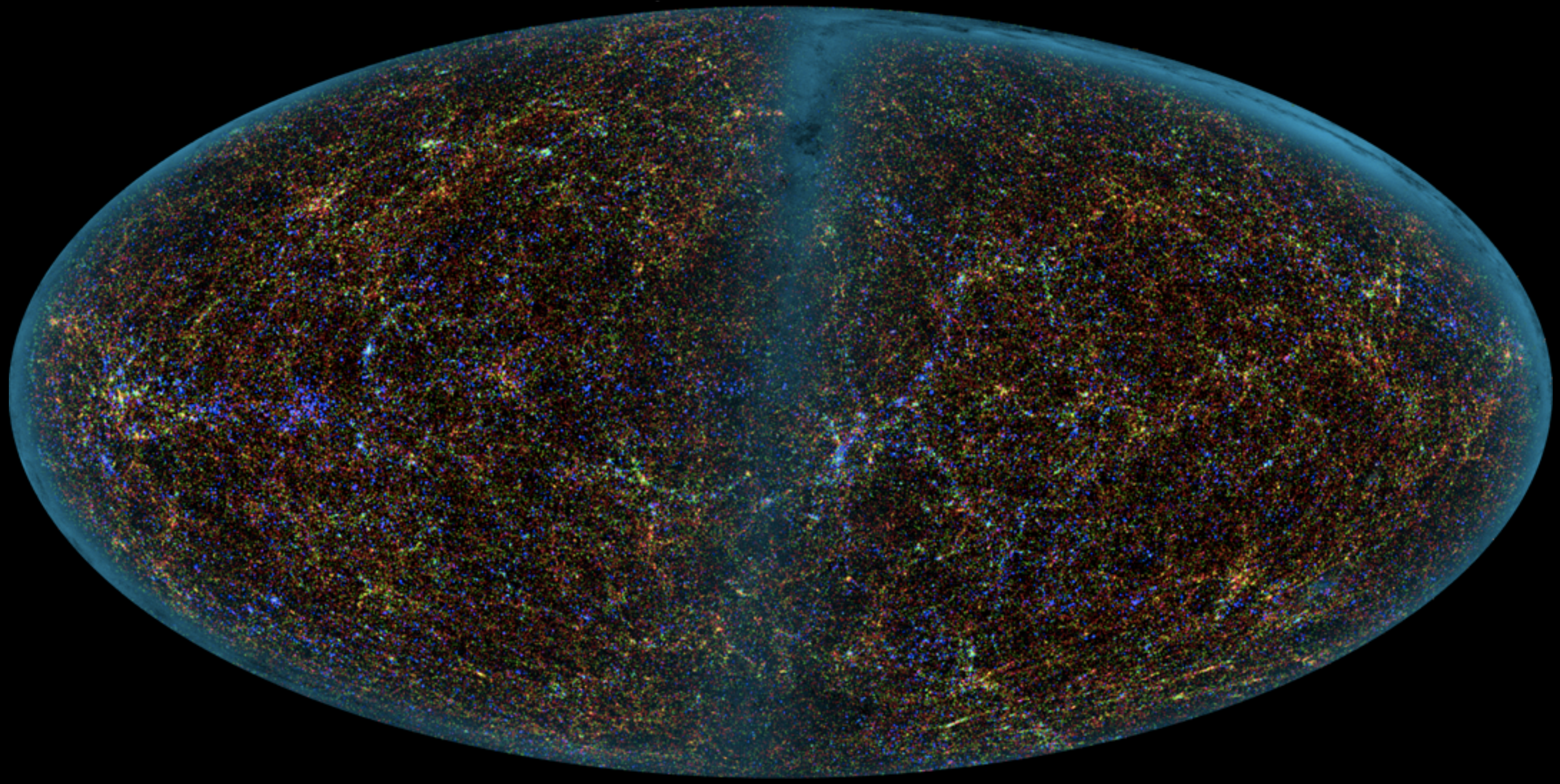
# Cosmology

## and Large Scale Structure



Today

Observational Tests

Luminosity Distance-redshift
- e.g., Type Ia SN

Angular size-redshift

Number Counts
- e.g., Galaxy N(m), N(z)

homework 2 due next time

17 September 2024

http://astroweb.case.edu/ssm/ASTR328/

# Observational Tests

## Five Classic Tests

| | | |
|---|---|---|
| • Luminosity-redshift relation | $D_L - z$ | Standard Candle |
| • Angular size-redshift relation | $D_A - z$ | Standard Rod |
| • Number-redshift relation | $N(z)$ | Source counts with redshift |
| • Number-magnitude relation | $N(m)$ | Source counts with magnitude |
| • Tolman test | $\Sigma(z)$ | Surface brightness not distance independent in Robertson-Walker geometry |

Other tests are possible. E.g., one could in principle make an age-redshift test - if one could confidently measure ages of objects at cosmic distances.

- Luminosity-redshift relation

Ideal case:
a **Standard Candle**
an object of constant, known luminosity ***L***

Measuring redshift-distance pairs $D_L, z$ measures cosmology through $H_0, q_0$

$$a(t) \approx 1 + H_0(t - t_0) - \frac{1}{2}q_0 H_0^2 (t - t_0)^2 + \dots$$

Then its apparent brightness is simply dimmed by its distance
as a consequence of the inverse square law in the appropriate geometry.

flux & luminosity

$$f = \frac{L}{4\pi D_L^2}$$

Luminosity distance

$$D_L = (1 + z)D_p$$

apparent & absolute magnitude

$$m - M = 5 \log D_L + 25$$

distance in Mpc

in practice, also have to worry about line of sight extinction $A$

$$m - M = 5 \log D_L + 25 + A$$

as a source can be dimmed by obscuration as well as remoteness

The line of sight extinction $A$ that corrects the distance modulus

$$m - M = 5 \log D_L + 25 + A$$

has been well mapped in the Milky Way

There are reasonably well calibrated maps of $A(\ell, b)$ where $\ell$ $b$ are Galactic longitude and latitude. You can look up the line-of sight extinction with resources like NED.

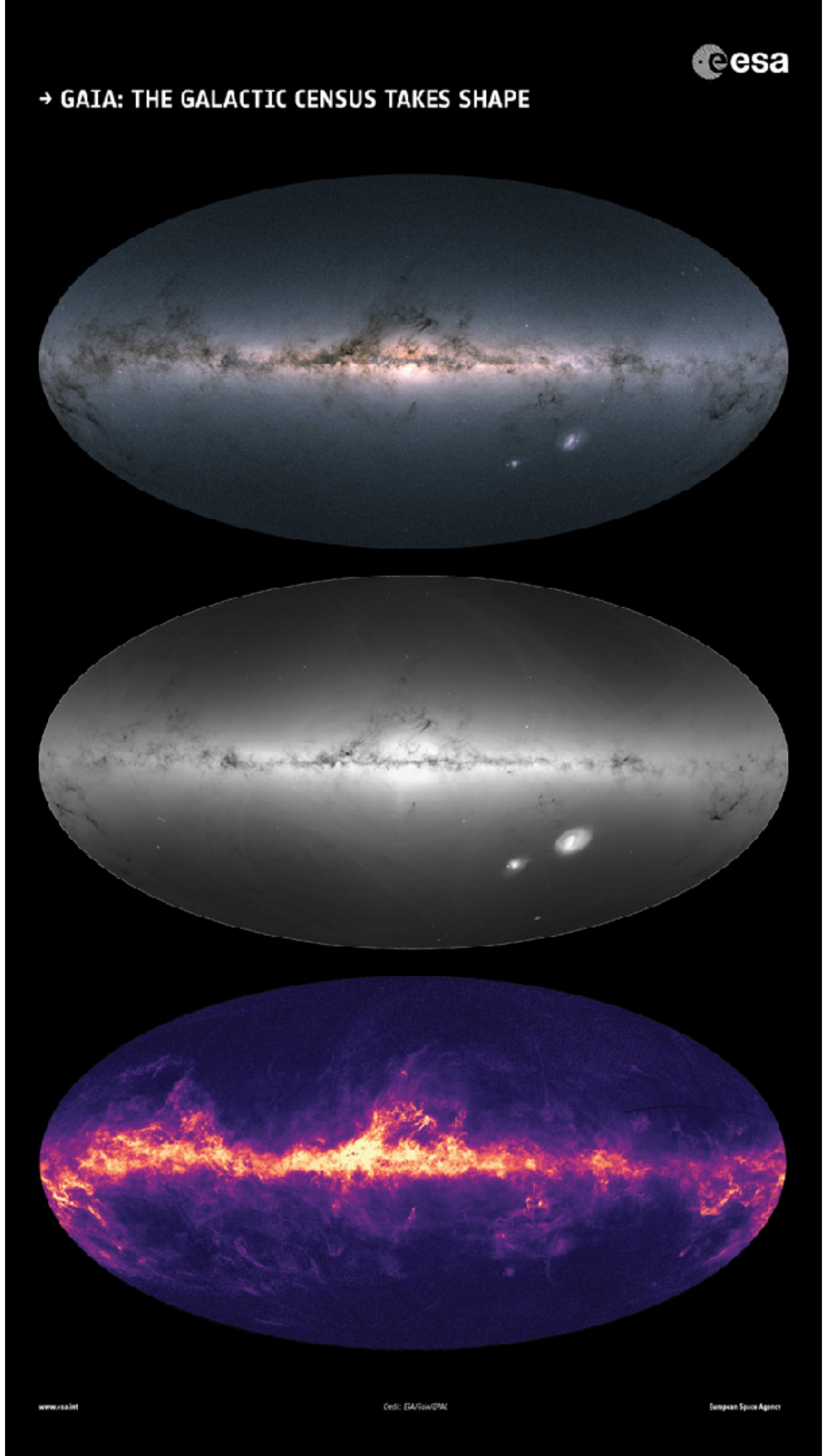
$A_{Gal}$ is wavelength dependent

**Galaxy Extinctions for NGC 3109**

View References in ADS (2) 1 of 1 (1 - 102 of 102)

| Bandpass<br>char | Central Wavelength<br>(um)<br>double | The Galactic extinction<br>(Mag)<br>double | Refcode of the publications<br>char |
|---|---|---|---|
| Landolt U | 0.35 | 0.289 | 2011ApJ...737..103S |
| Landolt B | 0.43 | 0.242 | 2011ApJ...737..103S |
| Landolt V | 0.54 | 0.183 | 2011ApJ...737..103S |
| Landolt R | 0.64 | 0.145 | 2011ApJ...737..103S |
| Landolt I | 0.80 | 0.100 | 2011ApJ...737..103S |
| CTIO U | 0.37 | 0.274 | 2011ApJ...737..103S |
| CTIO B | 0.43 | 0.243 | 2011ApJ...737..103S |
| CTIO V | 0.55 | 0.179 | 2011ApJ...737..103S |
| CTIO R | 0.65 | 0.141 | 2011ApJ...737..103S |
| CTIO I | 0.80 | 0.101 | 2011ApJ...737..103S |
| UKIRT J | 1.25 | 0.047 | 2011ApJ...737..103S |
| UKIRT H | 1.66 | 0.030 | 2011ApJ...737..103S |
| UKIRT K | 2.19 | 0.020 | 2011ApJ...737..103S |

This is only the dust in our own Galaxy. There can be additional dust in other galaxies, which is often hard to estimate, but in principle $A = A_{Gal} + A_{exgal}$.




