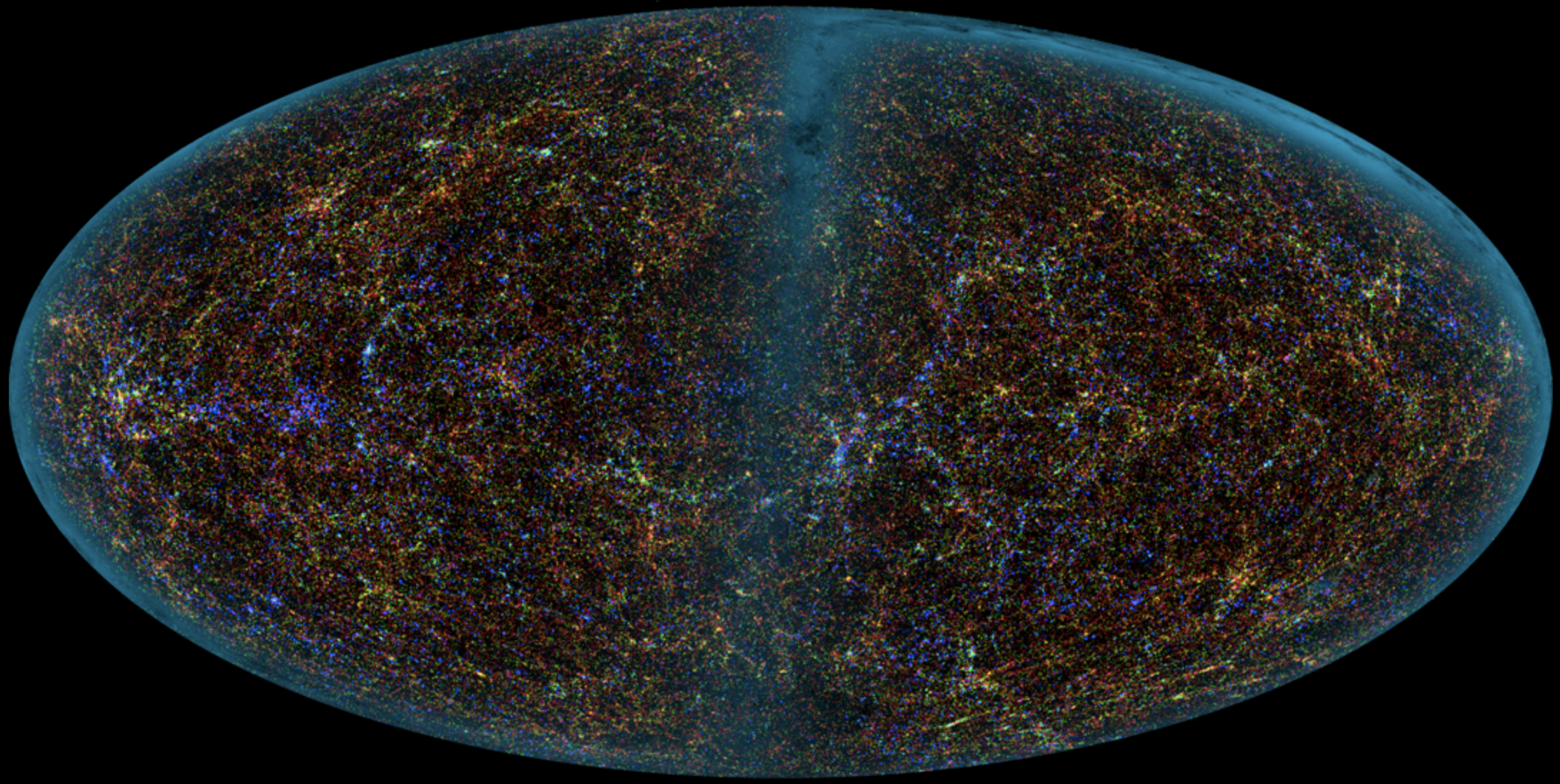
Cosmology

and Large Scale Structure



<u>Today</u>
Cosmological constraints
from the Power Spectrum
Cold Dark Matter

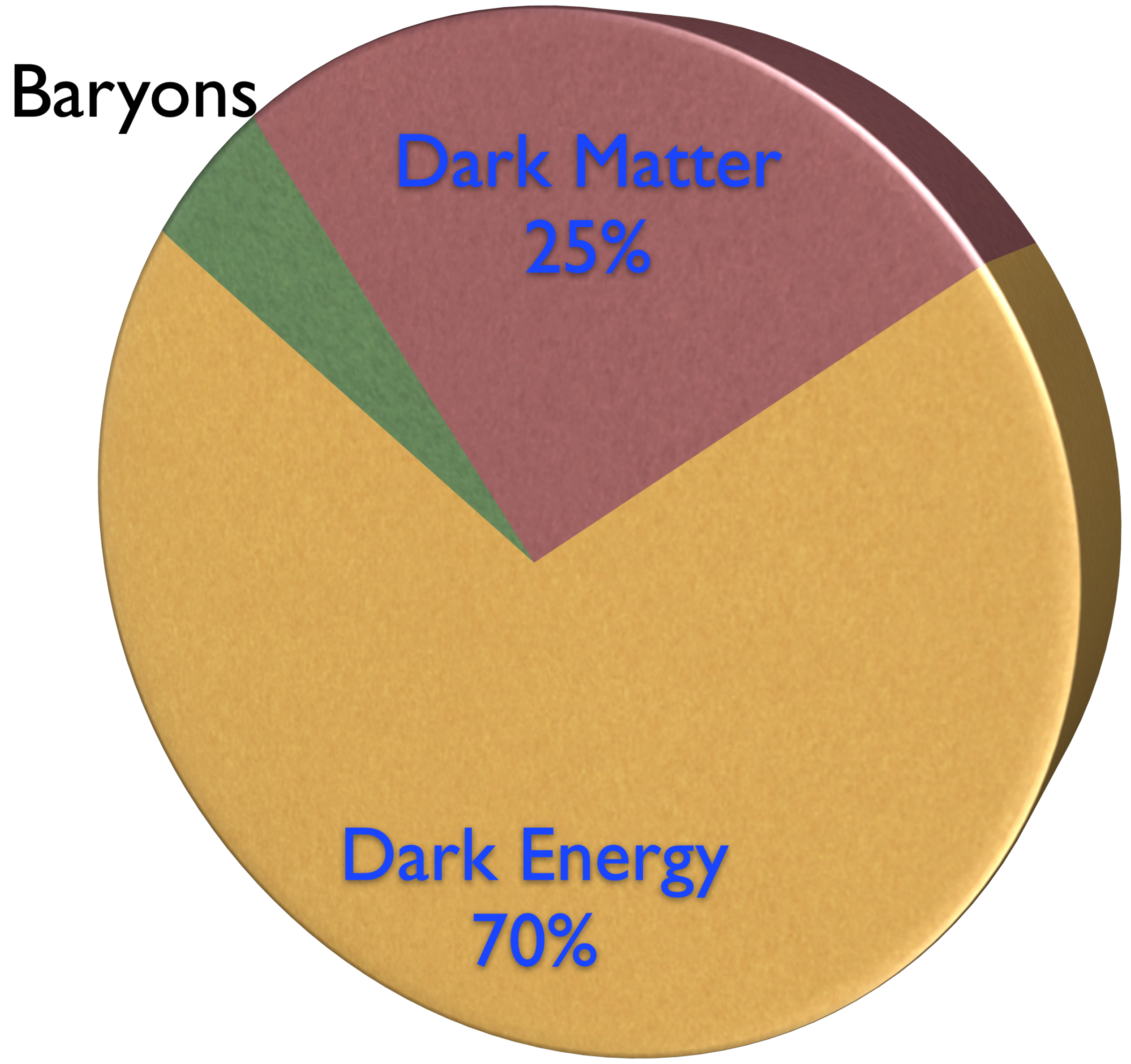
19 November 2024

http://astroweb.case.edu/ssm/ASTR328/

Empirical Pillars of the Hot Big Bang

1. Hubble Expansion
2. Big Bang Nucleosynthesis Ω_b
3. Cosmic Microwave Background

$\Omega_m = \Omega_b + \Omega_{DM}$



Auxiliary Hypotheses

- Dark matter Ω_{DM}
- Dark Energy Ω_Λ

Current mass-energy content of the universe

"Vanilla LCDM"

mass density Ω_{m_0} 0.30 give or take a bit

normal matter Ω_b 0.05 baryons - from BBN

mass that is *not* normal matter Ω_{CDM} 0.25 cold dark matter

cosmic background radiation Ω_r 5×10^{-5} photons plus 4×10^{-5} in neutrinos

neutrinos $0.001 < \Omega_\nu < 0.002$ for 3 neutrino flavors with $0.06 < \sum_{i=1}^{3} m_{\nu_i} < 0.12$ eV

upper limit from cosmic structure formation
lower limit from neutrino oscillations

dark energy Ω_{Λ_0} 0.70 energy density of vacuum

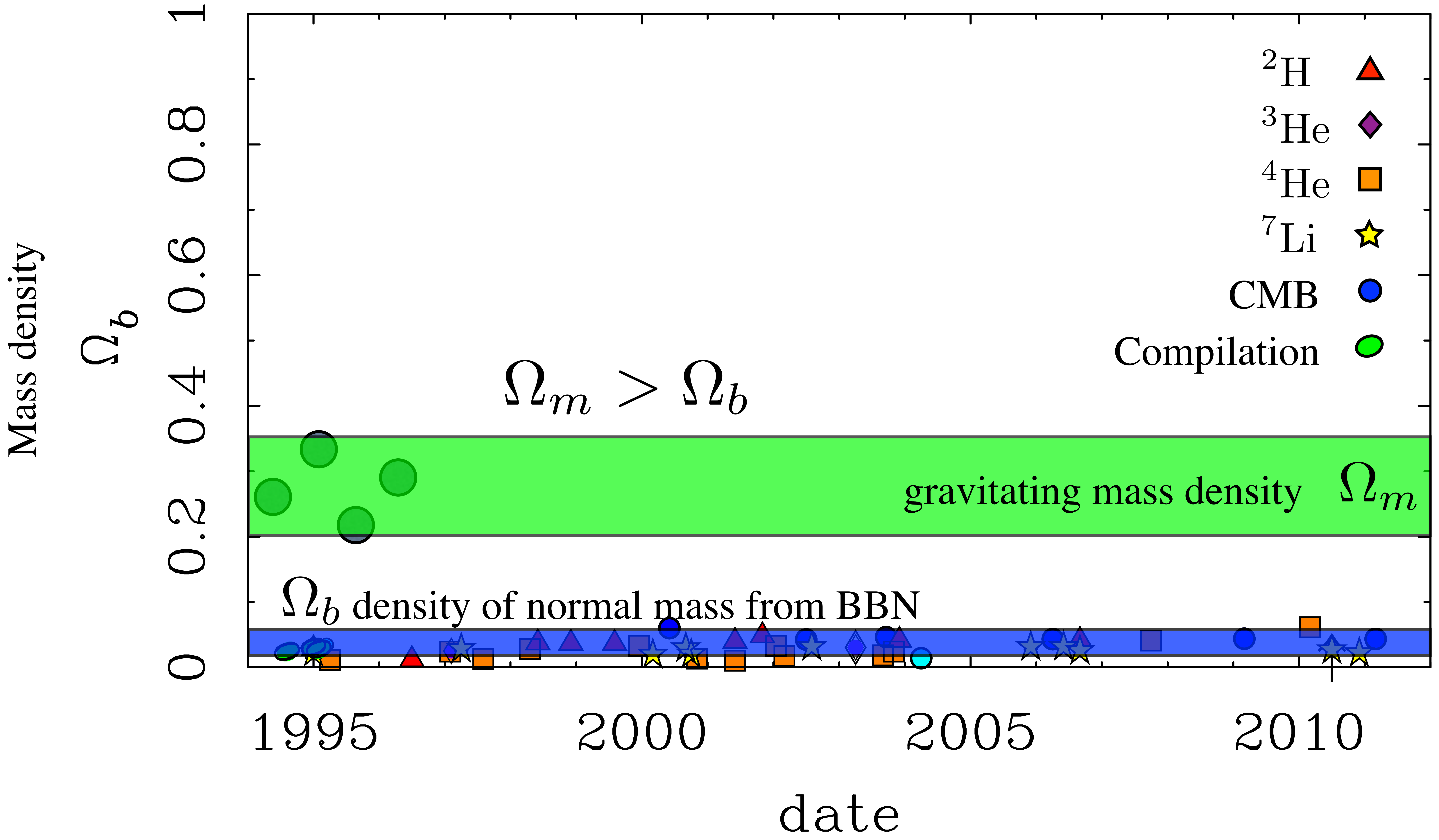
e.g, $\Omega_\nu = \frac{\sum m_\nu}{93 \text{ eV}}$

since $n_\nu = \frac{9}{11} n_\gamma$

$$\Omega_x = \frac{\rho_x}{\rho_{crit}} \qquad \rho_{crit} = \frac{3H_0^2}{8\pi G}$$

But estimates of Ω_m run higher: there's more mass than meets the eye

There is more gravitating mass than Big Bang Nucleosynthesis allows in normal matter.
Need **non-baryonic** dark matter.



