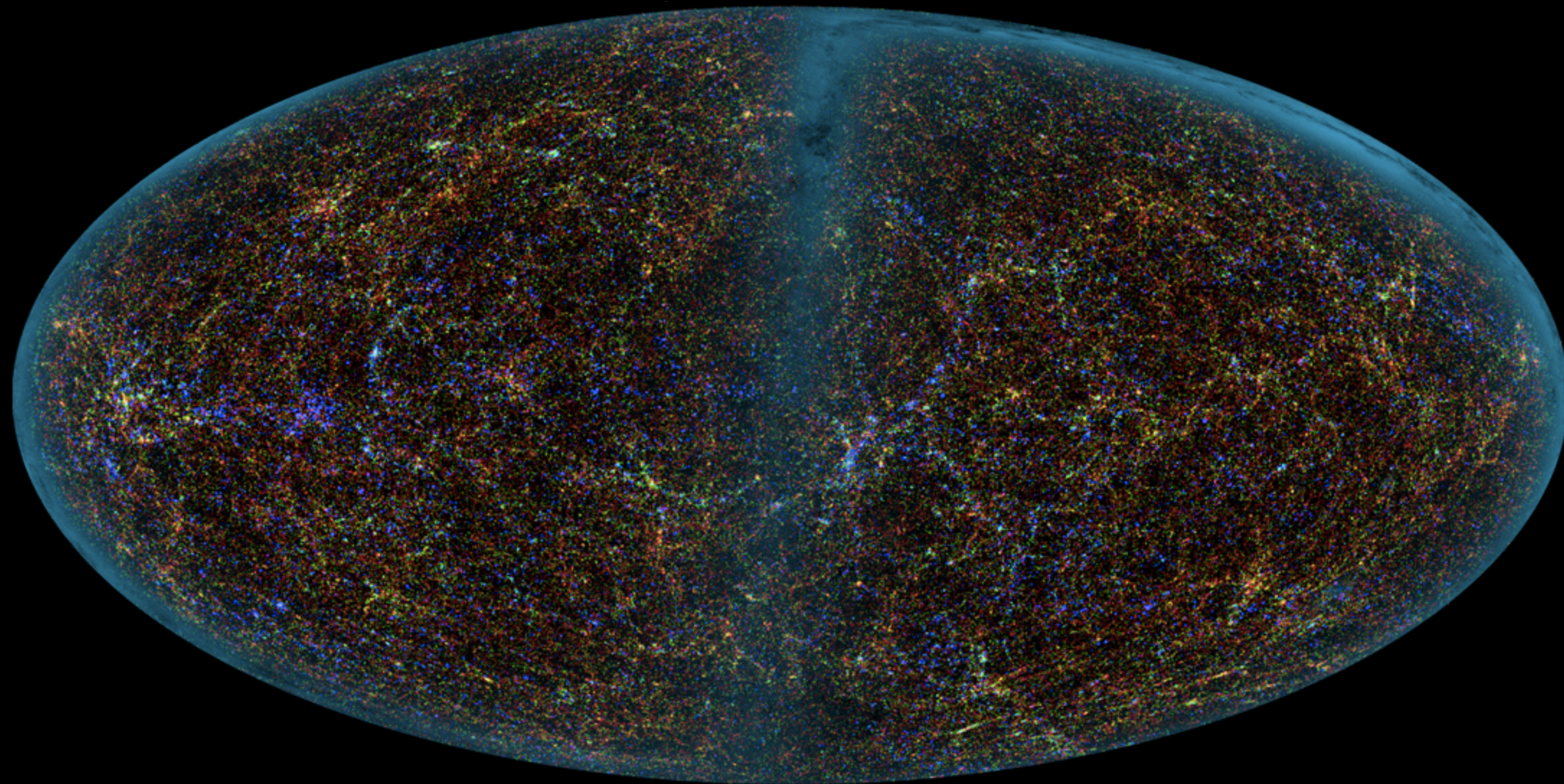


# Cosmology

## and Large Scale Structure



Today  
k-corrections

Distance Scale



# K-corrections

A correction to the magnitude of an object to account for the redshifting of its spectrum  $f(\lambda)$  through filter  $S_i(\lambda)$

$$K(z, T) = -2.5 \log \left[ (1+z) \frac{\int_{\lambda_1}^{\lambda_2} S_i(\lambda) f(\lambda) d\lambda}{\int_{\lambda_1}^{\lambda_2} S_i(\lambda) f[\lambda(1+z)] d\lambda} \right]$$

↗ spectral stretching     
 ↑ filter transmission window     
 ↖ change of spectrum through filter window

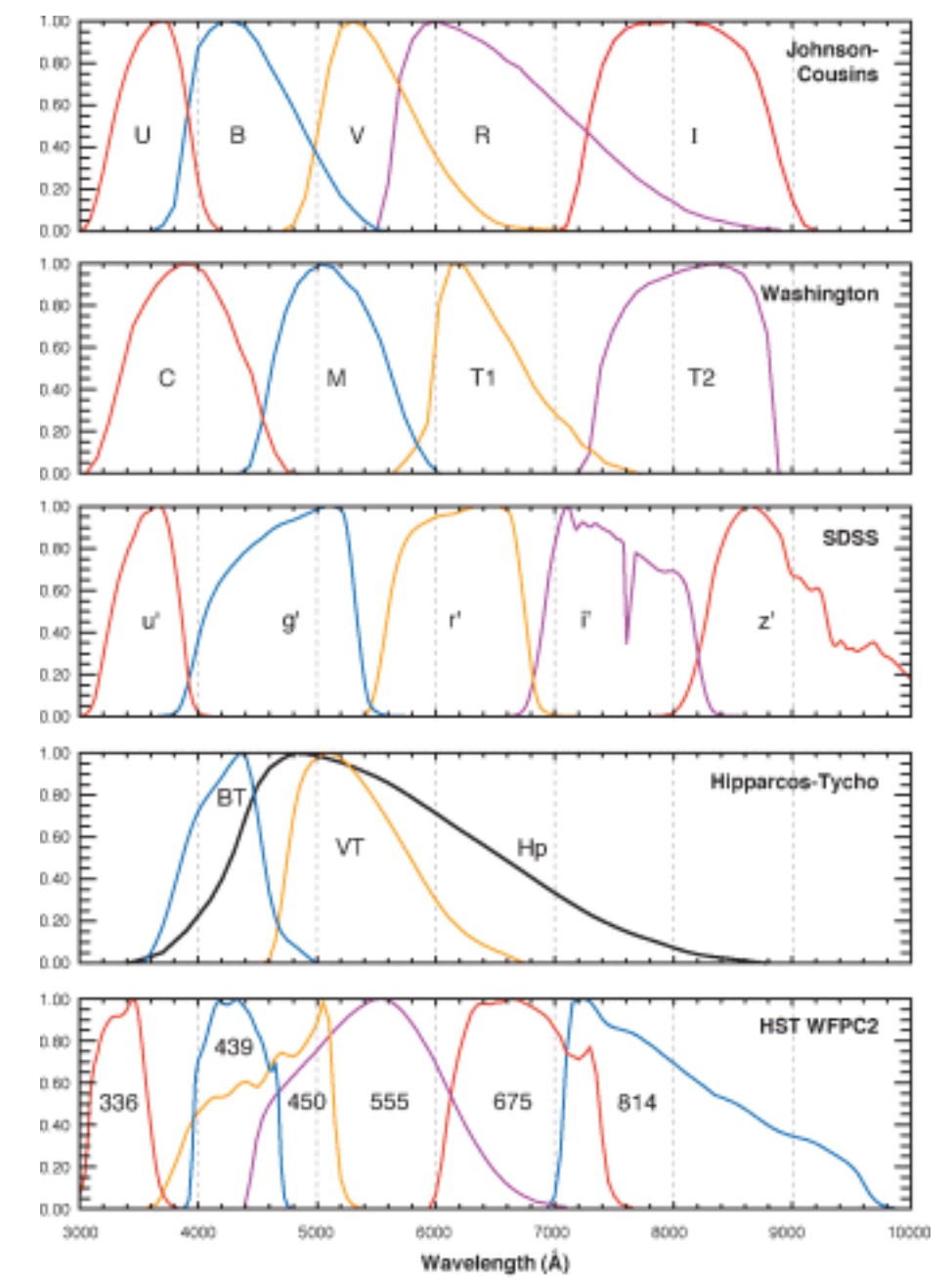
distance modulus becomes

$$m_i - M_i = 5 \log \left( \frac{D_L}{\text{Mpc}} \right) + 25 + A_i + K_i$$

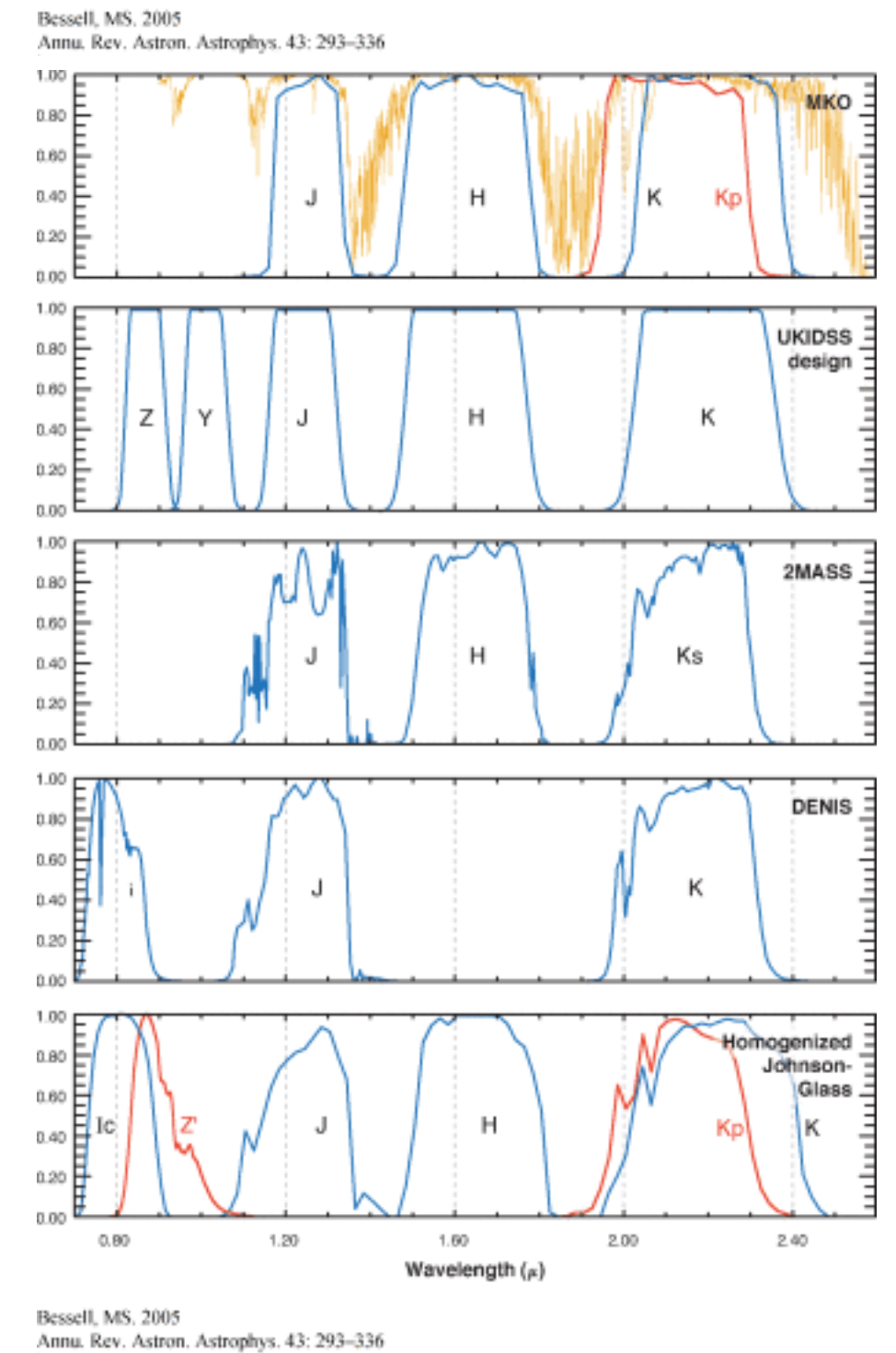
↑ specific to filter  $i$      
 ↑ extinction     
 ↖ K-correction

filter transmissions for...