**Gaia**

Brightness

Star counts

Dust (Zone of Avoidance)

- Luminosity-redshift relation

Ideal case:
a **Standard Candle**
an object of constant, known luminosity ***L***

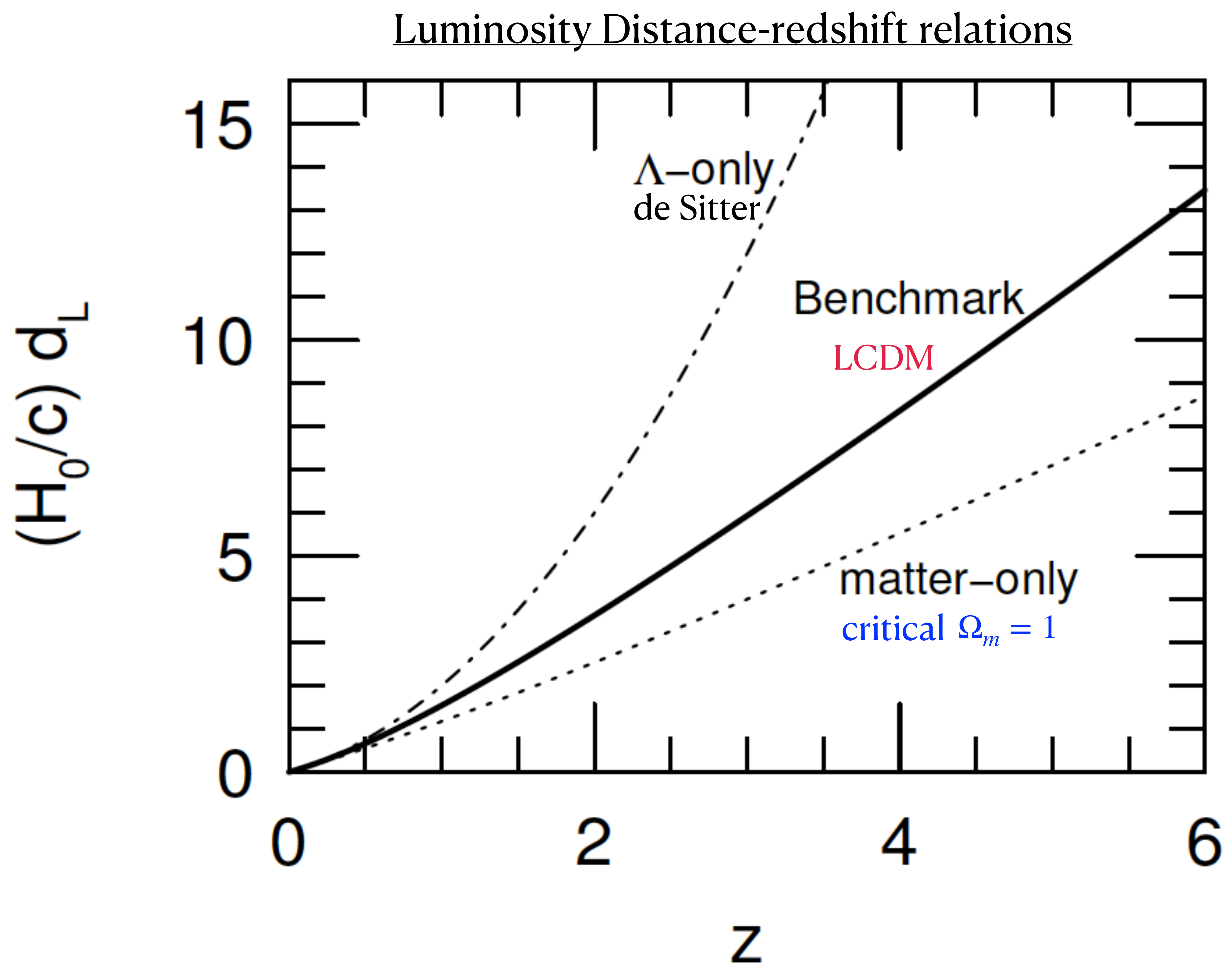
Example Standard Candles:

- Cepheids
- Tip of the Red Giant Branch
- Type Ia Supernovae

None of these Standard Candles are really standard, but they are standardizable — e.g., the Cepheid period-luminosity relation allows one to measure a distance-independent quantity (the period) as a proxy for the distance-dependent luminosity.

The trick is in the calibration.

- Luminosity-redshift relation




Note that the luminosity distance can easily exceed the Hubble length.

Figure 7.2: The luminosity distance of a standard candle with observed redshift $z$. The bold solid line gives the result for the Benchmark Model, the dot-dash line for a flat, lambda-only universe, and the dotted line for a flat, matter-only universe.

Ryden Fig. 6.2
(2nd ed)

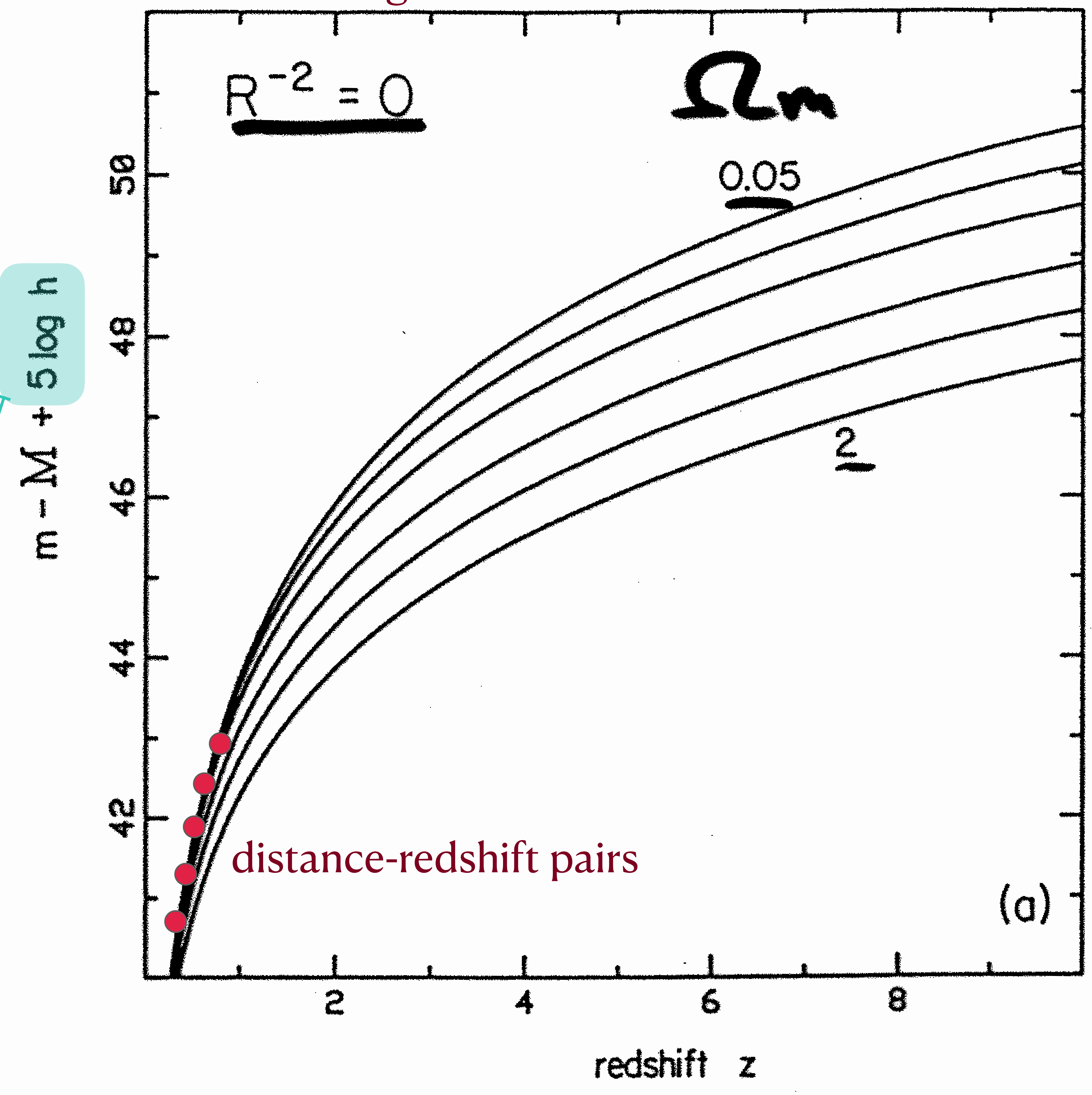
- Luminosity-redshift relation

Luminosity Distance-redshift relations

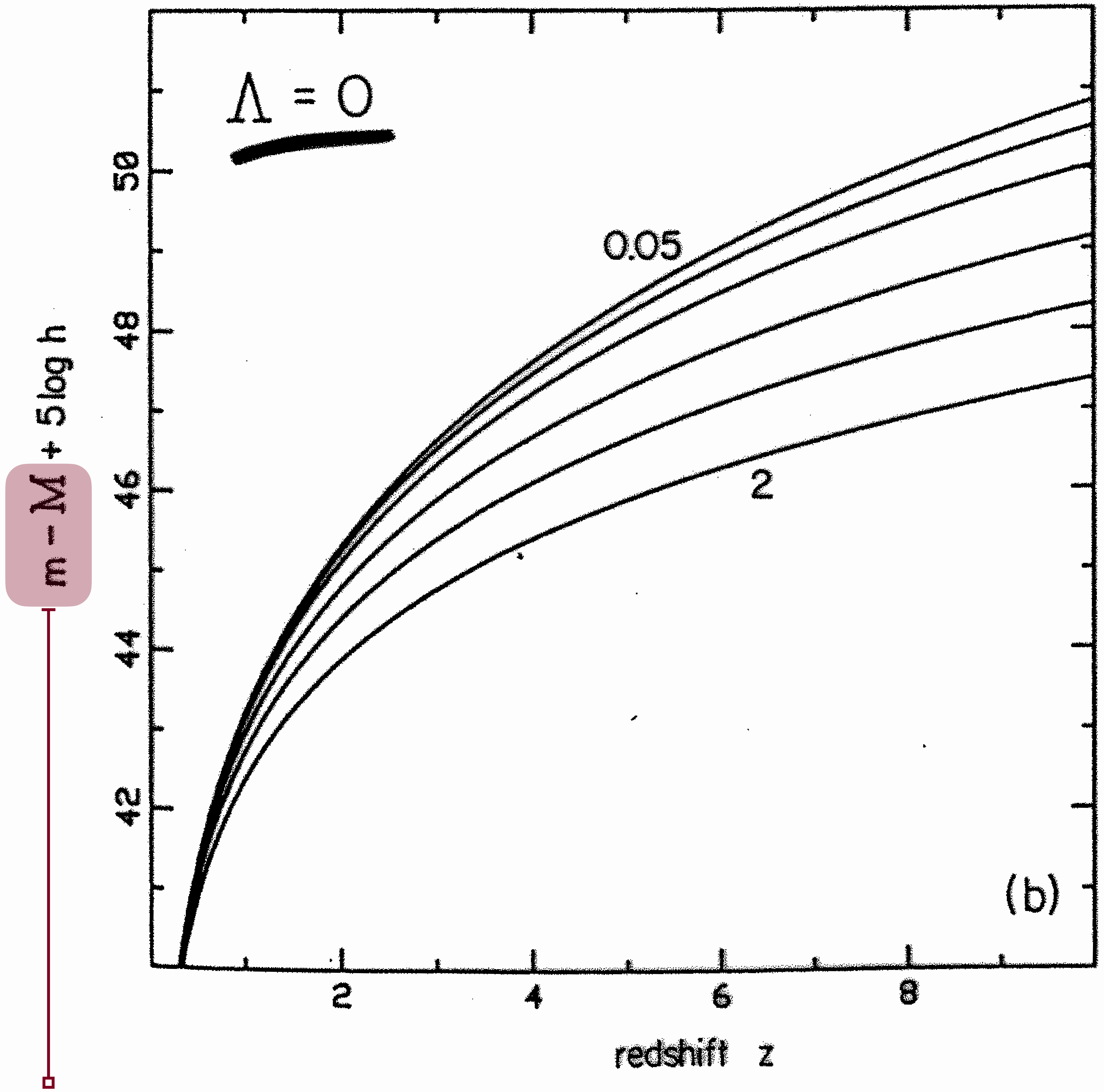
Figure 13.6. Bolometric distance modulus $m - M + 5 \log h$ as a function of redshift. The parameters are arranged as in figure 13.1.

