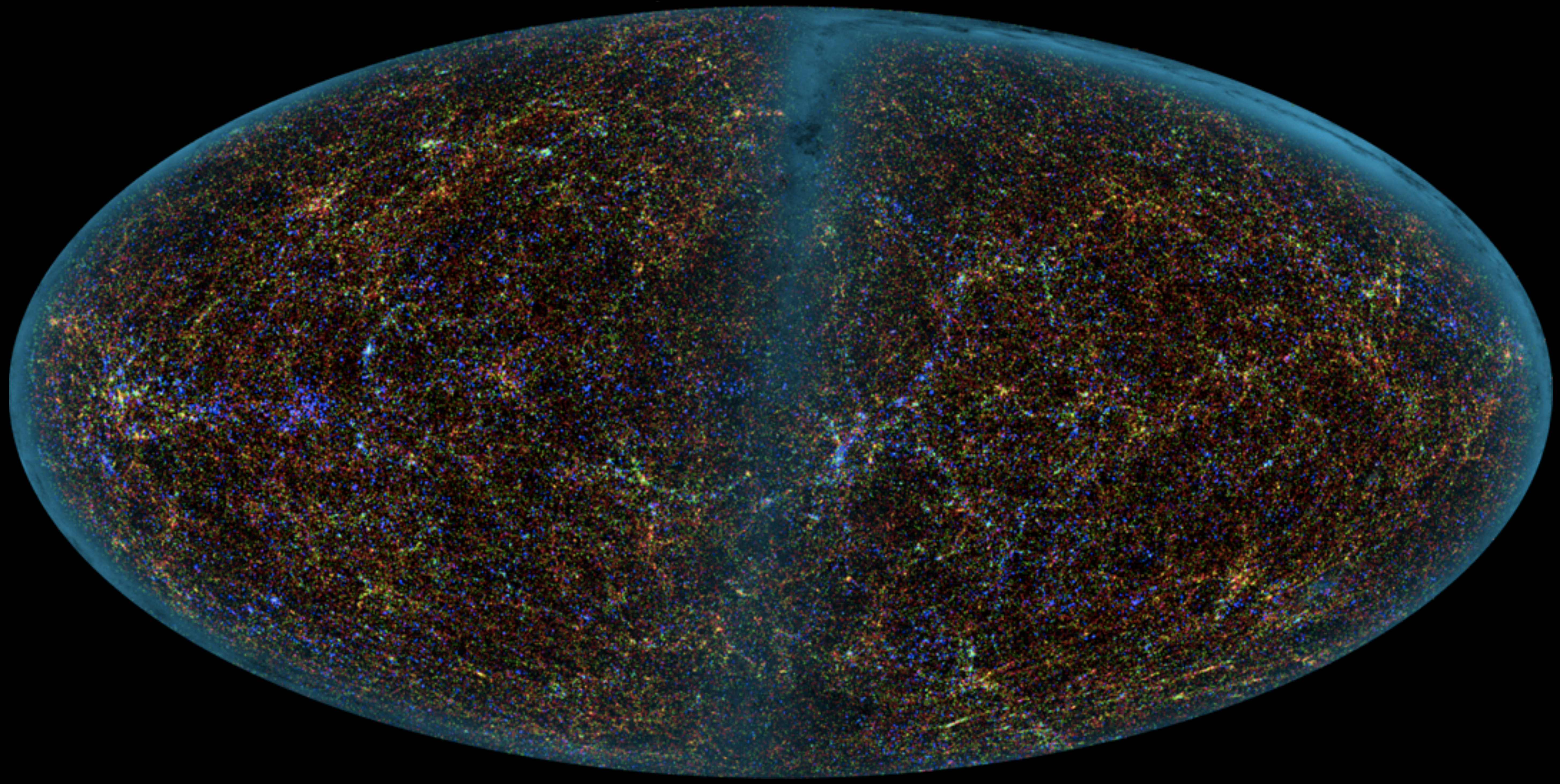
Cosmology

and Large Scale Structure



<u>Today</u>

Observational Tests

Time-redshift
Luminosity Distance-redshift
Angular Size Distance-redshift
relations in FLRW

homework 2 due Thu 9/19

12 September 2024

http://astroweb.case.edu/ssm/ASTR328/

comoving coordinates constant

Once we know (or assume) what kind of universe we live in, we specify the expansion history *a(t)*.

we know the expansion factor from the redshift

a photon propagating through the expanding universe traverses a distance element

$$d\ell = cdt = a(t)dr$$

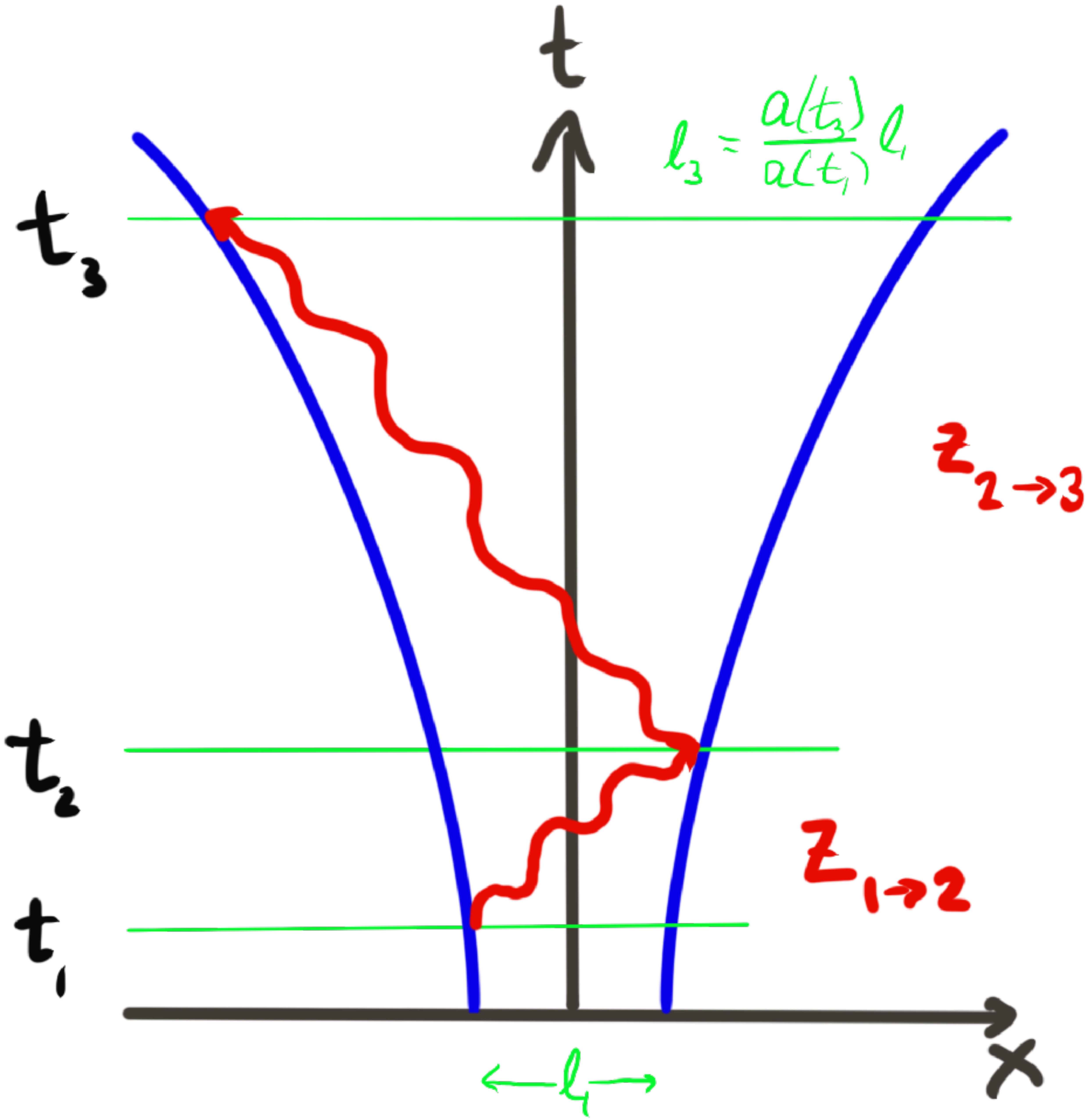
The comoving separation between two points is fixed, so

$$r = c\int_{t_1}^{t_2} \frac{dt}{a(t)} = c\int_{t_2}^{t_3} \frac{dt}{a(t)}$$

and

$$\frac{a(t_2)}{a(t_1)} = 1 + z_{1\to 2} \qquad \frac{a(t_3)}{a(t_2)} = 1 + z_{2\to 3}$$