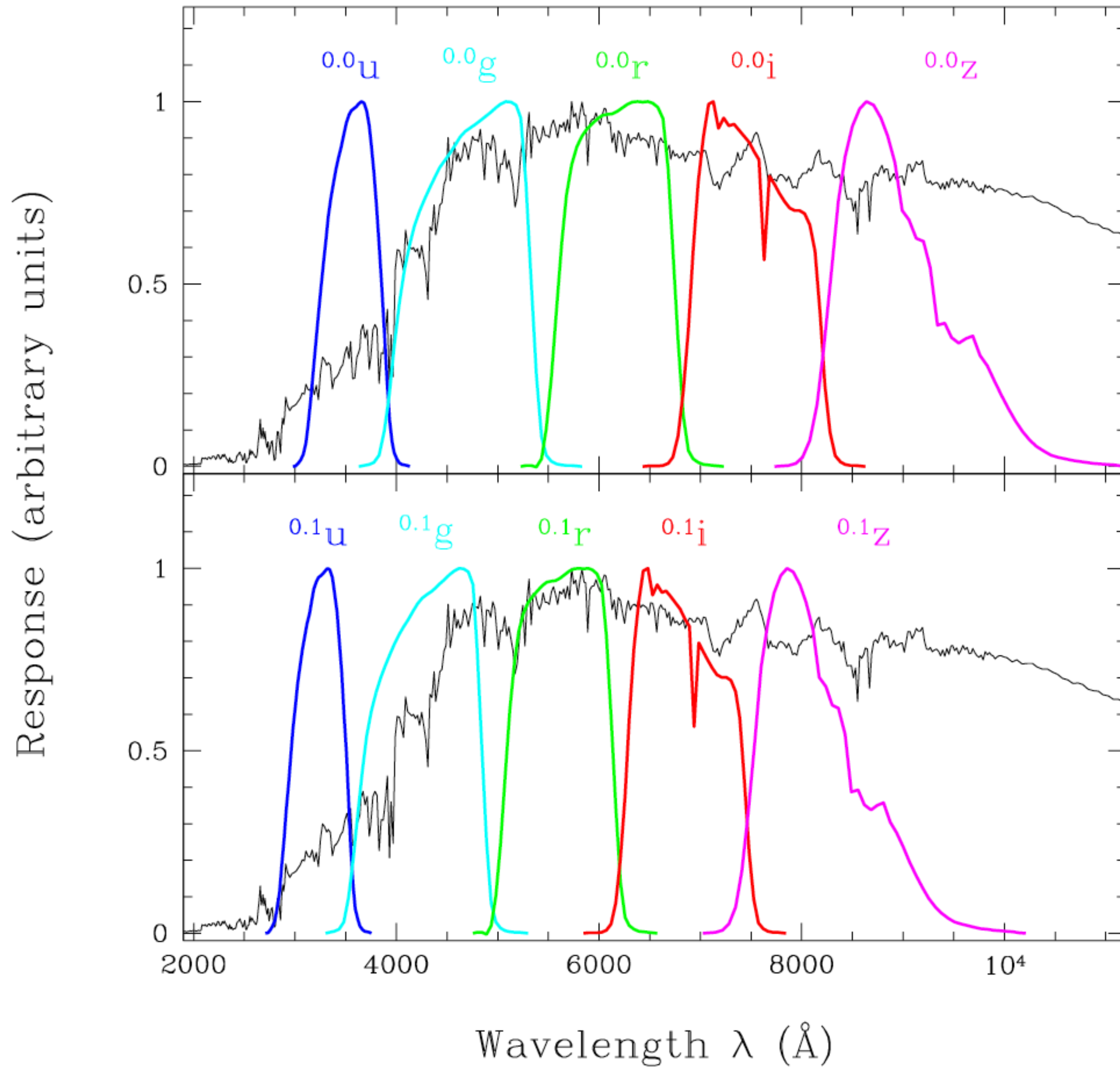
common optical filters



common near-infrared filters



# K-corrections



SDSS filters at  $z=0$

Note how the galaxy spectrum shifts through the fixed-bandwidth filters

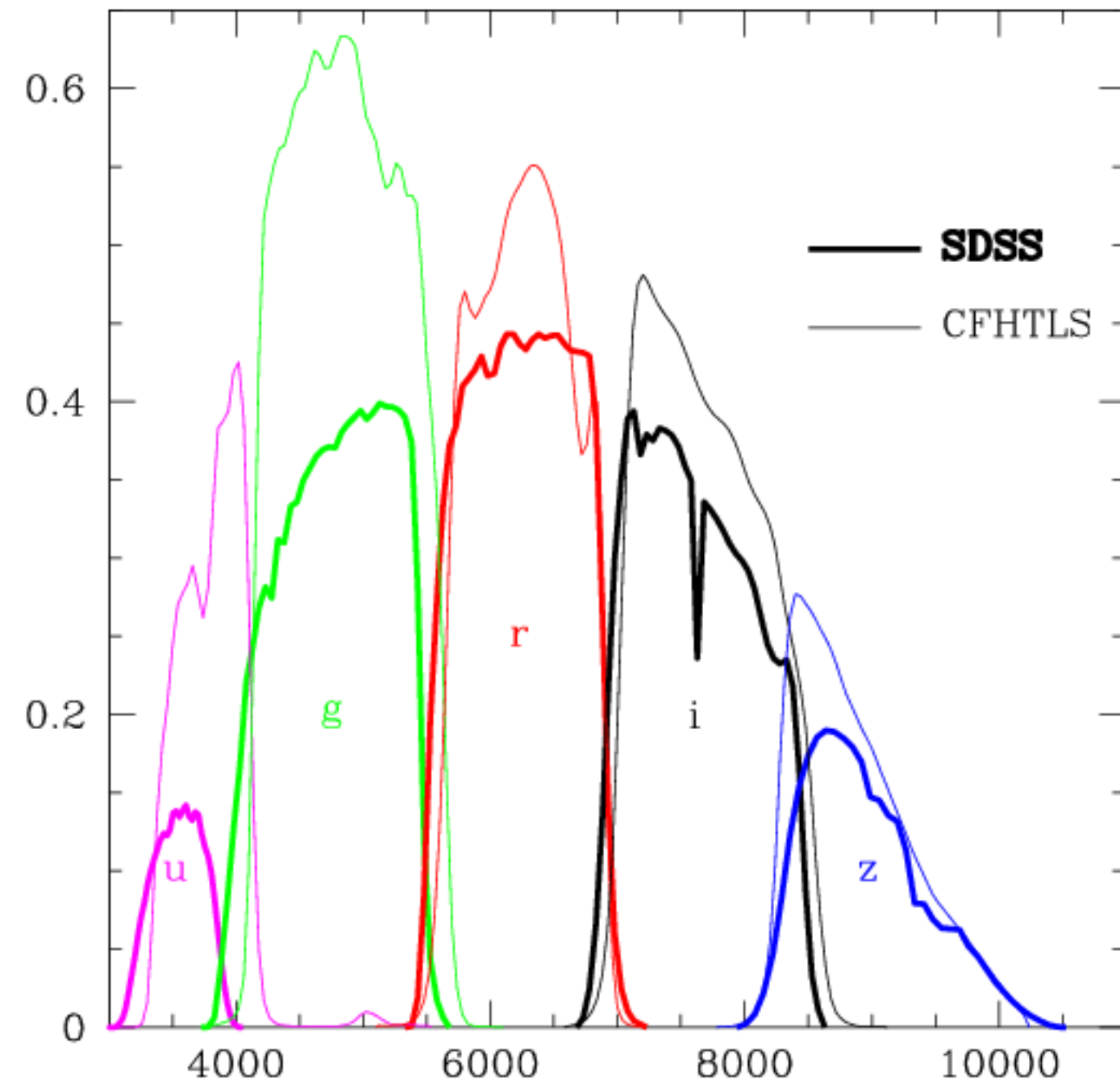
SDSS filters at  $z=0.1$

Blanton et al. (2002)

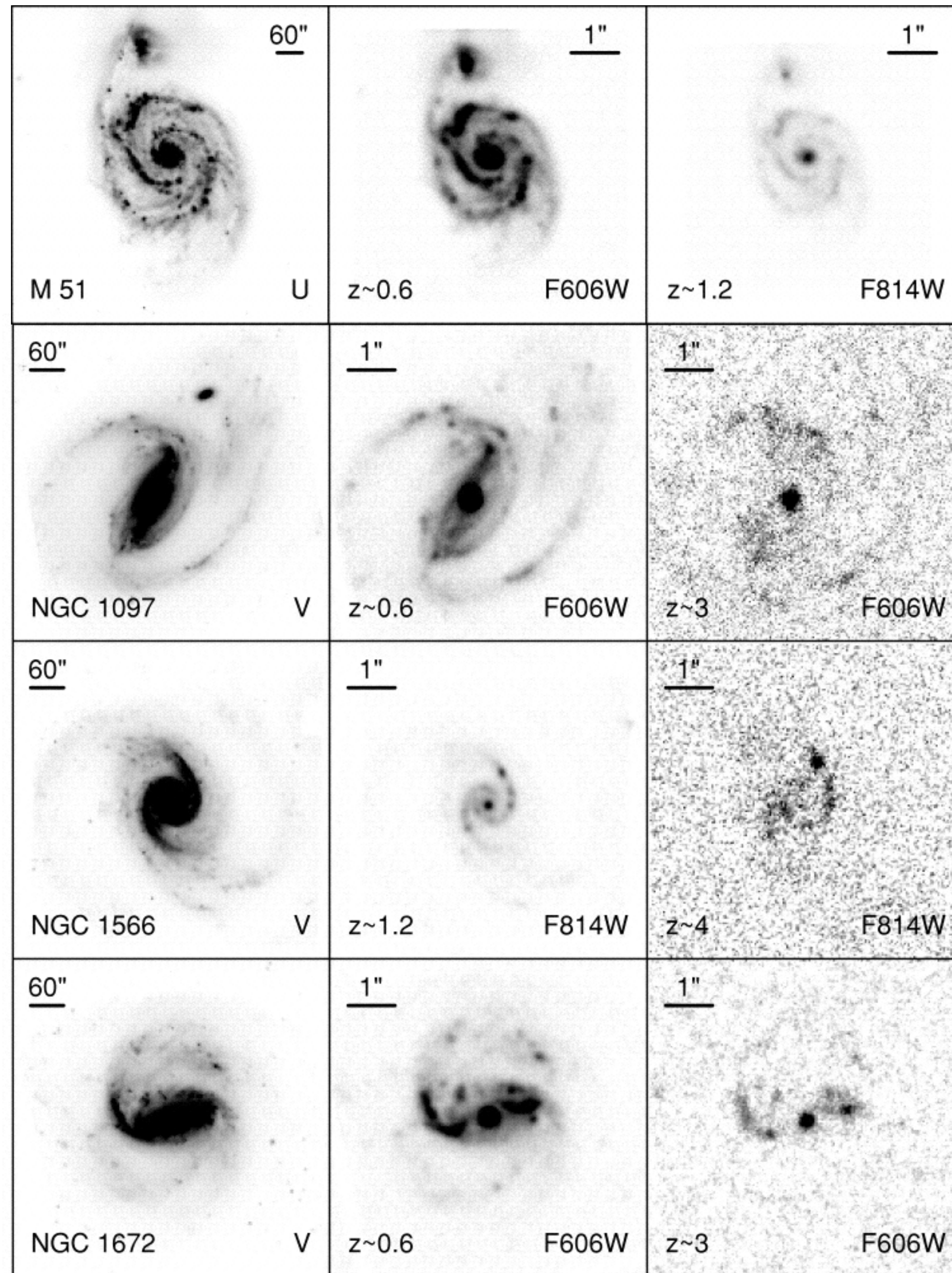
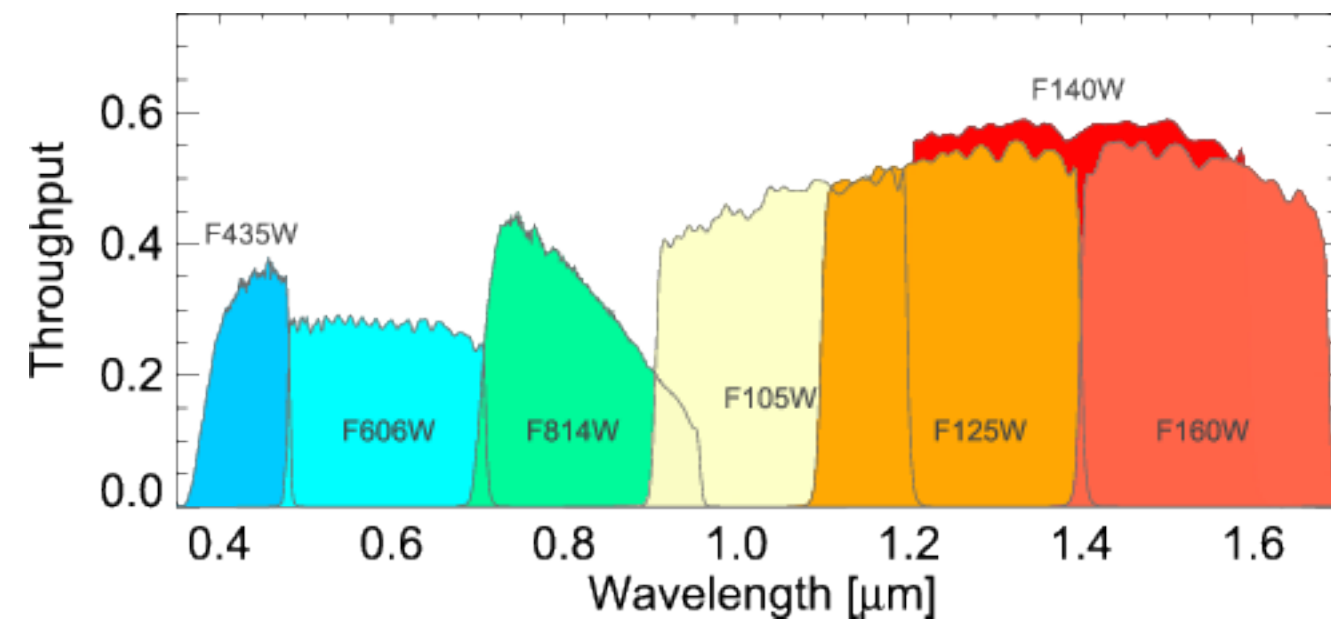
Fig. 2.— Demonstration of the differences between the unshifted SDSS filter system (0.0u , 0.0g , 0.0r , 0.0i , 0.0z ) in the top panel and the SDSS filter system shifted by 0.1 (0.1u , 0.1g , 0.1r , 0.1i , 0.1z ) in the bottom panel. Shown for comparison is a 4 Gyr-old instantaneous burst population from an update of the Bruzual A. & Charlot (1993) stellar population synthesis models. The K -corrections between the magnitudes of a galaxy in the unshifted SDSS system observed at redshift  $z = 0.1$  and the magnitudes of that galaxy in the 0.1-shifted SDSS system observed at redshift  $z = 0$  are independent of the galaxy's spectral energy distribution (and for AB magnitudes are equal to  $-2.5 \log_{10}(1 + 0.1)$  for all bands; Blanton et al. 2002a). This independence on spectral type makes the 0.1-shifted system a more appropriate system in which to express SDSS results, for which the median redshift is near redshift  $z = 0.1$ .