But estimates of Ω_m run higher: there's more mass than meets the eye

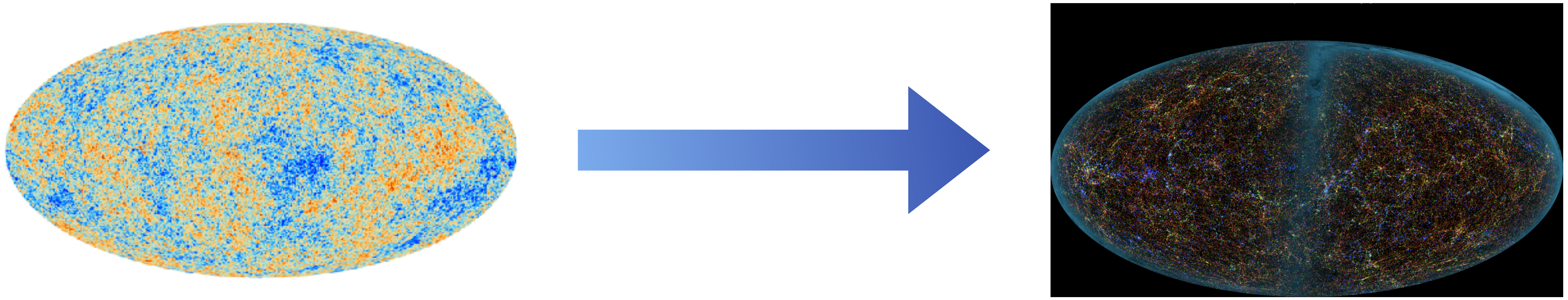
There are two compelling reasons why we need to invent **non-baryonic** cold dark matter:

1. There is more gravitating mass than Big Bang Nucleosynthesis allows in normal matter.

$$\Omega_m > \Omega_b$$

2. The need to grow large scale structure from very uniform initial conditions.

$$\delta(z = 1090) \sim 10^{-5} \;\rightarrow\; \delta(z = 0) \sim 1$$



Cosmologically, the only requirement to be CDM is

- dynamically cold (slow moving)
- non-baryonic (no E&M interactions)

could be

WIMPS

(or some other particle, but there are lots of extra particle-physics constraints on new particles)

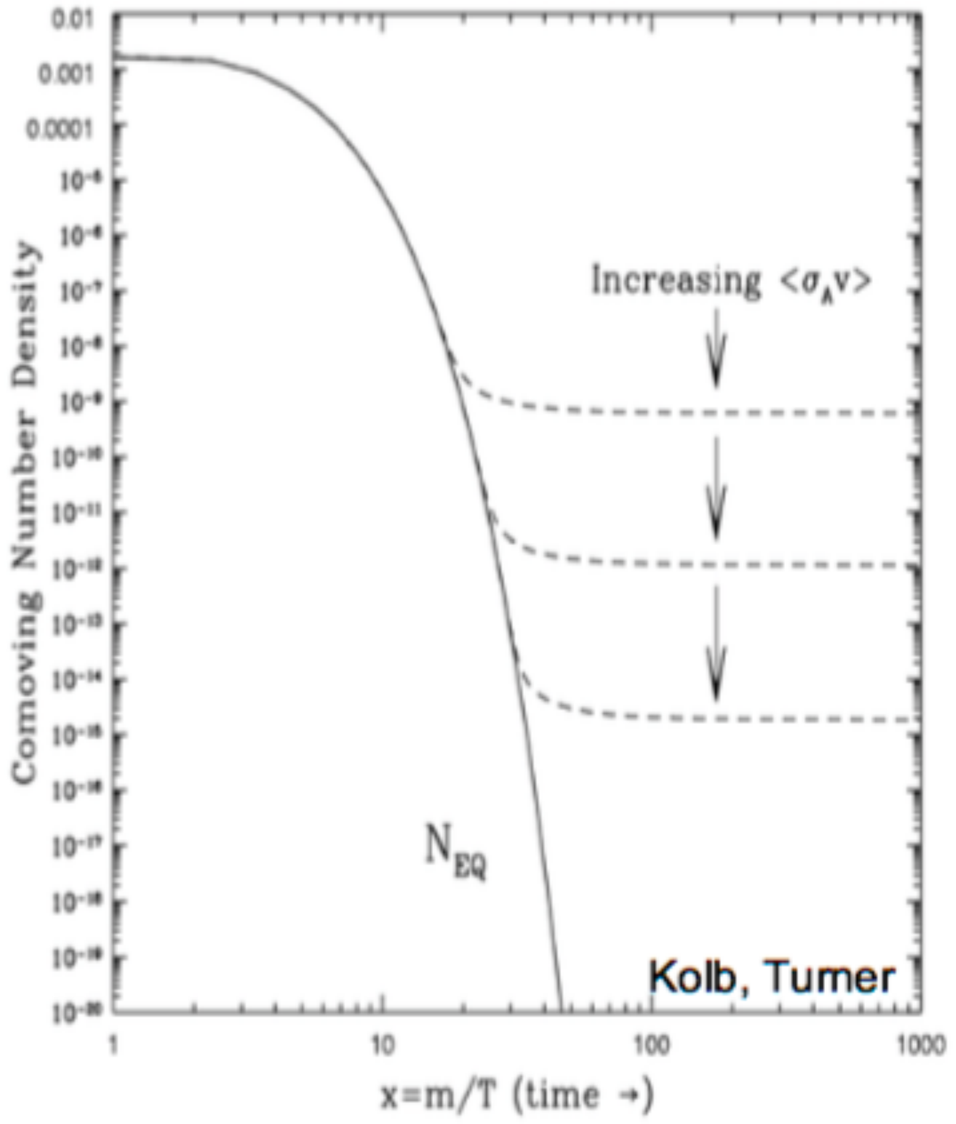
or

Black Holes

(masses of ~ 10^5 $M_\odot$ conceivable, but most mass ranges have been excluded by gravitational lensing observations)

WIMPs are considered the odds-on favorite CDM candidate because of the so-called `WIMP miracle': the relic density of a new weakly interacting particle is about right to explain the mass density.

THE WIMP MIRACLE

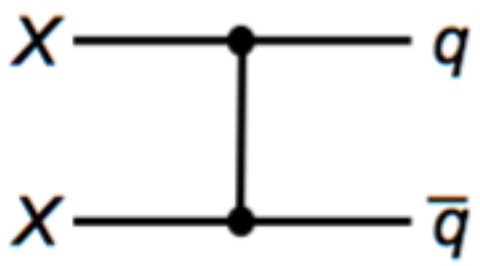


In the very early universe

- Assume a new (heavy) particle *X* is initially in thermal equilibrium

- Its relic density is

$$\Omega_X \propto \frac{1}{\langle \sigma v \rangle} \sim \frac{m_X^2}{g_X^4}$$



- $m_X \sim 100$ GeV, $g_X \sim 0.6 \rightarrow \Omega_X \sim 0.1$

- Remarkable coincidence: particle physics independently predicts particles with the right density to be dark matter

Originally expected $\sigma \sim 10^{-39}\ \mathrm{cm}^{-2}$, but only the thermal cross-section $\langle \sigma v \rangle$ matters here so it could be lower if compensated by the mean velocity.

Originally expected $m_X \sim 100\ \mathrm{GeV\ c^{-2}}$.

Lee-Weinberg Mass Window

The Lee-Weinberg limit refers to a lower limit of roughly 2 GeV on the mass of any possible heavy neutral lepton, i.e., any heavy neutrino. It does not exclude far lighter neutrinos and all currently known neutrino flavors have a mass less than an eV. The "heavy" lower limit was calculated by Benjamin Whisoh Lee and Steven Weinberg in 1977, their analysis sometimes called the Lee-Weinberg argument. A calculated upper limit termed the unitary bound, of a few TeV produces a range of possible heavy neutral leptons known as the Lee-Weinberg window. The fact that the total mass of such particles throughout the observable universe would be within the right order-of-magnitude to account for dark matter is termed the WIMP miracle, and the term WIMP is sometimes used specifically to mean such a lepton within this range.
Decades of searching for such particles has not detected any and there has been recent development of theories regarding why these limits may not apply to dark matter particles.

Originally expected $2\ \mathrm{GeV\ c^{-2}} < m_X < 140\ \mathrm{TeV\ c^{-2}}$

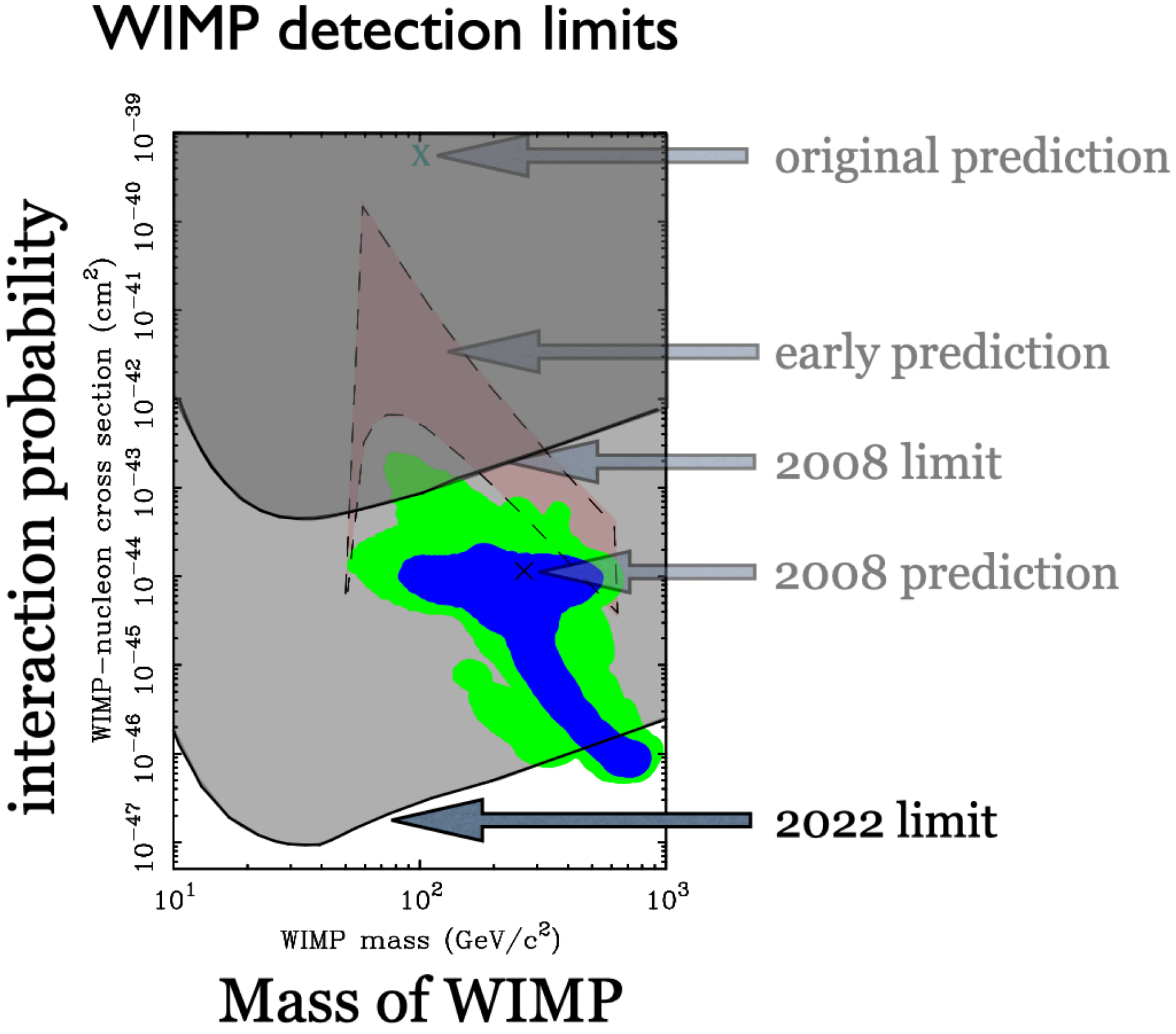
Lee-Weinberg limit — unitary bound

Smirnov & Beacom (2019) Phys. Rev. D 100, 043029

Direct detection experiments have repeatedly excluded predicted WIMP properties

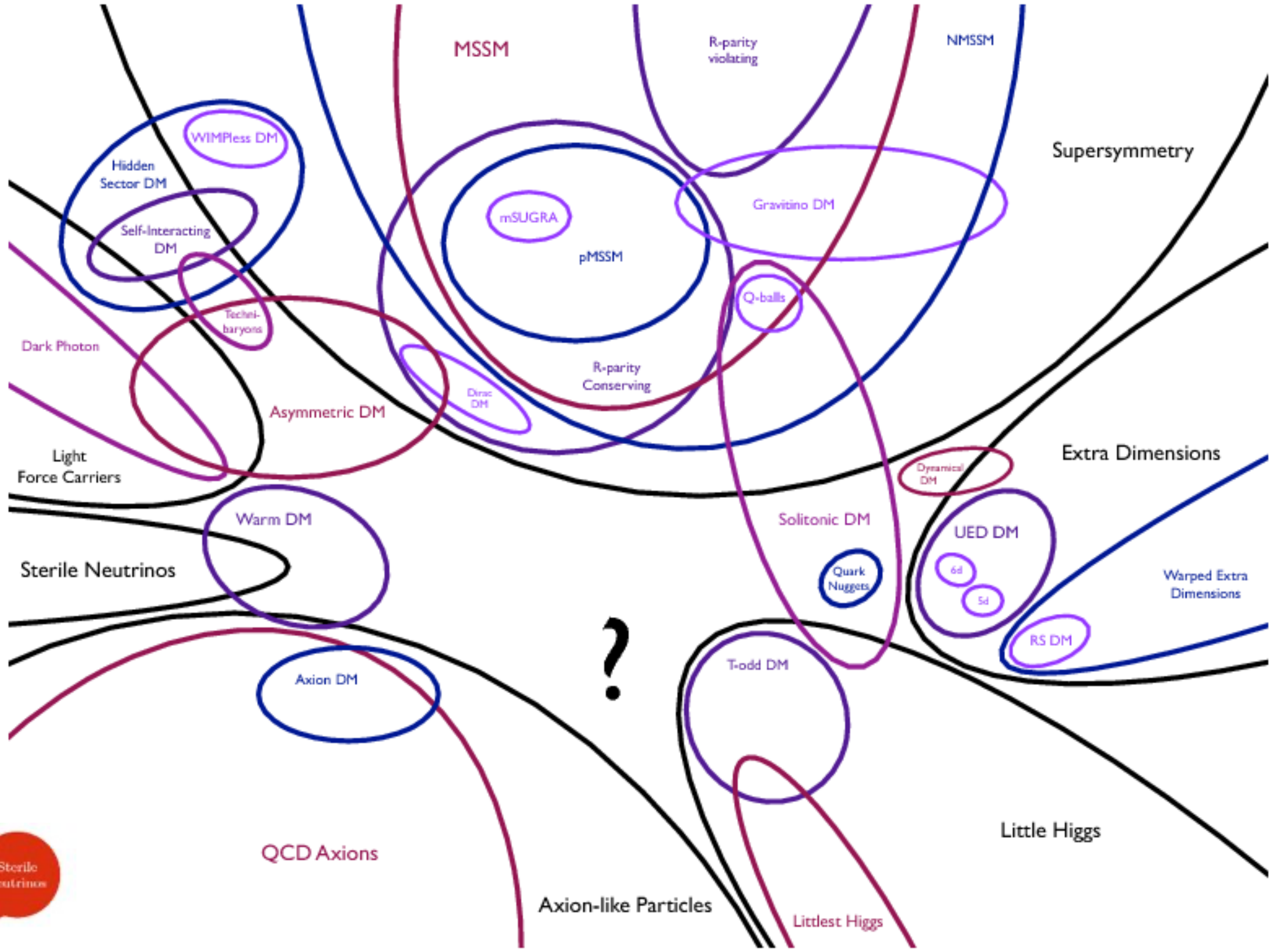
The original prediction of $\sigma \sim 10^{-39}$ is off scale, having been excluded long ago, BUT we can still get away with the "right" thermal cross-section $\langle \sigma v \rangle$ for the WIMP miracle if the mass is high enough for the velocity to be low.

Current data are exceedingly grim for the WIMP, but we stick with it out of habit and for lack of a better idea.