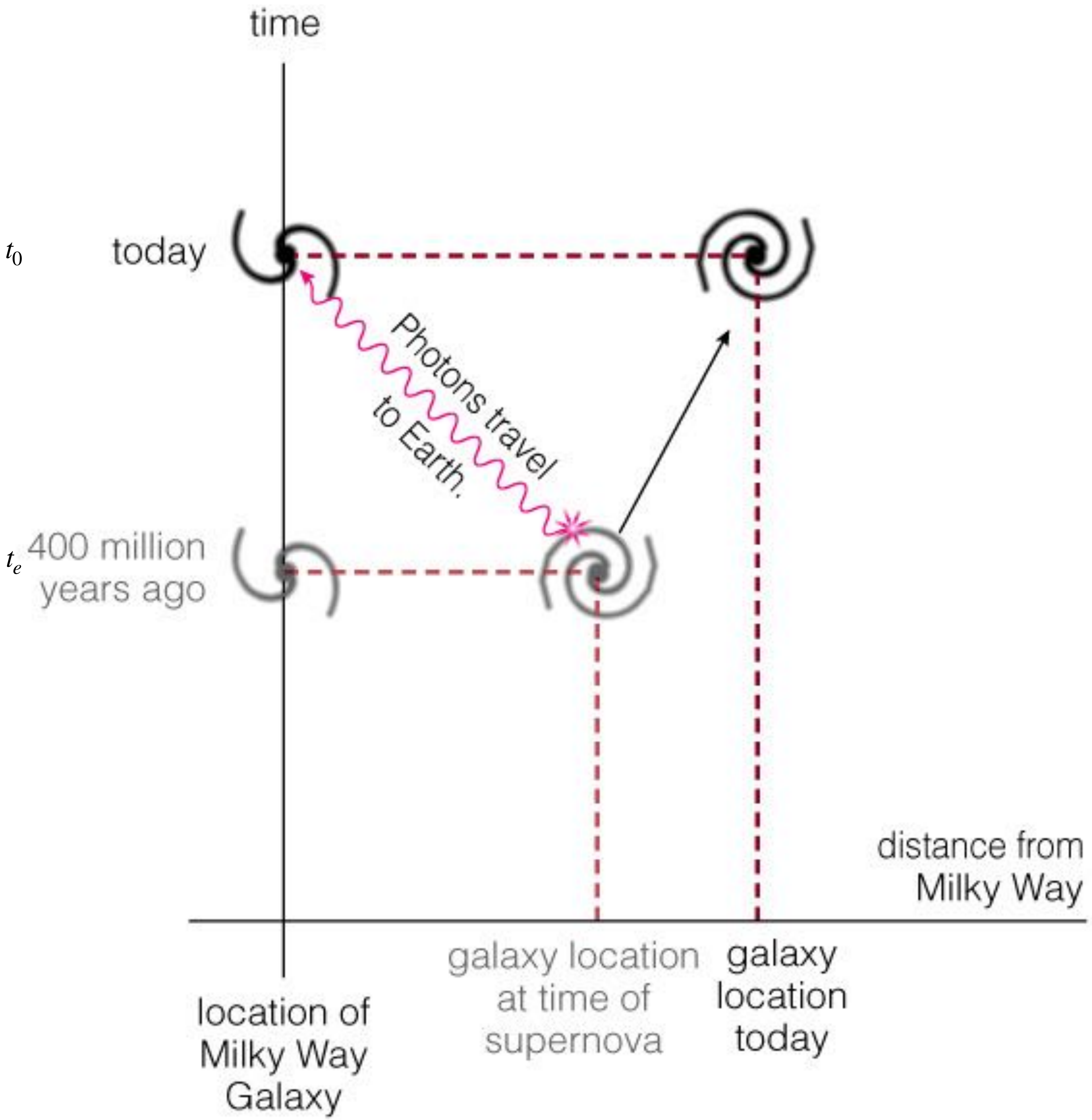
relates the redshift to the expansion factor



The proper distance

$$D_p(t_0) = c\int_{t_e}^{t_0} \frac{dt}{a(t)}$$

$$a(t) \approx 1 + H_0(t - t_0) - \frac{1}{2}q_0 H_0^2 (t - t_0)^2 + \dots$$



In terms of redshift,

$$D_p(z_e) = \frac{c}{H_0}\int_0^{z_e} \frac{dz}{E(z)}$$

For **zero** cosmological constant, there is an exact solution known as Mattig's equation:

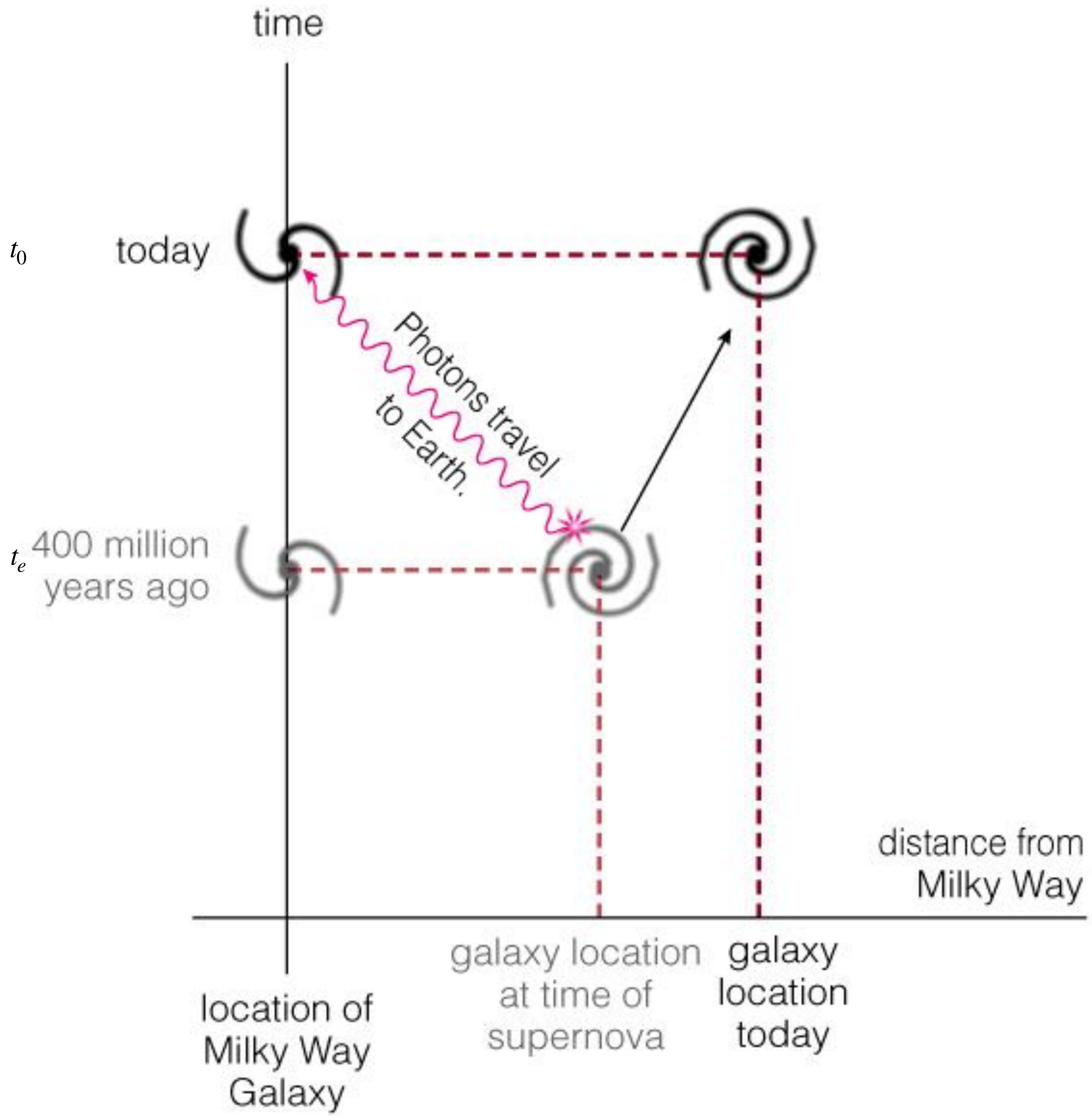
$$D_p(z) = \frac{2c}{H_0}\frac{[z\Omega_m + (\Omega_m - 2)(\sqrt{1 + z\Omega_m} - 1])}{\Omega_m^2(1+z)}$$