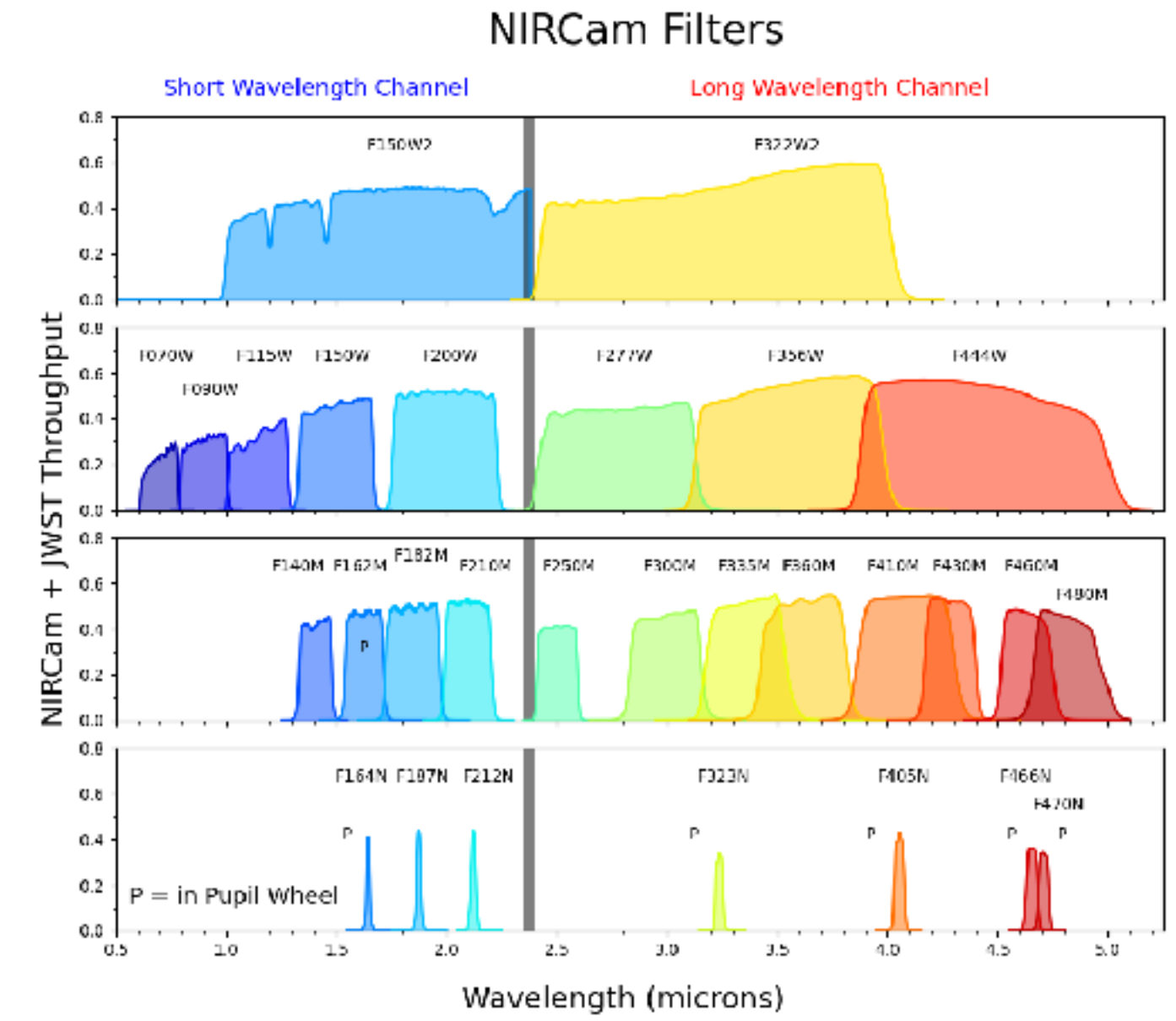
### SDSS ugriz filters



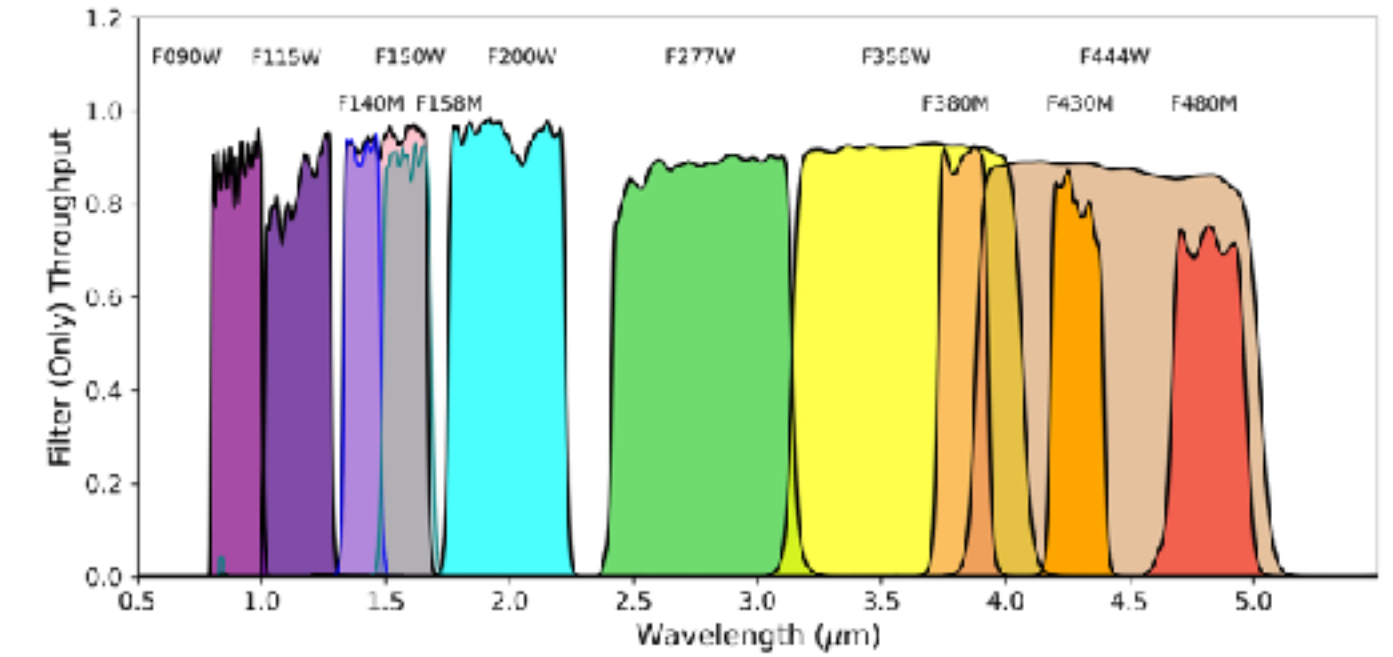
### HST/ACS and WFC3 filters



### JWST filters



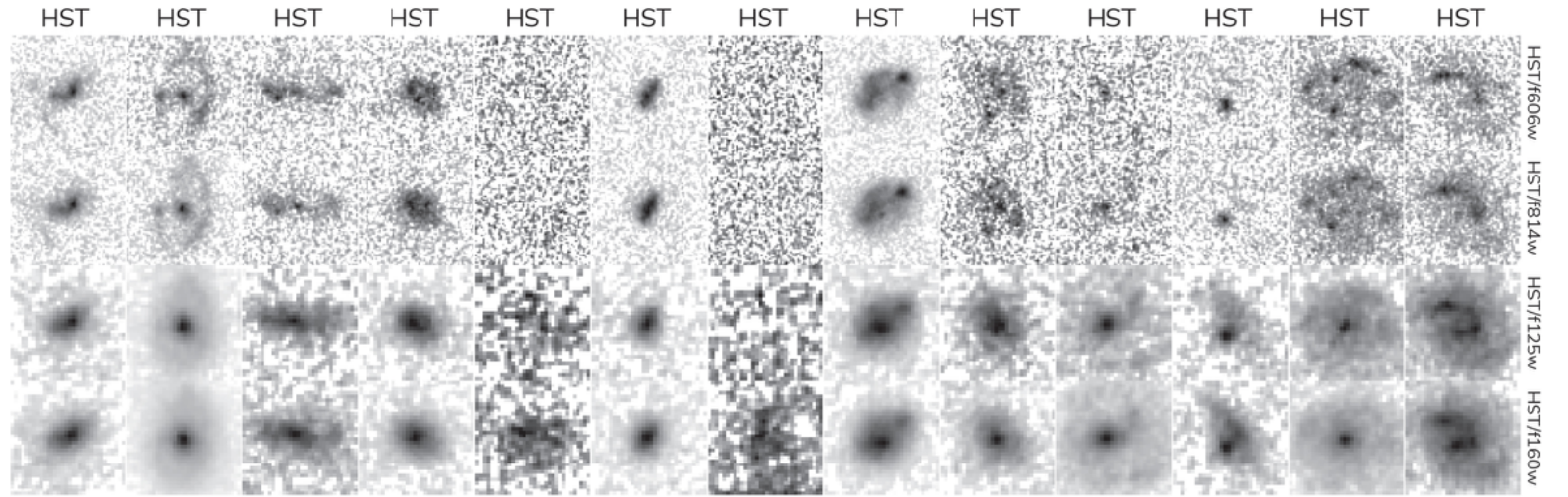
### NIRISS filters



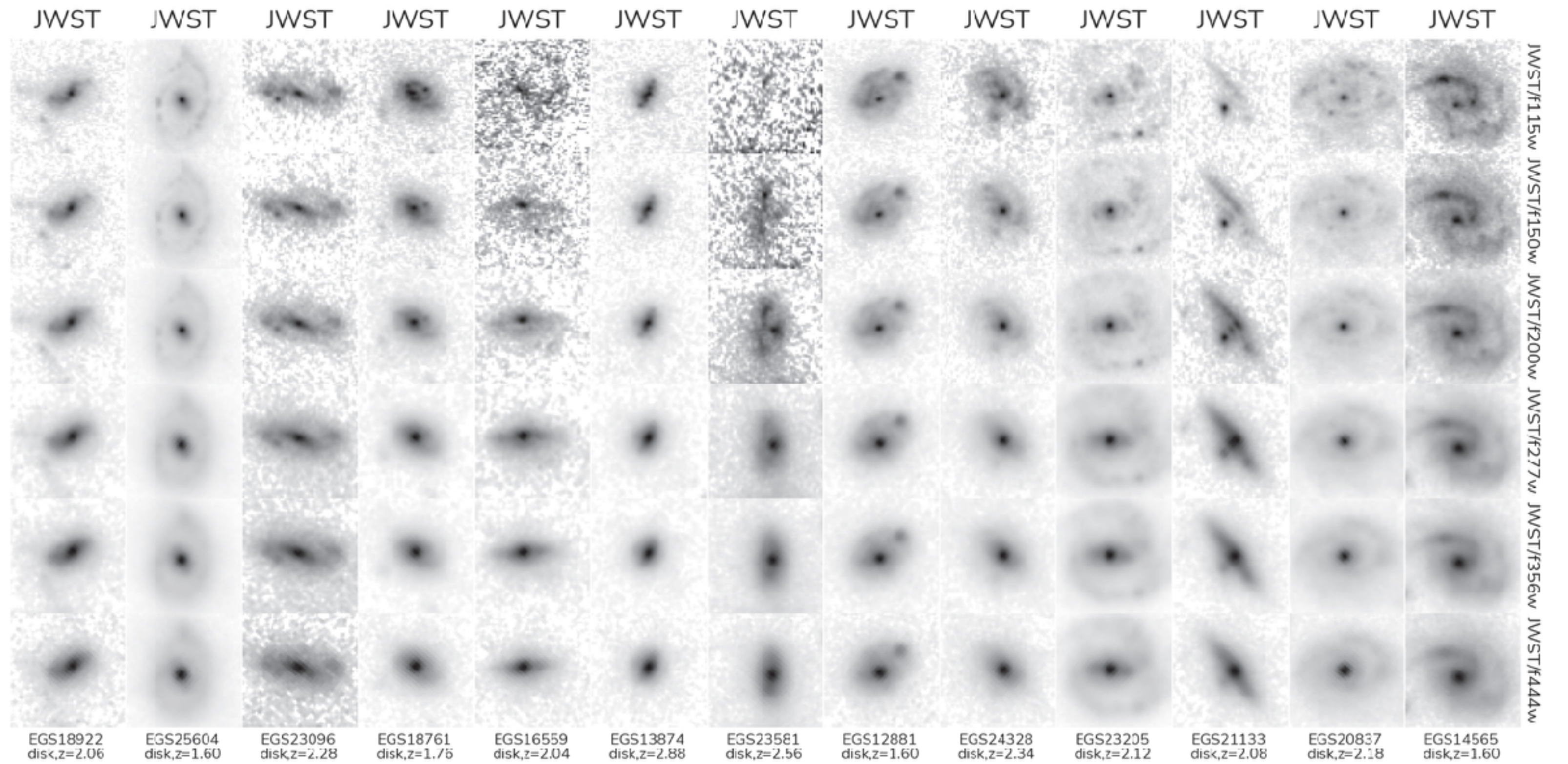


The JWST Hubble Sequence: The Rest-frame Optical Evolution of Galaxy Structure at  $1.5 < z < 6.5$

[Ferreira et al 2023, ApJ, 955, 94](#)



Lots of spiral galaxies out to  $z \approx 6$  that had previously thought to have been peculiar or “chain” galaxies





# Distance Scale

## Why do we need to get this right?

### Astrophysics:

- turn observed properties of objects (apparent magnitude, angular size) into intrinsic properties of objects (luminosity, physical size)

### Measure $H_0$ :

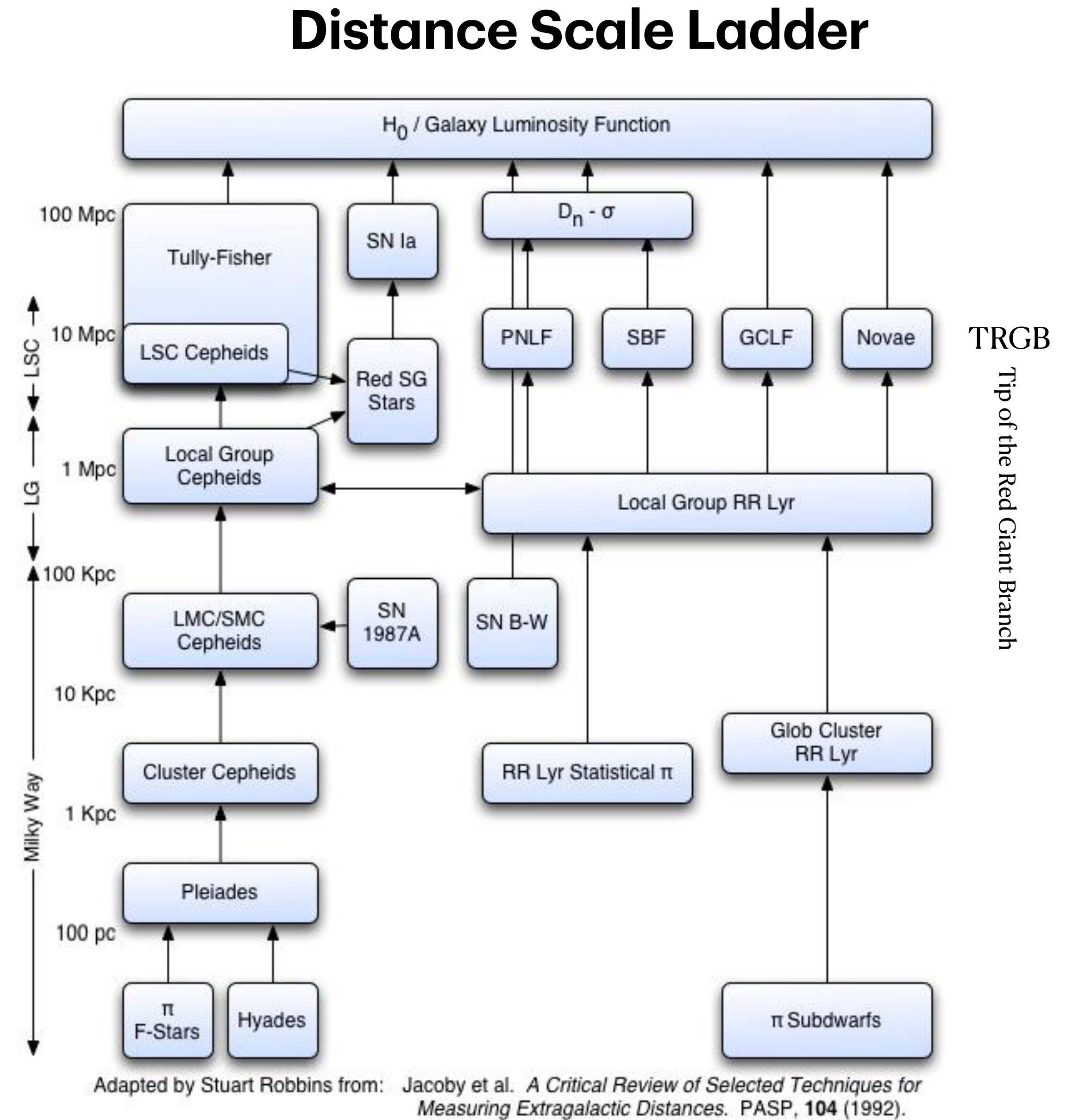
- Cosmological parameter, want local, independent confirmation of cosmological measurements at high redshift.