$h = \frac{H_0}{100}$ in km/s/Mpc

Flat cosmologies




Zero cosmological constant




$(m - M)$ is the distance modulus

Peebles Fig. 13.6

Example Standard Candle: • Type Ia Supernovae

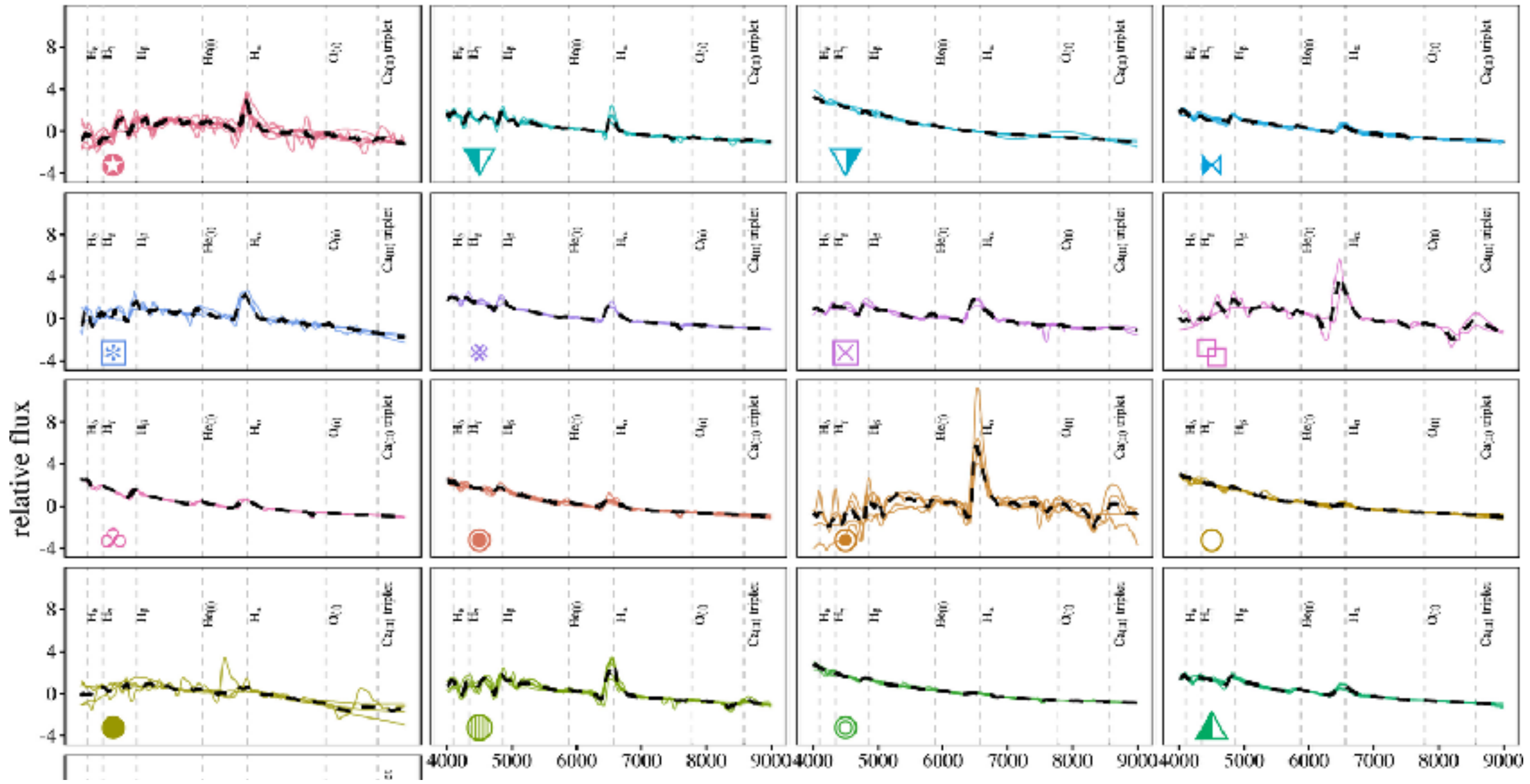
Supernovae classification:

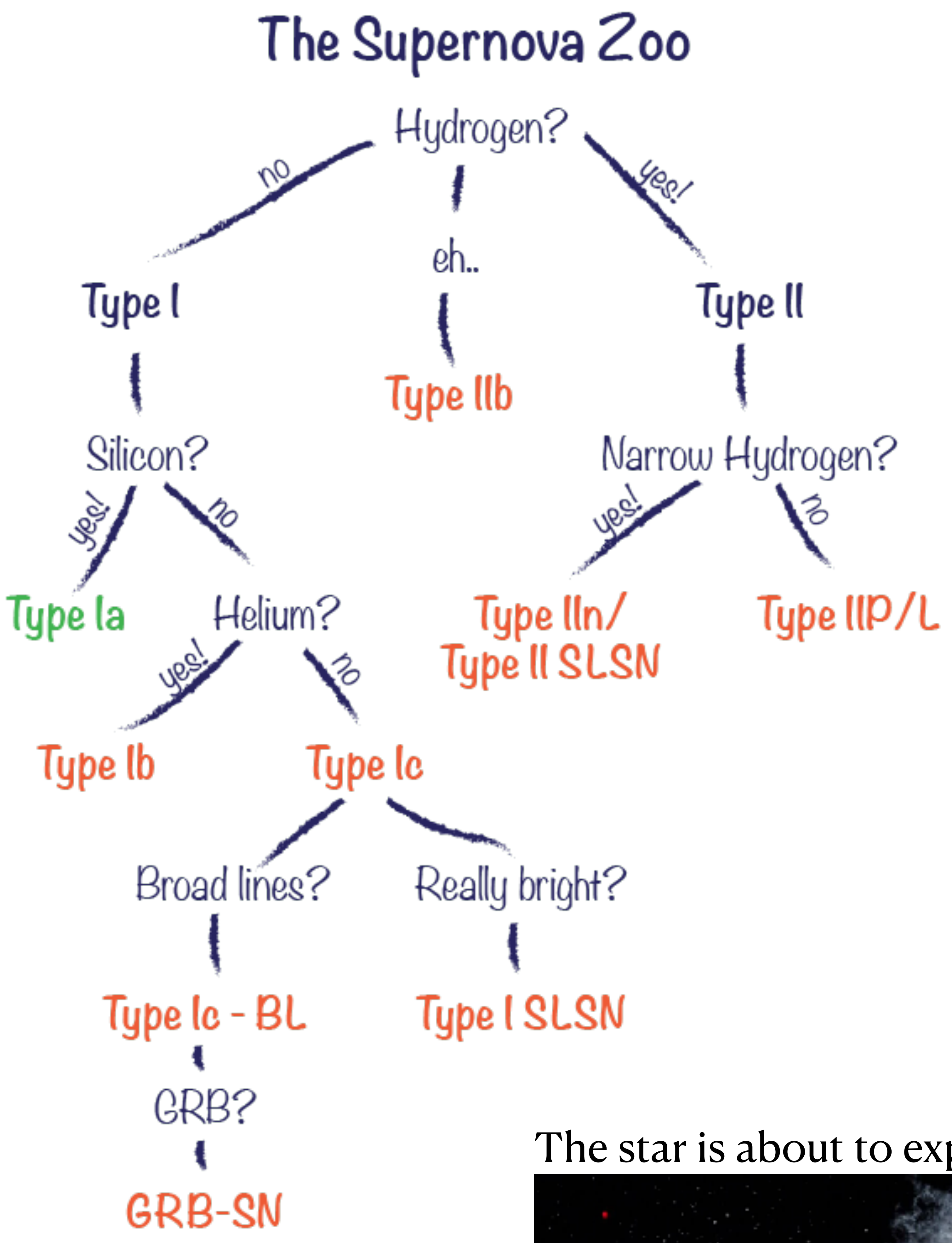
two physical mechanisms:

core collapse SN

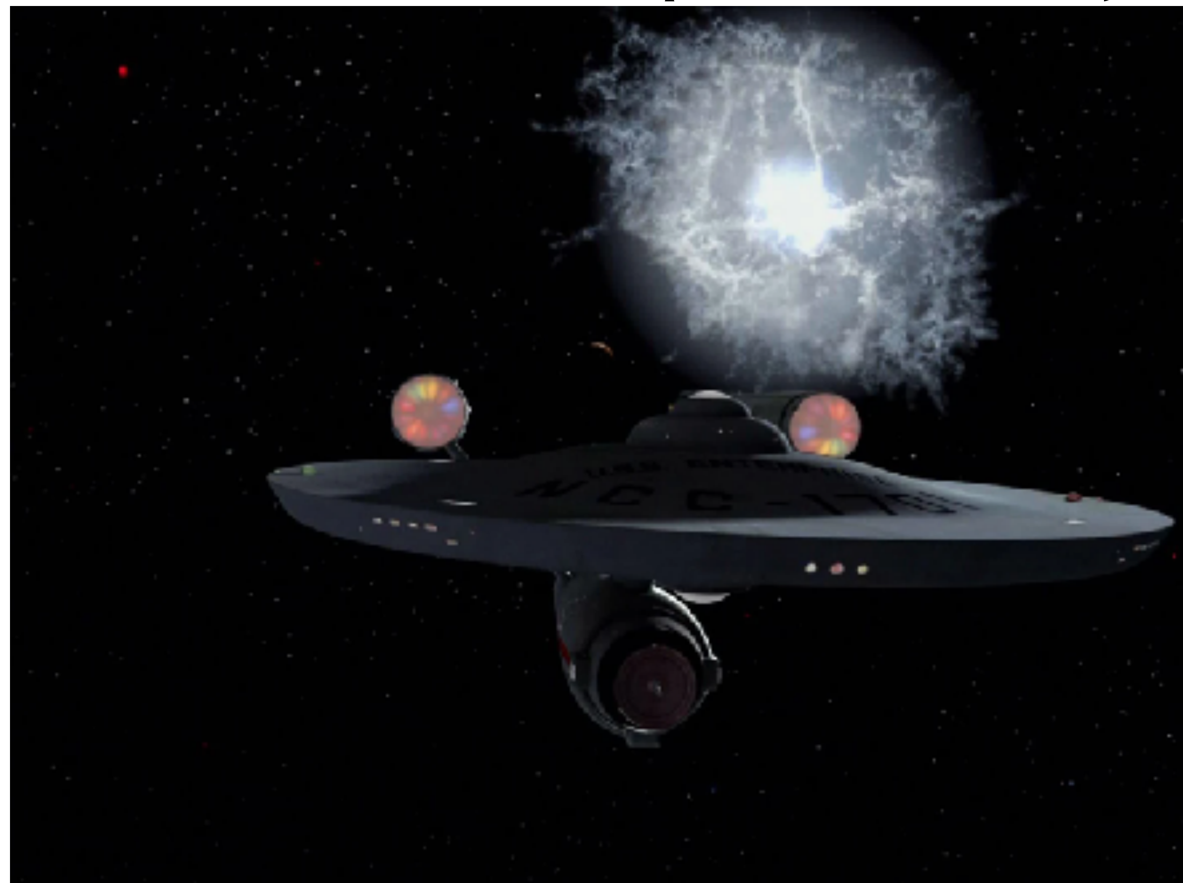
thermonuclear SN

supernovae are identified by their spectral characteristics




The star is about to explode! Run away!



