Cosmology — first exam review

Types: mythic, religious, philosophical, scientific

Philosophical possibilities:

	Finite	Indefinite	Infinite
	Aristotelean	Stoic / Victorian	Epicurean / Modern
	Geocentric	Geo / Milky Way-centric	No center
Olber's paradox:	finite, so no problem	stars trail off — run out of stuff	a real problem solved by cosmic expansion

Great Debate (1920):

Shapley	Curtis
✓ Milky Way big — we are not at its center	X Milky Way "small" — we happen to be near its center
X Spiral nebulae are gas clouds within the Milky Way	✓ Spiral Nebulae are other galaxies comparable to the Milky Way

Hubble '20s - 30s

- Andromeda & other spirals clearly external (Cepheid distance)
- The universe is expanding
- Galaxies are the building blocks of the universe
  wide array of mass & morphological type

Expansion of the universe solves stability problem (gravitational collapse) and resolves Olber's paradox

Principles:

Copernican Principle: nothing special about our location

Cosmological Principle: the universe is
- homogeneous
- isotropic

Perfect Cosmological Principle: CP + eternal
- looks the same to any observer at any time X

Since the universe is expanding, the light that reaches us from a remote cosmic source has been redshifted by the stretching of space, and the distance it has had to travel is greater than when it was emitted. This makes the effective distance for the purpose of dimming different from the current proper distance. It is thus useful to define

- Luminosity distance: $d_L = (1+z)\, d_p$

  Has the property that $f = \dfrac{L}{4\pi d_L^2}$

- Angular size distance: $d_A = \dfrac{d_p}{(1+z)}$

  Has the property that $\theta = \dfrac{\ell}{d_A}$

as in Euclidean space

Aside: Astronomical magnitude system

- apparent magnitude $m = -2.5 \log f + \xi$ is a logarithmic measure of flux
- absolute magnitude $M - M_\odot = -2.5 \log\left(\frac{L}{L_\odot}\right)$ is a measure of intrinsic luminosity (power)
- distance modulus $m - M = 5 \log d_L - 5$ is a measure of distance in parsecs

The absolute magnitude is defined to be the apparent magnitude an object would have if it were at a distance of 10 pc — less than the Kessel run

Governing Equations -

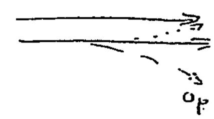
Friedmann equation
Acceleration equation
Robertson-Walker metric

Geometry is not Euclidean, only appears Euclidean in the limit of small sizes $r \to 0$

Robertson-Walker metric :

$$ds^2 = c^2 dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega\right] \qquad k = \begin{cases} +1 & \text{closed} \\ 0 & \text{flat} \\ -1 & \text{open} \end{cases}$$

Initially parallel light rays remain parallel in a flat ($k=0$) geometry. They converge in a closed geometry and diverge in an open geometry



Photons travel at the speed of light, so the events of emission and observation of a photon have a light-like separation : $ds = c$

Hence $\qquad c\,dt = a(t) \cdot f^{1/2}(r)$

If we know the cosmic expansion history $a(t)$, we can integrate over it to find the path-length travelled by a photon between two points of comoving separation r

$$c \int_{t_e}^{t_o} \frac{dt}{a(t)} = \int_0^r f^{1/2}(r)\, dr \qquad \left[= r \quad \text{in Ryden's notation} \right]$$

Acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{1}{3}\Lambda$$

mass density

Pressure – usually the energy-density of relativistic s

Eqn of state : $P = w\rho$.

$w = 0$	matter
$1/3$	radiation
-1	Λ

Friedmann eqn

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_m + \frac{\varepsilon_r}{c^2}\right) - \frac{kc^2}{(a R_0)^2} + \frac{c^2}{3}\Lambda \qquad \varepsilon_r = \alpha T_r^4$$

critical density – over/under between collapse and eternal expansion.

$$\rho_{crit} = \frac{3H_0^2}{8\pi G} \qquad \Omega_i = \frac{\rho_i}{\rho_{crit}}$$

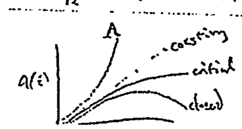
so defined $\quad \Sigma\,\Omega_i \equiv 1$

recall $H = \frac{\dot{a}}{a}$ and $a = \frac{1}{1+z}$

$$\left(\frac{H}{H_0}\right)^2 = \Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_k a^{-2} + \Omega_\Lambda$$

$$= \Omega_m (1+z)^3 + \Omega_r (1+z)^4 + \Omega_k (1+z)^2 + \Omega_\Lambda$$

$$= E^2(z)$$



The search for two numbers: H_0, Ω_{m_0}

Hubble parameter

$$H = \frac{\dot{a}}{a}$$ the Hubble parameter can vary over time

$$H_0 = \left(\frac{\dot{a}}{a}\right)_0$$ the Hubble constant is the expansion rate measured now ($t = t_u$; $a = 1$)

Density parameter

$$\Omega_m = \frac{\rho_m}{\rho_c}$$

the critical density

$$\rho_c = \frac{3H^2}{8\pi G}$$

is the over/under between a universe the recollapses/expands forever (in the absence of dark energy)

If one can measure H_0 & Ω_{m_0} ($+ \Omega_{\Lambda_0}$ as it turns out)

then the expansion history $a(t)$ of the universe is specified by the Friedmann eqn

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = H_0^2\left[\Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_k a^{-2} + \Omega_\Lambda\right]$$

and one can work out the age - redshift relation $t(z)$

from $a(t)$ and $a = \frac{1}{1+z}$

note that $t = t_u \approx \frac{K}{H_0}$ now $K > 1$ if $q_0 < 1$

$K < 1$ if $q_0 > 1$

$$t \to 0 \text{ as } a \to 0 \text{ \& } z \to \infty$$

In the absence of a theory specifying $a(t)$,

can make Taylor expansion

$$a(t) \approx 1 + H_0(t - t_u) - \frac{1}{2} q_0 H_0^2 (t - t_u)^2 + \ldots$$

where $t - t_u$ is the lookback time

& q_0 is the deceleration parameter

$$q = -\frac{a\ddot{a}}{\dot{a}^2}$$

Five Classic Tests

$D_L - z$	standard candles
$D_A - z$	standard rods
$N(z)$	number counts with redshift
$N(m)$	number counts with magnitude
Tolman test	cosmological dimming

Surface brightness is distance-independent in Euclidean geometry, but suffers strong dimming cosmologically.

$$\Sigma \sim \frac{f}{\theta^2} \sim \frac{D_L^{-2}}{D_A^{-2}} \sim (1+z)^{-4} \qquad !$$

This Tolman test does not distinguish between cosmologies, it only ~~tests~~ that the geometry is non-Euclidean as expected.

The other four can distinguish between different q_0, always in the sense that cosmologies that decelerate a lot (high q_0) have expanded less than those which don't (low q_0). So distances and volumes are always bigger when q_0 is smaller.