- Once measured, can use it as a distance indicator (Hubble distance:  $d=v/H_0$ )

### Measure peculiar motions in the universe:

- $v_{\text{obs}} = H_0 d + v_{\text{pec}}$
- if we know distance *independent* of redshift, we can look for large scale velocity structure in the universe, test the assumption of isotropy, and measure  $\Omega_m$ .

## Important Complications:

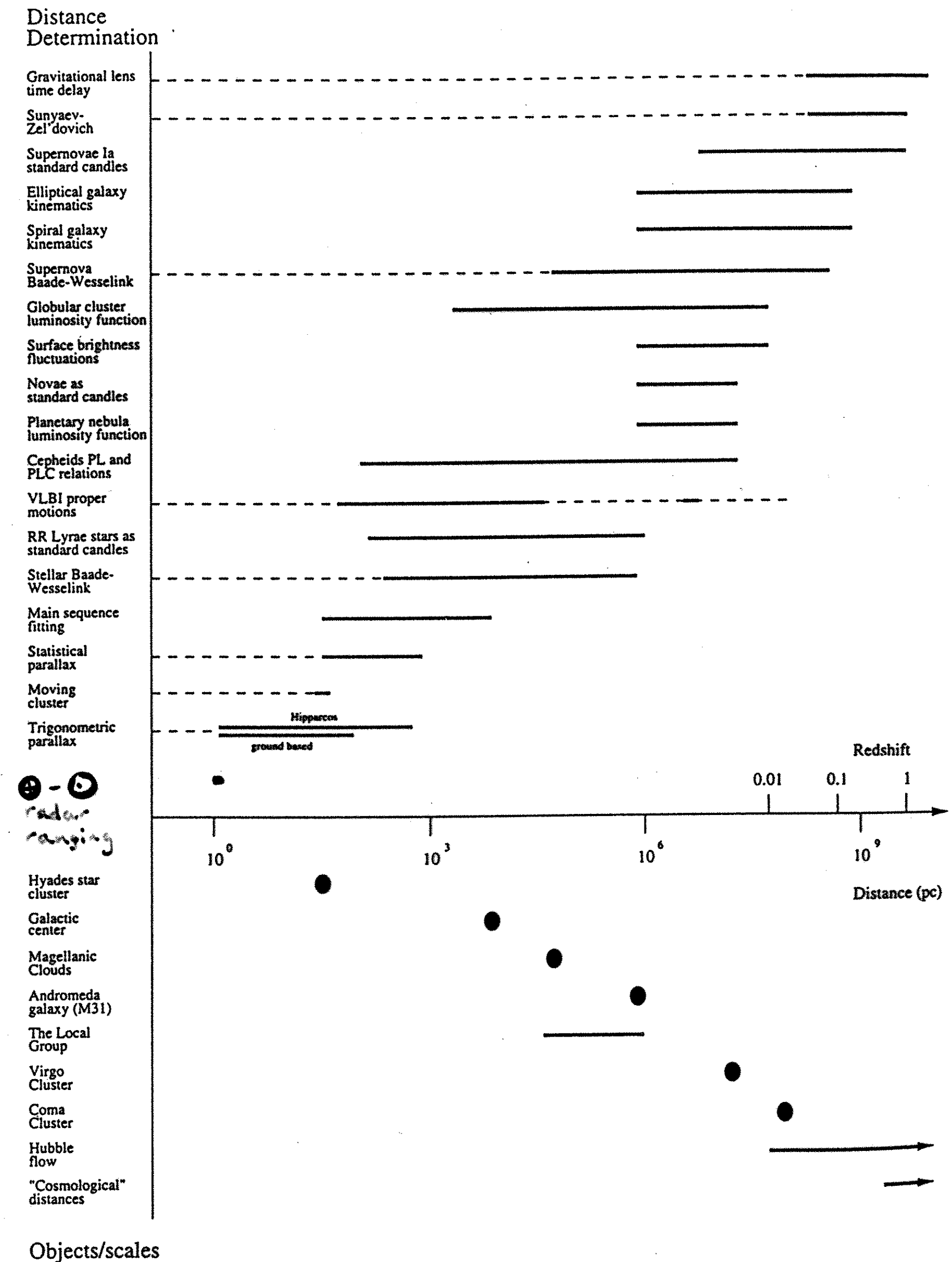
- An accurate measure of  $H_0$  means getting out to a distance where  $v_{\text{pec}} \ll H_0 d$ .
- Local galaxies do *not* have useful Hubble distances, due to [peculiar motions](#) and [Virgo-centric flow](#).
- Distances *within* clusters (ie with accuracies of +/- few Mpc) are *not knowable* via Hubble's law.
- Need *several* distance estimators to reduce systematic errors between methods.