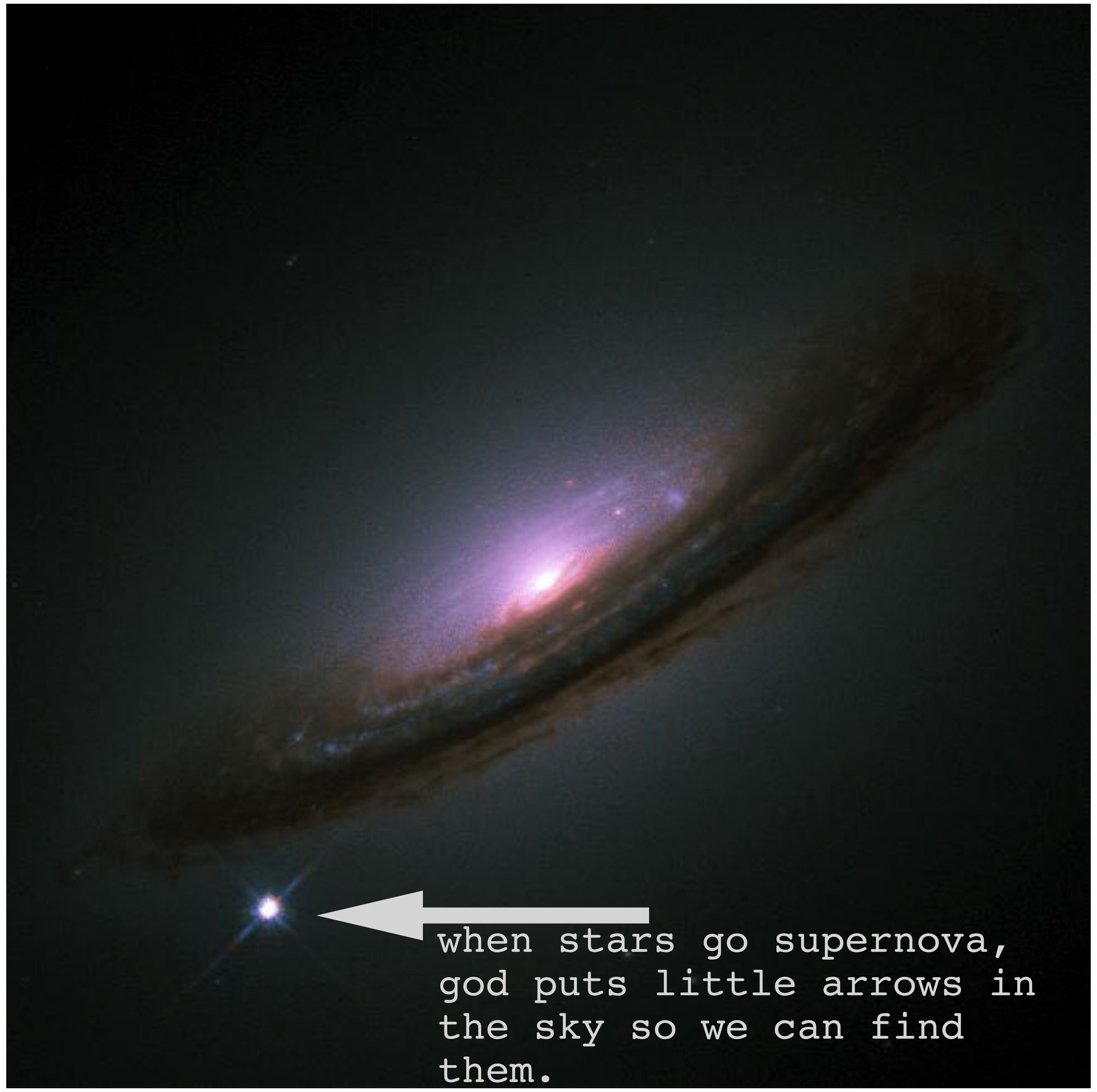
Example Standard Candle:

- Type Ia Supernovae

thermonuclear SN

Exploding white dwarf.

When a mass accretion event pushes a white dwarf over the Chandrasekhar limit (1.4 $M_\odot$), the sudden compression results in the fusion of carbon & oxygen, detonating the remnant in its entirety.




Explosion once Chandrasekhar limit exceeded, with an consistent(?) energy of $10^{51}$ ergs.

# Example Standard Candle:

- Type Ia Supernovae

  thermonuclear SN

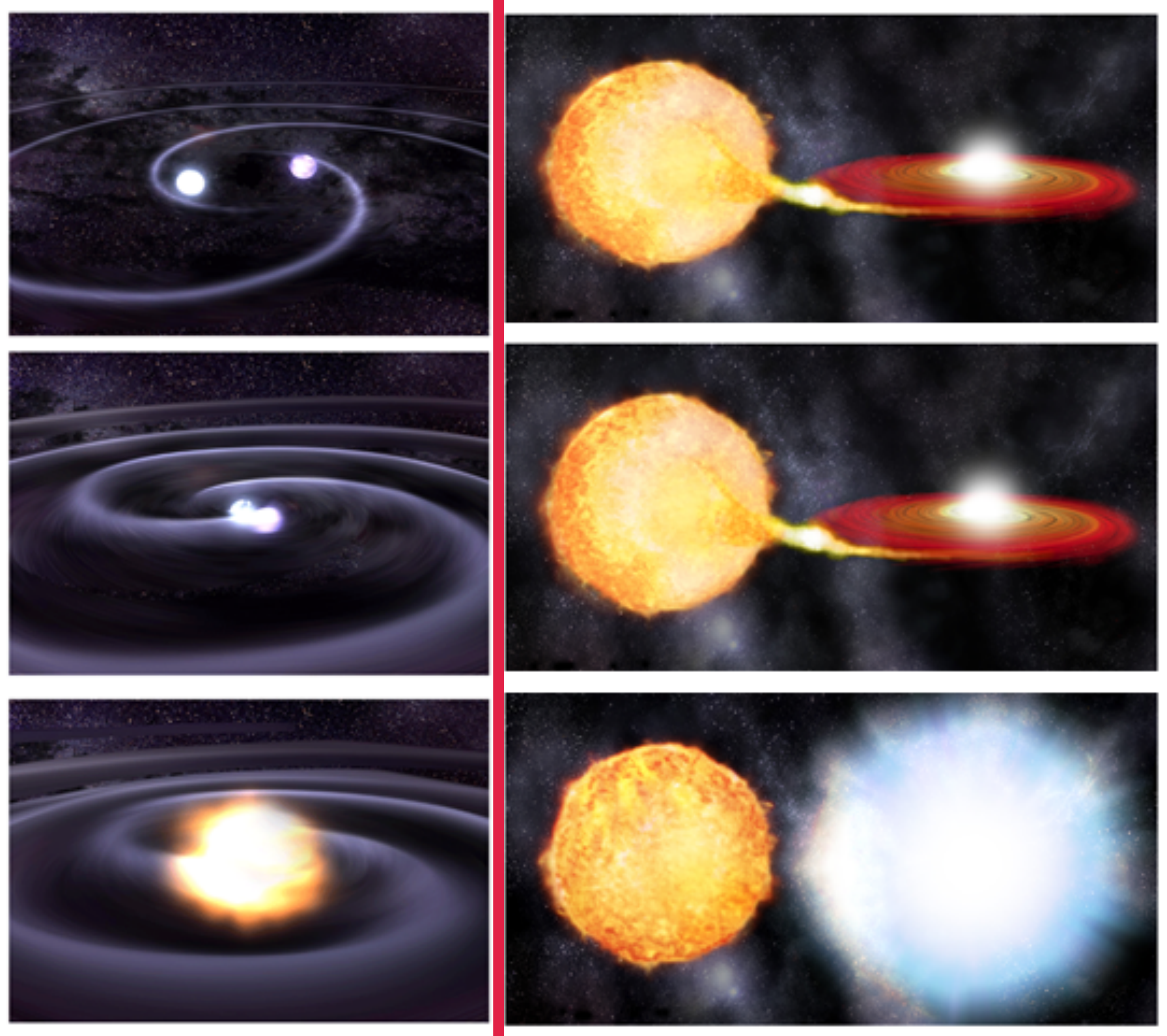
Exploding white dwarf.

When a mass accretion event pushes a white dwarf over the Chandrasekhar limit (1.4 $M_\odot$), the sudden compression results in the fusion of carbon & oxygen, detonating the remnant in its entirety.

Two mechanisms

merger of double degenerate white dwarfs

accretion onto single degenerate white dwarf



Energy release depends on mass of merging white dwarfs - not a good standard candle.

Explosion once Chandrasekhar limit exceeded, with an consistent(?) energy of $10^{51}$ ergs.

Is it really a standard candle?

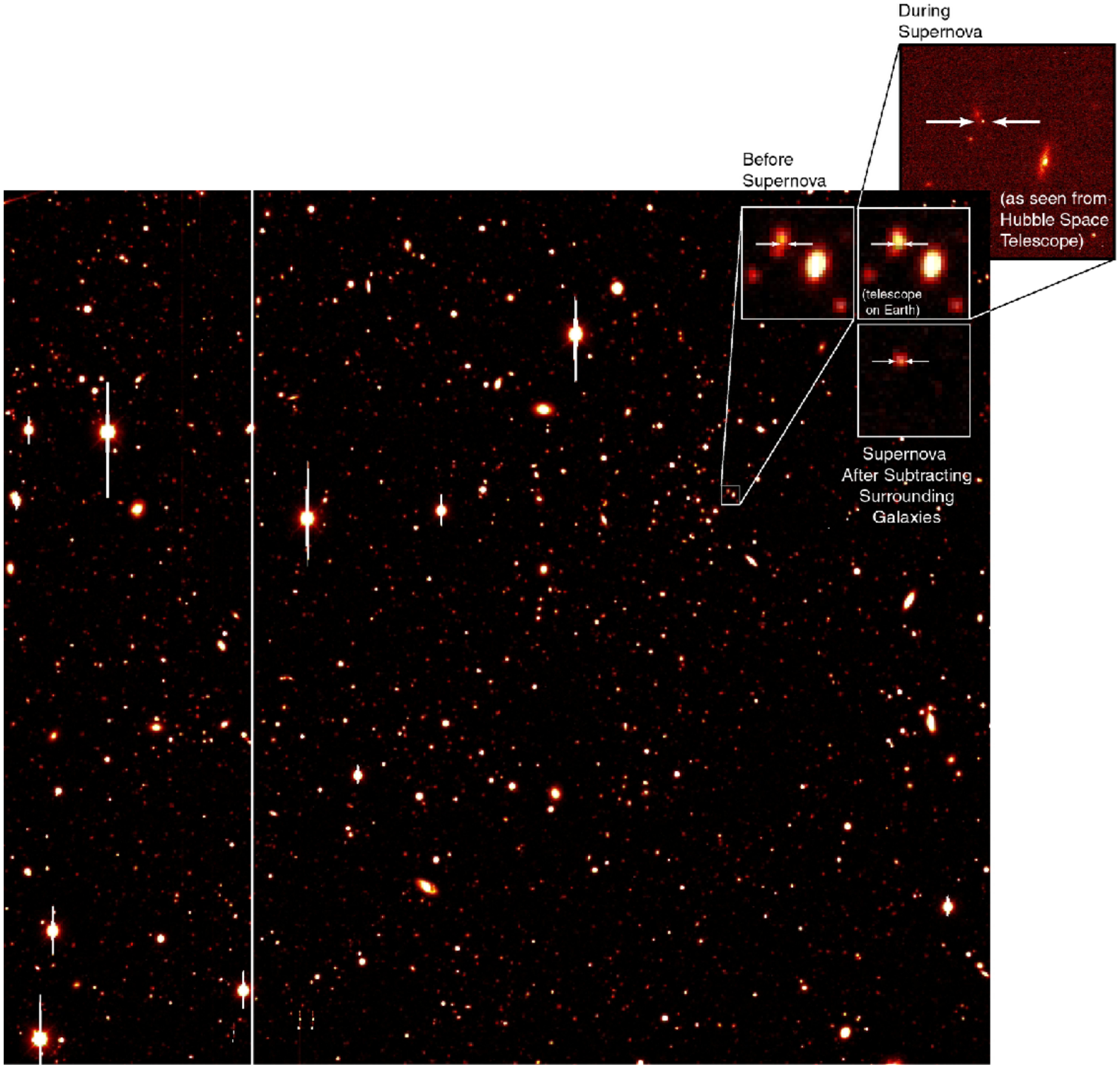
Example Standard Candle:

- Type Ia Supernovae

Survey wide swath of sky, imaging repeatedly over many nights, looking for change. If you look at enough galaxies, you'll see SN go off.

