A neon lamp produces	an absorption line spectrum

→ The answers to part a are actually given in the multiple choice! (question 13 in Pink Exam)

Yes, the last two problems are worth 12 points each - that's 4 extra credit points!