# Distance Scale

- Solar System
  - earth-sun distance
- Trigonometric Parallax
  - statistical & secular parallax; moving clusters
- Main Sequence Fitting
- Bright Star Standard Candles
  - Cepheids, RR Lyraes, TRGB
- Secondary Distance Indicators
  - Type Ia SN, Tully-Fisher, Fundamental Plane, SB Fluctuations
- Absolute Methods
  - Gravitational lens time delay, SZ effect, water masers

## Distance Scale Ladder



distance modulus  $m - M = 5 \log(d) - 5$

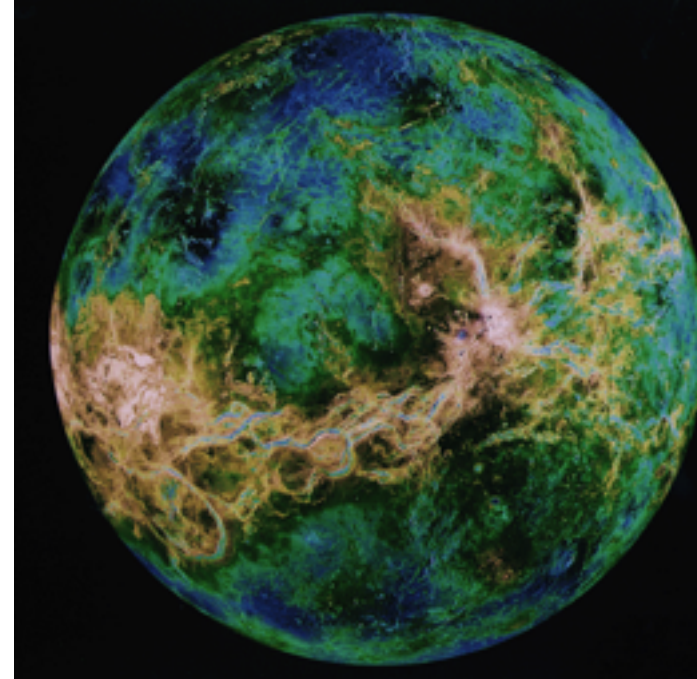
# Distance Scale

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- Absolute Methods
  - Gravitational lens time delay, SZ effect, water masers
- Trigonometric methods absolute
  - same as land surveys - use Pythagoras!
- Secondary Distance Indicators
  - Generally relate a distance dependent quantity (luminosity or size) to a distance independent quantity that is correlated with it.
  - e.g., Cepheid P-L relation: the period  $P$  is used as an indicator of the luminosity  $L$
- Absolute Methods
  - make use of physics that is distance-independent
  - e.g., the speed of light is constant, but light must traverse a different path for each image in a gravitational lens, so measuring the time delay between images constrains the distance through  $c\Delta t$ .



# Distance Scale

1 AU = 149597870.7 km (IAU definition, 2012)



Radar map of Venus

Prior to the Magellan mission, the biggest uncertainty in the AU was the thickness of Venus's atmosphere: one measured the distance via radar reflection off the ground, but the angular size from the reflection of clouds high up in the atmosphere.

- Solar System

- earth-sun distance
- measure
  - sun-venus angular separation  $\theta$  at maximum elongation (45 - 47°; varies due to eccentricity)
    - known with great accuracy via orbital periods
  - earth-venus distance  $d_{EV}$ 
    - measure via radar reflection
- solve for earth-sun distance (1 AU)
- Historically, use period ratio to relate  $\sin \theta$  to observed  $\theta$ .
  - Gauss's gravitational constant extremely well measured

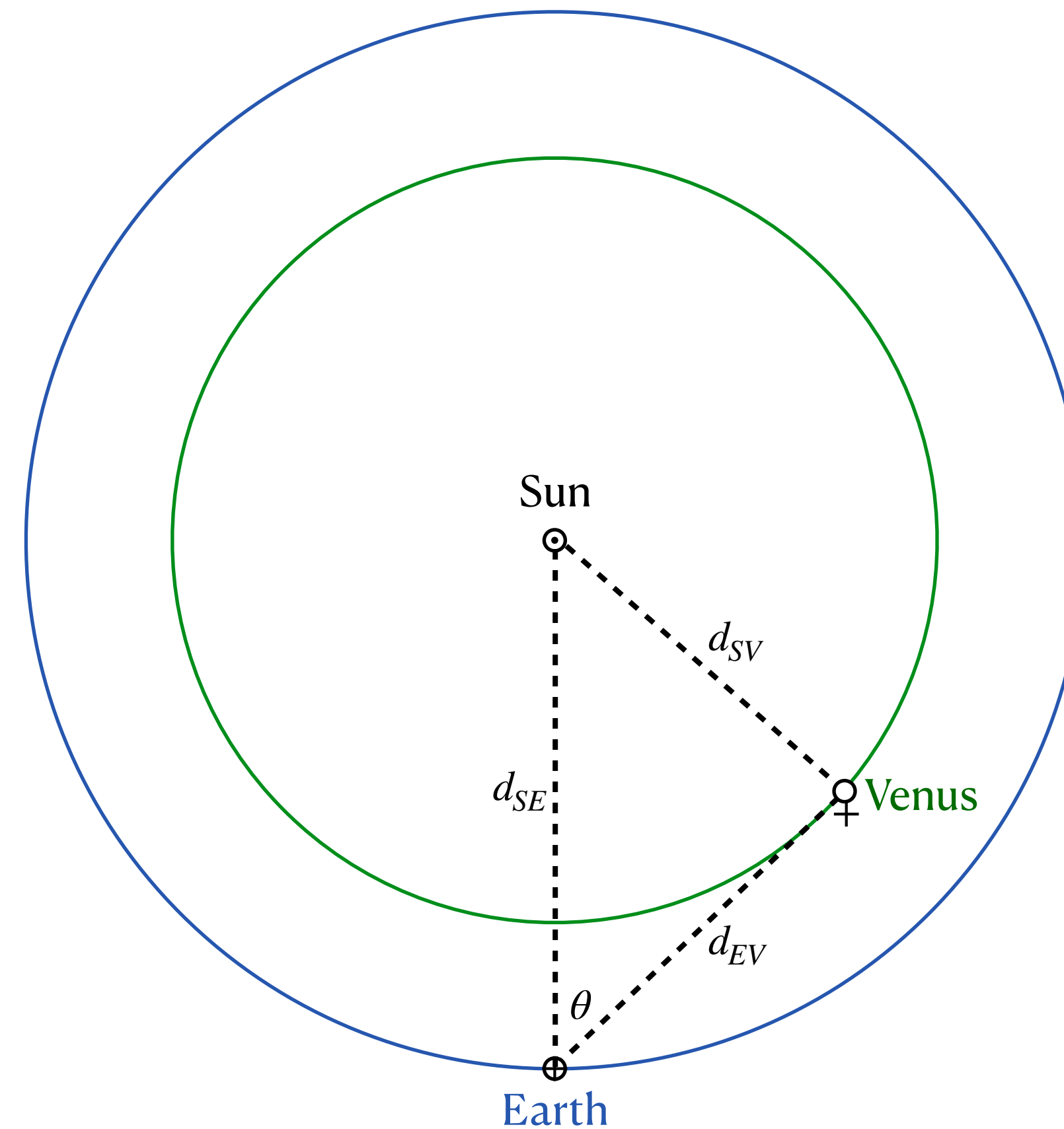
- $k = \frac{2\pi}{P(aM)^{1/2}} = 0.01720209895 \text{ rad/day}$

- in modern parsing,

- $GM_{\odot} = 1.32712440018(9) \times 10^{20} \text{ m}^3 \text{ s}^{-2}$

Experimental measurements of  $G$  alone are considerably less accurate:

$$G = 6.67430(15) \times 10^{-11} \text{ kg}^{-1} \text{ m}^3 \text{ s}^{-2}$$



$$\cos \theta = \frac{d_{EV}}{d_{SE}} \qquad \sin \theta = \frac{d_{SV}}{d_{SE}} = \left( \frac{P_V}{P_E} \right)^{2/3}$$

Thanks, Kepler!



# Distance Scale

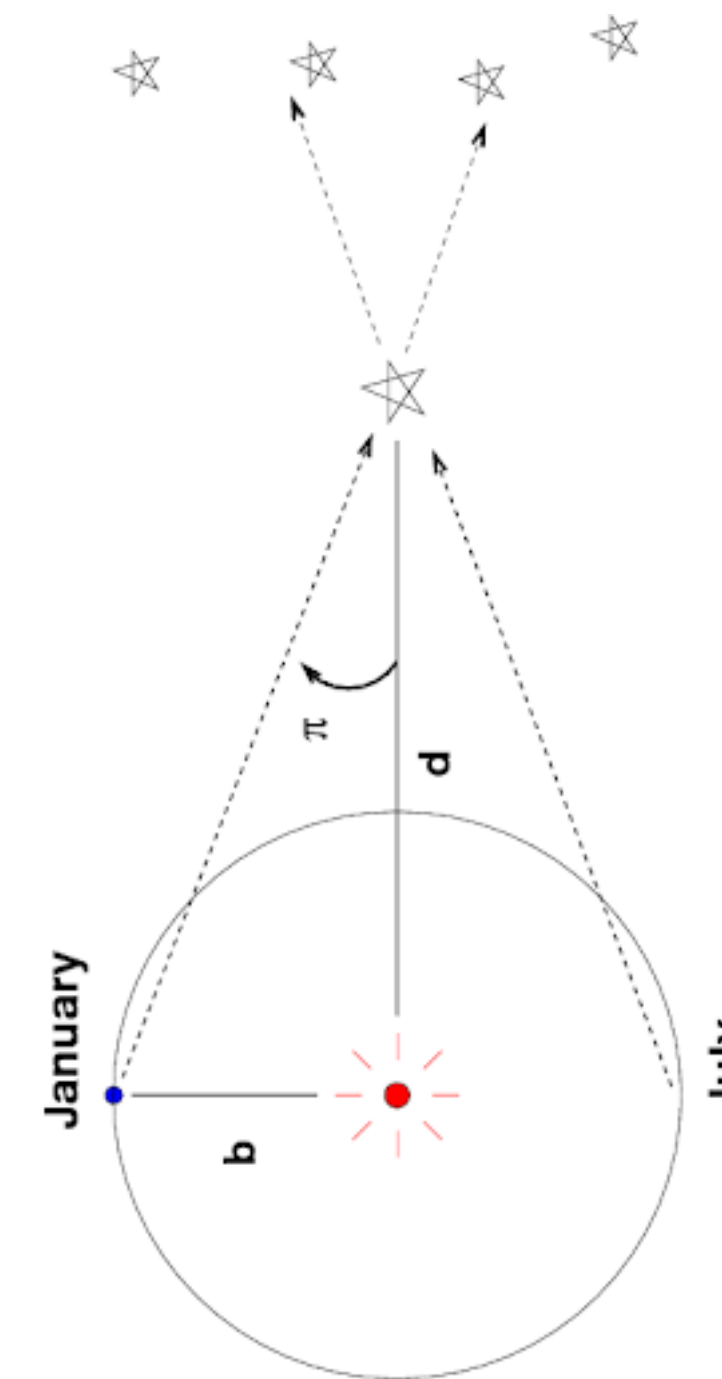
- Trigonometric Parallax
  - use Earth's orbit as baseline
  - measure angular shift in position of a star relative to background stars

$$d_* = \frac{1}{\pi}$$

$d$  in pc for  $\pi$  in arcseconds  
(1 pc is defined by a parallax angle of 1")

