ASTR121
Homework 1 −Solutions

Ch. 19, Prob. 32.

We are given that $d = 2 \times 10^6$ AU.

a. 1 pc = 206265 AU

$$d = \frac{2 \times 10^6 \text{ AU}}{206265 \text{ AU}/\text{pc}} = 9.69 \text{ pc}$$

b. $p = \frac{1}{d} = \frac{1}{9.69} = 0.1''$.

This is a measurable parallax angle with modern technology. The parallax accuracy of the Hipparcos satellite was 0.001 ".

Ch. 19, Prob. 33.

$$d = \frac{1}{p} = \frac{1}{0.153''} = 6.54 \text{ pc}$$

Extra : upper limit : $d_+ = \frac{1}{0.143''} = 6.99$ pc; lower limit : $d_- = \frac{1}{0.163''} = 6.13$ pc

So, with an uncertainty of 0.01 ", the range of distances becomes 6.13 pc − 6.99 pc. And for an uncertainty of 0.1", the range of distances becomes 3.95 pc − 18.87 pc.

Ch. 19, Prob. 34.

We are given that $p = 0.255''$, $\mu = 8.67''/\text{yr}$, and $V_r = 246$ km / s.

a. $V_t = 4.74\,\mu d = 4.74\,\frac{\mu}{p} = 4.74\,\frac{8.67''/\text{yr}}{0.255''} = 161$ km / s

b. $V = \sqrt{V_r^{\,2} + V_t^{\,2}} = \sqrt{246^2 + 161^2} = 294$ km / s

c. Since V_r is positive, it implies that the star is moving away from the Sun.

Ch. 19, Prob. 37.

For this problem, we want to use the Doppler shift equation : $\frac{\lambda - \lambda_0}{\lambda_0} = \frac{V_r}{c}$, where λ is the observed wavelength, λ_0 the rest frame wavelength, and V_r the radial velocity. This equation simplified becomes : $\frac{\lambda}{\lambda_0} - 1 = \frac{V_r}{c}$.

a. We are given that $\lambda = 656.41$ nm and $\lambda_0 = 656.28$ nm. Solving for V_r :

$$V_r = \left(\frac{\lambda}{\lambda_0} - 1\right) c = \left(\frac{656.41}{656.28} - 1\right) 3 \times 10^8 = 5.94 \times 10^4 \text{ m/s} = 59.4 \text{ km/s}$$

b. The star is moving away from us. Since the observed wavelength is greater than the rest wavelength, the spectrum is redshifted. By convention, velocity is positive for recession and negative for approach.

c. We are given that $\lambda_0 = 486.13$ nm. Solving for λ :

$$\lambda = \left(\frac{V_r}{c} + 1\right) \lambda_0 = \left(\frac{5.94 \times 10^4}{3 \times 10^8} + 1\right) 486.13 = 486.23 \text{ nm}$$

d. The answers do not depend on the distance. They only depend on the velocity of the star.

Ch. 19, Prob. 38.

To do this, we first need to utilize the small angle formula :

$$D = \frac{\alpha d}{206265}$$

$$\frac{D}{t} = \frac{\alpha}{t}\,\frac{d}{206265}\left[\frac{\text{pc}}{\text{year}}\right], \text{ where } t = 1 \text{ year}$$

$$V_t = \mu d\left[\frac{\text{pc}}{206265 \text{ yr}}\right] = \mu d\left[\frac{3.086 \times 10^{13} \text{ km}}{206265\,(3.16 \times 10^7 \text{ s})}\right] = 4.74\,\mu d\,[\text{km/s}]$$

Ch. 19, Prob. 40.
Since the two stars have the same apparent brightness, star B, being further away, must be intrinsically more luminous. Since $L \propto bd^2$ where L is the luminosity,
b the apparent brightness,
and d the distance. Therefore:

$$\frac{L_B}{L_A} = \frac{b_B\, d_B^{\,2}}{b_A\, d_A^{\,2}} = \frac{d_B^{\,2}}{d_A^{\,2}} = \left(\frac{d_B}{d_A}\right)^2 = \left(\frac{48}{12}\right)^2 = 4^2 = 16;$$ So star B is 16 times more luminous.

Ch. 19, Prob. 41.
Since the two stars have the same luminosity, star D, being closer, must appear brighter than star C. Since $b \propto \frac{1}{d^2}$, b is the apparent brightness and d the distance. Therefore:

$$\frac{b_D}{b_C} = \frac{d_C^{\,2}}{d_D^{\,2}} = \left(\frac{d_C}{d_D}\right)^2 = \left(\frac{60}{12}\right)^2 = 5^2 = 25;$$ So star D appears 25 times brighter.

Ch. 19, Prob. 45.
We are given that for this particular star, $d = 2.97$ pc. We are also told that this star is 60 times dimmer than what the naked eye can see: $b_{star} = \frac{1}{60}\, b_{eye}$. We also know that the brightness depends only on the distance $\left(b \propto \frac{1}{d^2}\right)$, since it's the same star, L remains the same. So:

$$\frac{b_{star}}{b_{eye}} = \frac{d_{eye}^{\,2}}{d_{star}^{\,2}} = \left(\frac{d_{eye}}{d_{star}}\right)^2$$

$$\frac{1}{60} = \left(\frac{d_{eye}}{2.97}\right)^2 \Longrightarrow d_{eye} = 0.38 \text{ pc} = 7.91 \times 10^4 \text{ AU}$$

This distance is about 2000 times the orbit of Pluto around the Sun. That would be very close for a star since that distance is within the Oort cloud.

Ch. 19, Prob. 48.
We are given that $M = 0$, $m = 14$. The distance modulus:

$m - M = 5 \log(d) - 5$
$14 - 0 = 5 \log(d) - 5$
$19 = 5 \log(d)$
$d = 6.3$ kpc

Ch. 19, Prob. 52.
We have the equation: $B - V = m_B - m_V = 2.5 \log\left(\frac{b_V}{b_B}\right)$.

a. This equation is correct because it relates the difference in the B and V magnitudes to the brightness ratio of B to V. It can be derived from the magnitude – brightness ratio relation in Box 19 – 3 in the textbook.

b. Bellatrix: $B - V = 2.5 \log(0.81) = -0.23$
The Sun: $B - V = 2.5 \log(1.87) = 0.68$
Betelgeuse: $B - V = 2.5 \log(5.55) = 1.86$
These results imply that the smaller the color index, the bluer (or hotter) the star.

Extra Credit:
We are given that $m = 12.1$ and $p = 0.222''$.

a. $m - M = 5 \log(d) - 5 = 5 \log\left(\frac{1}{p}\right) - 5 = 5 \log\left(\frac{1}{0.222}\right) - 5 = -1.73$
$M = -(-1.73 - m) = -(-1.73 - 12.1) = 13.83$

b. $M_{sun} = 4.83$
$M_{sun} - M = 2.5 \log\left(\frac{L}{L_{sun}}\right)$
$4.83 - 13.83 = 2.5 \log\left(\frac{L}{L_{sun}}\right) = -9$
$\frac{L}{L_{sun}} = 2.5 \times 10^{-4}$; Therefore, this star is 2.5×10^{-4} times as luminous as the Sun.