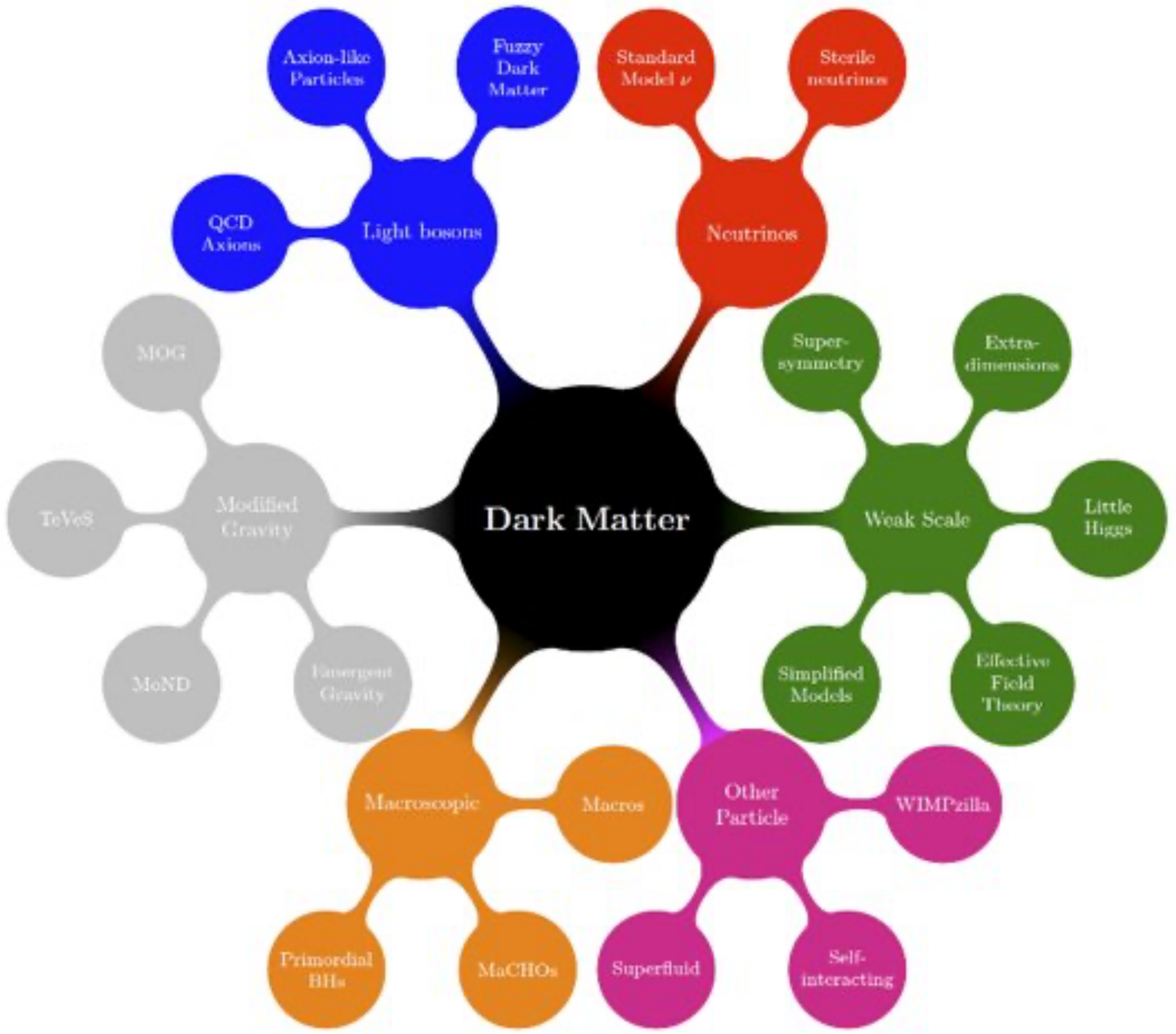
and yet others... (Tim Tait)

Lots of particle candidates for CDM:
WIMPs
Axions
Light dark matter
wimpzillas
etc.



Can imagine other candidates as well:

Warm DM
Self-interacting DM
etc.



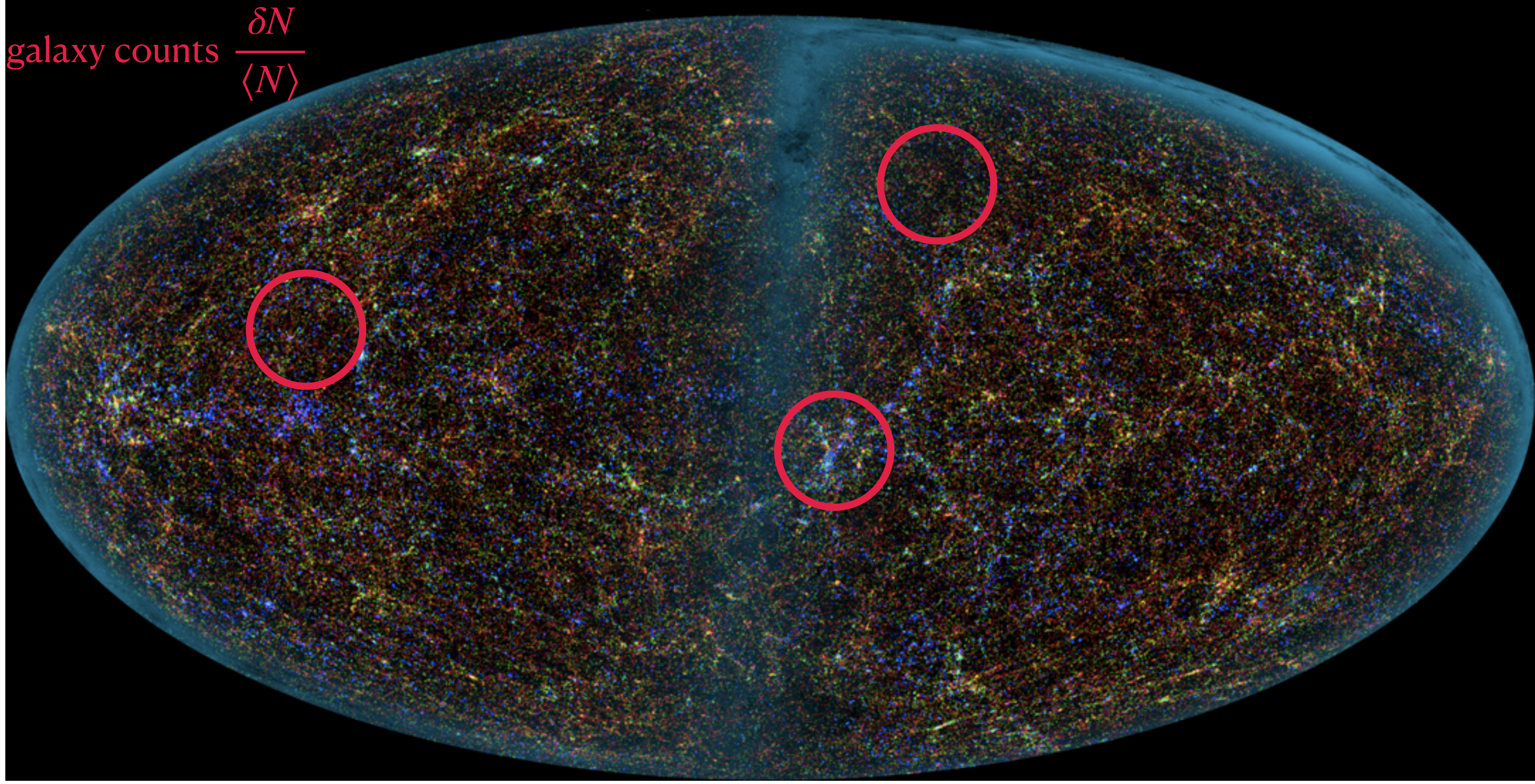
Measurements of the gravitating mass density

- Cluster M/L
  - measure M/L of a cluster, combine with measured luminosity density of universe.
- Weak lensing
  - measure shear over large scales
- Peculiar Velocity Field
  - measure deviations from Hubble flow
- Power spectrum of galaxies — $z = 0$
- CMB fits — $z = 1090$

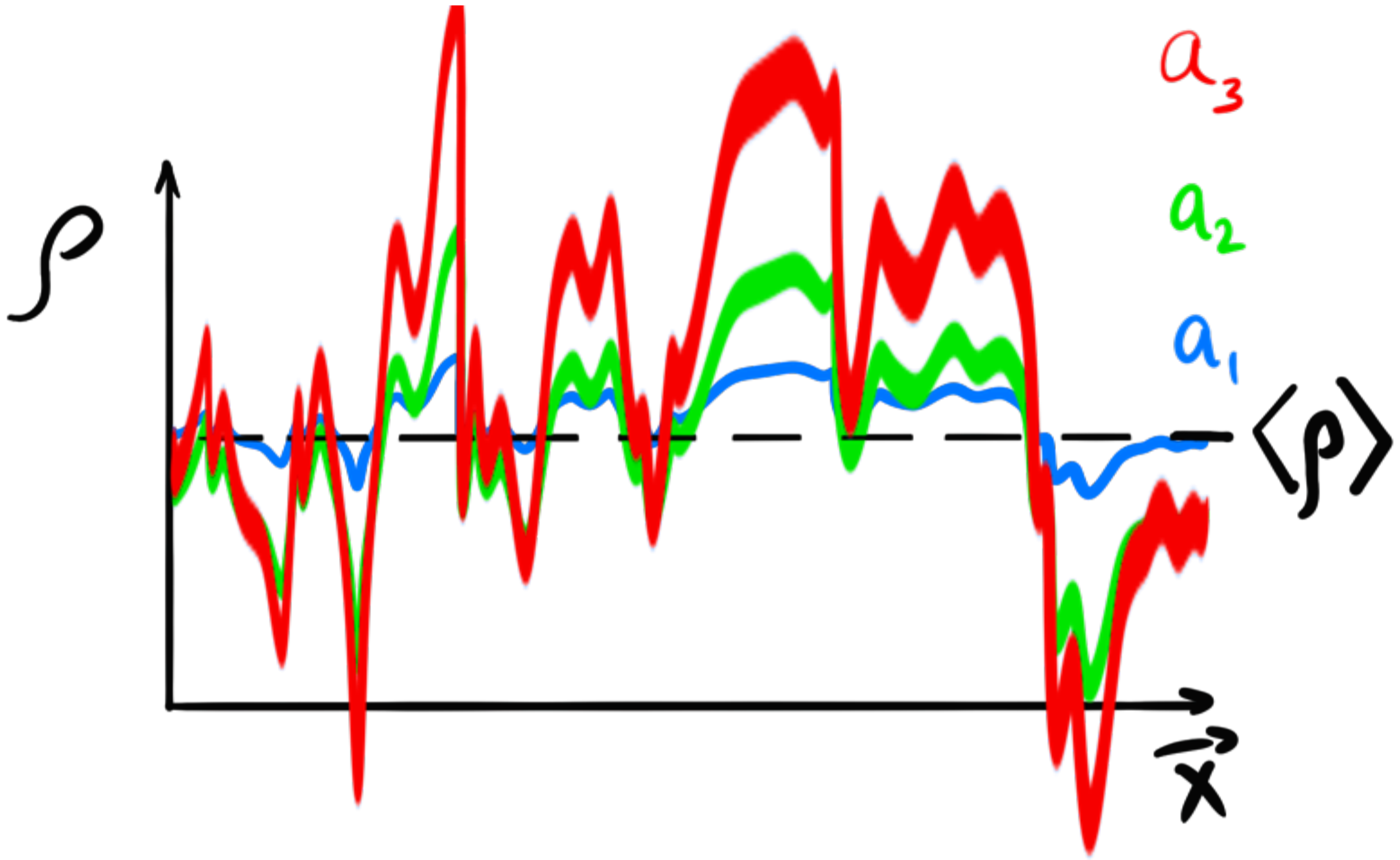
Structure formation basics:

Density perturbations $\delta = \frac{\rho - \langle\rho\rangle}{\langle\rho\rangle}$ grow as $\delta(t) \sim a(t)$.

In the early universe, $\langle\rho\rangle = \rho_{\text{crit}}$.



over-density δ grows linearly with time



You can't get here from there

The factor of 100 offset in density and temperature fluctuations is a prime motivation for non-baryonic **cold dark matter** — a substance for which perturbations δ can grow sufficiently large while not leaving an imprint of corresponding magnitude on the CMB.

Radiation and baryon plasma tightly coupled at recombination, so a fluctuation in density is reflected by one in temperature: $\frac{\delta\rho}{\rho} \propto \frac{\Delta T}{T}$.

Large Scale Structure

Quantified with the **correlation function** $\xi(r)$ which is the Fourier transform of the **power spectrum** $P(k)$.

The correlation function is the excess probability of finding a galaxy near another galaxy over that in a random distribution.

$$\frac{dN}{N} = [1 + \xi(r)]dV \qquad \xi(r) = \frac{V}{(2\pi)^3}\int P(k)e^{-\vec{k}\cdot\vec{r}}\, d^3k$$

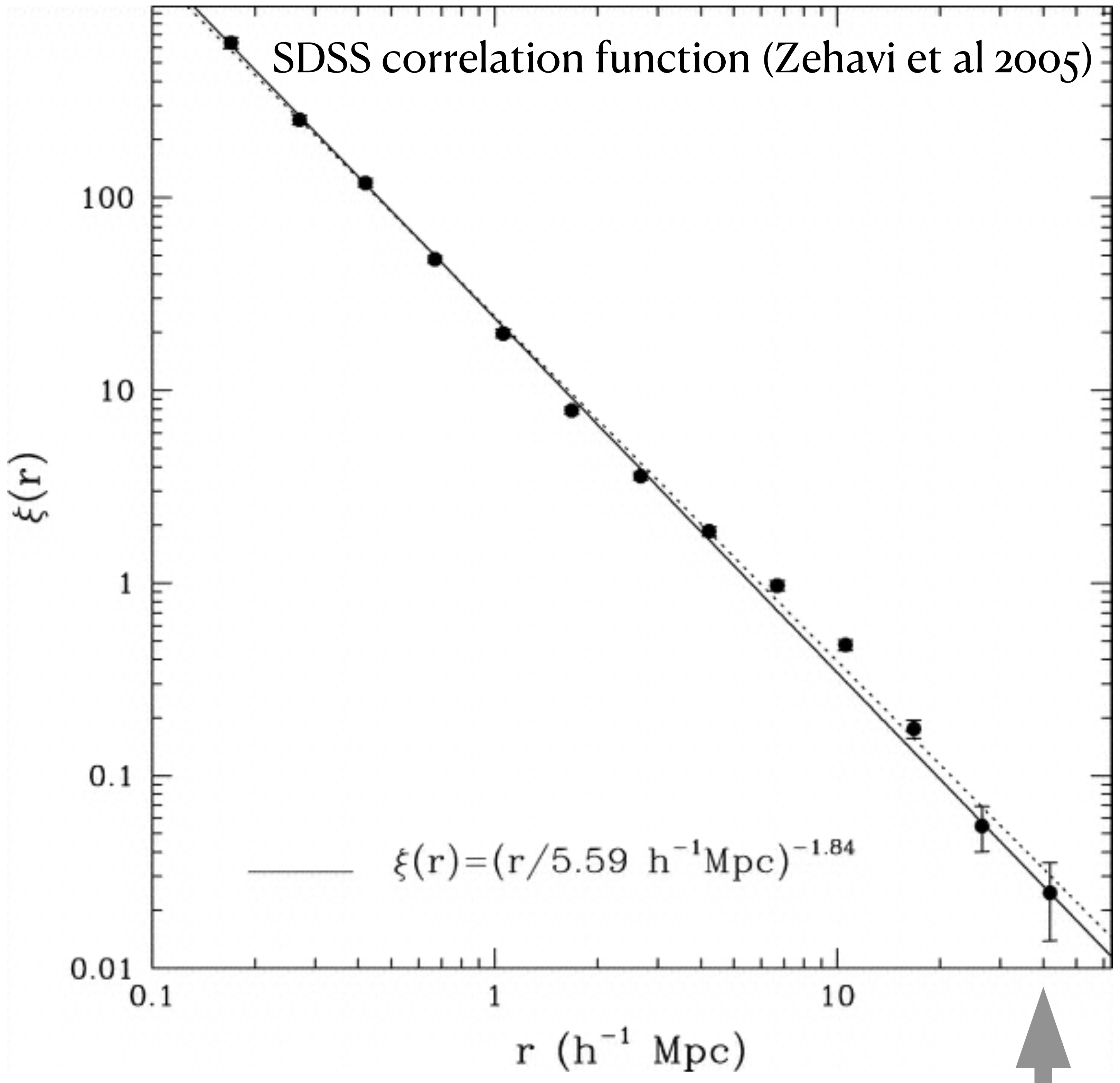
$$P(k) \propto |\delta(k)|^2 \propto k^n \qquad \xi(r) \propto r^{-(n+3)}$$

Harrison-Zeldovich spectrum has $n = 1$, which is a Gaussian random field.
Inflation predicts $n \approx 1$, but different flavors of Inflationary theory predict slightly different values depending on the shape of the Inflationary potential (the Inflaton).