206,265 arcseconds in one radian, so  
206,265 AU in one pc

$$1 \text{ pc} = 3.086 \times 10^{13} \text{ km}$$



$$\pi \approx \tan \pi = \frac{b}{d_*}$$

$$b = d_{SE} = 1 \text{ AU}$$

small angle approximation excellent here



# Distance Scale

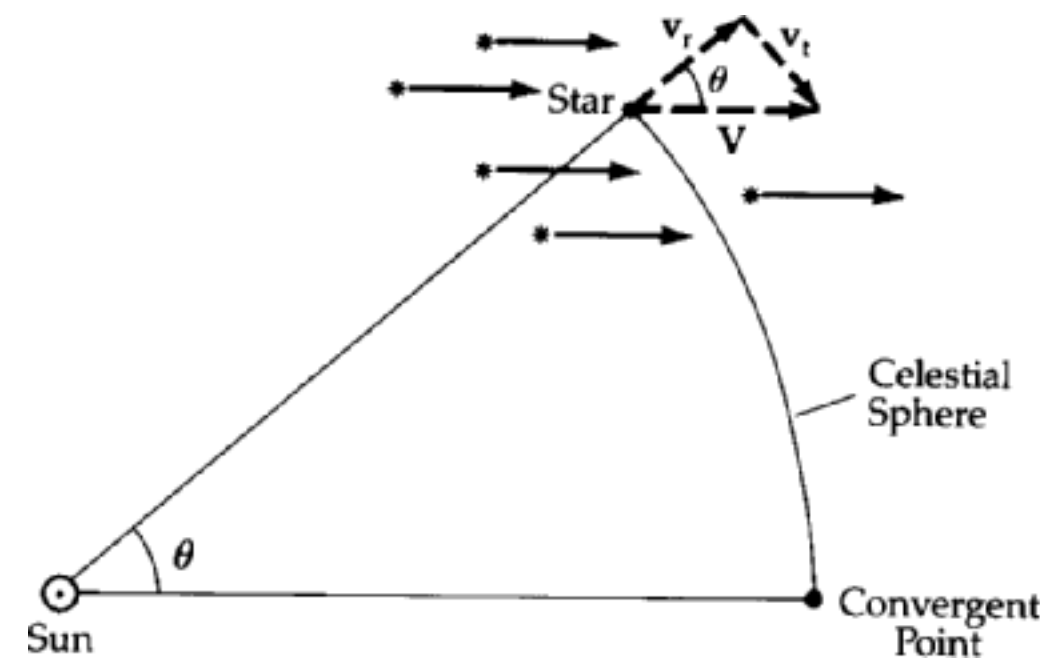
- Moving Clusters
  - convergent point method

$$1 \text{ AU/yr} = 4.74 \text{ km/s}$$

$$V_{\tau} = 4.74 \frac{\mu}{\pi}$$

$$V = \sqrt{V_r^2 + V_{\tau}^2}$$

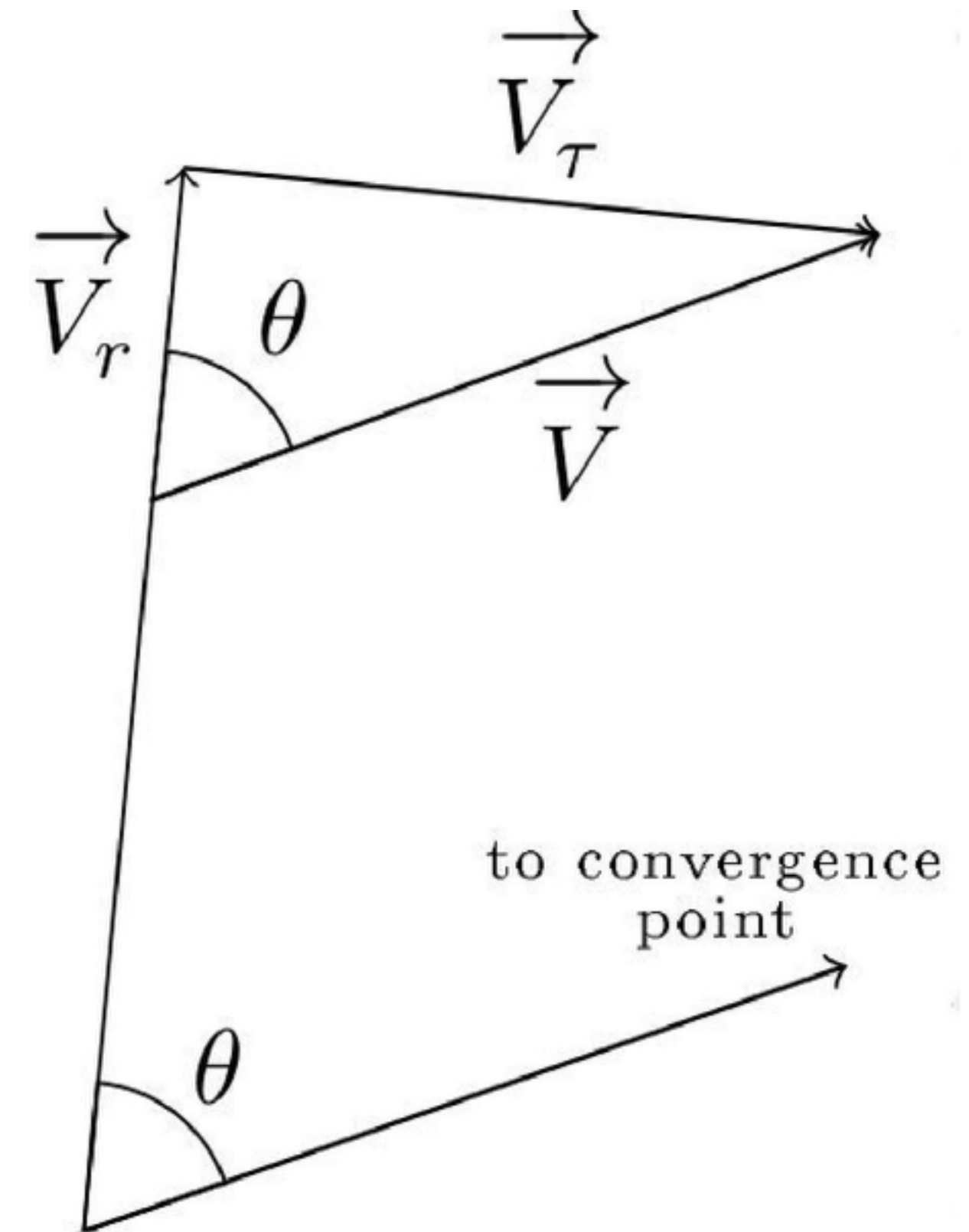
$$\frac{1}{d} = \pi = \frac{4.74\mu}{V \tan \theta}$$



$\mu$  is the proper motion (arcsec/yr)  
 $\pi$  is the parallax (arseconds)

$$V_r = V \cos \theta$$

$$V_{\tau} = V \sin \theta = 4.74 \frac{\mu}{\pi}$$



Works on clusters of stars where it is possible to perceive their joint motion on the sky

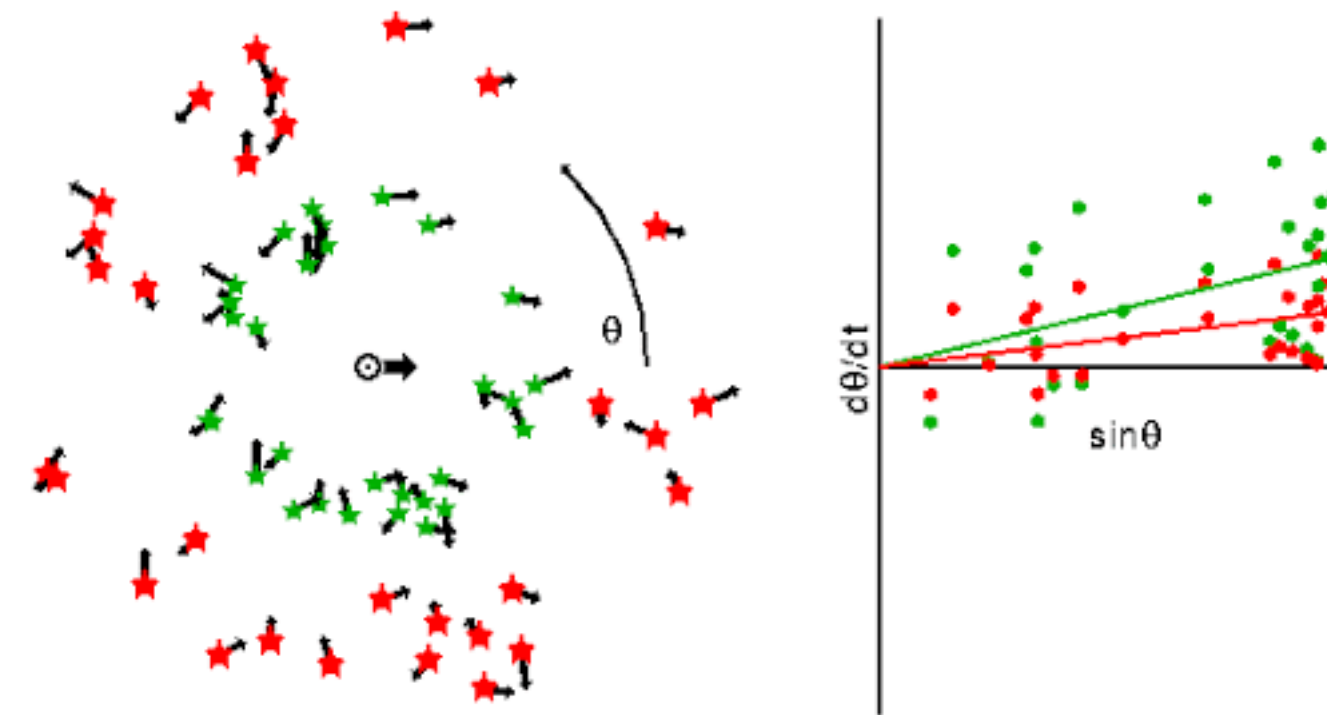


# Distance Scale

- Secular Parallax
  - The Sun moves wrt the Local Standard of Rest
  - Motion of the sun provides a baseline

$$d = \frac{V_{\odot}}{m} = \frac{4.16}{m}$$

where the odd constant 4.16 is the Solar motion in au/yr.