Supernova Cosmology Project

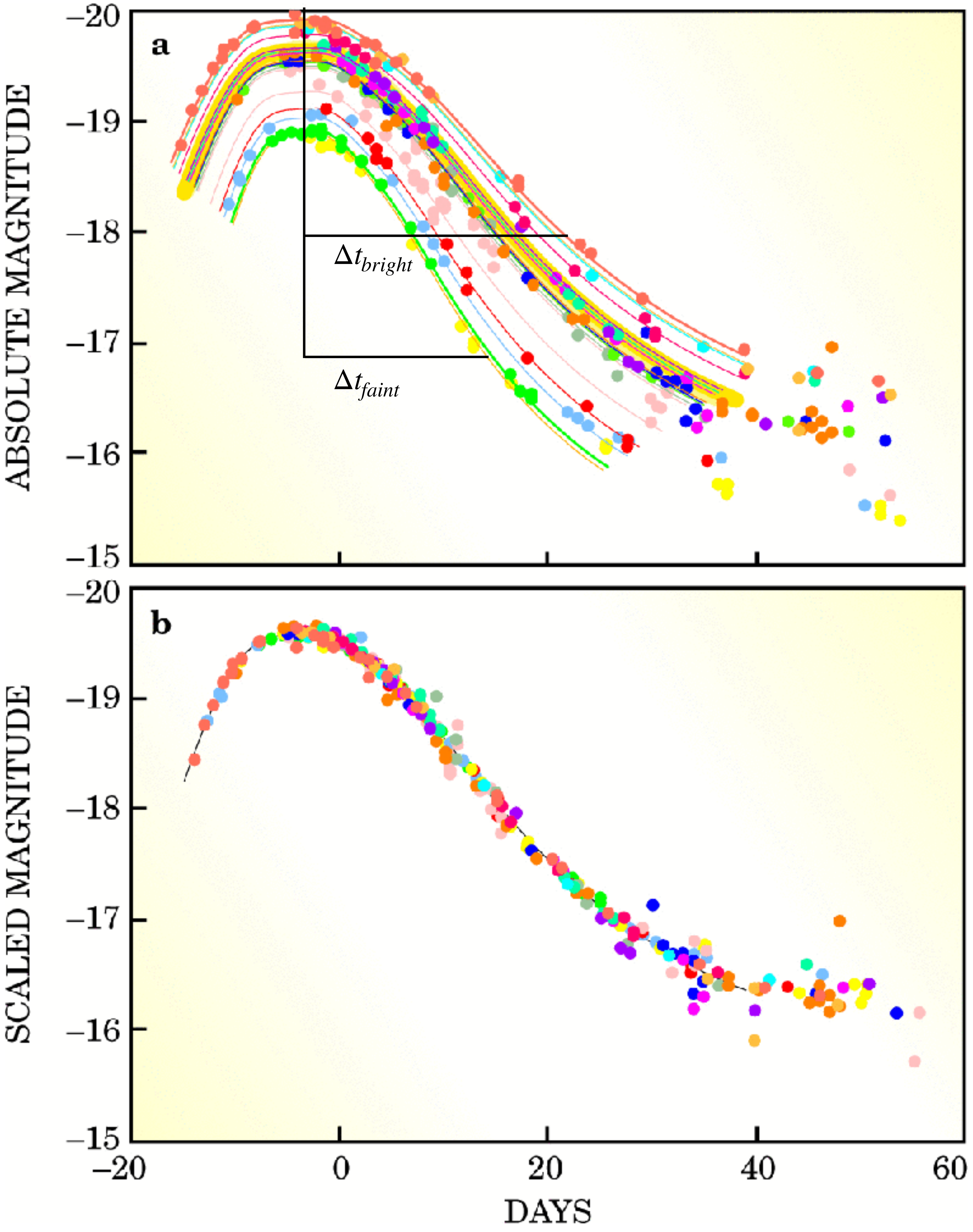
Perlmutter et al. (1998)



$$M_{max,corr} = M_{max,obs} + f(\Delta t)$$

Type Ia SN not quite standard candles, but standardizable: the peak brightness correlates with rest-frame decay time.

$$m_{max,corr} = M_{max,corr} - 5\log H_0 + 25 + 5\log\left(cz(1 + \frac{1-q_0}{2}z)\right)$$

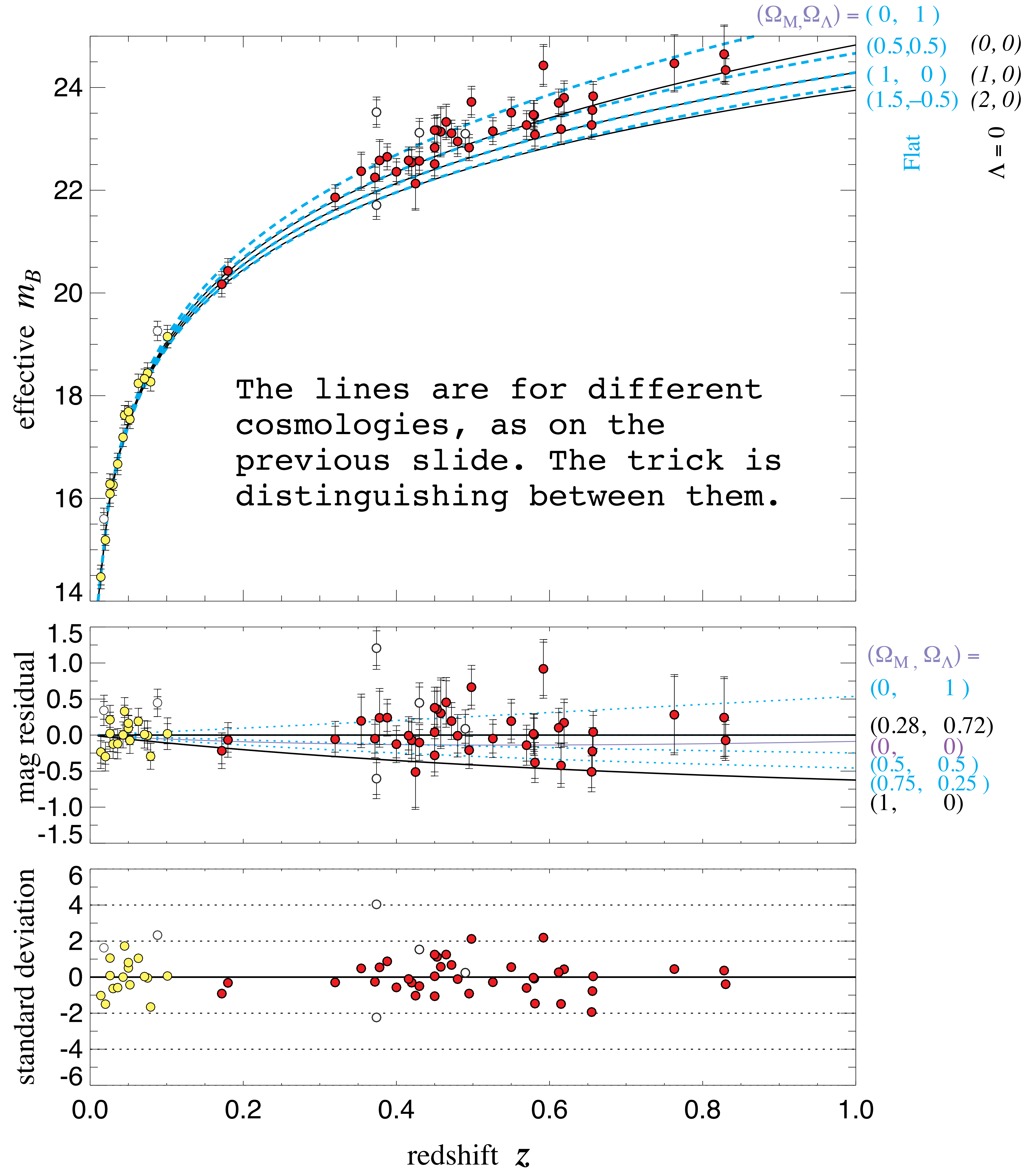


- # Luminosity-redshift relation

<u>Hubble diagram</u>
apparent magnitude vs. redshift
equivalent to distance modulus for standard candle
(M constant)

This example for Type Ia SN from the
Supernova Cosmology Project
(won the Nobel Prize in Physics in 2011)