In general, there is no analytic solution, but can approximate with the Taylor expansion:

$$D_p(z) = \frac{c}{H_0}\left[z - \frac{1}{2}(1+q_0)z^2\right]$$

Where the leading term is Hubble's Law

$$D_p(z) = \frac{cz}{H_0}$$



For time rather than distance

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

Friedmann equation

$$H = \frac{\dot{a}}{a}$$

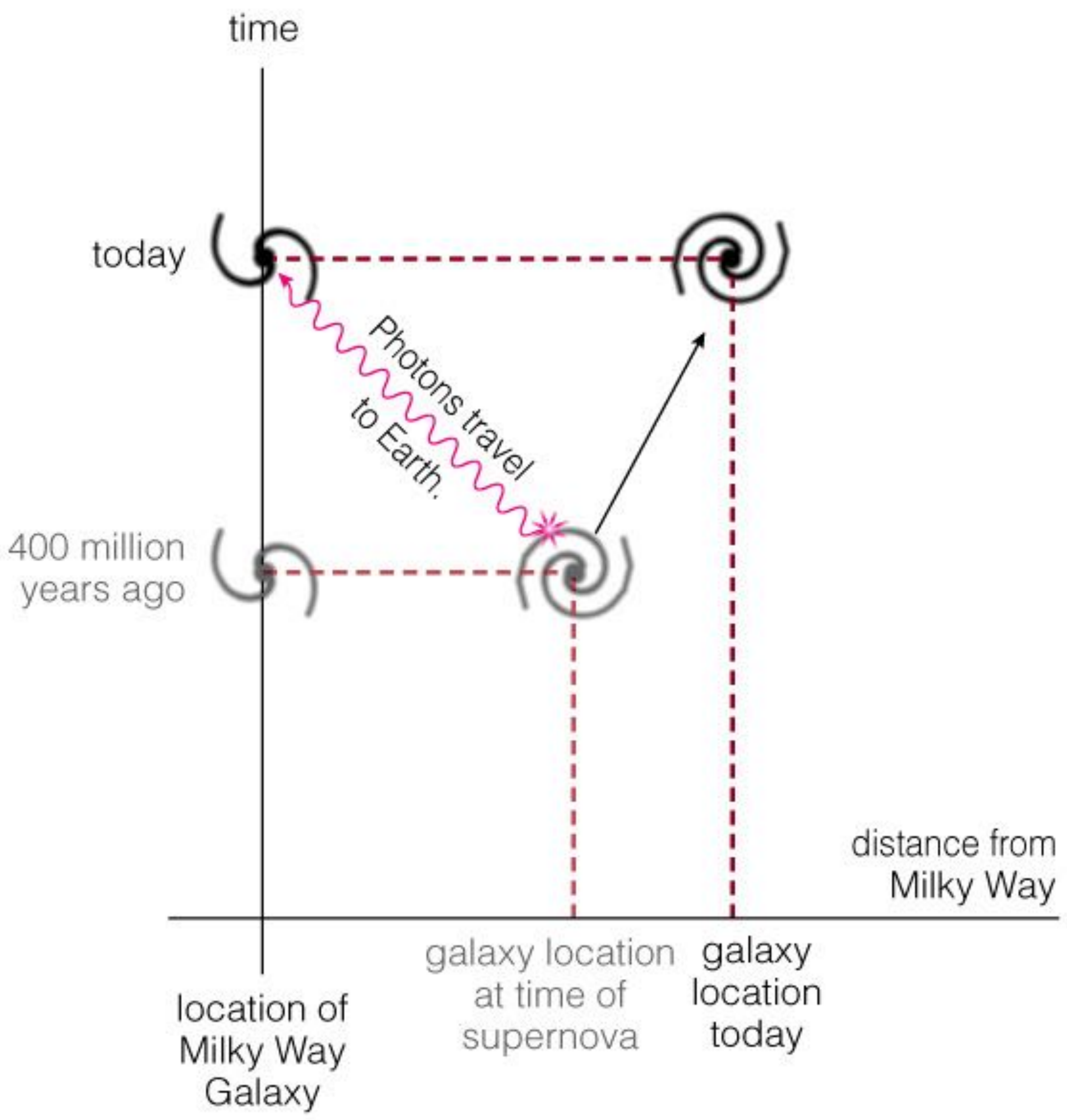
$$\frac{\dot{a}}{a} = H_0 E(a)$$

$$a = (1+z)^{-1}$$

$$E(z) = \sqrt{\Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}}$$

$$H_0 \int_{t_e}^{t_o} dt = \int_{a_e}^{1} \frac{da}{aE(a)} = \int_0^{z_e} \frac{dz}{(1+z)E(z)}$$

note that limits of the integral get flipped



$$H_0(t_0 - t_e) = \int_0^{z_e} \frac{dz}{(1+z)E(z)}$$

$(t_0 - t_e)$ is the **Lookback time** - the time since the photon was emitted.

The age of the universe is obtained by setting

$$t_e = 0\ ;\ z \to \infty$$

$$H_0 t_0 = \int_0^\infty \frac{dz}{(1+z)E(z)}$$

which can be approximated by

$$t_0 \approx \left(\frac{2}{3H_0}\right)(0.7\Omega_{m_0} + 0.3 - 0.3\Omega_{\Lambda_0})^{-0.3}$$

There is no deep theory in this last formula.
It is just a fitting formula that approximates the answer to a few %.

Similarly, the redshift-age of a matter dominated universe can be approximated as

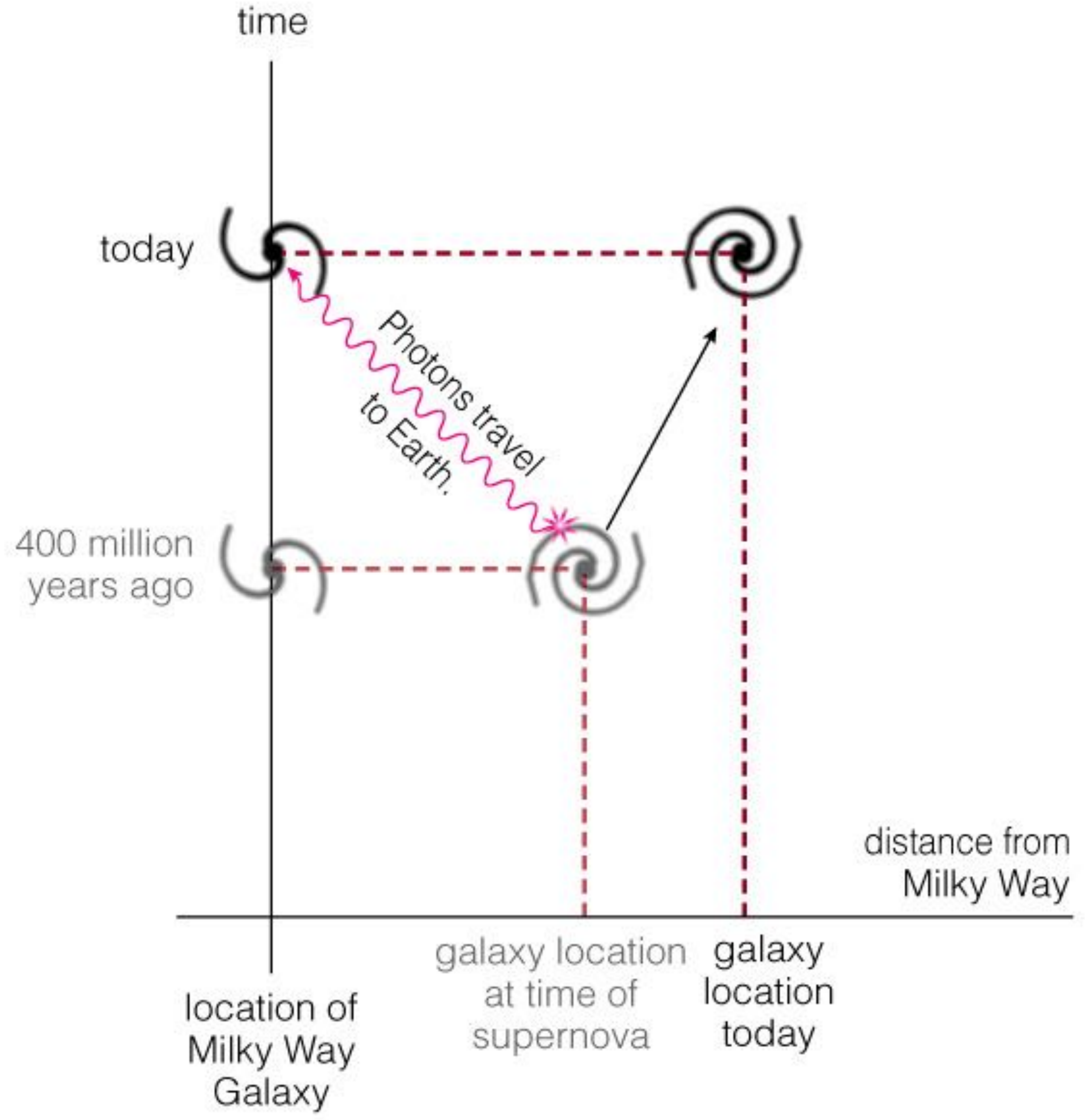
$$\frac{1}{t(z)} \approx H(z)[1 + \frac{1}{2}\Omega_m^{0.6}(z)]$$ Peacock (3.46)