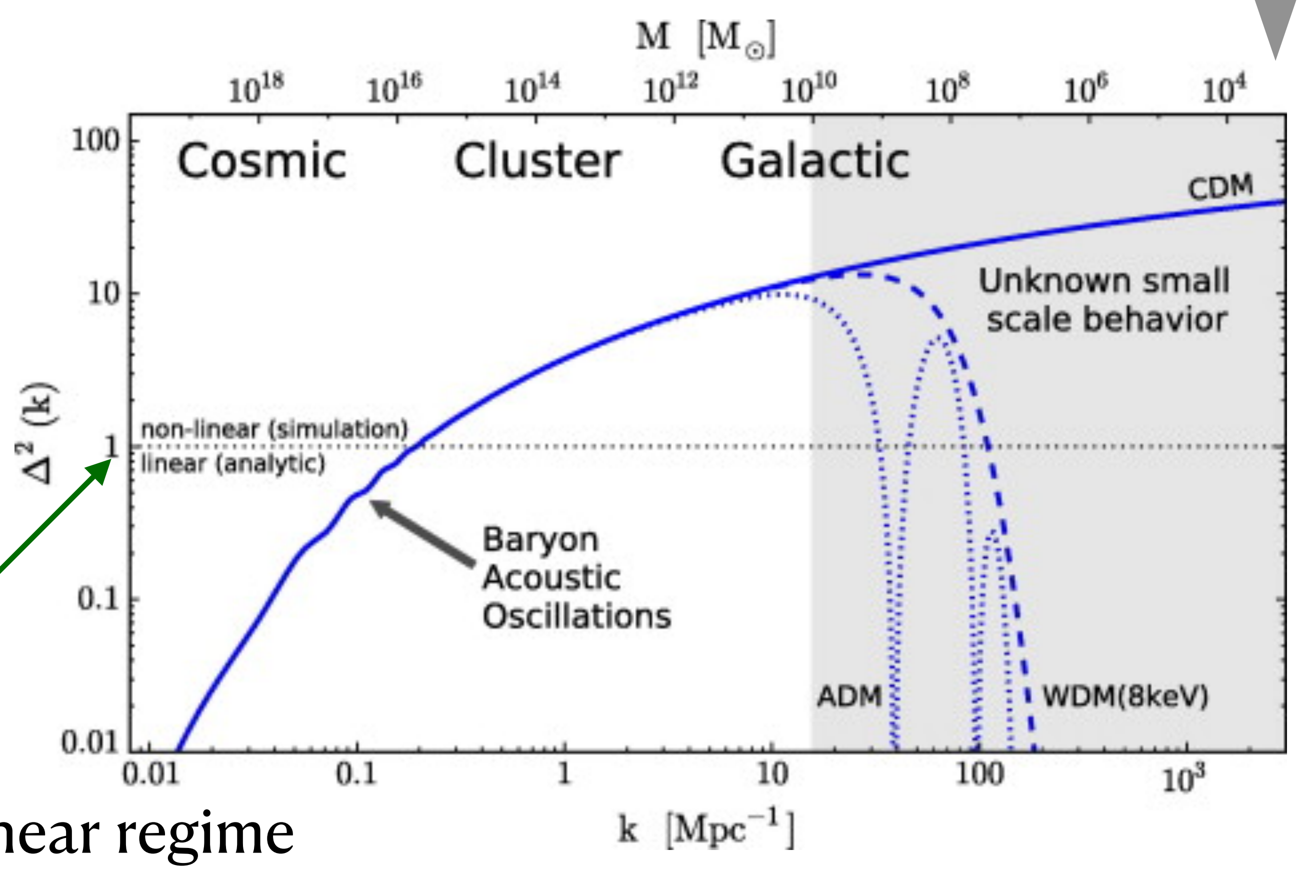
Planck measures $n = 0.965 \pm 0.004$

The shape of the power spectrum is set by n
The amplitude of the power spectrum is set by σ_8

SDSS correlation function (Zehavi et al 2005)



Power Spectrum

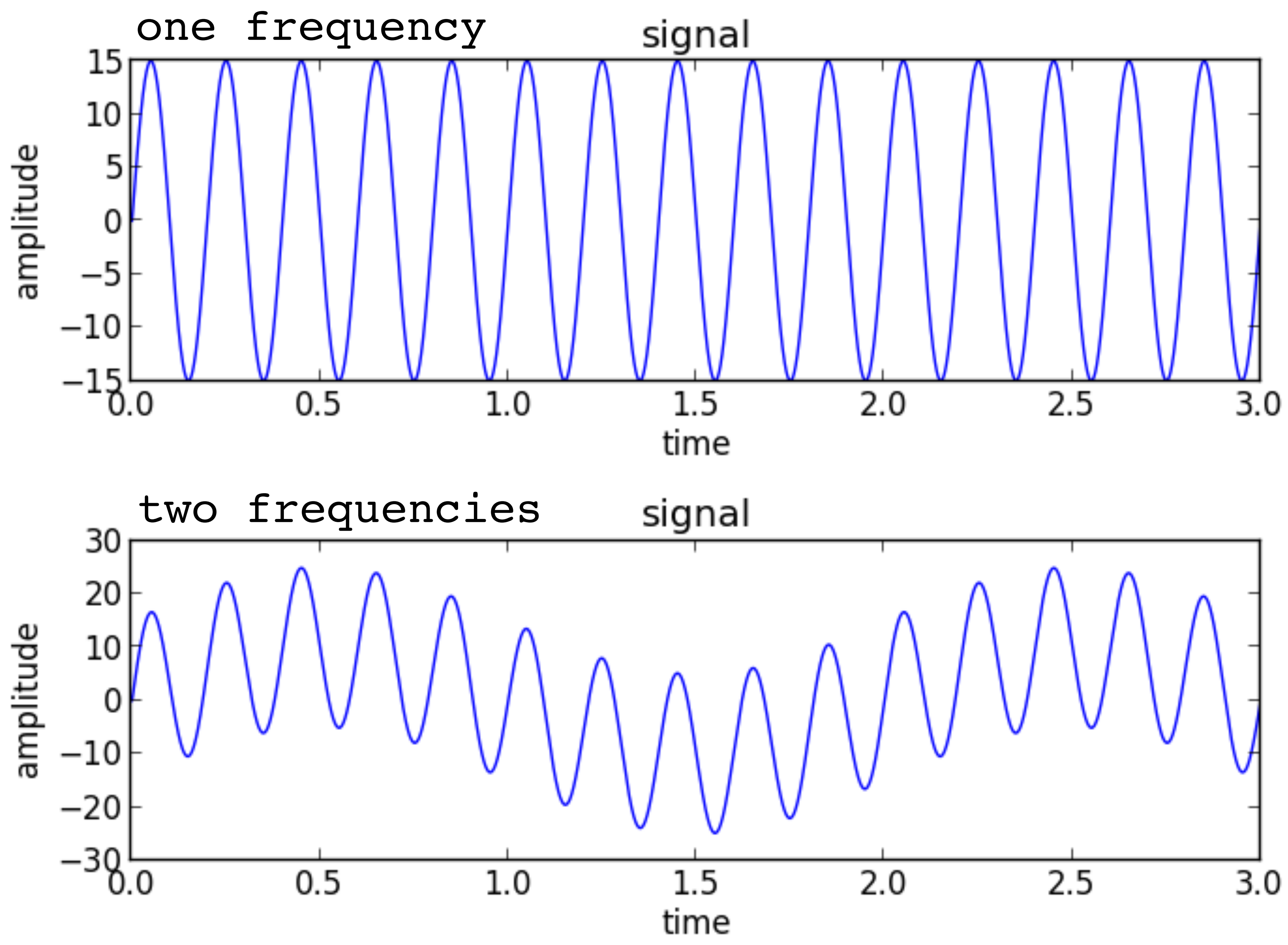


$\delta > 1$ marks the transition to the non-linear regime where perturbation theory no longer applies.

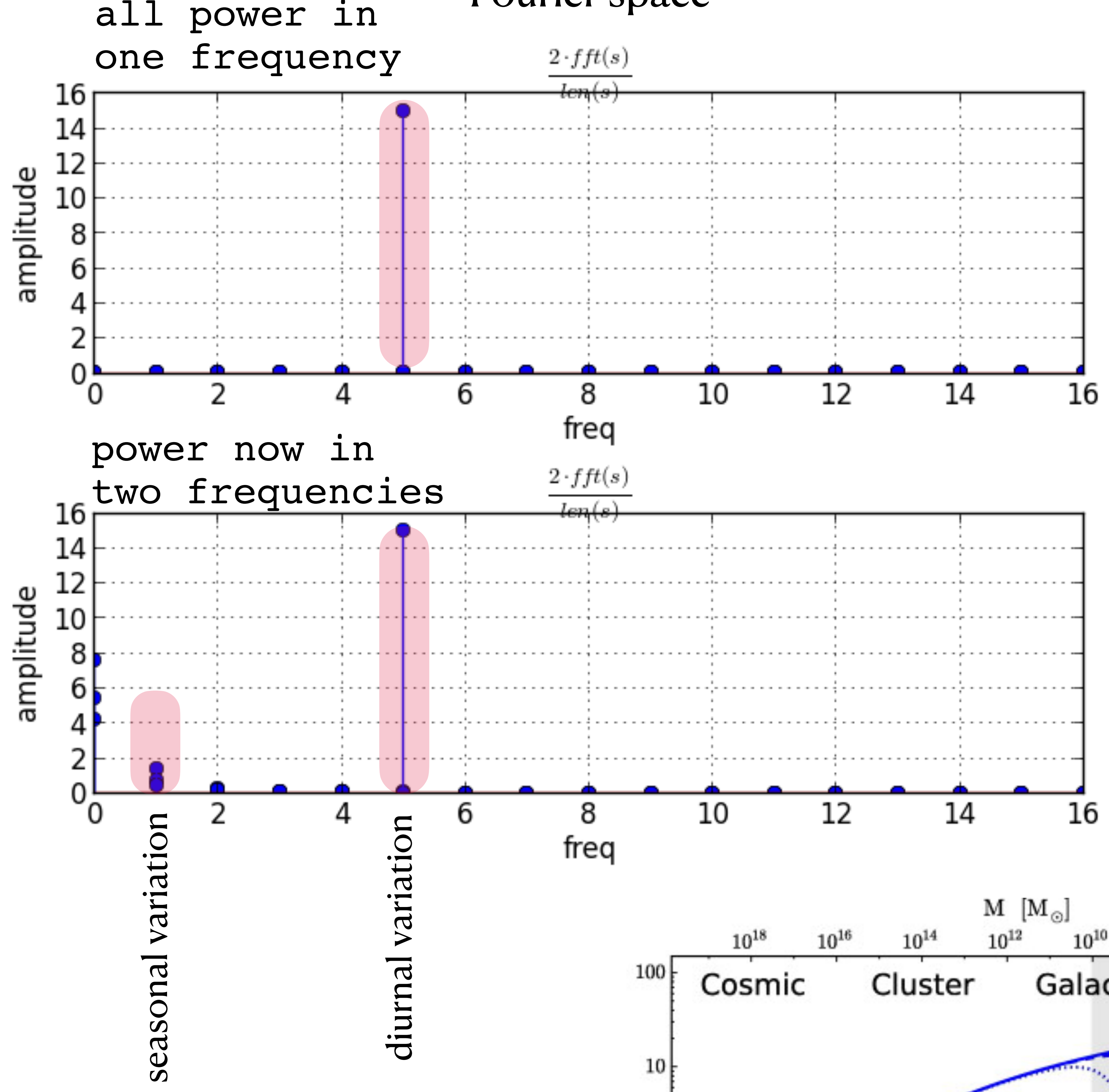
Superposition of two sinusoids

(e.g., diurnal and annual temperature variation)

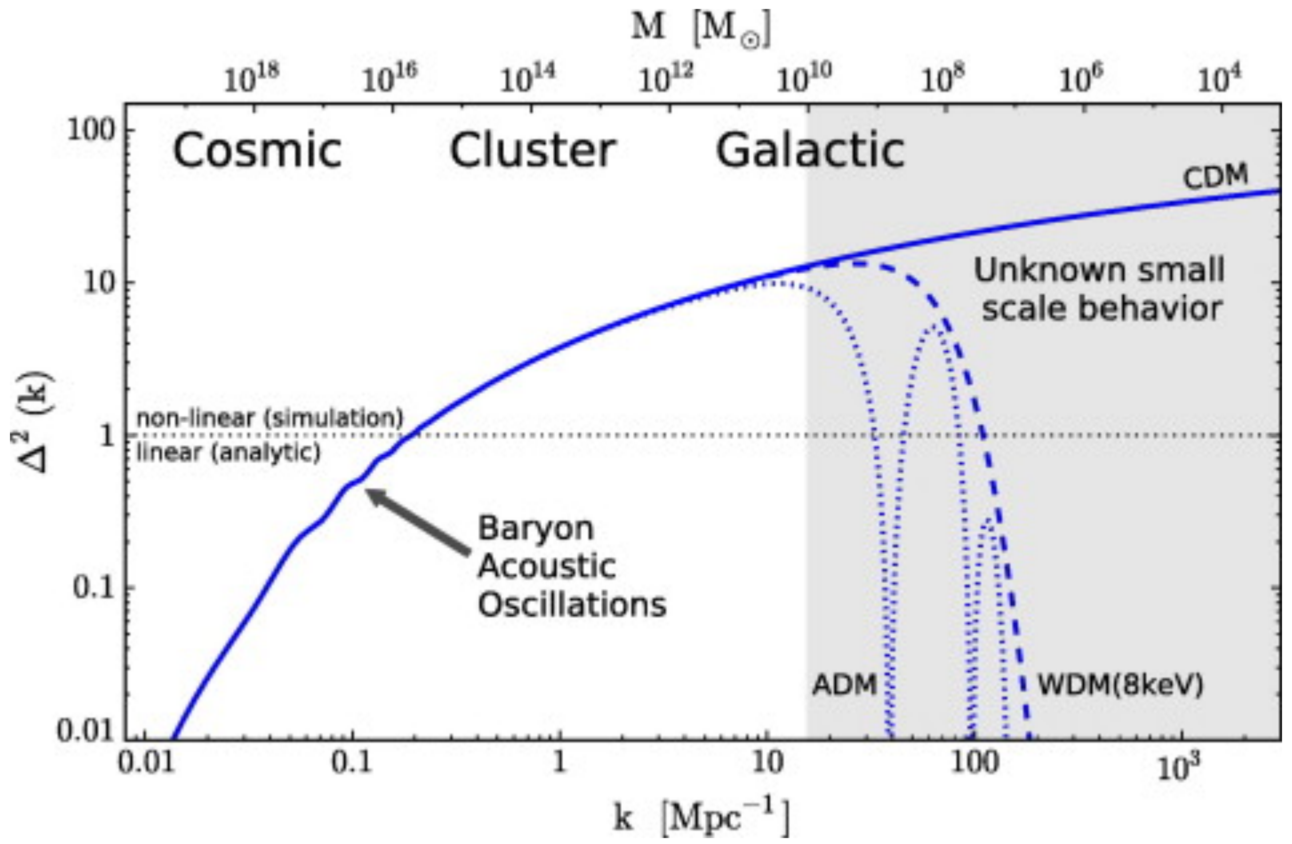
Real space



Fourier space



So a smooth power spectrum has contributions from all frequencies, but also picks out which are more common.



