ASTR121
Homework 3 − Solutions

Ch. 22, Prob. 37.
a. For this white dwarf, $M = 1\,M_{Sun}$, $R = 1\,R_{Earth}$.

$$\rho_{WD} = \frac{M}{V} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{1.989 \times 10^{30}\ \mathrm{kg}}{\frac{4}{3}\pi\,(6.38 \times 10^6)^3} = 1.8 \times 10^9\ \frac{\mathrm{kg}}{\mathrm{m}^3} = 1.8 \times 10^7\ \frac{\mathrm{g}}{\mathrm{cm}^3}.$$

b. Material is ejected from the star at the escape velocity. Therefore:

$$v_{esc} = \sqrt{\frac{2GM}{R}} = \sqrt{\frac{2\,(6.67 \times 10^{-11})\,(1.989 \times 10^{30})}{6.38 \times 10^6}} = 6.5 \times 10^6\ \frac{\mathrm{m}}{\mathrm{s}}.$$

Ch. 22, Prob. 39.
a. If we want to observe a supernova event from its very beginning, we want to observe supergiant stars that are about to explode.

b. Supergiants have luminosity class of type I. So from appendix 4 and 5, the only stars that meet this criterium are Rigel, Betelgeuse, Antares, and Deneb.

Ch. 22, Prob. 46.
The strong Silicon absorption line indicates that this is a type Ia supernova. From Figure 22 − 20, type Ia SNe peak at absolute magnitude of $M = -19$. Given that the apparent magnitude $m = +16.5$, we can use the distance modulus to find d.

$$m - M = 5\log(d) - 5$$

$$\log(d) = \frac{16.5 + 19 + 5}{5} = 8.1$$

$$d = 1.3 \times 10^8\ \mathrm{pc} = 130\ \mathrm{Mpc}.$$

Ch. 23, Prob. 35.
a. A typical neutron star has mass of $3\,M_{Sun}$ and radius of 10 km.

$$\frac{\rho_{N\,star}}{\rho_{neutron}} = \frac{\frac{M_{N\,star}}{\frac{4}{3}\pi R_{N\,star}^3}}{\frac{M_{neutron}}{\frac{4}{3}\pi R_{neutron}^3}} = \frac{M_{N\,star}\,R_{neutron}^3}{M_{neutron}\,R_{N\,star}^3} = \frac{(3 \times 1.989 \times 10^{30})\,(10^{-15})^3}{(1.7 \times 10^{-27})\,(10^4)^3} = 3.5$$

b. The neutron star density is greater than the density of the neutron, implying that the neutrons are overlapping. The neutrons in the center of the star are at such high densities that they form a superfluid. (See Figure 23−8.)

Ch. 23, Prob. 40.
a. There could be three reasons why we do not see the pulsar. One is that the beams of radiation simply do not cross our line of sight to the pulsar and thus we cannot detect the pulses. Another reason is that the pulsar has spun down over the last 1 million years and is no longer a spinning neutron star. The third reason might be that the magnetic field of the star is very weak, though how that could happen is less clear.

b. Here we will use Wien's law:

$$\lambda_{nm} = \frac{2.9 \times 10^6}{T} = \frac{2.9 \times 10^6}{6 \times 10^5} = 4.8\ \mathrm{nm} \dashrightarrow \text{X-rays}$$

Since the star itself emits most strongly in the X−ray, the star's signal overwhelms any pulsation signal from the beams.

c. Here we will use the Stefan−Boltzmann law:

$$L = 4\pi R^2\,\sigma T^4 \dashrightarrow R^2 = \frac{L}{4\pi\sigma T^4} = \frac{(0.046)\,(4 \times 10^{26})}{4\pi\,(5.67 \times 10^{-8})\,(6 \times 10^5)^4} \dashrightarrow R = 14\ \mathrm{km}.$$

Since neutron stars are typically 20 km across, a radius of 14 km is consisitent with the star being a neutron star. The radius of a white dwarf, the next smallest stellar object, is much bigger.

Ch. 24, Prob. 34.
For this problem, we will use Kepler's third law:

$$(M_1 + M_2) = \frac{a^3}{p^2}$$ where a and p are in units of AU and years, respectively.

$p = 7.75\ \text{hr} = 8.85 \times 10^{-4}$ yrs; $a = 2.8\, R_{Sun} = 1.32 \times 10^{-2}$ AU.

$$(M_1 + M_2) = \frac{(1.32 \times 10^{-2})^3}{(8.85 \times 10^{-4})^2} = 2.9\, M_{Sun}$$

Since neutron stars have mass greater than $1.4\, M_{Sun}$, then 2.9 is a reasonable estimate of the total mass of this neutron star binary system.

Ch. 24, Prob. 36.
a. $d = vt$. Since the distance traveled by an object in a circular orbit is the circumference of the orbit:

$$a = \frac{vt}{2\pi} = \frac{(457 \times 10^3)(0.32 \times 24 \times 3600)}{2\pi} = 2.0 \times 10^9\ \text{m}.$$

b. Newton's version of Kepler's third law:

$$p^2 = \left[\frac{4\pi^2}{G(M_1 + M_2)}\right] a^3, \text{ assuming } M_1 + M_2 = M_1$$

$$M_1 = \frac{4\pi^2 a^3}{G p^2} = \frac{4\pi^2 (2.0 \times 10^9)^3}{G(0.32 \times 24 \times 3600)^2} = 6.19 \times 10^{30}\ \text{kg} = 3.1\, M_{Sun}$$

Ch. 24, Prob. 39.

$$R_{sch} = \frac{2GM}{c^2} = \frac{2(6.67 \times 10^{-11})}{(3 \times 10^8)^2} M = 1.5 \times 10^{-27} M$$

a. $M = 5.79 \times 10^{24}$ kg.

$$R_{sch} = 1.5 \times 10^{-27} M = 1.5 \times 10^{-27} (5.79 \times 10^{24}) = 8.7 \times 10^{-3}\ \text{m}$$

$$\rho_{BH} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{5.79 \times 10^{24}}{\frac{4}{3}\pi (8.7 \times 10^{-3})^3} = 2.1 \times 10^{30}\ \frac{\text{kg}}{\text{m}^3}$$

b. $M = 1.989 \times 10^{30}$ kg.

$$R_{sch} = 1.5 \times 10^{-27} M = 1.5 \times 10^{-27} (1.989 \times 10^{30}) = 3.0 \times 10^3\ \text{m}$$

$$\rho_{BH} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{1.989 \times 10^{30}}{\frac{4}{3}\pi (3.0 \times 10^3)^3} = 1.8 \times 10^{19}\ \frac{\text{kg}}{\text{m}^3}$$

c. $M = 2.4 \times 10^{39}$ kg.

$$R_{sch} = 1.5 \times 10^{-27} M = 1.5 \times 10^{-27} (2.4 \times 10^{39}) = 3.6 \times 10^{12}\ \text{m}$$

$$\rho_{BH} = \frac{M}{\frac{4}{3}\pi R^3} = \frac{2.4 \times 10^{39}}{\frac{4}{3}\pi (3.6 \times 10^{12})^3} = 12.3\ \frac{\text{kg}}{\text{m}^3}$$

Extra Credit:

Given: $a = 2.8\, R_{Sun} = 1.96 \times 10^{12}$ mm. At a rate of $\frac{3\ \text{mm}}{7.75\ \text{hr}}$, it would take

$$\frac{1.96 \times 10^{12}}{\frac{3\ \text{mm}}{7.75\ \text{hr}}} = 5.1 \times 10^{12}\ \text{hr}$$ or 5.8×10^8 years for this pair of neutron stars to collide.