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

$$H = \frac{\dot{a}}{a}$$

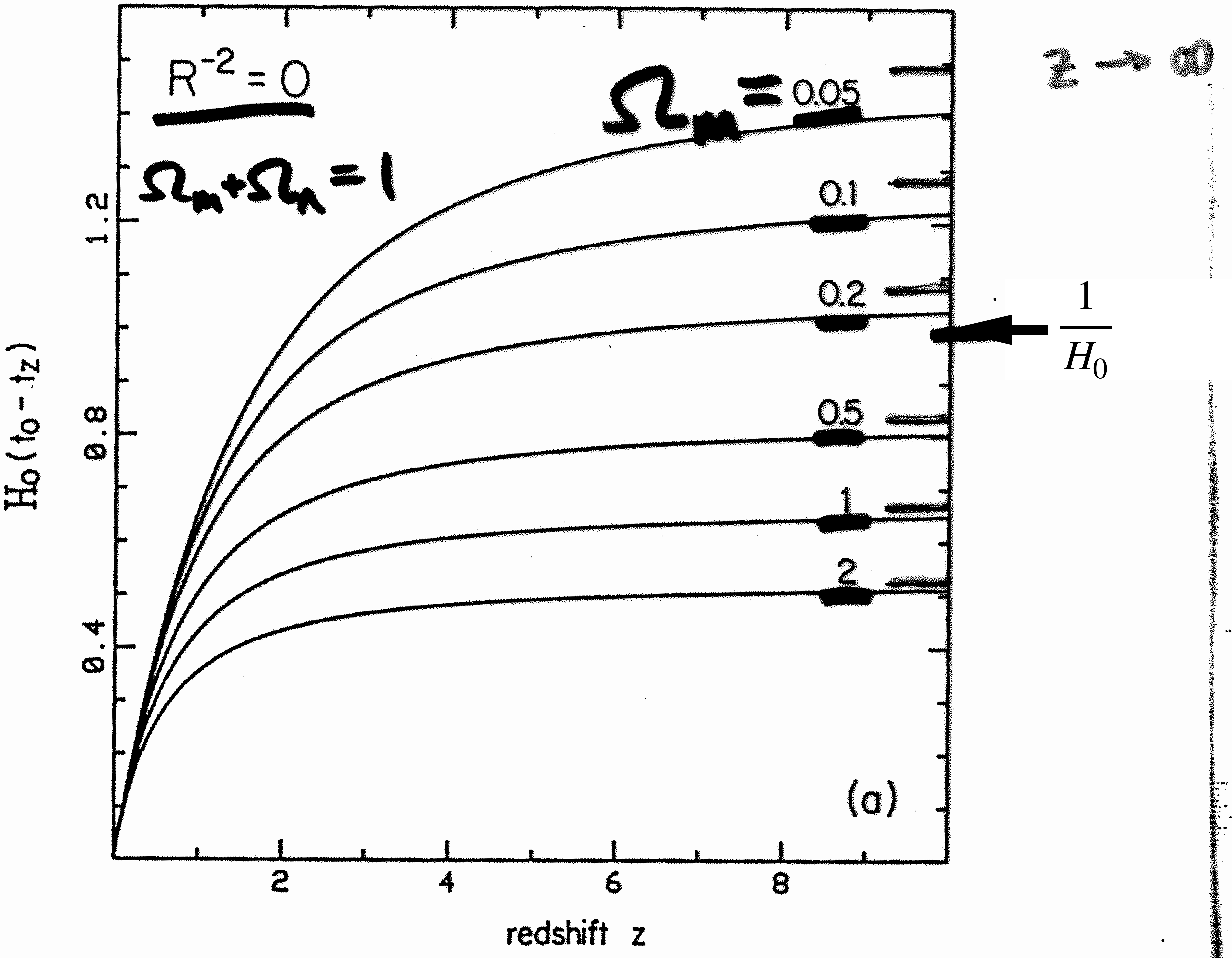
$$a = (1+z)^{-1}$$

$$E(z) = \sqrt{\Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}}$$



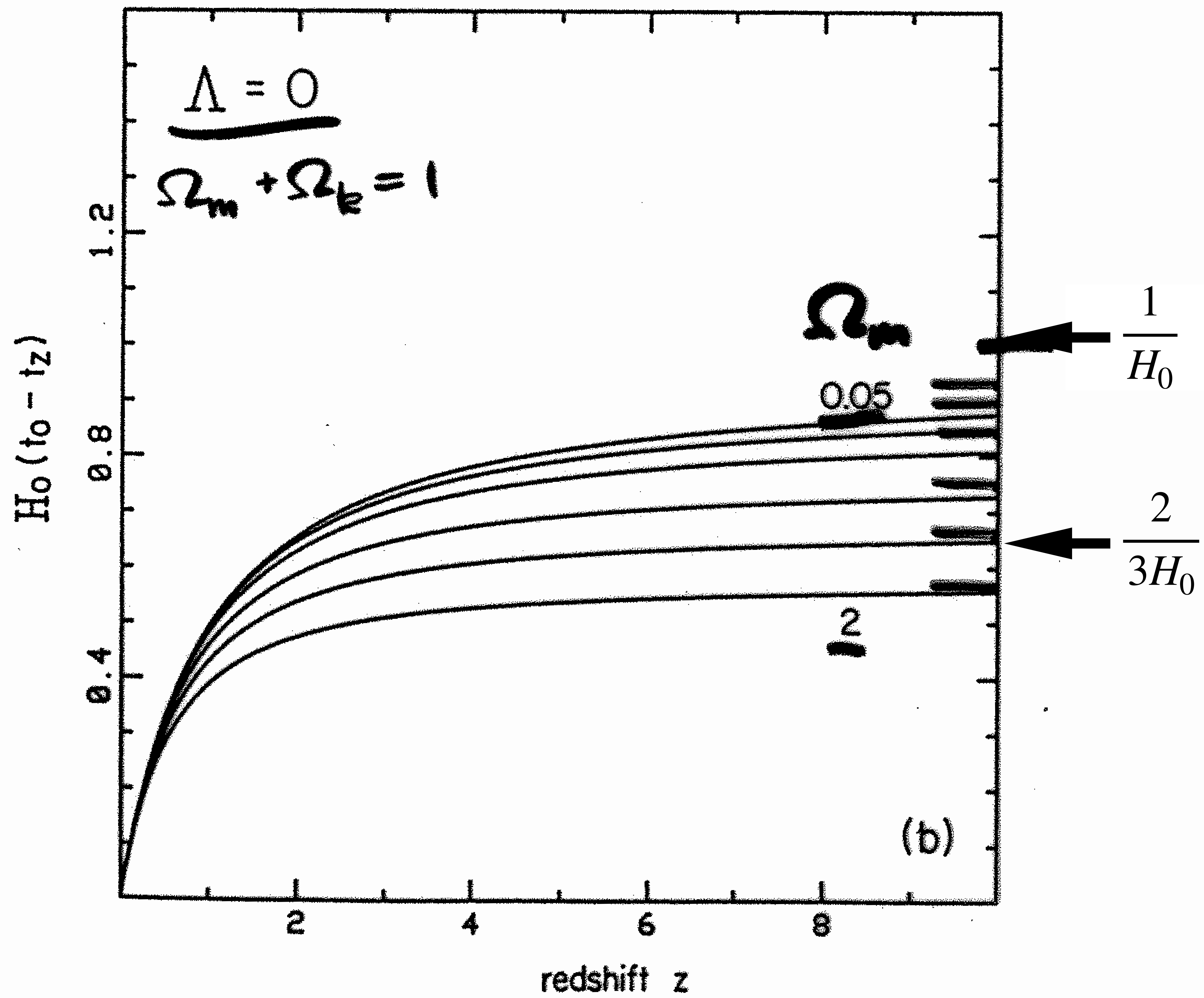
Peebles, Principles of Physical Cosmology

Figure 13.1. Lookback time as a function of redshift. The long dashes on the right-hand axis show the age t_o of the universe computed from $z \to \infty$. In panel (a) space curvature is negligible, and in panel (b) the cosmological constant, Λ, is negligibly small. The curves are labeled by the density parameter, Ω.