- # Power spectrum of galaxies

$$\delta \equiv \frac{\delta\rho}{\rho}$$

The power spectrum is commonly used to quantify large scale structure.
It is the related to the 2 point correlation function via Fourier transform.

2 point correlation function: $$\xi(r) = \langle \delta(\vec{x}) \cdot \delta(\vec{x} + \vec{r}) \rangle$$

The 2 point correlation function is the probability of finding one galaxy near another in excess over a random distribution.

Power spectrum: $$P(k) = \langle |\delta_k|^2 \rangle$$ where $$k = \frac{2\pi}{\lambda}$$

where k is the wavenumber corresponding to the scale λ

Fourier transform:

$$\xi(\vec{r}) = \frac{V}{(2\pi)^3} \int |\delta_k|^2 e^{-i\vec{k}\cdot\vec{r}} d^3k$$ averaged over volume V

($|\delta_k|^2$ ← $P(k)$)

the choice of "window function"
- how the volume is defined -
is a technical detail that matters

- # Power spectrum of galaxies

$$\delta \equiv \frac{\delta\rho}{\rho} \qquad k = \frac{2\pi}{\lambda}$$

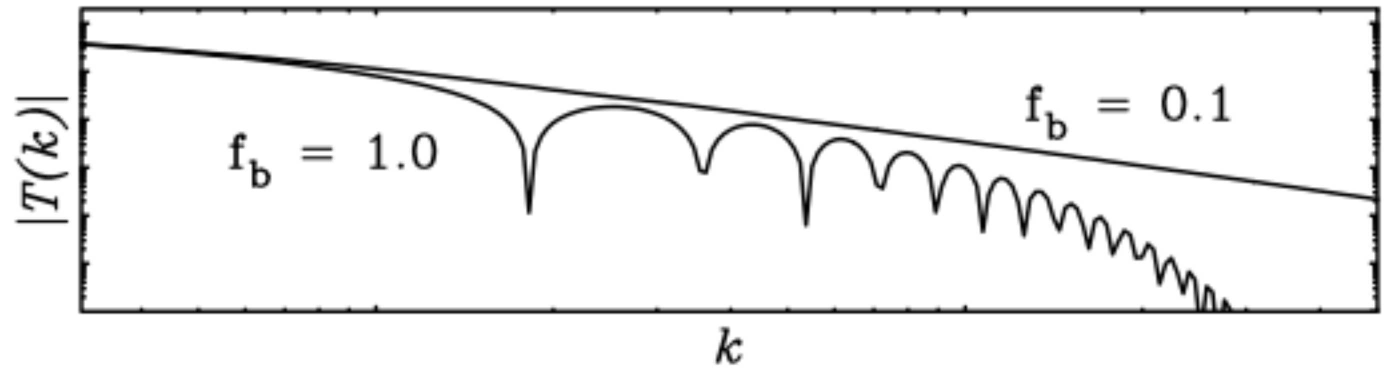
Power law power spectrum: $$P(k) = \langle |\delta_k|^2 \rangle \propto k^n$$

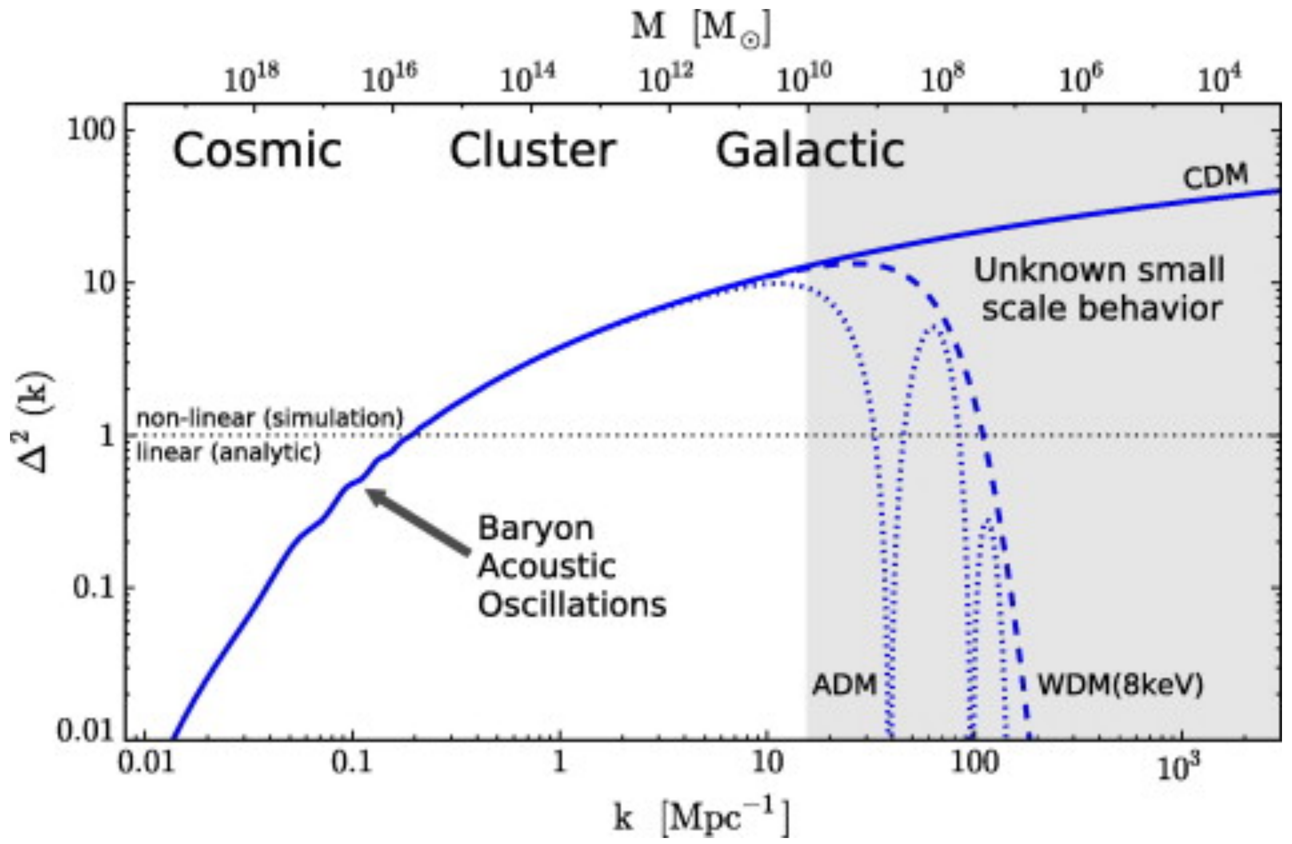
where n = 1 is scale free, with the same power on all scales.
This is observed to be nearly the case on large scales that have not yet collapsed. It is modulated on small scales by structure formation.

One way to think of it is the rms variation at each scale λ

$$M \sim \lambda^3 \qquad \delta_{\rm rms} \propto M^{-(n+3)/6}$$

cold dark matter smooths baryon acoustic oscillations





There is more rms variance on small scales, so more power there.
[On very large scales, the universe is homogeneous, so no variance.]

By convention, the normalization is set on a scale of 8 Mpc, where

$$\frac{\delta N_{gal}}{N_{gal}} = 1$$ with corresponding mass variance σ_8

Planck measures $\sigma_8 = 0.811 \pm 0.006$

From an accident report in the *Boston Driver's Handbook*:
"The guy was all over the road. I had to swerve several times before I hit him."

The power spectrum of SCDM missed badly:
too much power on small scales;
too little power on large scales.

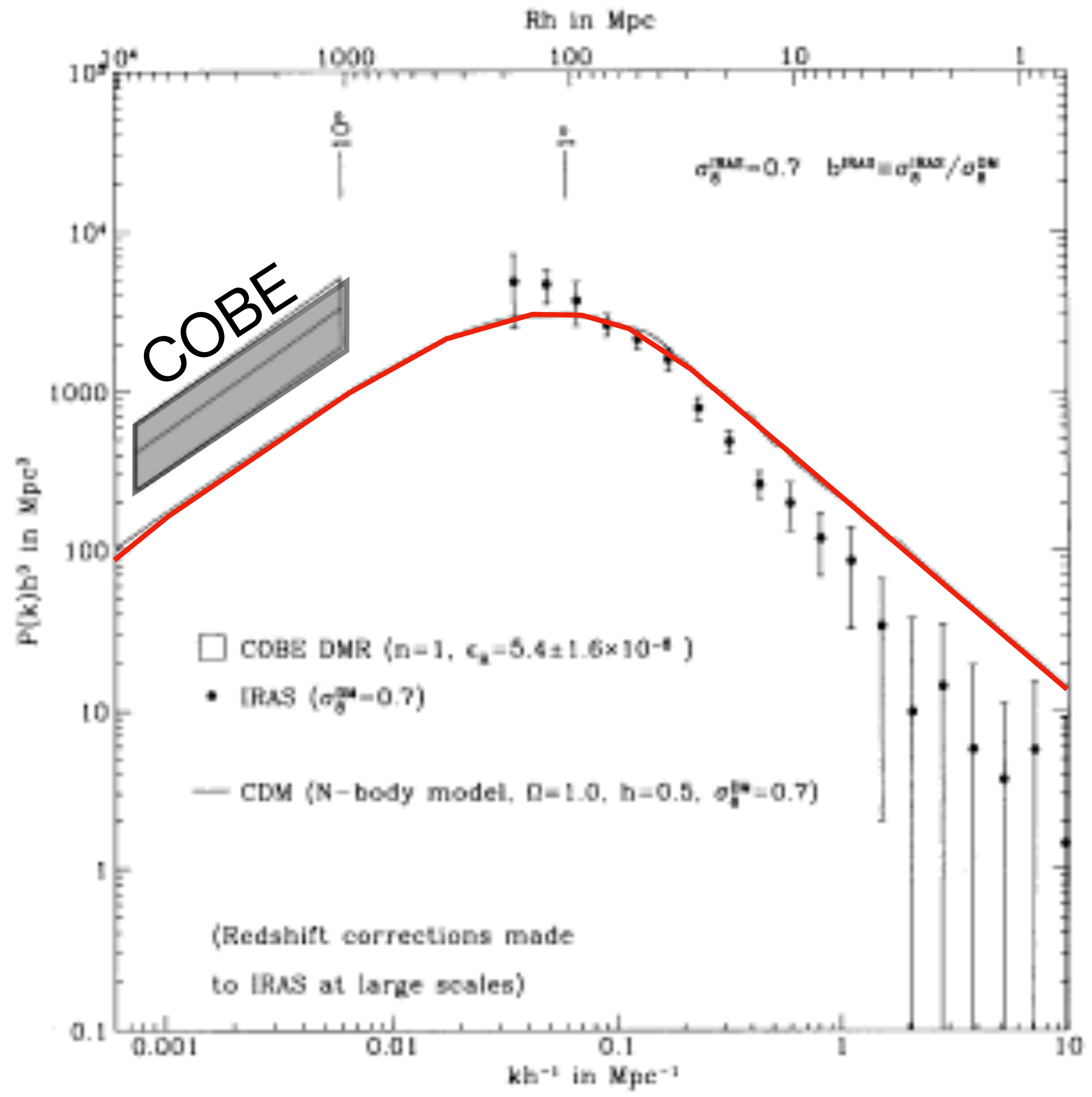
<u>SCDM</u> ("Standard" CDM)

$$\Omega_m = 1$$
$$H_0 = 50$$

$\Omega_m h = 0.5$ expected

$\Omega_m h \approx 0.2$ observed

Schramm (1993) also expressed concern
about the existence of quasars at $z \approx 4$ (!)