Supernova Cosmology Project



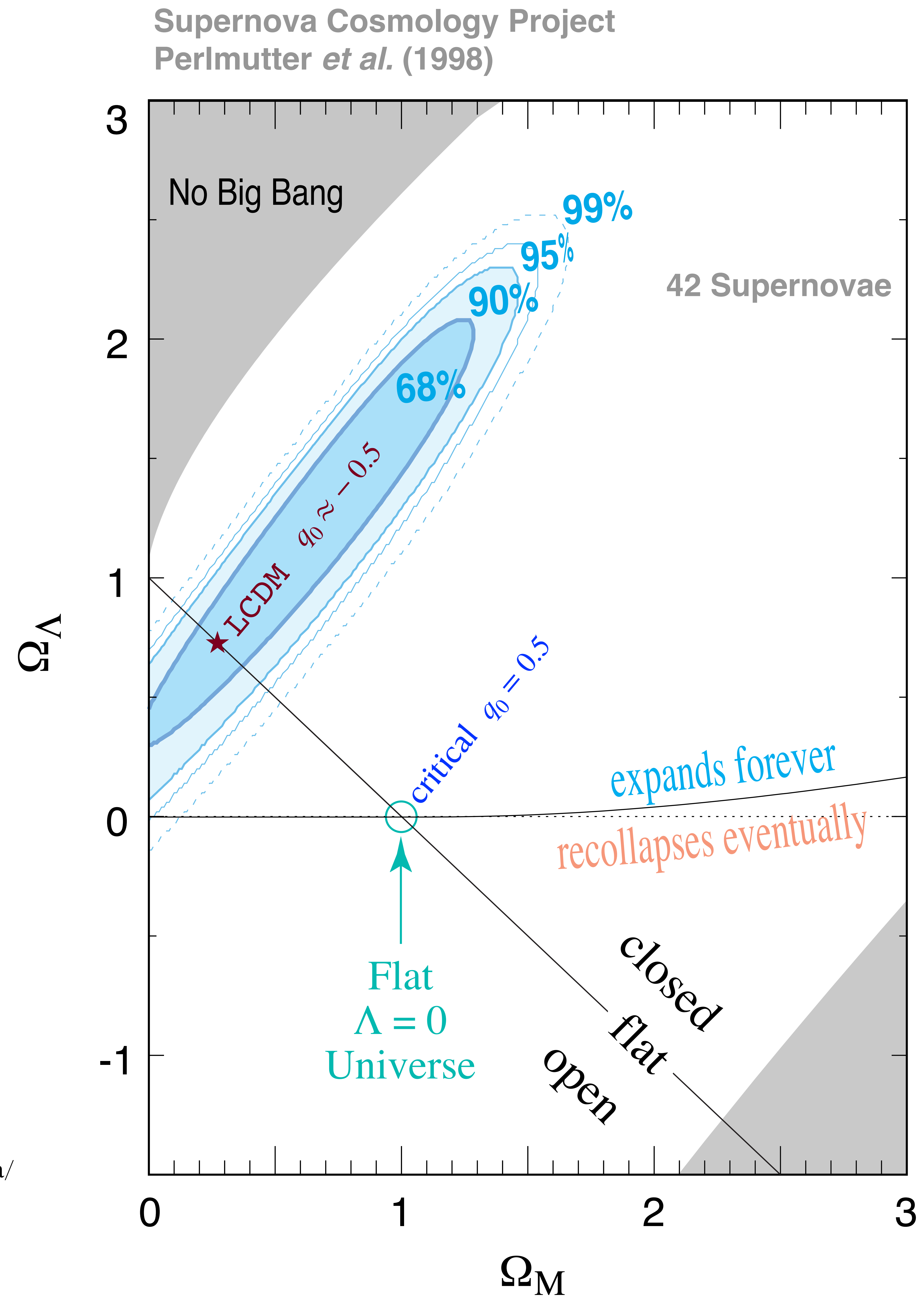

**Perlmutter, *et al.* (1998)**

- Luminosity-redshift relation

The Type Ia SN constraint practically excluded non-zero cosmological constant, but it did not provide a strong constraint on $\Omega_m$ and $\Omega_\Lambda$ individually.

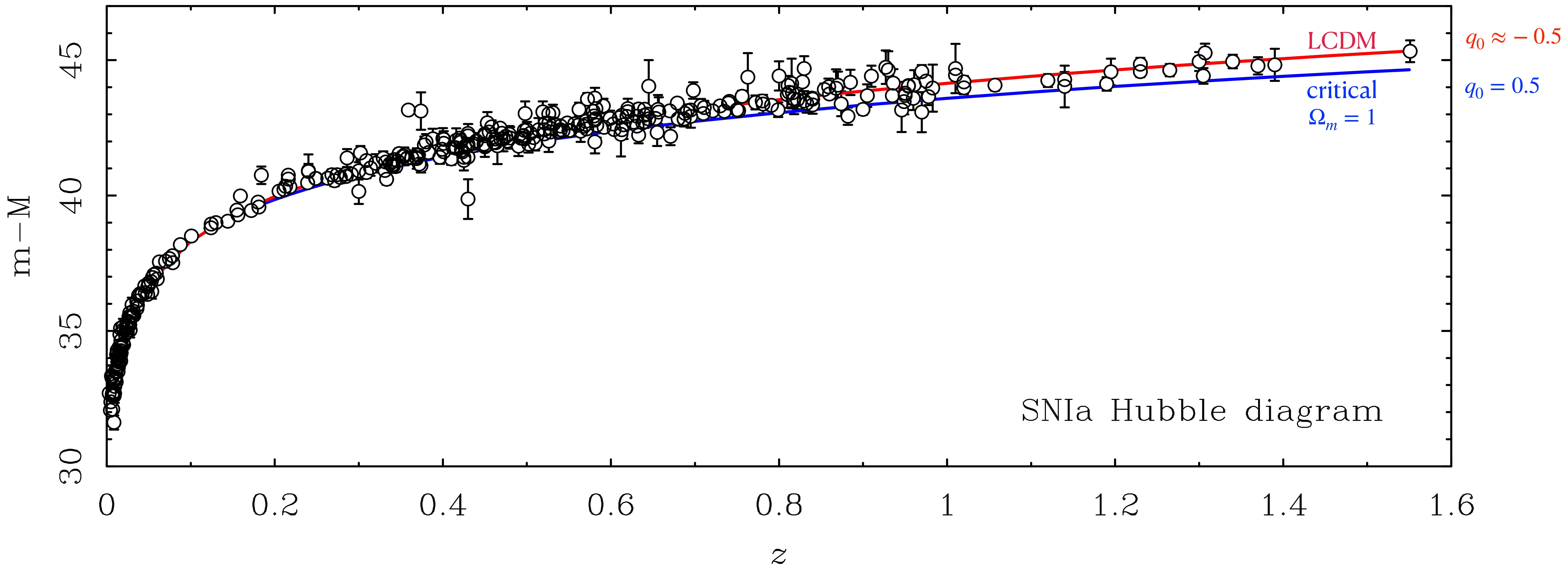
LCDM depended on the inclusion of other information, like independent measures of the mass density and the assumption of a flat geometry.

There is a LOT more to the history of this subject; see https://tritonstation.com/2019/01/28/a-personal-recollection-of-how-we-learned-to-stop-worrying-and-love-the-lambda/



- Luminosity-redshift relation

<u>Type Ia SN Hubble diagram</u>
(more recent data)

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- Angular size-redshift relation

Ideal case:
a **Standard Rod**
an object of constant, known size $\ell$

angular extent & size

$$\theta = \frac{\ell}{D_A}$$

Angular size distance

$$D_A = \frac{D_p}{(1+z)}$$

Note that

$$D_A = \frac{D_L}{(1+z)^2}$$

*ANGULAR-DIAMETER DISTANCE* 137

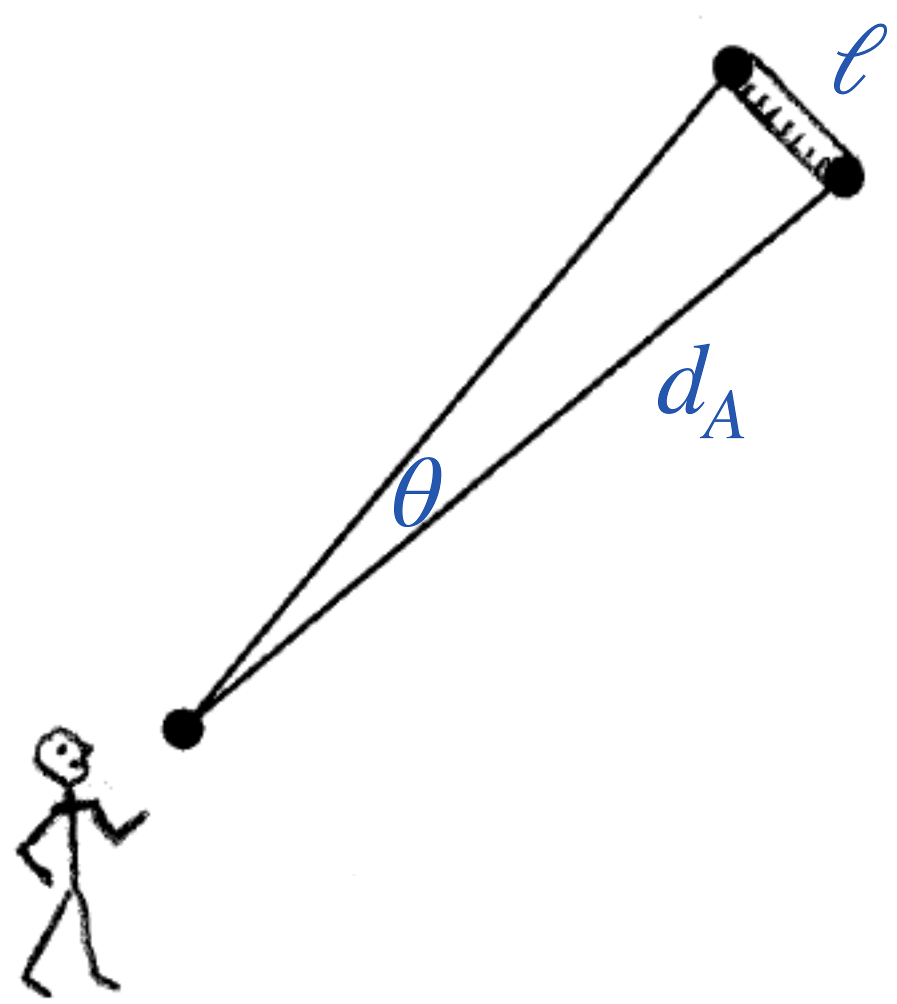
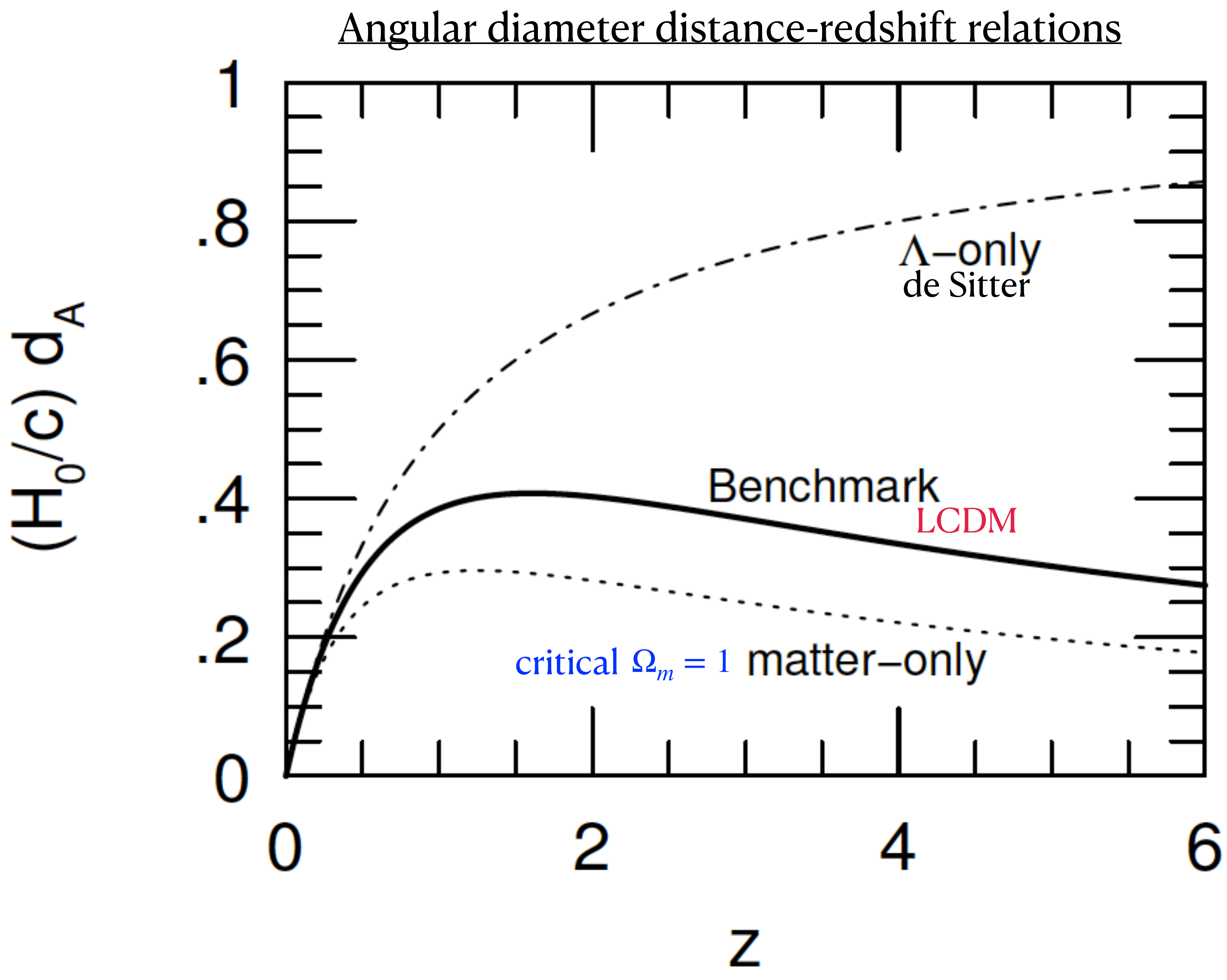



Figure 7.3: An observer at the origin observes a standard yardstick, of known proper length $\ell$, at comoving coordinate distance $r$.