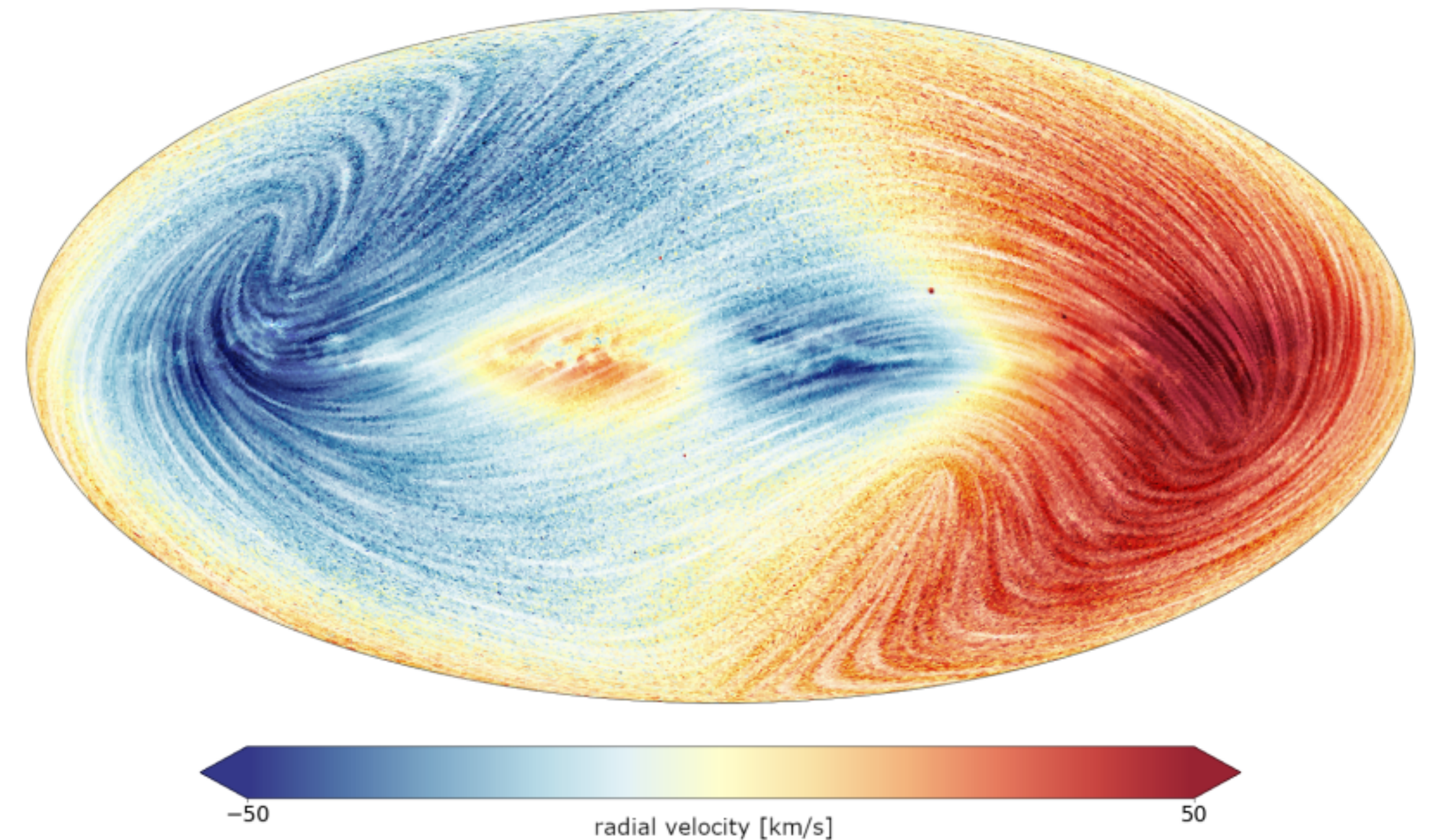
The diagram above shows two sets of stars, with two mean distances. The green stars show a small mean distance, while the red stars show a large mean distance. Because of the Solar motion (20 km/sec relative to the average of nearby stars) there will be an average proper motion away from the point of the sky the Solar System is moving towards. This point is known as the *apex*. Let the angle to the apex be  $\theta$ . Then the proper motion  $\mu = d\theta/dt$  will have a mean component proportional to  $\sin\theta$ , shown by the lines in the plot of  $d\theta/dt$  vs  $\sin\theta$ . The slope of this line is  $m$ .



# Distance Scale

- Statistical Parallax
  - Stars move.
  - Can determine mean baseline for a specified stellar type.
  - Assuming motion is random, so proper motion and radial motion are on average the same,

$$d = \frac{\langle V_r \rangle}{\langle \mu \rangle} = \frac{\text{scatter in radial velocities}}{\text{scatter in proper motions}}$$



This sky map shows the velocity field of the Milky Way for ~26 million stars. The colours show the radial velocities of stars along the line-of-sight. Blue shows the parts of the sky where the average motion of stars is towards us and red shows the regions where the average motion is away from us. The lines visible in the figure trace out the motion of stars projected on the sky (proper motion).

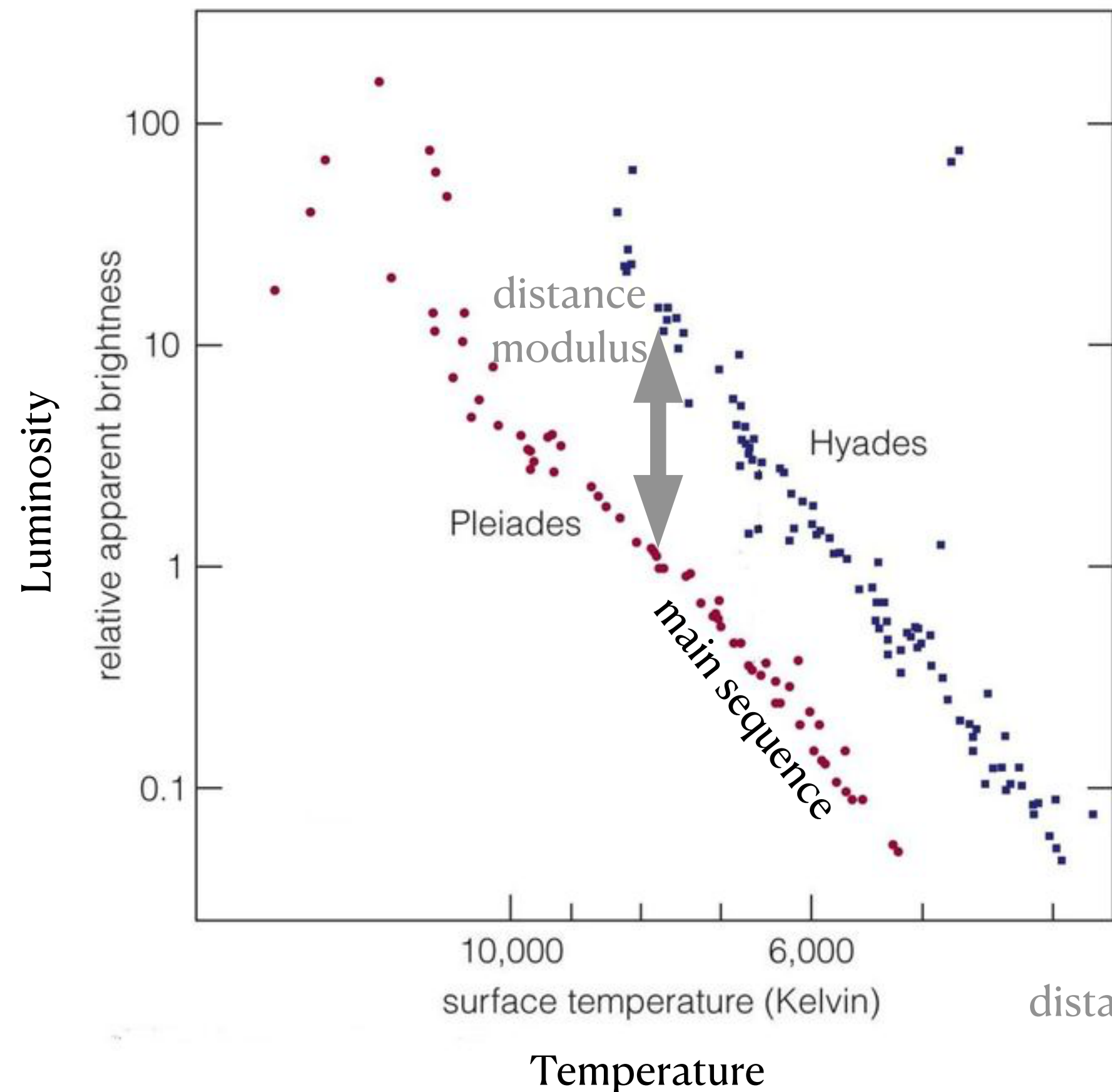


# Distance Scale

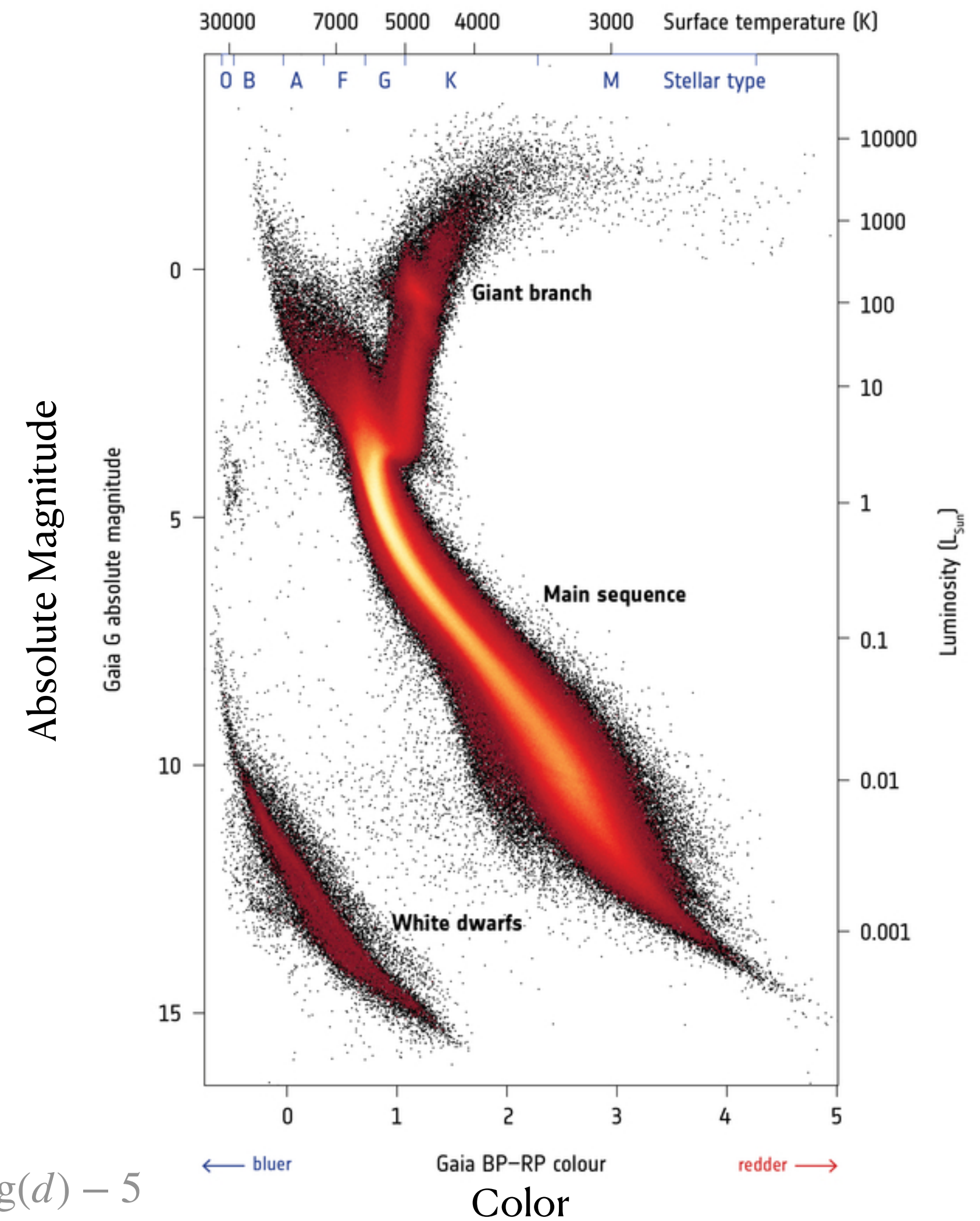
ESA's Gaia mission provides parallax distances for over 4 million stars within 1.5 kpc

→ **GAIA'S HERTZSPRUNG-RUSSELL DIAGRAM**  
aka HR diagram, color-magnitude diagram

- Main Sequence Fitting
  - absolute calibration by parallax
  - apply to more distant clusters



distance modulus  $m - M = 5 \log(d) - 5$



Most stars are main sequence, but other types are well represented (35,000 white dwarfs!)