SCDM
$\Omega_m h = 0.5$
$\sigma_8 = 0.7$

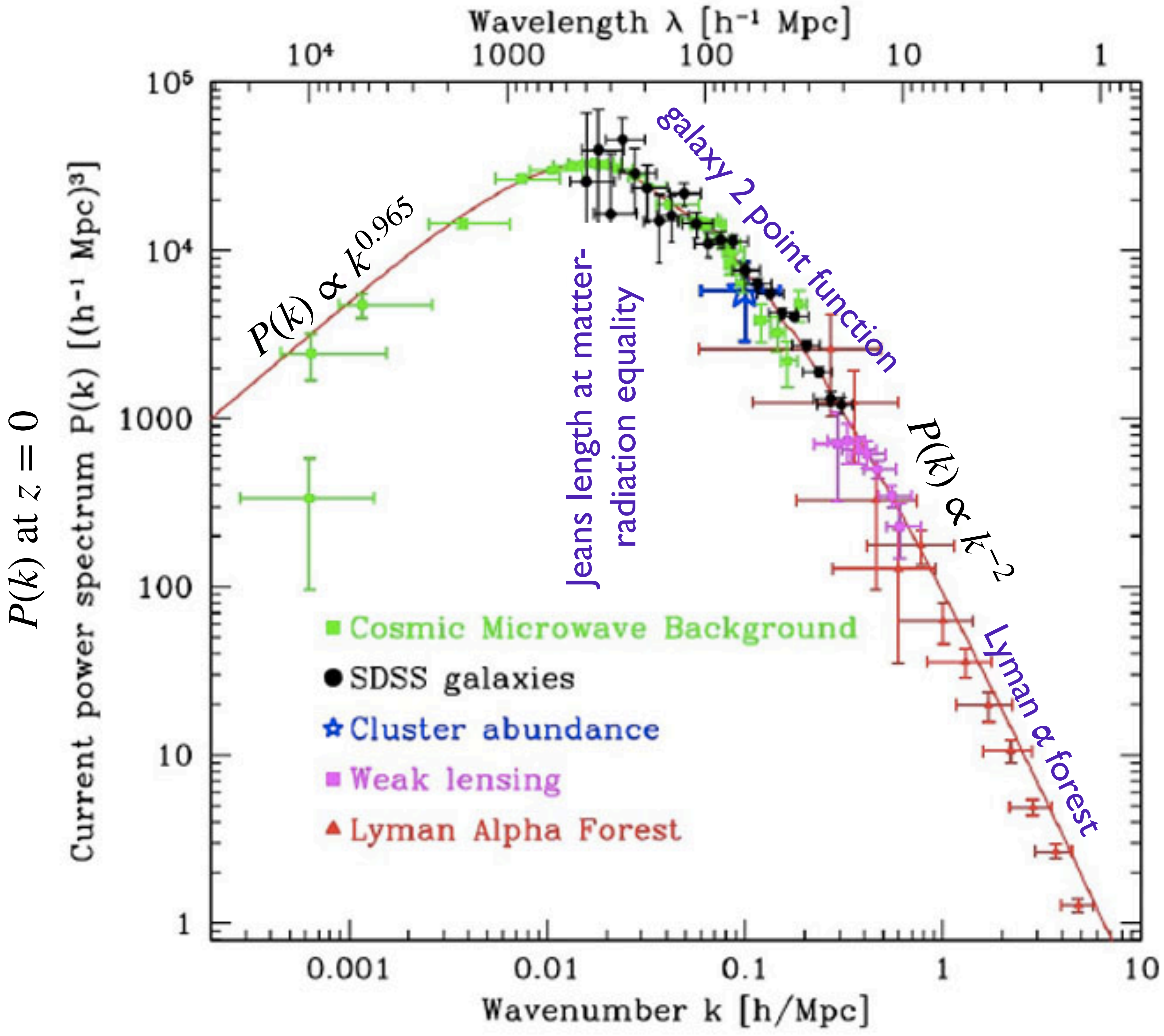
FIG. 10.—Solid curve is the real space power spectrum of the full nonlinear CDM *N*-body simulation (as in Fig. 3) normalized to the real space variance of *IRAS* galaxies ($\sigma_8 = 0.7$). The points are the *IRAS* redshift space $\tilde{P}(k)$ from Fig. 4, rescaled by eq. (17) with $\Omega = 1$ and $b = 1$; this is then, apart from the effects of the convolution in eq. (14), an approximation to the power spectrum of *IRAS* galaxies in *real* space on large scales if the *IRAS* galaxies are unbiased. The box indicates the power spectrum inferred from the *COBE* DMR measurements, assuming a $n = 1$ spectral index and $\epsilon_H = (5.4 \pm 1.6) \times 10^{-6}$ (Smoot et al. 1992; Wright et al. 1992). Note that when the CDM model is normalized to the *IRAS* variance, it produces excessive power on small scales while simultaneously failing to produce sufficient power on large scales to match the *COBE* results.

Fisher et al. (1993) ApJ, 402, 42

All this is solved by LCDM - provided that we are no longer concerned about the flatness/coincidence problem.

Planck estimates: $n = 0.965 \pm 0.004$

$$\sigma_8 = 0.811 \pm 0.006$$



Jeans length at matter-radiation equality

$$\lambda_J = c_s \sqrt{\frac{\pi}{G\rho}}$$

sound speed of photon-baryon fluid

$$c_s^2 = \frac{\partial P}{\partial \rho} = \frac{1}{3} c^2$$

Eqn of state for photons,

$$P = \frac{1}{3} \rho c^2$$

(P here is pressure)

imprints standard rod on surface of last scattering.

at smaller scales, things go non-linear from gravitational collapse, pressure, dissipation, feedback, etc. Described by a **Transfer function**

$$T(k) \equiv \frac{\delta_k(z=0)}{D(z)\, \delta_k(z)}$$

where D(z) is the linear growth factor - what it would have been without all these nasty non-linear effects.

Large Scale Structure

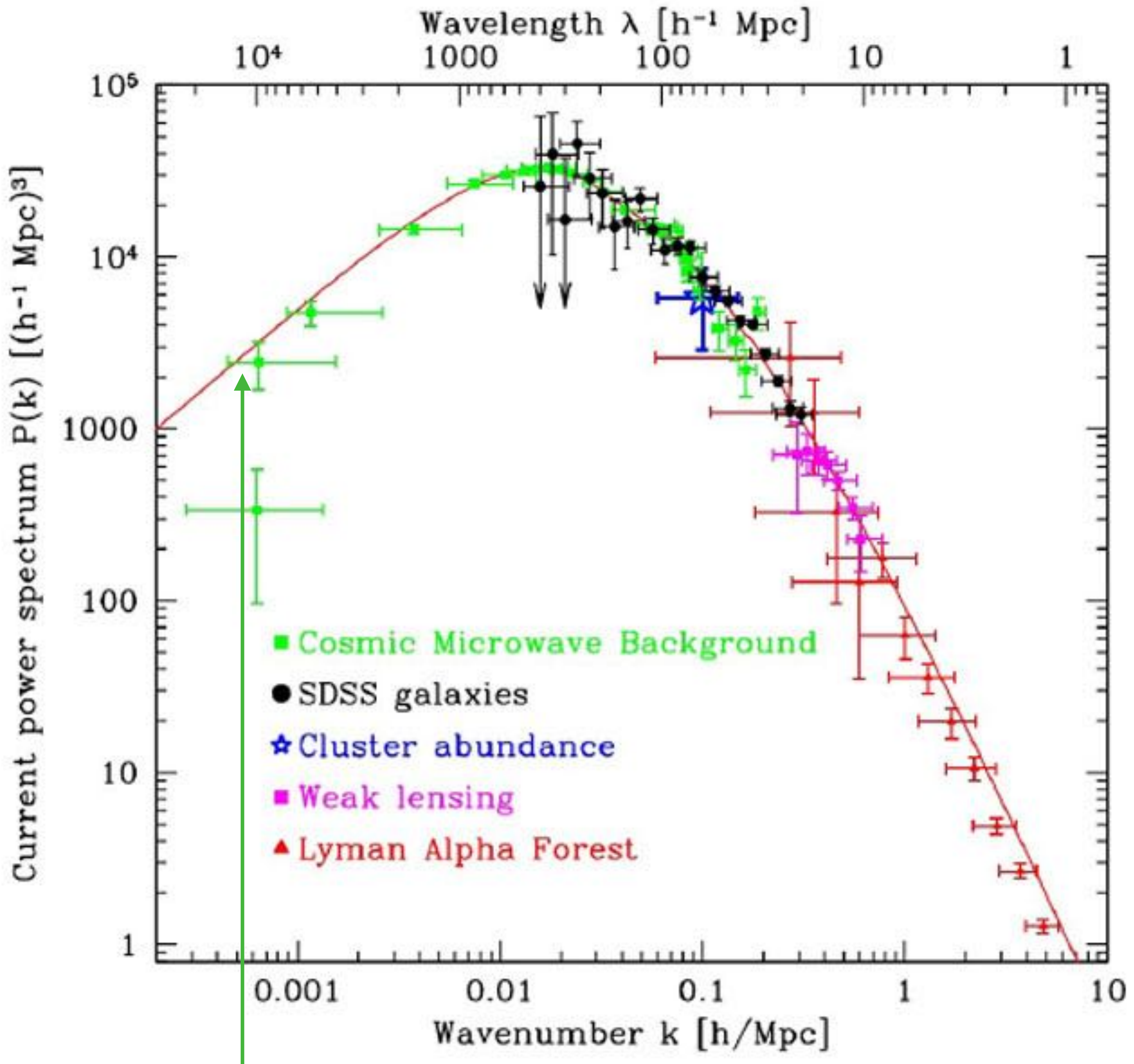
Quantified with the **correlation function** $\xi(r)$ which is the Fourier transform of the **power spectrum** $P(k)$.

The correlation function is the excess probability of finding a galaxy near another galaxy over that in a random distribution.

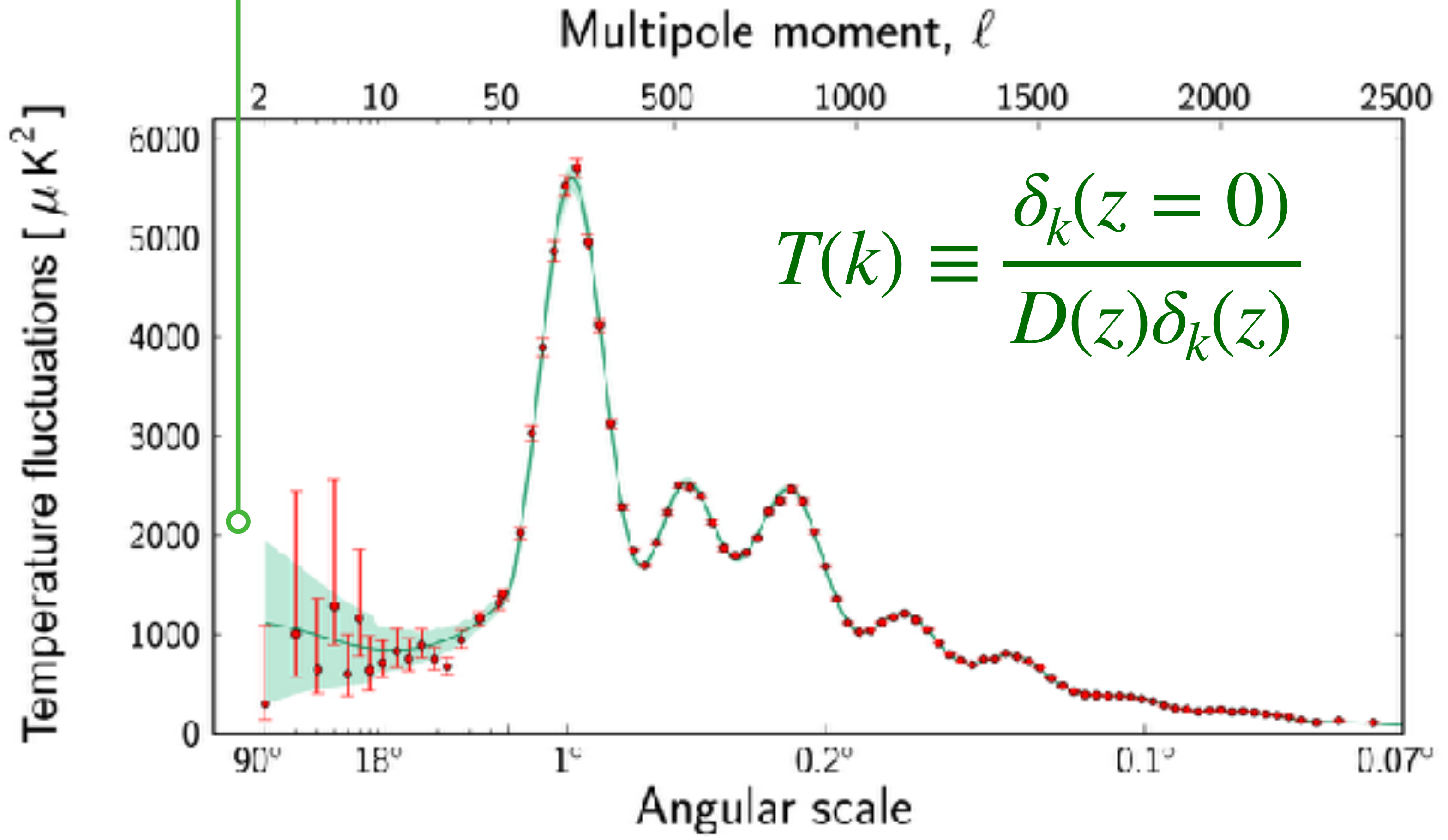
$$\frac{dN}{N} = [1 + \xi(r)]dV \qquad \xi(r) = \frac{V}{(2\pi)^3}\int P(k)e^{-\vec{k}\cdot\vec{r}}\, d^3k$$

$$P(k) \propto |\delta(k)|^2 \propto k^n \qquad \xi(r) \propto r^{-(n+3)}$$

Harrison-Zeldovich spectrum has $n = 1$, which is a Gaussian random field. Inflation predicts $n \approx 1$, but different flavors of Inflationary theory predict slightly different values depending on the shape of the Inflationary potential (the Inflaton). Planck measures $n = 0.965 \pm 0.004$



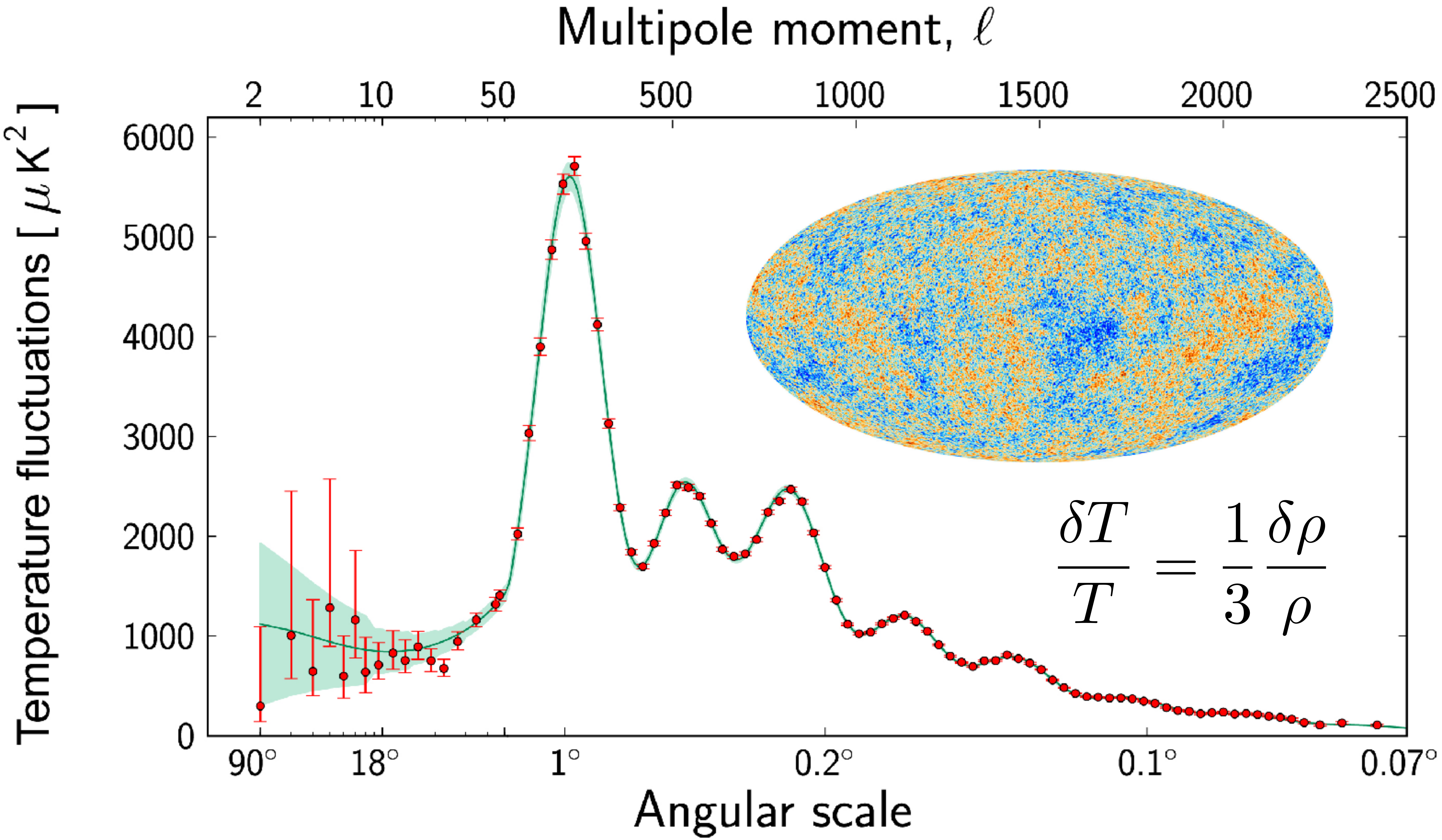
CMB power spectrum at z = 1000 in linear regime mapped to galaxy power spectrum at z=0 by calculation of $D(z)$



$$T(k) \equiv \frac{\delta_k(z=0)}{D(z)\delta_k(z)}$$

CMB power spectrum

Detailed shape of the acoustic power spectrum depends sensitively on cosmic parameters. First and foremost, the location of the first peak measures the angular diameter distance to the surface of last scattering. This is the best evidence that the universe is very nearly flat: $\Omega_k = -0.011 \pm 0.006$ (Planck X 2018)