• Angular size-redshift relation



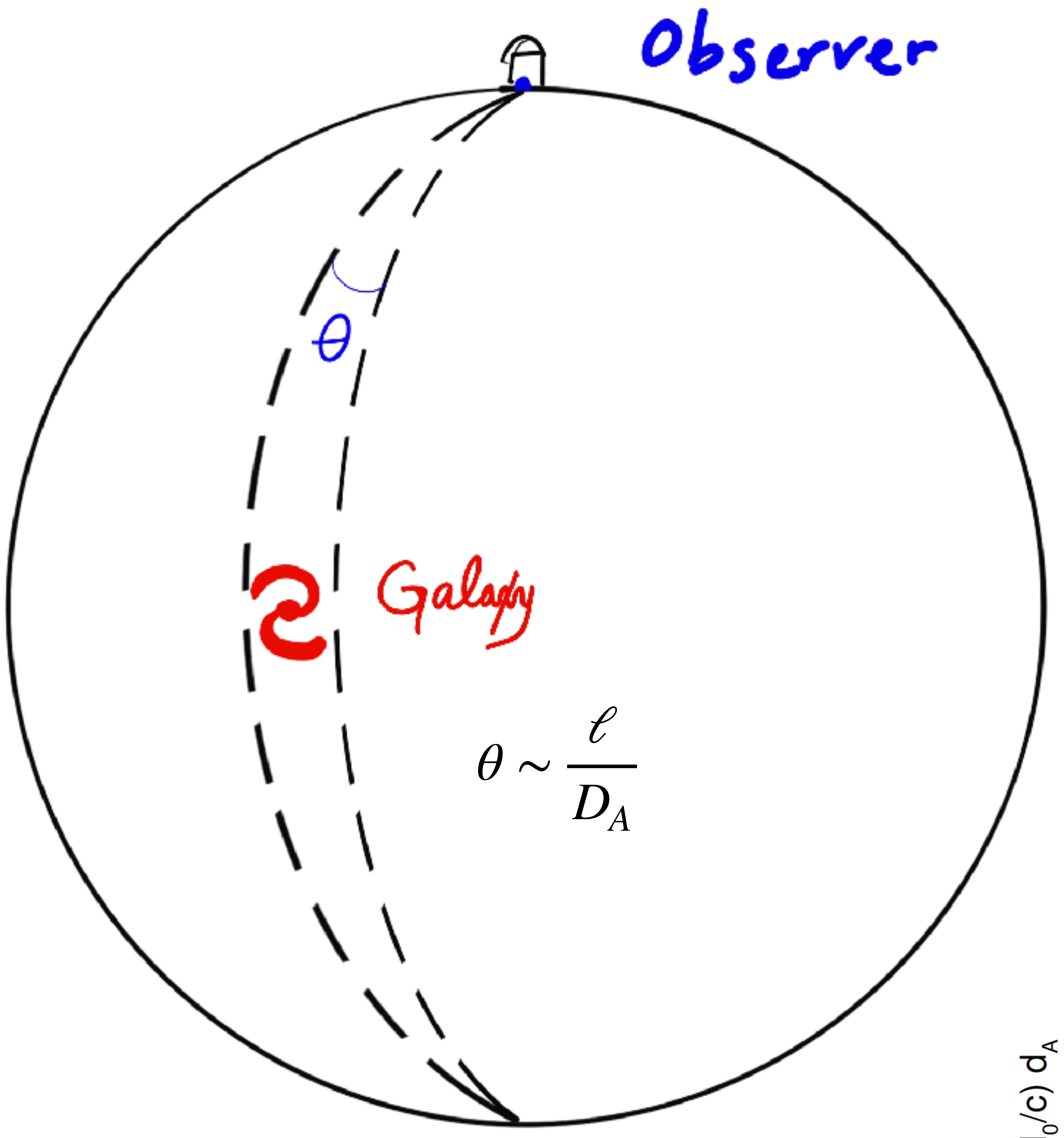

Note that the angular diameter distance never exceeds the Hubble length.

Sometimes has a maximum!

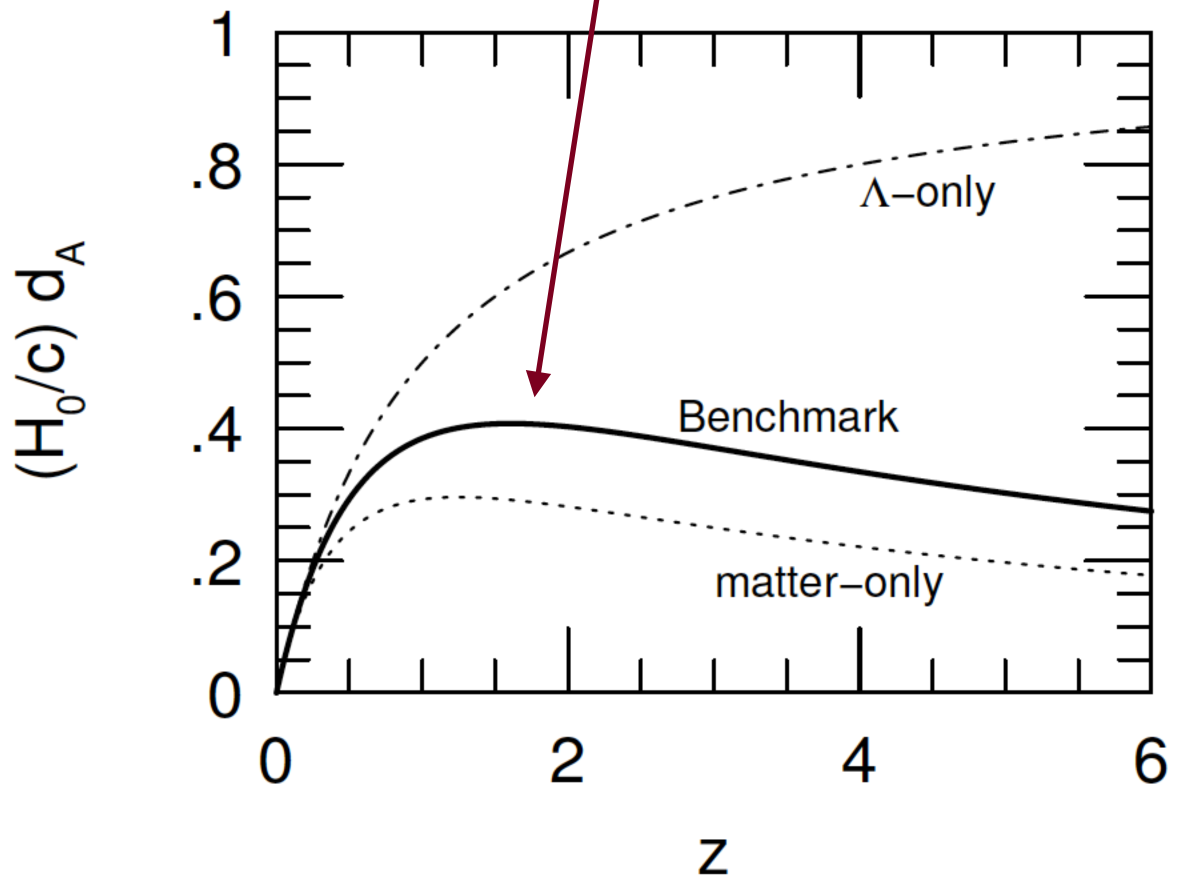
Figure 7.4: The angular-diameter distance for a standard yardstick with observed redshift $z$. The bold solid line gives the result for the Benchmark Model, the dot-dash line for a flat, lambda-only universe, and the dotted line for a flat, matter-only universe.

Ryden Fig. 6.4 (2nd ed)

Angular size can have a minimum in non-Euclidean geometries because of the divergence of light rays. Beyond the distance corresponding to this minimum size, objects start to look bigger again!




For LCDM, the minimum angular size occurs around $z \approx 1.6$ at $D_A \approx 1.75$ Gpc