# Distance Scale

- Bright Star Standard Candles
  - Cepheids, RR Lyraes
  - calibrate by
    - parallax
    - main sequence fitting of clusters containing these stars

Luminosity of variable stars correlate with oscillation period

$$L = 4\pi R^2 \sigma T_e^4$$

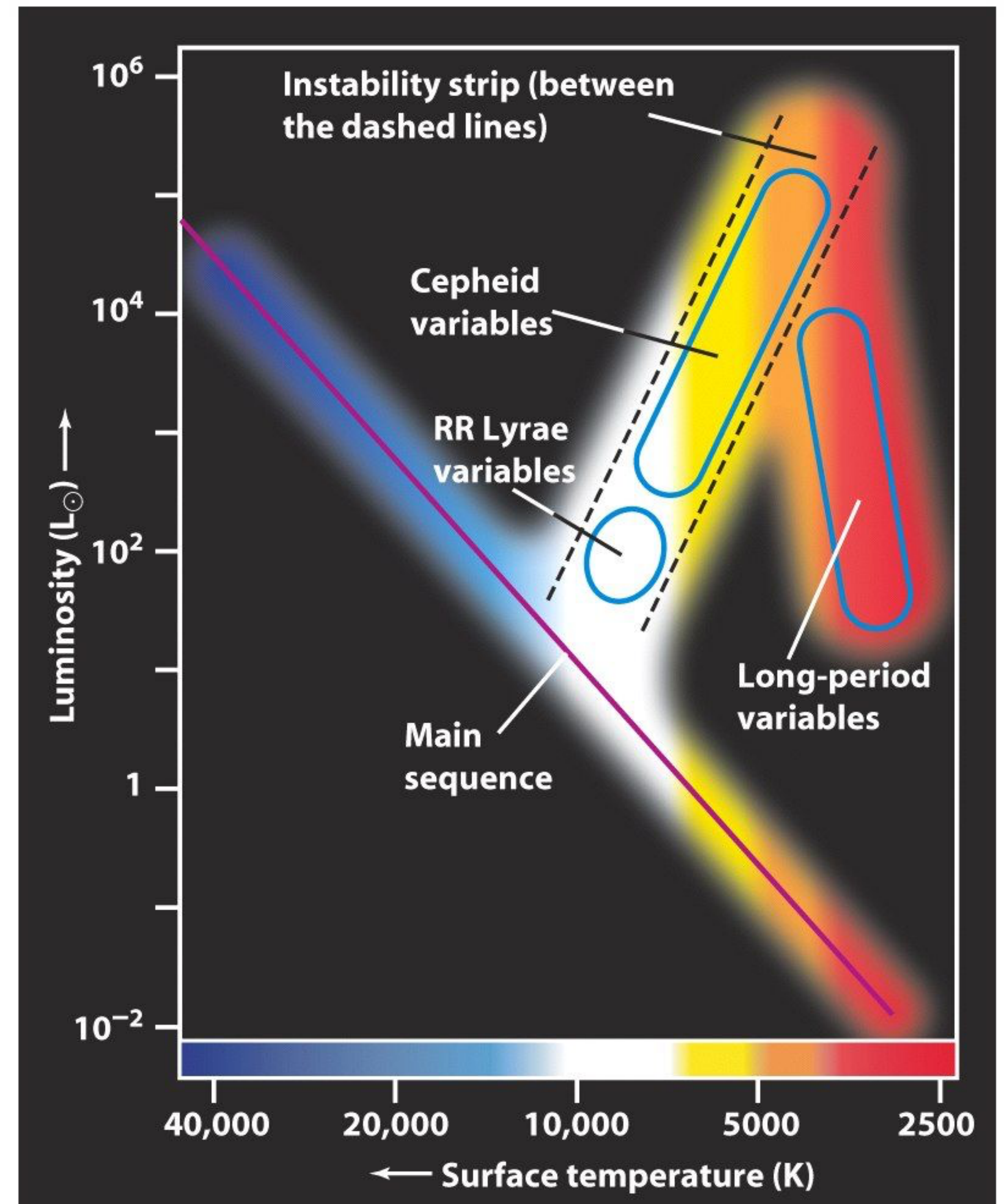
use luminosity and effective surface temperature to infer radius

Baade-Wesselink method

$$\int_{R_1}^{R_2} dR = -p \int_{t_1}^{t_2} V_{\text{los}} dt$$

$p \approx 1.4$  corrects line of sight velocity to radial velocity, accounting for limb darkening

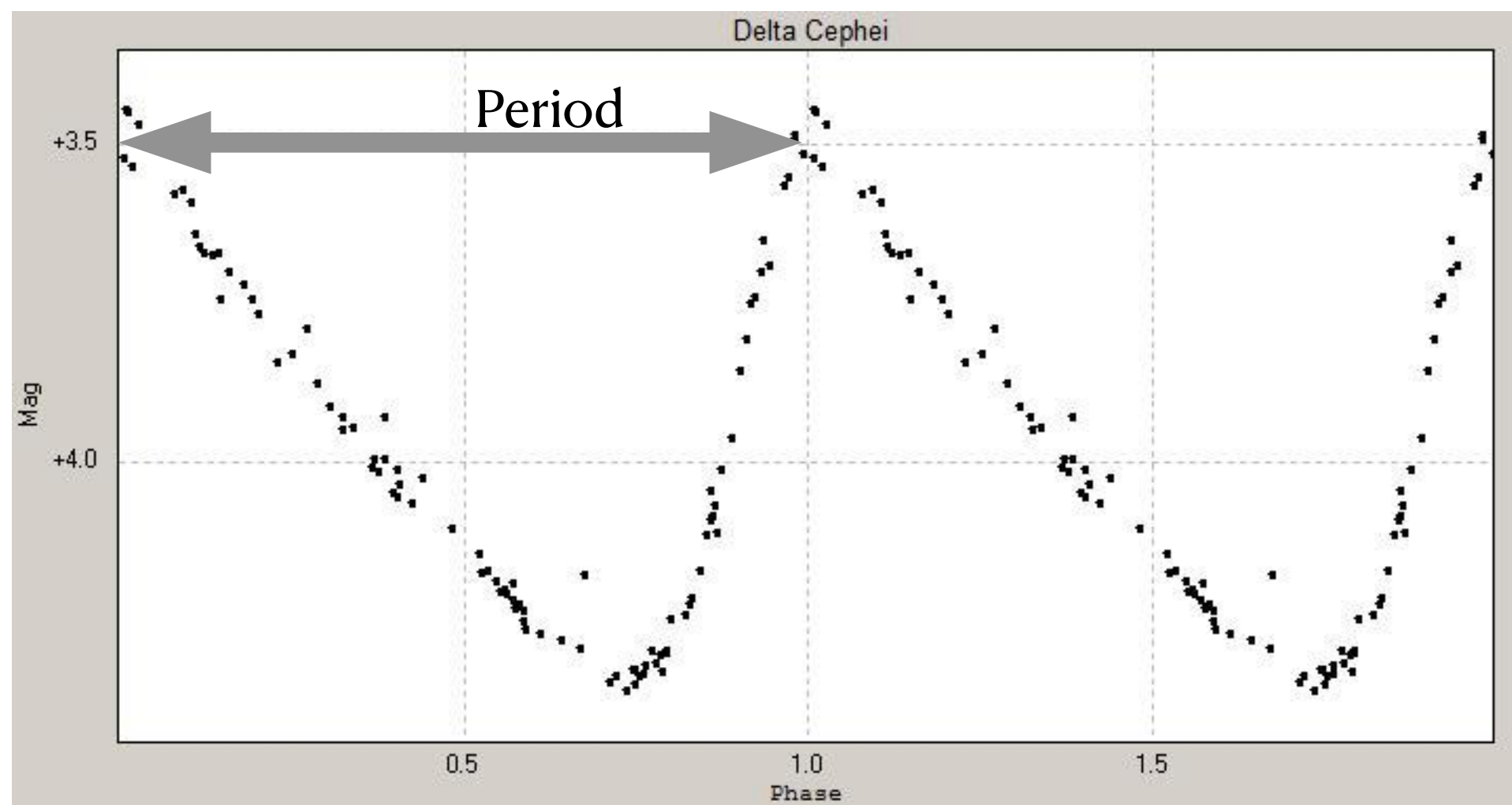
Instability strip in the HR diagram (not the same as the giant branch)



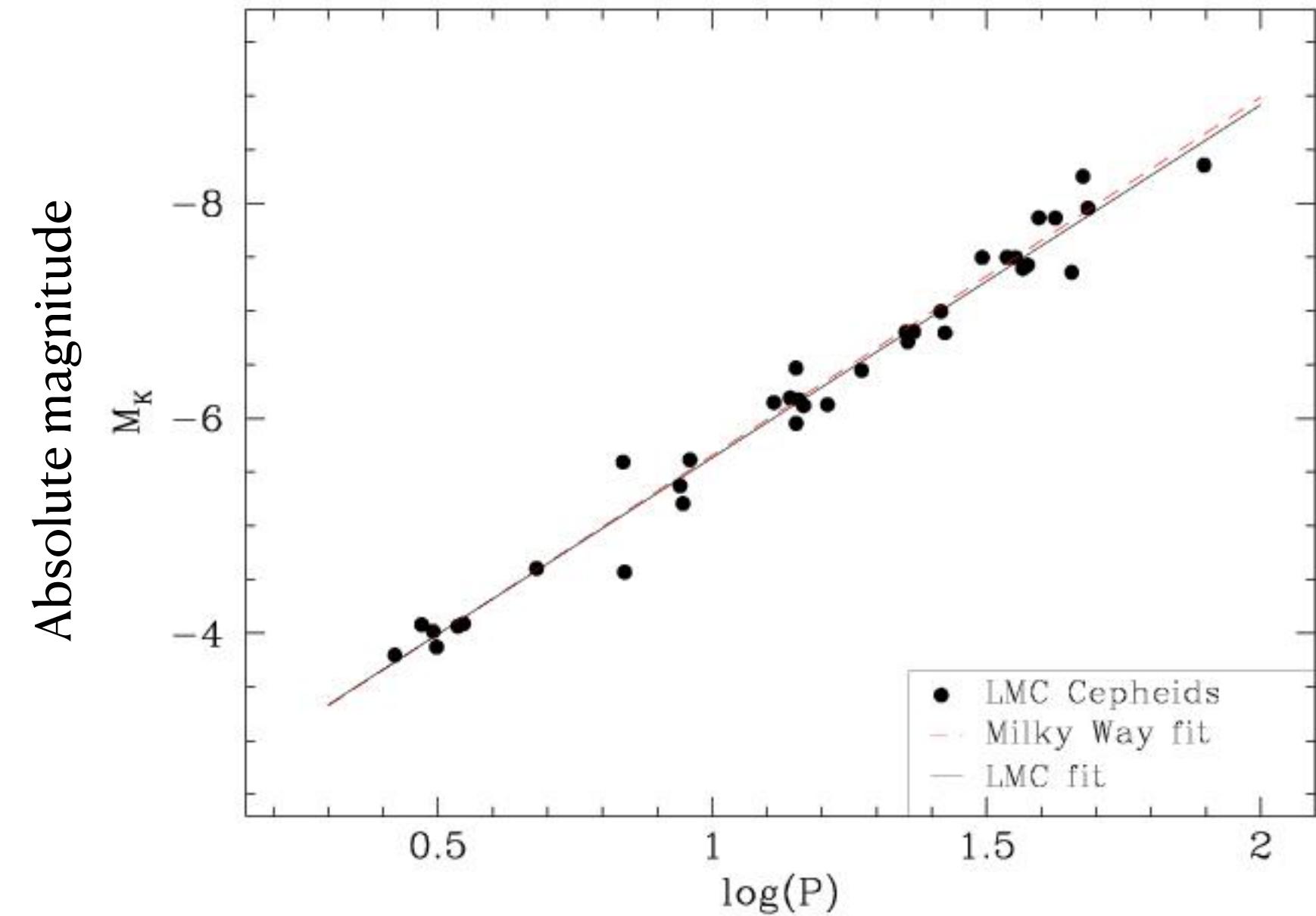


# Distance Scale

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    - parallax
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Cepheid P-L relation



Period

Bright Cepheids have long periods;  
faint Cepheids have short periods.

Discover through repeated observation.  
Measure period, infer luminosity from P-L relation.  
Apply inverse square law, accounting for extinction  $A$ :

$$m_K - M_K = 5 \log(d) - 5 + A_K$$

calibration band-pass dependent

metallicity dependent