CMB power spectrum

Etc. pole

$\ell = 1,2,3$

$\ell = 4,5,6$

$\ell = 7,8,9$

$\ell = 10,11,12$

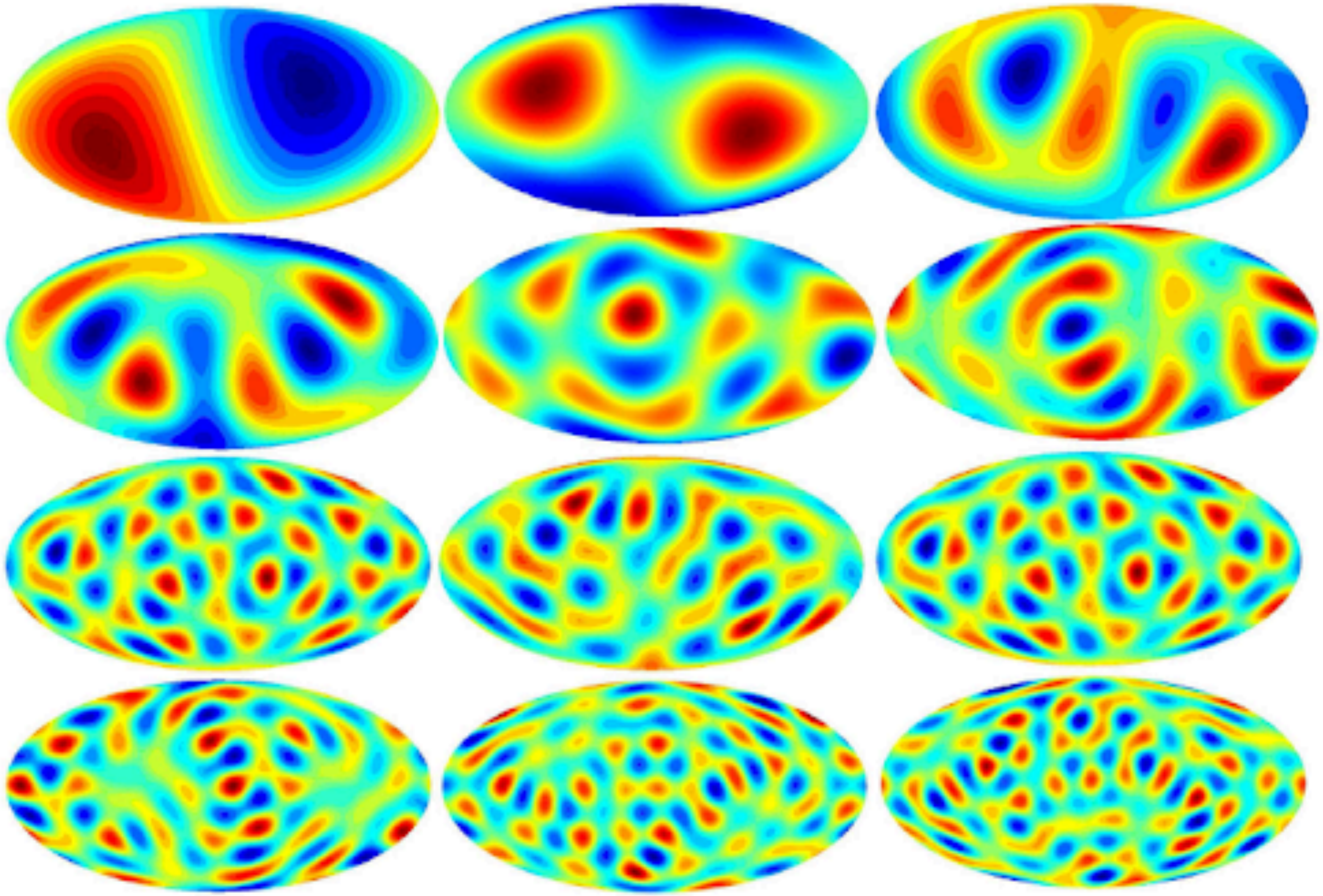
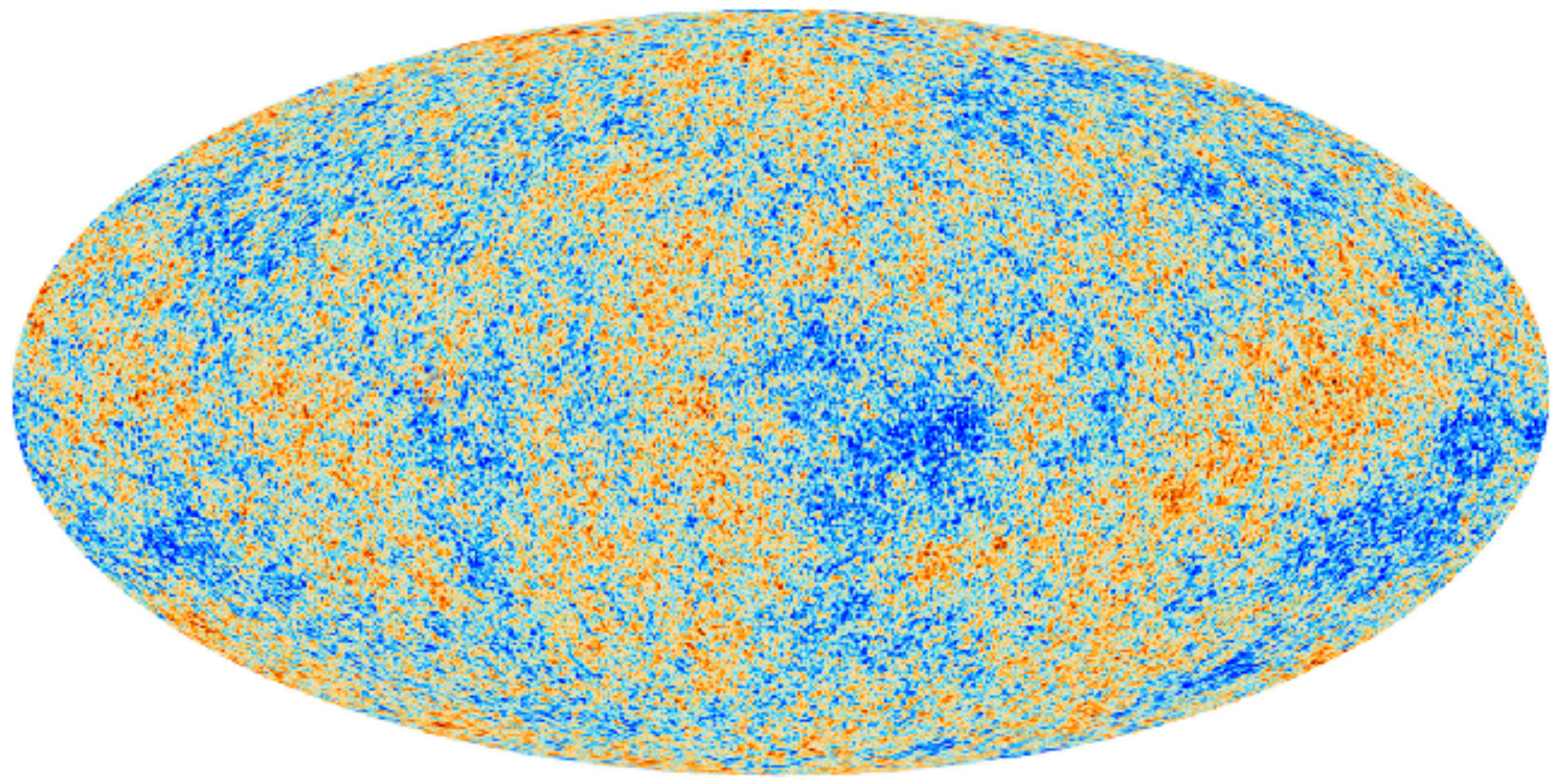


Figure 6: Randomly generated skies containing only a single multipole ℓ. Staring from top left: $\ell = 1$ (dipole only), 2 (quadrupole only), 3 (octupole only), 4, 5, 6, 7, 8, 9, 10, 11, 12. Figure by Ville Heikkilä.



Spherical harmonics provide a convenient way to decompose the fluctuations observed on the sky

$$\frac{\Delta T}{T}(\theta,\phi) = \sum_{\ell=0}^{\infty}\sum_{m=-\ell}^{\ell} A_{\ell m} Y_{\ell m}$$

with Fourier transform

$$A_{\ell m} = \int_{\rm sky} \frac{\Delta T}{T}(\theta,\phi) Y^*_{\ell m} d\Omega$$

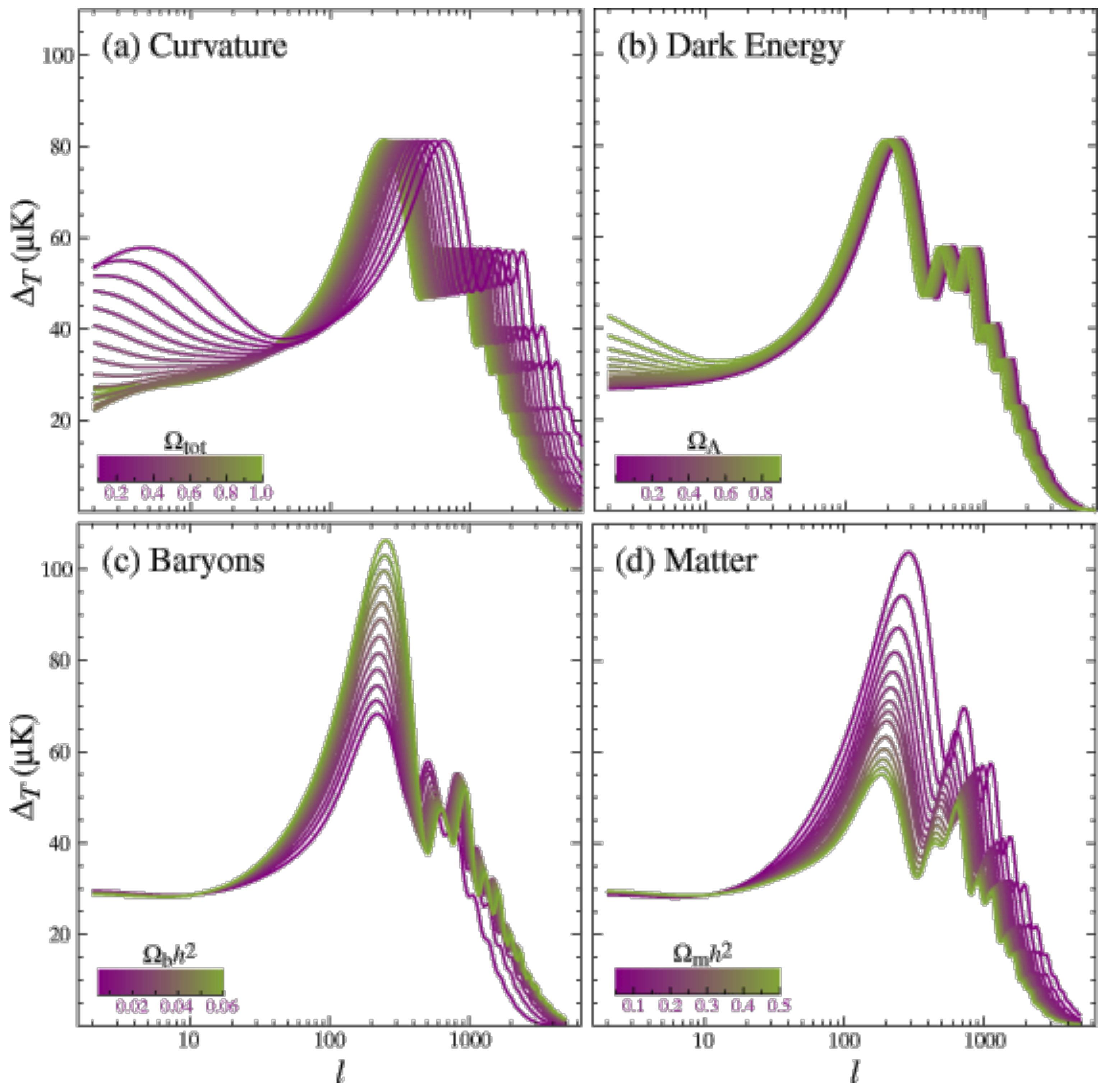
giving the power in fluctuations on an angular scale $\frac{\pi}{\ell}$

$$C_\ell = \frac{1}{2\ell+1}\sum_m A_{\ell m}A^*_{\ell m} = \langle |A_{\ell m}|^2 \rangle$$

note: $1^\circ = 0.0175$ radians so one degree corresponds to $\frac{\pi}{\ell} = 0.0175$, hence $\ell = 180$.
Multipole ℓ varies inversely with angular scale.

CMB power spectrum

Detailed shape of the acoustic power spectrum depends sensitively on cosmic parameters.



Best-fit cosmology obtained from multi-parameter fit. Well constrained, but not unique - lots of parameter degeneracy.