Zero cosmological constant

These cosmologies have only decelerated, so must have ages less than one Hubble time.



Peebles, Principles of Physical Cosmology

Solutions from Felten & Isaacman (1986) Reviews of Modern Physics, **58**, 689

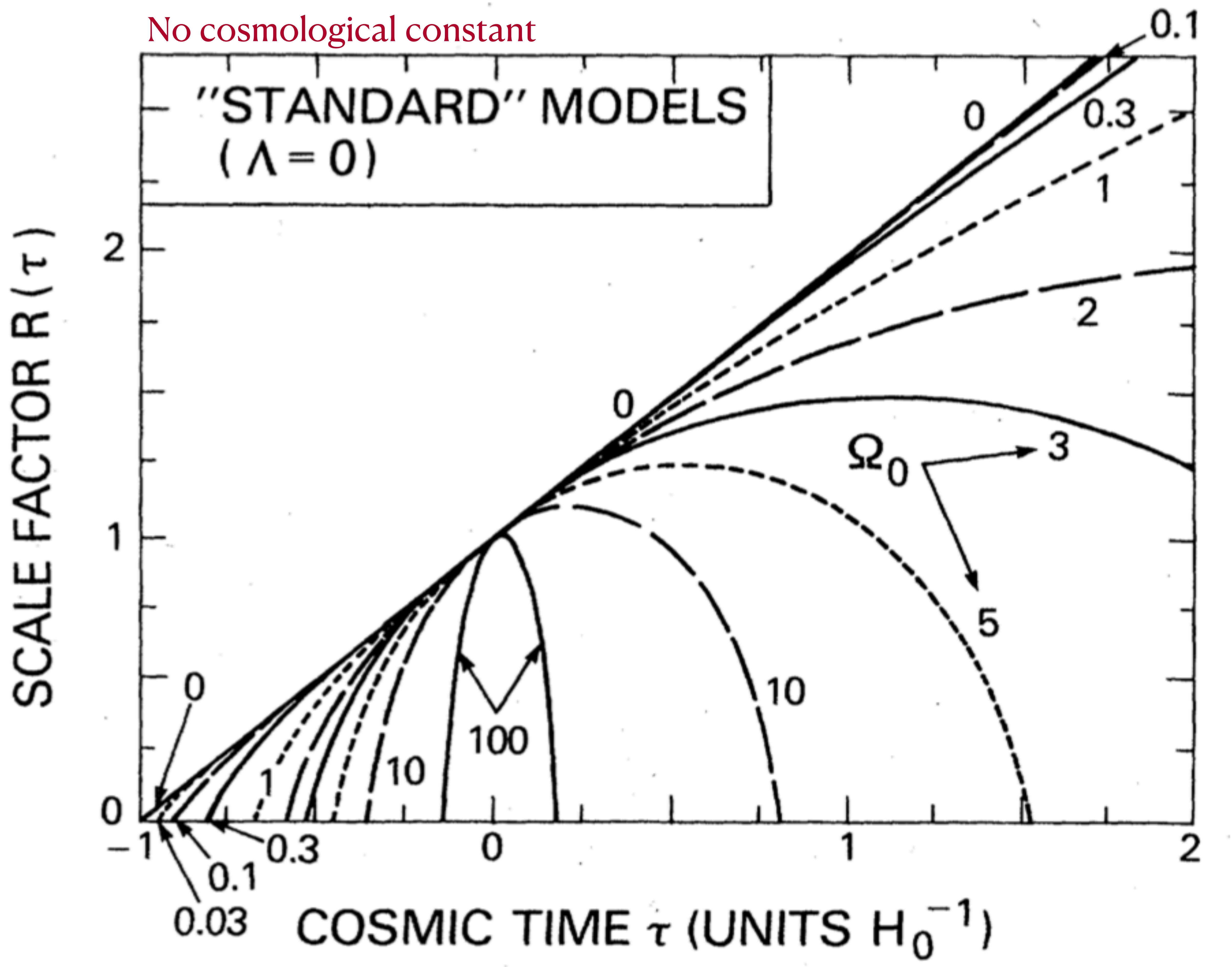


FIG. 3. "Standard" Friedmann models. The family of scale factors $R(\tau)$ for the "standard models" ($\Lambda=0$). The free parameter, shown on the curves, is Ω_0. As shown by the τ intercepts, all models have ages ≤ 1 ($\leq H_0^{-1}$ yr).

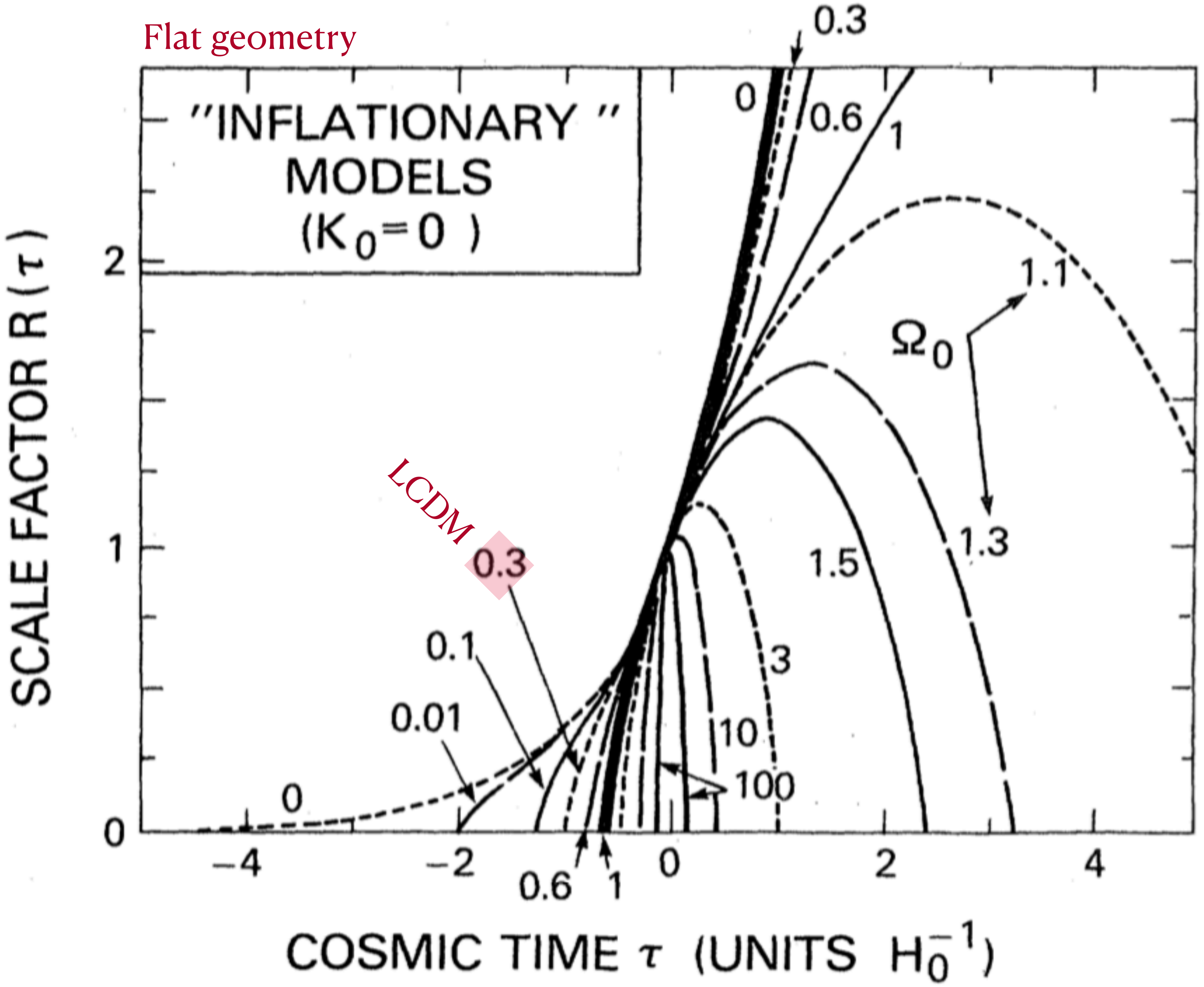


FIG. 2. "Inflationary" Friedmann models. The family of scale factors $R(\tau)$ for models satisfying the "inflationary constraint" (three-space curvature $K_0=0$). The free parameter, shown on the curves, is Ω_0. The cosmological constant Λ is determined from Ω_0 by Eq. (14).

