ASTR121
Homework 2 −Solutions

Ch. 19, Prob. 55.
For a star with equal H and HeI line strengths, we want to see where those two curves meet in Figure 19 – 12. This occurs for a B4 star, with surface temperature of 20000 K.

Extra : A star with equal Fe I and Ca I line strengths has a spectral type of K9 and a surface temperature of 3900 K. A star with equal He I and He II line strengths is of O7 spectral type and has a surface temperature of 40000 K.

Ch. 19, Prob. 56.
Given : $R_P = 2\,R_Q$, $T_P = 4000$ K, $T_Q = 8000\text{ K} = 2\,T_P$

$$\frac{L_P}{L_Q} = \left(\frac{R_P}{R_Q}\right)^2 \left(\frac{T_P}{T_Q}\right)^4 = \left(\frac{2\,R_Q}{R_Q}\right)^2 \left(\frac{T_P}{2\,T_P}\right)^4 = 2^2 \left(\frac{1}{2}\right)^4 = \frac{1}{4}$$

Therefore, star Q has a greater luminosity by a factor of 4.

Ch. 19, Prob. 57.
Given : $L_X = 12\,L_Y$, $T_X = 3500$ K, $T_Y = 7800$ K

$$\frac{L_X}{L_Y} = \left(\frac{R_X}{R_Y}\right)^2 \left(\frac{T_X}{T_Y}\right)^4$$

$$\frac{12\,L_Y}{L_Y} = \left(\frac{R_X}{R_Y}\right)^2 \left(\frac{3500}{7800}\right)^4$$

$$\left(\frac{R_X}{R_Y}\right) = \sqrt{\frac{12}{\left(\frac{3500}{7800}\right)^4}} = 17.2$$

Therefore, star X has a greater radius by a factor of 17.2.

Ch. 19, Prob. 64.
From the orbits of the stars, we are given that $a = 10$ AU, $P = 5$ years. We are also given that the mass ratio of the two stars are $\frac{M_1}{M_2} = 4$.

a. $M_1 + M_2 = \frac{a^3}{P^2} = \frac{10^3}{5^2} = 40\,M_{Sun}$
b. $M_1 = 4\,M_2$; $M_1 + M_2 = 40\,M_{Sun}$
Therefore, $4\,M_2 + M_2 = 40\,M_{Sun}$, $M_2 = 8\,M_{Sun}$ and $M_1 = 32\,M_{Sun}$.

Ch. 21, Prob. 31.
Given : $L_{Sun} = 3.9 \times 10^{26}\ \frac{J}{s}$, $\tau_{Sun} = 1.2 \times 10^{10}$ years

From Box 21 – 2, we are told that $L = \frac{E}{\tau}$ where E is the total energy produced over the star’s lifetime and τ is the lifetime of the star. The total energy is given as $E = fMc^2$, where f is the fractional mass of the star that is spent to produce the energy. Therefore,

$$L_{Sun} = \frac{fM_{Sun}\,c^2}{\tau_{Sun}},$$

implying $$f = \frac{L_{Sun}\,\tau_{Sun}}{M_{Sun}\,c^2} = \frac{(3.9 \times 10^{26}\ \frac{J}{s})\,(1.2 \times 10^{10}\ \text{yr})\,(3.16 \times 10^7\ \frac{s}{yr})}{(1.989 \times 10^{30}\ \text{kg})\,(3 \times 10^8\ \frac{m}{s})^2} = 8.3 \times 10^{-4}$$

Only 0.7 % of hydrogen mass is converted into energy. So the fraction of the initial hydrogen that is converted into helium is $f_{H\,burnt} = \frac{f}{0.007} = 0.12$. The total mass of hydrogen burnt over the Sun's lifetime is $M_{H\,burnt} = f_{H\,burnt}\, M_{Sun} = (0.12)\,(1.989 \times 10^{30}\ \text{kg}) = 2.4 \times 10^{29}$ kg.

Ch. 21, Prob. 32.
a. The main sequence stars given are all bright stars with high surface temperatures. These stars must be younger than the Sun. Based on Figure 21 – 12, these stars would have evolved off the main sequence if they formed at the same time as the Sun. Since they are still main sequence stars, they must be younger

b. Given only the spectral type, you cannot tell whether αCentauri A is older or younger than the Sun. It takes about 10^{10} years for a G2 star to evolve off the main sequence. We can only say that it is not dramatically older than the Sun because it is still on the main sequence.

Ch. 21, Prob. 34.
a. $\frac{\tau}{\tau_{Sun}} = \left(\frac{M}{M_{Sun}}\right)^{-2.5} = 9^{-2.5} = 4.1 \times 10^{-3}$; the main sequence lifetime of this star is 4.9×10^{7} yrs

b. $\frac{\tau}{\tau_{Sun}} = 0.25^{-2.5} = 32$; the main sequence lifetime of this star is 3.8×10^{11} yrs

Ch. 21, Prob. 36.
Given : $F_{future} = 2000\, F_{now}$, $T_{now} = 14\,°C = 14 + 273 = 287$ K

The Stefan – Boltzmann equation tells us : $\frac{F_{future}}{F_{now}} = \left(\frac{T_{future}}{T_{now}}\right)^{4}$, $2000 = \left(\frac{T_{future}}{287}\right)^{4}$;
$T_{future} = (287)\left(\sqrt[4]{2000}\right) = 1919$ K.

Extra Credit :
Stars are mostly made of H and He to begin with. Dropping the abundance of metals from ~1 % to 0.01 % weakens the lines due to metals, but hardly affects the hydrogen lines.