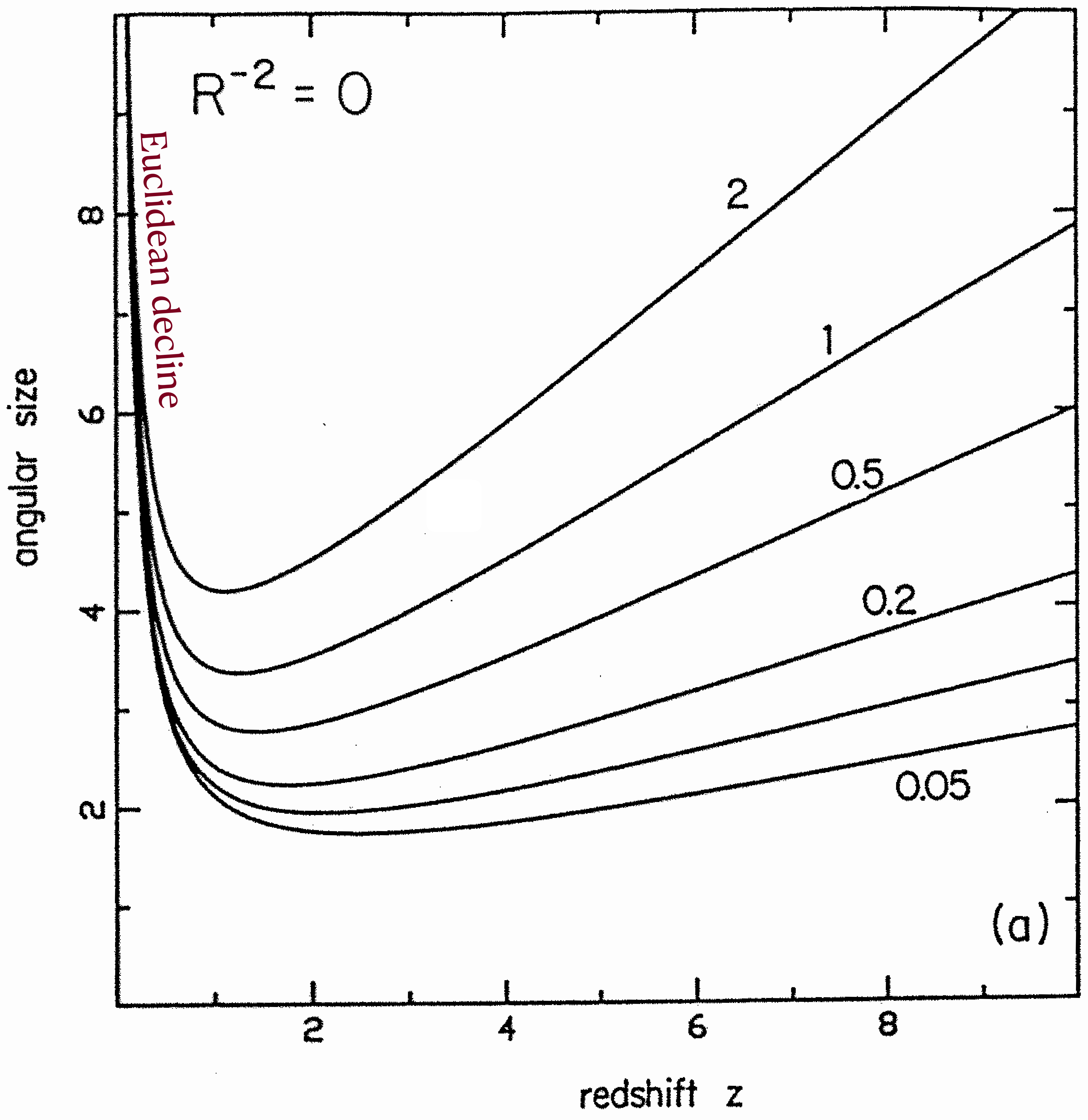
# • Angular size-redshift relation

Peebles Fig. 13.5

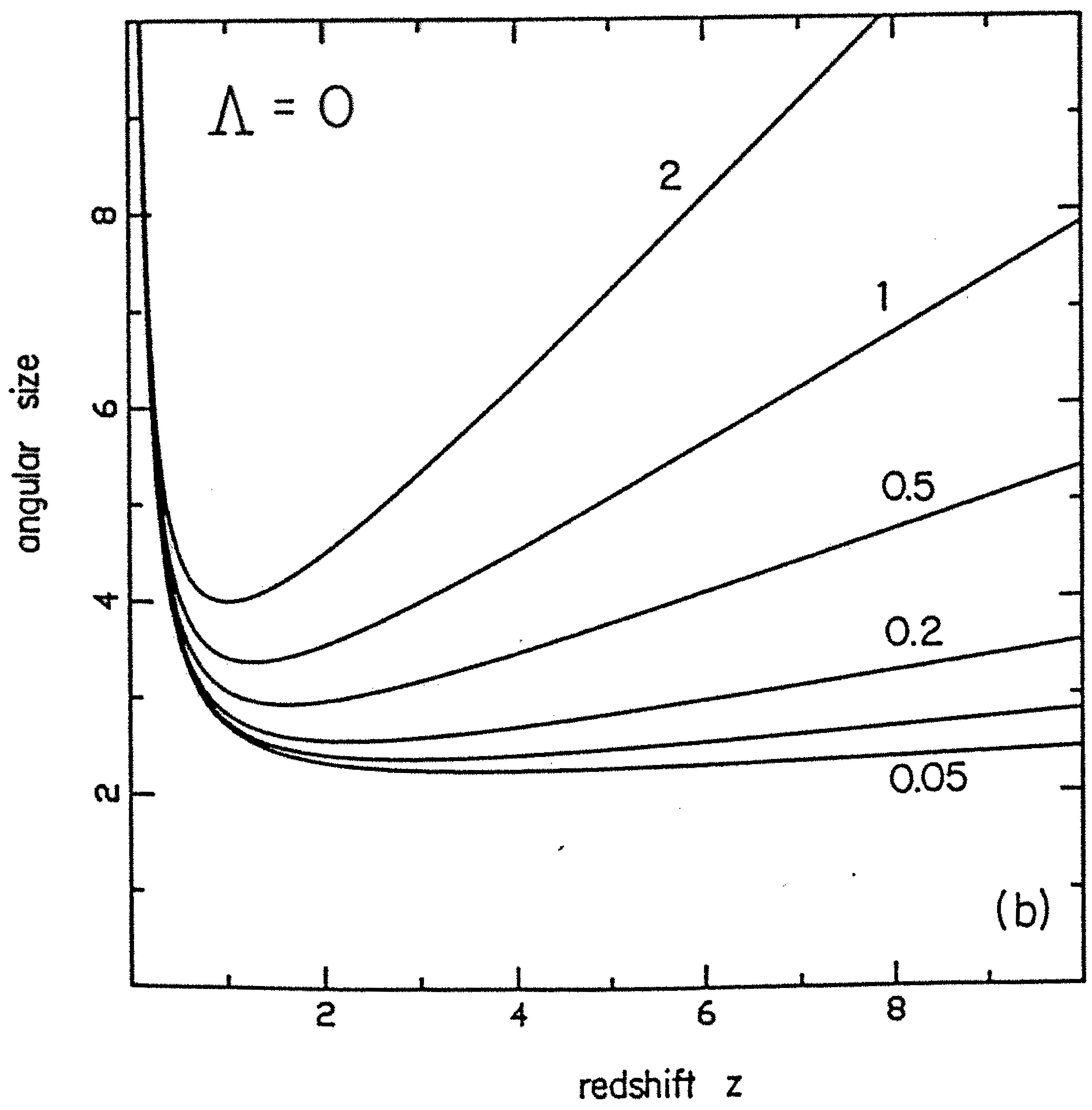
Angular Size-redshift relations

Figure 13.5. Angular size as a function of redshift. The vertical axis is the factor $F_\theta = (1+z)/H_o a_o r(z)$ in equation (13.47). The parameters are arranged as in figure 13.1.

Flat cosmologies




Zero cosmological constant




Initially objects decline in angular size with increasing distance, but this trend reverses at high redshift in the Robertson-Walker geometry!

Cosmic Volume • for number counts N(m), N(z)

Robertson-Walker metric

$$ds^2 = -c^2dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

Volume

$$V = a^3 \int_0^r \frac{r^2 dr}{\sqrt{1-kr^2}} \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\phi$$

An open universe ($\Omega_m < 1$) is "bigger" than a closed universe ($\Omega_m > 1$)

integrates to

$$V = \frac{4\pi}{3}(ar)^3 f(r) \qquad f(r) = \begin{cases} \frac{3}{2}\left[\frac{\sin^{-1} r}{r^3} - \frac{\sqrt{1-r^2}}{r^2}\right] \\ 1 \\ \frac{3}{2}\left[\frac{\sqrt{1+r^2}}{r^2} - \frac{\sinh^{-1} r}{r^3}\right] \end{cases} \quad \text{for} \quad k = \begin{cases} 1 \\ 0 \\ -1 \end{cases}$$

## Cosmic Volume

- for number counts N(m), N(z)

in terms of the proper distance

$$D_p = a(t)\int_0^r \frac{dr}{\sqrt{1-kr^2}}$$

we can make the Taylor expansion

Sandage (1988, ARA&A, 26, 561)

$$V = \frac{4\pi}{3}D_p^3\left[1 - \frac{k}{5}\left(\frac{D_p}{R}\right)^2 + \mathcal{O}\left(\frac{D_p}{R}\right)^4\right]$$

$\frac{k}{R^2}$ is the curvature of space

Note that the volume increases as the curvature becomes more negative, so a closed universe is "small" and an open universe is "big"

# Cosmic Volume

Sandage 1988, ARAA, 26, 561



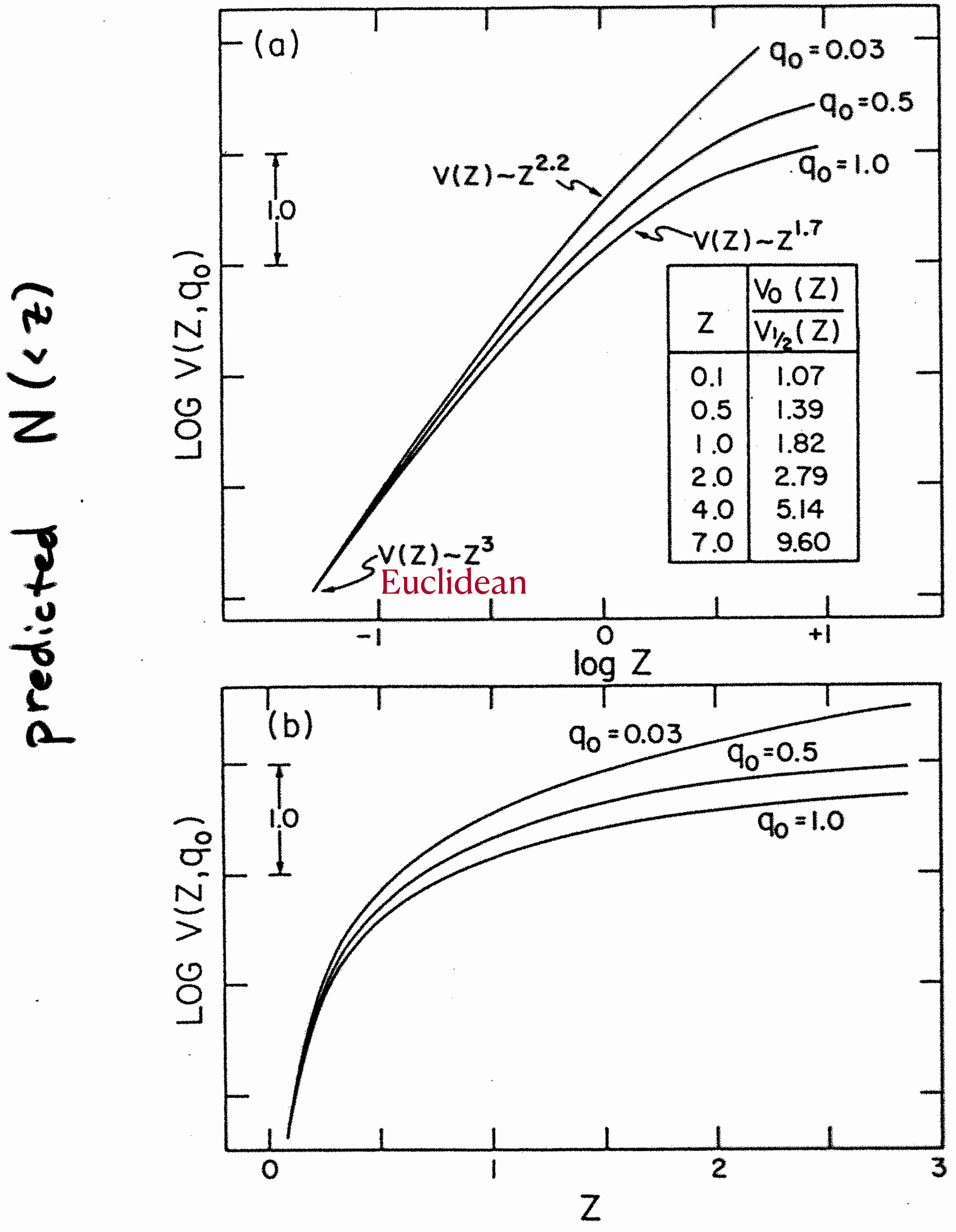



*Figure 1* Theoretical $N(z, q_0)$ relations for three values of $q_0$. Plotted is the integral count, i.e. the total number of galaxies in a complete (volume-limited) sample that have redshifts smaller than $z$. Parts (*a*) and (*b*) are the same function but plotted as log $z$ (*a*) and $z$ (*b*).

Note: decelerating cosmologies have "less" volume than Euclidean. The long(N)-m diagram should have a slope < 0.6.

as $q_0 \downarrow$, $V \uparrow$

Note: in 1988, we didn't consider $q < 0$ to be a physical possibility