Solutions from Felten & Isaacman (1986) Reviews of Modern Physics, **58**, 689

Can in principle have solutions in which there was no Big Bang in the past, depending on the value of Lambda.

Ω_m, Ω_Λ, Ω_k all non-zero

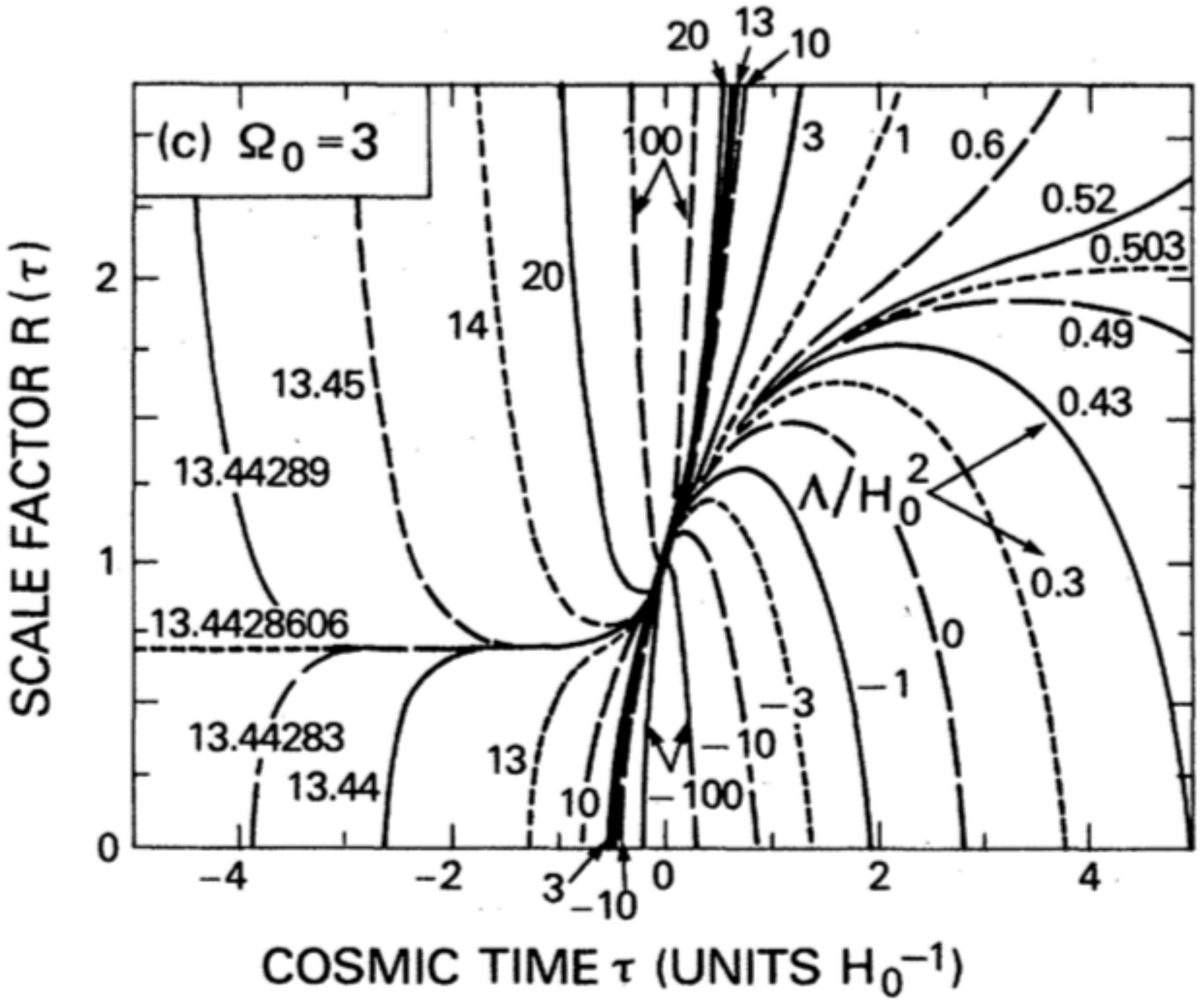


FIG. 1. Solutions of the Friedmann equation. Three families of scale factors $R(\tau)$ for Friedmann (zero-pressure) universes, with three fixed values of the present density parameter Ω_0: (a) $\Omega_0=0.1$; (b) $\Omega_0=1$; (c) $\Omega_0=3$. The free parameter, shown on the curves, is the cosmological constant Λ in units of H_0^2, where H_0 is the present Hubble parameter. The time τ is measured in units of the Hubble time H_0^{-1} and is taken $=0$ at present. The scale factor $R(\tau)$ is normalized to unity at present: $R_0=1$. For further discussion see the text.

