ASTR121
Homework 4 − Solutions

Ch. 20, Prob. 29.
At 1 kpc, 15 % of the light gets through. At 2 kpc, 2.25 % (15 % of 15 %) gets through. At 3 kpc, 0.34 % (15 % of 2.25 %) gets through.

Ch. 20, Prob. 33.
From the Stefan – Boltzmann equation :

$$\frac{L_{then}}{L_{now}} = \left(\frac{R_{then}}{R_{now}}\right)^2 \left(\frac{T_{then}}{T_{now}}\right)^4$$

$$\frac{R_{then}}{R_{now}} = \frac{\sqrt{\frac{L_{then}}{L_{now}}}}{\left(\frac{T_{then}}{T_{now}}\right)^2} = \frac{\sqrt{1000}}{\left(\frac{1000}{5800}\right)^2} = 1064$$

$$R_{then} = 1064\, R_{now} = 7.4 \times 10^8 \text{ km} = 5 \text{ AU}$$

Ch. 20, Prob. 39.
Over the last 10^4 years, the supernova expanded radially for 75 light – years.

$$\text{speed} = \frac{\text{distance}}{\text{time}} = \frac{75 \text{ ly}}{10^4 \text{ yr}} = \frac{75 \text{ ly} \left(3 \times 10^8 \frac{m}{s}\right) \left(3.16 \times 10^7 \frac{s}{yr}\right)}{10^4 \text{ yr} \left(3.16 \times 10^7 \frac{s}{yr}\right)} = 2.25 \times 10^3 \frac{\text{km}}{\text{s}} = 0.0075\, c$$

Ch. 25, Prob. 33.
According to the text, the Sun ' s orbital period around the center of the Galaxy is 2.2×10^8 years. So over its lifetime, the Sun has completed $\frac{4.56 \times 10^9}{2.2 \times 10^8} = 20.7$ orbits.

Ch. 25, Prob. 36.
Given : $v = 400 \frac{km}{s}$; $r = 2 \times 10^4$ pc $= 6.17 \times 10^{17}$ km; Assume circular orbit.

a. $\text{time} = \frac{\text{distance}}{\text{speed}} = \frac{2\pi r}{v} = \frac{2\pi (6.17 \times 10^{17} \text{ km})}{400 \frac{km}{s}} = 9.7 \times 10^{15}\ s = 3.1 \times 10^8$ years.

b. $M = \frac{rv^2}{G} = \frac{(6.17 \times 10^{20}\ m)(4 \times 10^5 \frac{m}{s})^2}{6.67 \times 10^{-11}} = 1.5 \times 10^{42}$ kg.

Ch. 25, Prob. 39.
Given : $P^2 = \frac{4\pi^2 a^3}{G(M + M_{Sun})}$, assume circular orbit.

Since the mass of the galaxy is much greater than that of the Sun, $M + M_{Sun} = M$. For a circular orbit, $a = r$. From the previous problem, the period of an object in circular orbit is $P = \frac{2\pi r}{v}$. Substituding all this into the given equation, we get :

$$\left(\frac{2\pi r}{v}\right)^2 = \frac{4\pi^2 r^3}{GM} \;\text{-->}\; \frac{4\pi^2 r^2}{v^2} = \frac{4\pi^2 r^3}{GM} \;\text{-->}\; M = \frac{rv^2}{G}$$

Ch. 25, Prob. 41.
a. $R_{Sch} = \frac{2\,GM}{c^2} = \frac{2\,G\,(3.7 \times 10^6\, M_{Sun})}{c^2} = \frac{2\,(6.67 \times 10^{-11})(3.7 \times 10^6)(1.989 \times 10^{30})}{(3 \times 10^8)^2} = 1.1 \times 10^{10}$ m

$= 1.1 \times 10^7$ km $= 0.07$ AU

b. $D = 8$ kpc $= 2.5 \times 10^{20}$ m; $d = 2\, R_{Sch} = 2.2 \times 10^{10}$ m

$$\alpha = \frac{206265\, d}{D} = \frac{205265\,(2.2 \times 10^{10})}{2.5 \times 10^{20}} = 1.8 \times 10^{-5} \text{ arcsec}$$

c. $D = 120\,AU = 1.8 \times 10^{13}$ m; $d = 2\,R_{Sch} = 2.2 \times 10^{10}$ m

$$\alpha = \frac{206265\,d}{D} = \frac{205265\,(2.2 \times 10^{10})}{1.8 \times 10^{13}} = 252\text{ arcsec}$$

Yes, it would be discernible to the naked eye.

Suppose you have a space ship that can travel at 0.999 c. Should you wish to visit Canopus (313 light – years distant), how much will you age on the way there?

For this problem, we will use the Lorentz formula for time dilation:

$$t = \frac{t_0}{\sqrt{1 - \left(\frac{v}{c}\right)^2}},$$ where t is time measured in the rest frame and t_0 is the proper time of the moving frame. In this problem, how much you will age in the moving frame is the proper time. So, that's what we want to solve for. The time it takes to get there as measured by someone in the rest frame (in this case, the frame of the Earth) is t. Therefore:

$$t = \frac{\text{distance}}{\text{speed}} = \frac{313\text{ ly}}{0.999\,c} = \frac{313\,(c)\,(1\text{ year})}{0.999\,c} = 313.3\text{ years}$$

$$t_0 = t\sqrt{1 - \left(\frac{v}{c}\right)^2} = 313.3\sqrt{1 - (0.999)^2} = 14\text{ years}.$$

Extra Credit:

MOND stands for Modfied Newtonian Dynamics. It is different than normal Newtonian gravity in that it introduces a term that dominates the gravity equation at large distances. This theory explains the leveling off of galaxy rotation curves without invoking dark matter, the existence of which is still questionable. For more information on MOND, visit Dr. McGaugh's MOND page at http://www.astro.umd.edu/~ssm/mond/.