ASTR121
Homework 6 − Solutions

Ch. 26, Prob. 41.
From the plot, we find that the galaxy is about 20 kpc in radius. At this distance, the orbital velocity is approximately 275 km / s.

a. $p = \frac{d}{v} = \frac{2\pi R}{v} = \frac{2\pi\,(20 \times 3.086 \times 10^{16}\ \text{km})}{275} = 1.4 \times 10^{16}\ \text{s} = 4.5 \times 10^{8}\ \text{yr}.$

b. The enclosed mass is given by :

$$M = \frac{R v^2}{G} = \frac{(20 \times 10^3 \times 3.086 \times 10^{16}\ \text{m})\,(2.75 \times 10^5\ \text{m/s})^2}{6.67 \times 10^{-11}} = 7.0 \times 10^{41}\ \text{kg} = 3.5 \times 10^{11}\ M_{Sun}$$

Ch. 28, Prob. 30.
Given : $\frac{d}{d_0} = 0.2$; the 0 subscript denotes present − day values.

a. The cosmological redshift of an object is linearly related to the expansion of space in between galaxies. So :

$$\frac{d}{d_0} = \frac{\lambda_0}{\lambda} = \frac{1}{1+z} = 0.2 \text{ --> solving for } z\text{, we get } z = 4$$

b. $\frac{d}{d_0} = \frac{1}{1+z} = \frac{1}{9} = 0.11$

$$\rho = \frac{\text{Mass}}{\text{Volume}} \propto \frac{\text{Mass}}{d^3} \propto \frac{\text{Mass}}{(0.11\, d_0)^3} \propto 729\, \frac{\text{Mass}}{{d_0}^3} = 729\, \rho_0$$

c. The comological constant is the same throughout time. So $\rho_{\Lambda,\, z=5} = \rho_{\Lambda,\, z=2} = \rho_{\Lambda,\, z=0}$

Ch. 28, Prob. 32.

a. $T_0 = \frac{1}{H_0} = \frac{1}{50\ \text{km/s/Mpc}} = \frac{1}{50 / (3.086 \times 10^{19})\ \text{s}} = 2.0 \times 10^{10}\ \text{yr}$

b. $T_0 = \frac{1}{H_0} = \frac{1}{75\ \text{km/s/Mpc}} = \frac{1}{75 / (3.086 \times 10^{19})\ \text{s}} = 1.3 \times 10^{10}\ \text{yr}$

c. $T_0 = \frac{1}{H_0} = \frac{1}{100\ \text{km/s/Mpc}} = \frac{1}{100 / (3.086 \times 10^{19})\ \text{s}} = 9.8 \times 10^{9}\ \text{yr}$

d. The age of globular clusters can place constraints on the Hubble constant because they are one of the oldest objects in the Universe. Since they are within the Universe, they cannot be older than the Universe itself.

Ch. 28, Prob. 35.
Given : T = 2.725 K

From Wien ' s Law, $\lambda = \frac{0.0029}{T} = \frac{0.0029}{2.725} = 1.06 \times 10^{-3}\ \text{m} = 1.06\ \text{mm}$

Ch. 28, Prob. 37.
Given : $\rho_{rad} = \frac{4\sigma T^4}{c^3}$; T = 272.5

a. The temperature of 272.5 is a factor of 100 times greater than what it is now. Since the radiation density is proportional to T^4, $\rho_{rad,\, T=272.5} = 100^4\, \rho_{rad,\, T=2.725} = 4.6 \times 10^{-23}\ \frac{\text{kg}}{\text{m}^3}$

b. If the matter density stays the same, then $\rho_{rad,\ T=272.5}$ is greater than ρ_m, so the Universe would be radiation dominated.

Ch. 28, Prob. 42.
Given : $\Omega_\Delta = 0.73$

a. $q_{z=0.5} = \frac{1}{2} - \frac{3}{2}\left[\frac{0.73}{0.73 + (0.27)\ (1.5)^3}\right] = -0.167$

b. $q_{z=1} = \frac{1}{2} - \frac{3}{2}\left[\frac{0.73}{0.73 + (0.27)\ (2)^3}\right] = 0.121$

c. The definition of q implies that a positive sign means deceleration, and a negative sign means the opposite, acceleration. From the calculations above, it appears that the Universe was decelerating at $z = 1$, but at $z = 0.5$, it accelerates.

Ch. 29, Prob. 27.
Given : $T = 3000$ K, $\rho_m = 10^{-18}$ kg / m^3; assume $m = m_H = 1.67 \times 10^{-27}$ kg

$$L_J = \sqrt{\frac{\pi\, k\, T}{m\, G\, \rho_m}} = \sqrt{\frac{\pi\ (1.38 \times 10^{-23})\ (3000)}{(1.67 \times 10^{-27})\ (6.67 \times 10^{-11})\ (10^{-18})}} = 1.08 \times 10^{18}\ \text{m} = 114\ \text{ly}$$

$$M = \rho V = \rho_m \left(\frac{4}{3}\pi r^3\right) = 10^{-18} \left(\frac{4}{3}\pi\right) \left(\frac{1.08 \times 10^{18}\ \text{m}}{2}\right)^2 = 6.6 \times 10^{35}\ \text{kg} = 3.3 \times 10^5\ M_{Sun}$$

Ch. 29, Prob. 30.
Following the method used in Chapter 28, problem 30, we know that $\frac{d}{d_0} = 0.168$

$$\rho \propto \frac{1}{d^3} \propto \frac{1}{(0.168\, d_0)^3} = 211\, \rho_0$$

Extra Credit :
The Anthropic Principle states that any valid theory of the Universe must be consistent with our existence at this particular time and place in the Universe. In other words " If something must be true for us, as humans, to exist, then it is true simply because we exist."