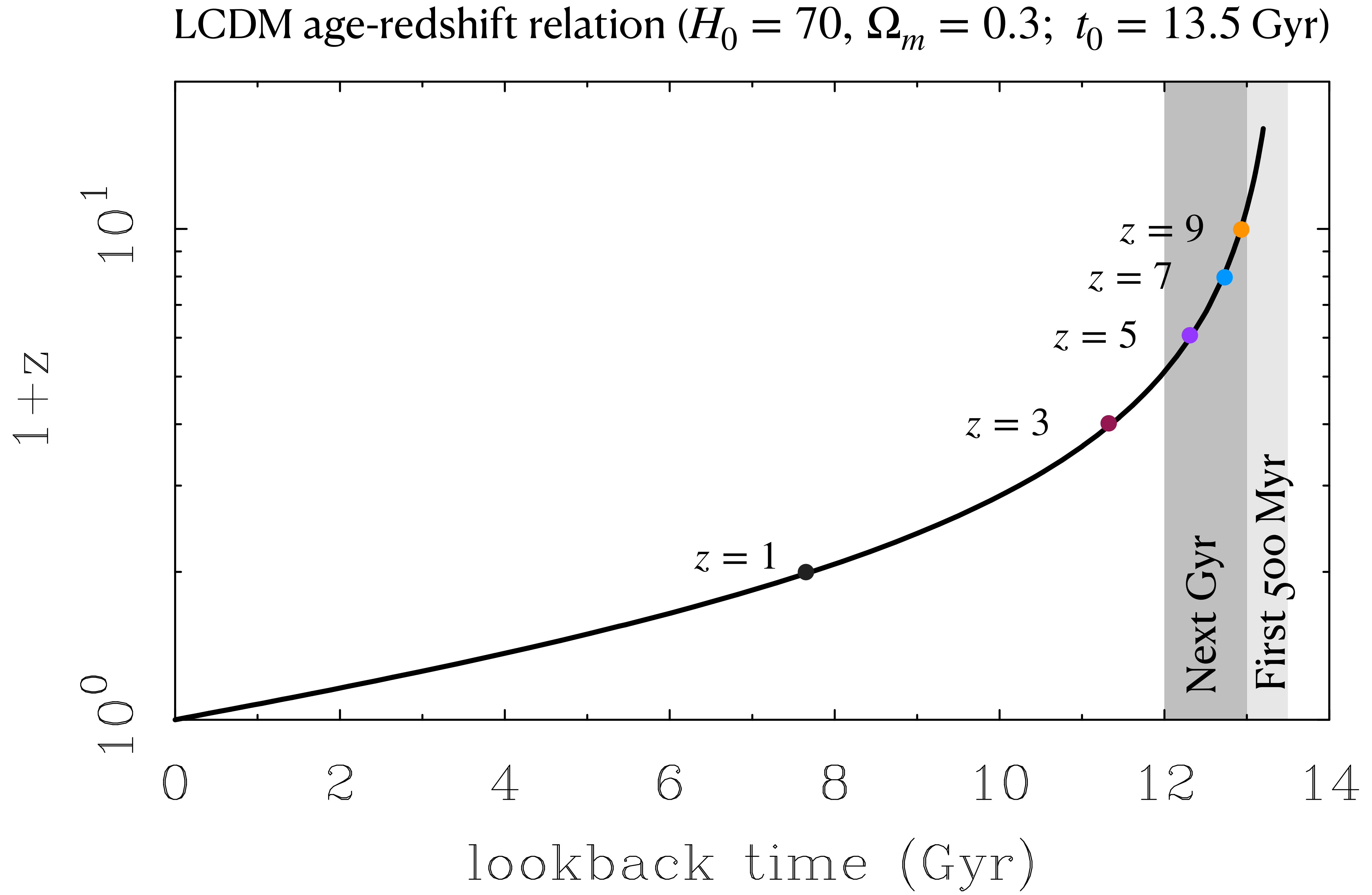
Observational Tests

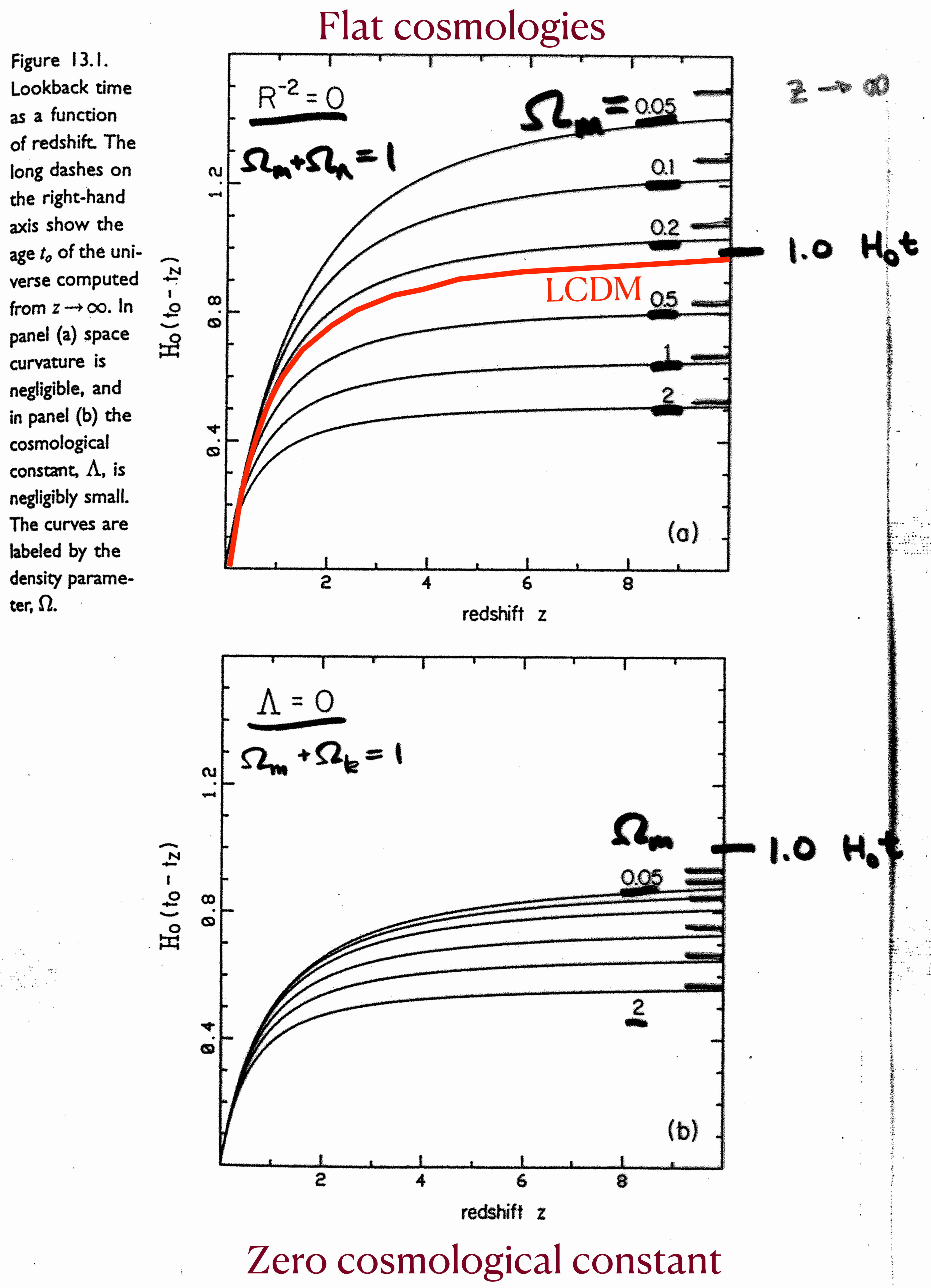
Five Classic Tests

• Luminosity-redshift relation	$D_L - z$	Standard Candle
• Angular size-redshift relation	$D_A - z$	Standard Rod
• Number-redshift relation	$N(z)$	Source counts with redshift
• Number-magnitude relation	$N(m)$	Source counts with magnitude
• Tolman test	$\Sigma(z)$	Surface brightness not distance independent in Robertson-Walker geometry

Other tests are possible. E.g., one could in principle make an age-redshift test - if one could confidently measure ages of objects at cosmic distances.