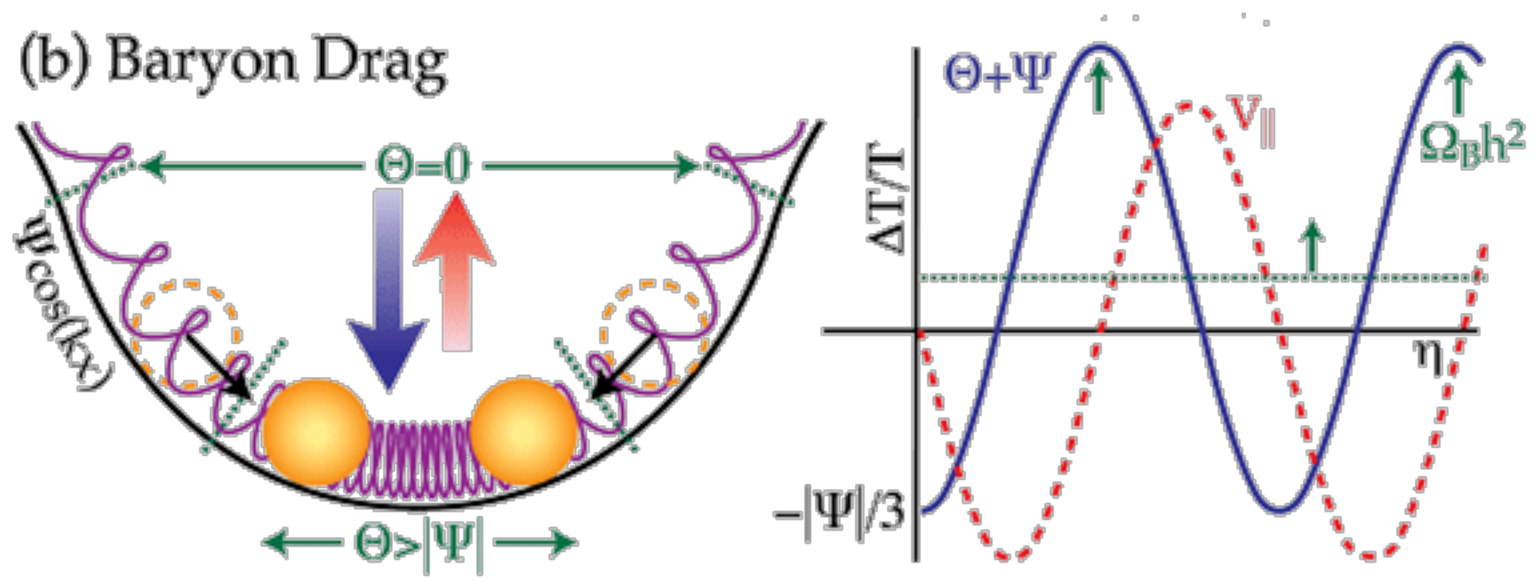
Compression and rarefaction nearly cancel out, but don't quite. Left with

$$\frac{\delta T}{T} = \frac{1}{3}\frac{\delta \rho}{\rho}$$

Damped and driven oscillator

Baryons damp oscillations, like a kid dragging his feet on a swing.
pure damping spectrum in limit of all baryons

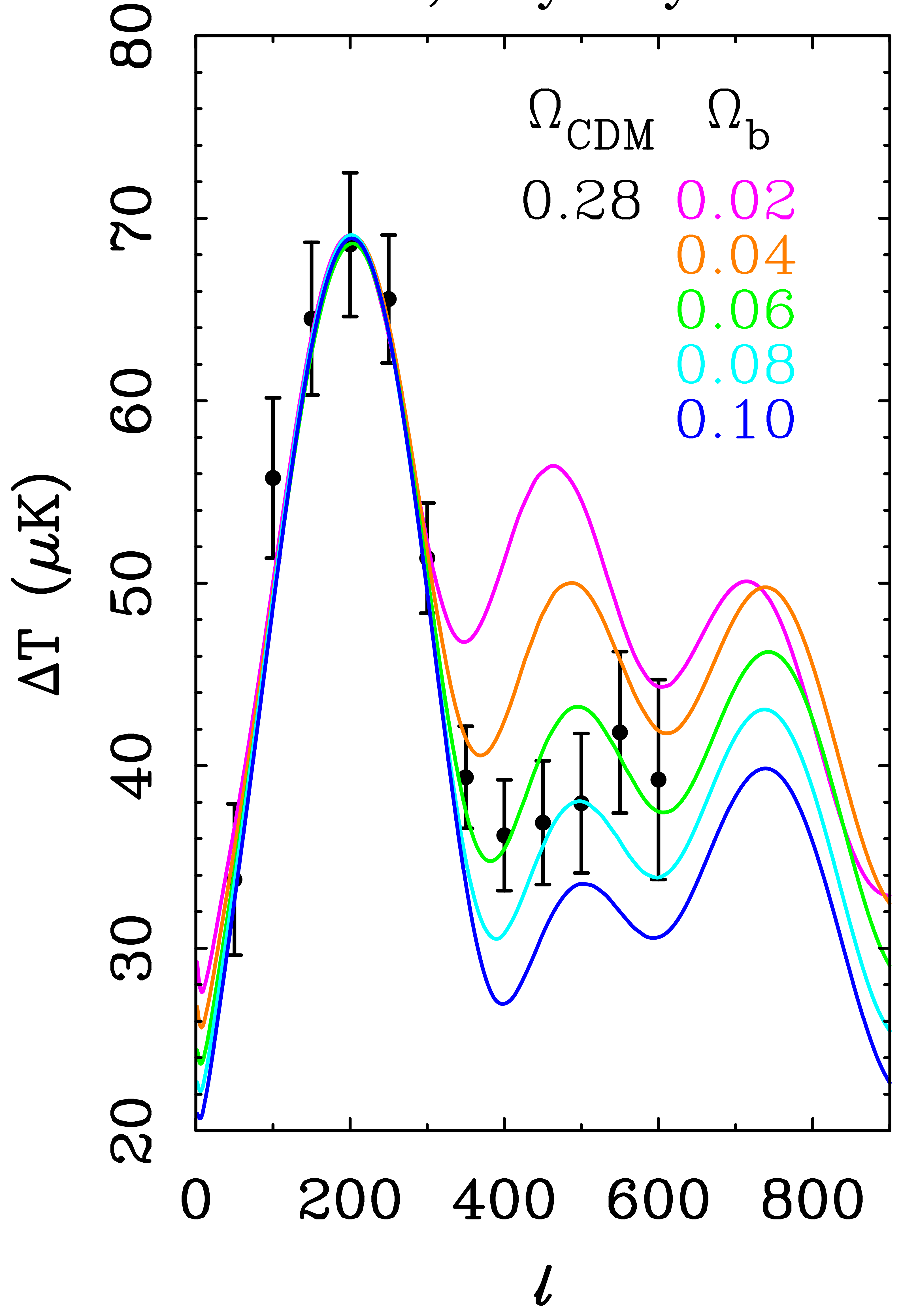
Dark matter helps drive oscillations, like a parent pushing the kid.



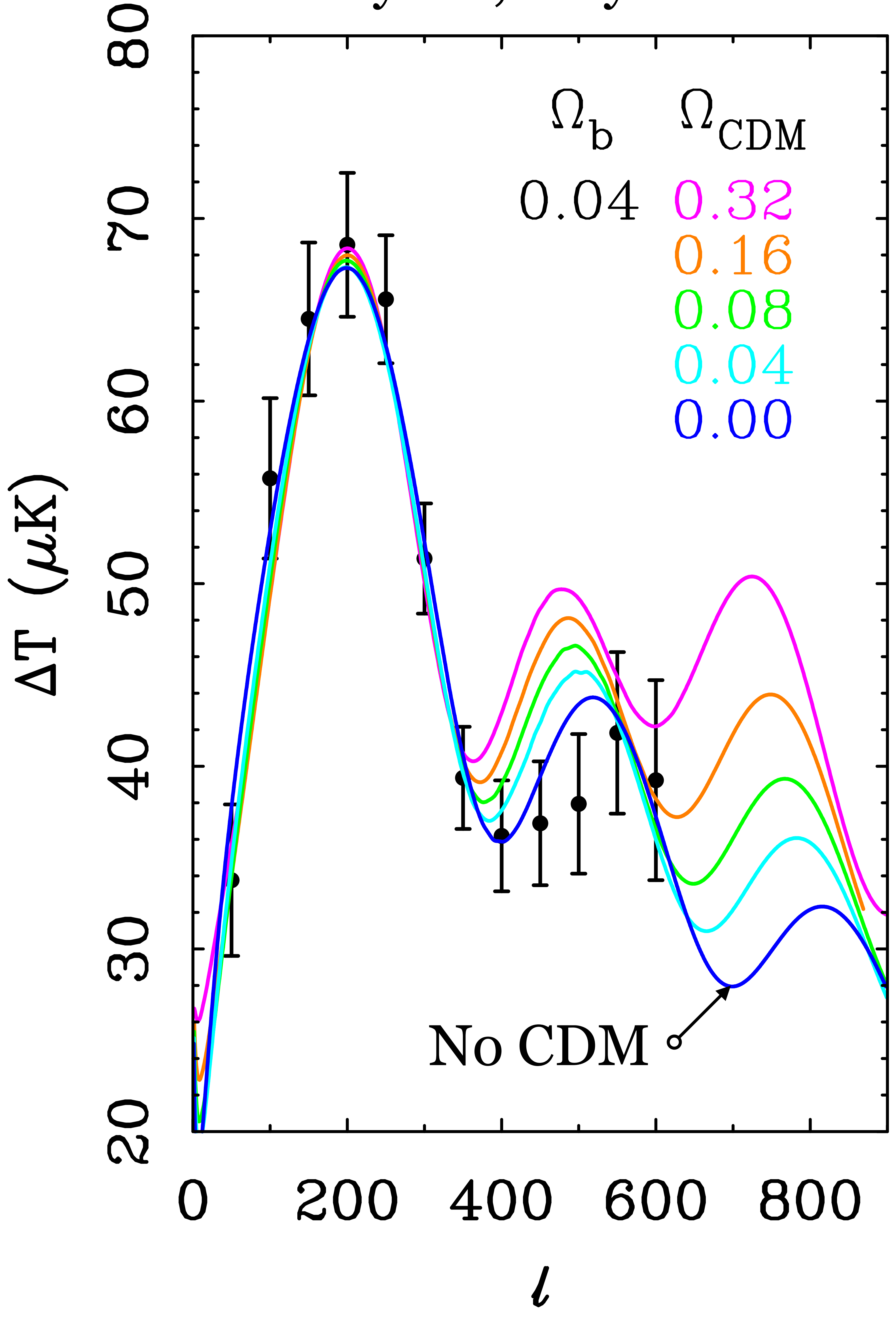
Wayne Hu provides a nice CMB tutorial at http://background.uchicago.edu/index.html

CMB dependence on the density of baryonic and non-baryonic matter

Fix CDM; vary baryons



Fix baryons; vary CDM

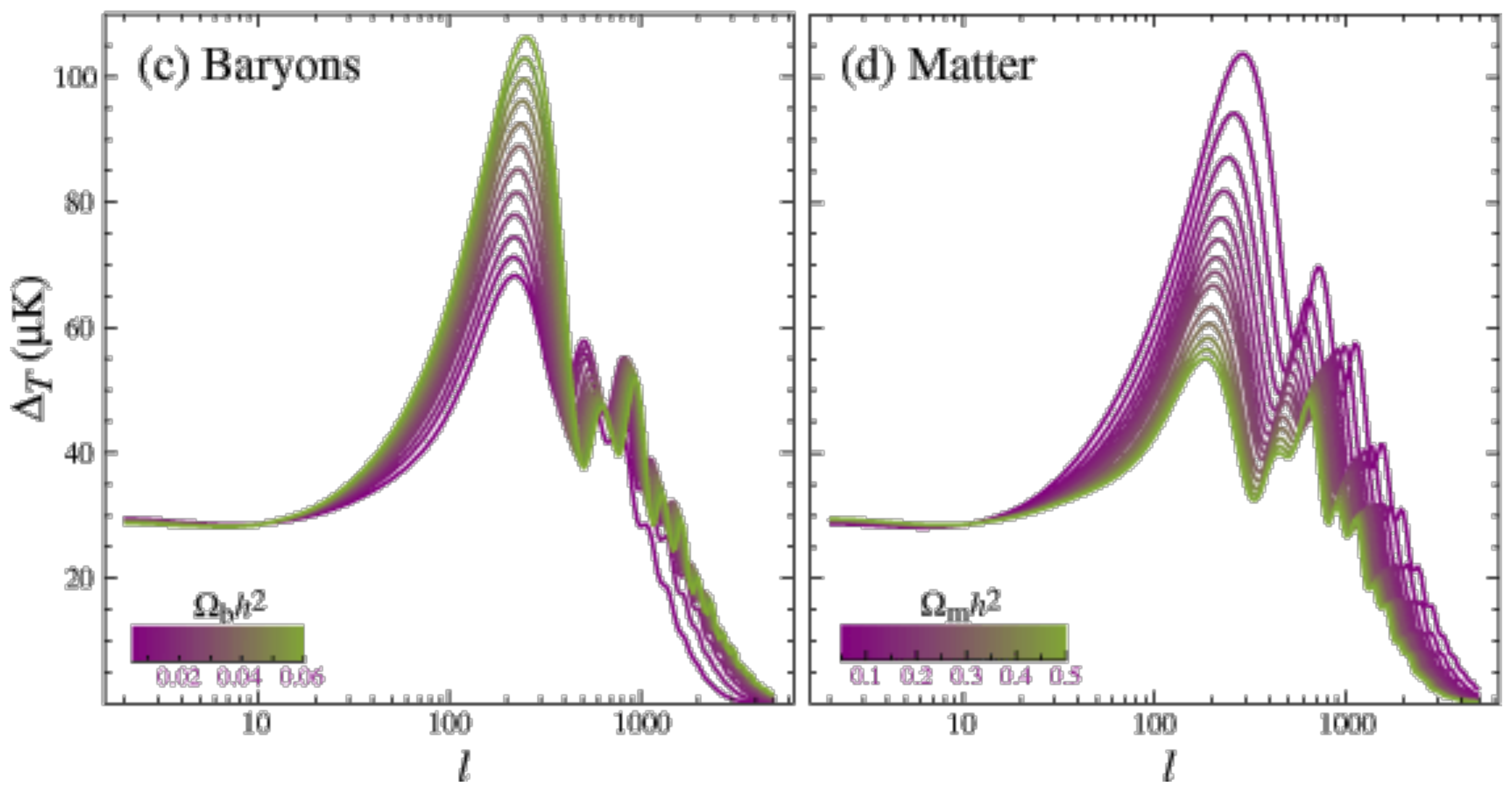


Damped and driven oscillator

Baryons damp oscillations, like a kid dragging his feet on a swing.

pure damping spectrum in limit of all baryons

Dark matter helps drive oscillations, like a parent pushing the kid.



There have long been two compelling reasons why we need to invent **non-baryonic** cold dark matter:

1. There is more gravitating mass than Big Bang Nucleosynthesis allows in normal matter.

$$\Omega_m > \Omega_b$$

2. The need to grow large scale structure from very uniform initial conditions.

$$\delta(z = 1090) \sim 10^{-5} \;\rightarrow\; \delta(z = 0) \sim 1$$

A third compelling reasons to retain **non-baryonic** cold dark matter is the amplitude of the third peak in the acoustic power spectrum of the CMB

$$A_3 \approx A_2$$

This show the effects of a driving term that exceeds the damped spectrum of baryons alone.

CMB power spectrum