- Age-redshift relation





Age-redshift test enabled by JWST observations

arXiv:2409.03829

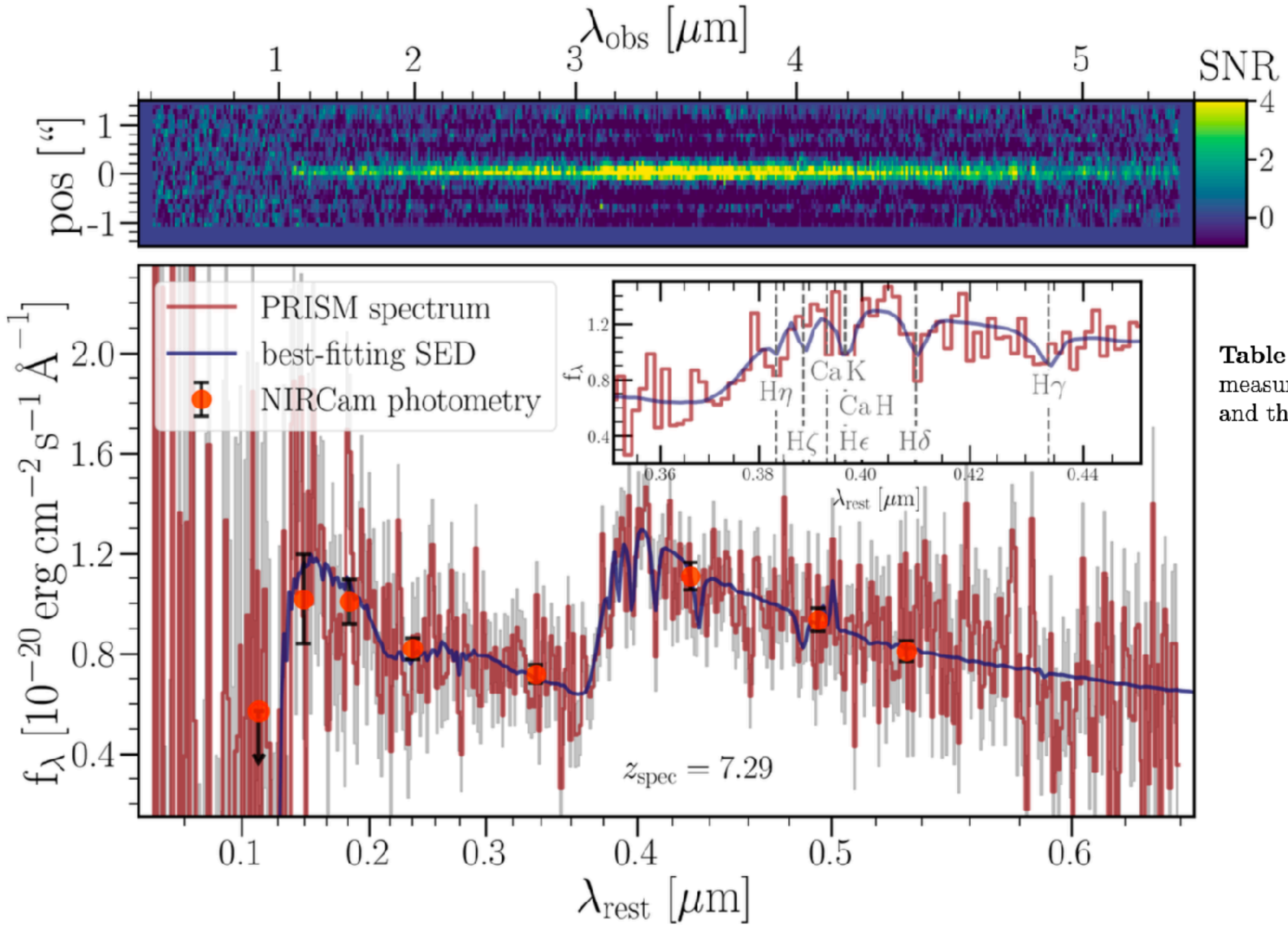


Figure 1. NIRSpec/PRISM Spectrum of RUBIES-UDS-QG-z7. Top: 2D SNR spectrum. Bottom: 1D spectrum of RUBIES-UDS-QG-z7 in red, with 1σ uncertainties in gray. The NIRCam photometry is shown as orange dots and the best-fitting SED from prospector in blue (see Section 3). A zoom-in to the region around 0.4μm is shown in the inset panel, where we highlight the position of various absorption features. Note also that the best-fitting SED latches on the Balmer absorption lines.

Table 2. Physical properties of RUBIES-UDS-QG-z7, as measured with **Prospector** for the fiducial model (free Z) and the high metallicity (high-Z) fit.

quantity	fiducial	high-Z
z_{spec}	$7.287^{+0.007}_{-0.006}$	$7.290^{+0.005}_{-0.006}$
$\log(M_*/M_\odot)$	$10.23^{+0.04}_{-0.04}$	$10.19^{+0.04}_{-0.04}$
$\log(\Sigma_{*,c}/M_\odot\,kpc^{-2})$	$10.85^{+0.11}_{-0.12}$	$10.80^{+0.11}_{-0.12}$
SFR_{10} [$M_\odot\,yr^{-1}$]	$0.64^{+0.83}_{-0.60}$	$1.08^{+1.55}_{-0.98}$
SFR_{50} [$M_\odot\,yr^{-1}$]	$0.83^{+11.11}_{-0.76}$	$2.13^{+5.54}_{-1.92}$
SFR_{100} [$M_\odot\,yr^{-1}$]	$0.84^{+20.16}_{-0.78}$	$48.89^{+21.12}_{-13.04}$
A_V [mag]	$0.34^{+0.08}_{-0.09}$	$0.27^{+0.09}_{-0.07}$
t_{50} [Gyr]	$0.20^{+0.07}_{-0.02}$	$0.16^{+0.03}_{-0.02}$
t_{90} [Gyr]	$0.12^{+0.01}_{-0.01}$	$0.07^{+0.01}_{-0.01}$
$\log(Z/Z_\odot)$	$-0.94^{+0.05}_{-0.04}$	$0.07^{+0.08}_{-0.11}$

age-redshift test now possible with JWST

- Tolman Test

Also referred to as $(1+z)^4$ dimming.

<u>Surface brightness dimming</u>

No surface brightness dimming in Euclidean geometry

$$\Sigma \sim \frac{f}{\theta^2} \sim \frac{D^{-2}}{D^{-2}} \sim \text{constant}$$

Lots of surface brightness dimming in Robertson-Walker geometry

$$\Sigma \sim \frac{f}{\theta^2} \sim \frac{D_L^{-2}}{D_A^{-2}} \sim \frac{D_p^{-2}(1+z)^{-2}}{D_p^{-2}(1+z)^2} \sim (1+z)^{-4}$$

Surface brightness dims as a strong function of redshift!

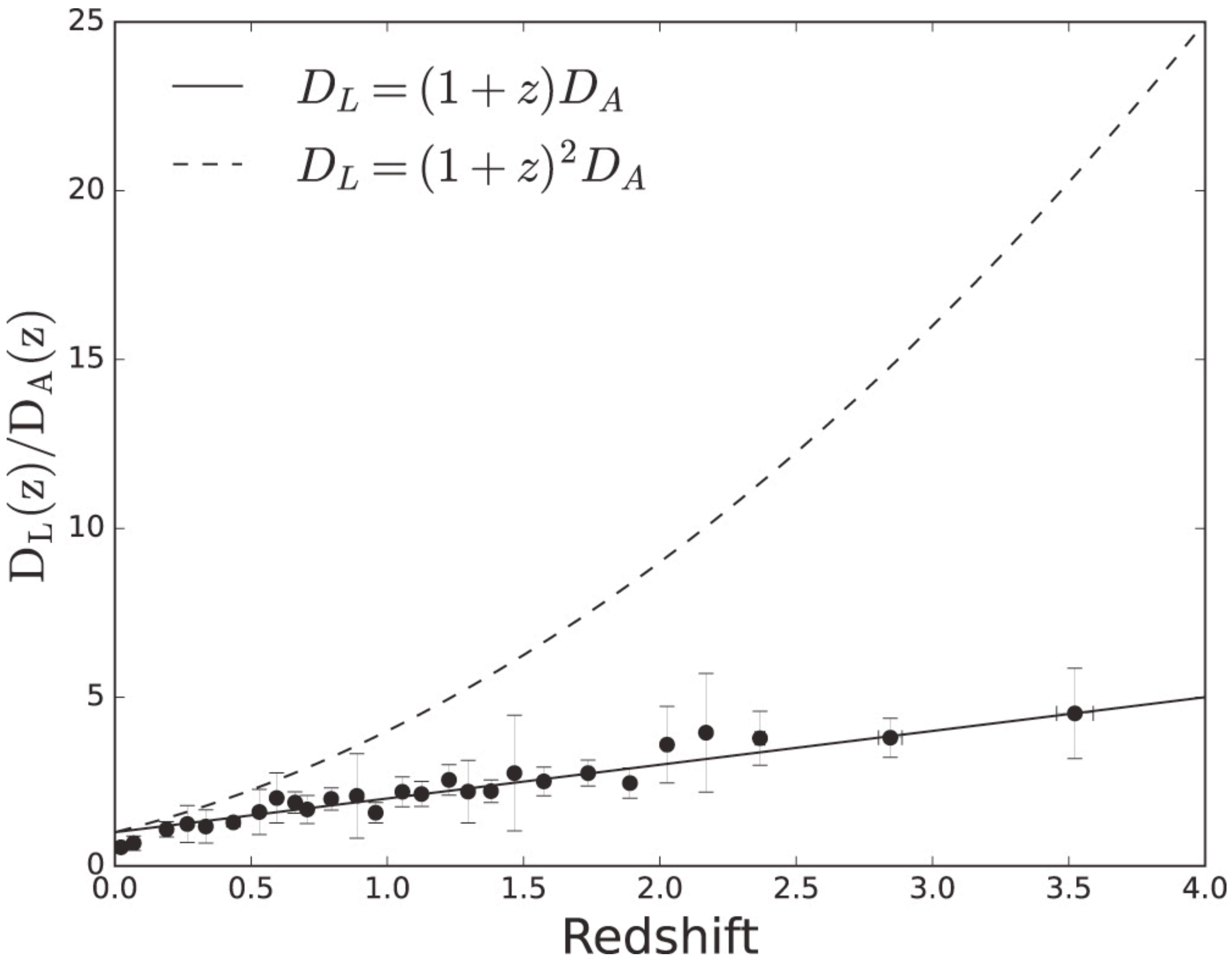
The Tolman test is a sanity check:
it does not distinguish between FLRW models: the same amount of dimming occurs in all.
In practice, it is hard to distinguish from evolutionary effects.

- Angular size-redshift relation

Li (2023) Fig. 3

angular sizes of compact radio sources

- Angular size-redshift relation



Li (2023) Fig. 4

angular sizes of compact radio sources