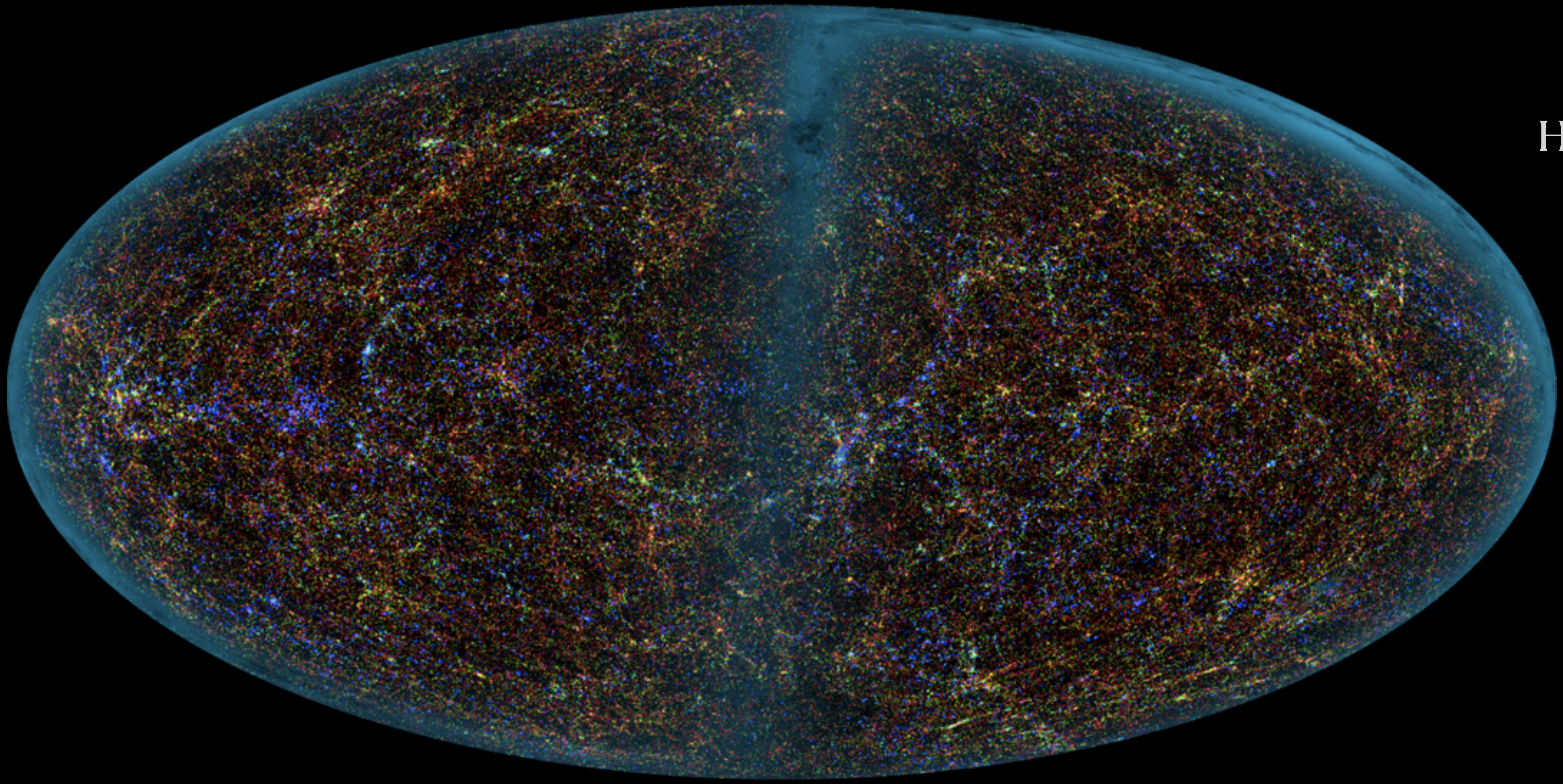
Cosmology

and Large Scale Structure



<u>Today</u>
Hubble Expansion

29 August 2024

http://astroweb.case.edu/ssm/ASTR328/

The Stability Problem - gravity wants to collapse everything into a black hole

Richard Bentley
(1662 – 1742)

Bentley-Newton correspondence

Exchange of letters in 1693,
long after the apple dropped (1664).

Bentley: would not a finite assemblage of stars collapse from their mutual gravity?

Newton: if the matter was evenly diffused through an infinite space, it would never convene into one mass.

Bentley: can such a system remain stable?

Newton: such an assemblage, even if infinite, is like an array of needles standing upright on their points, ready to fall one way or another.

Newton: this frame of things could not always subsist without divine power to conserve it.

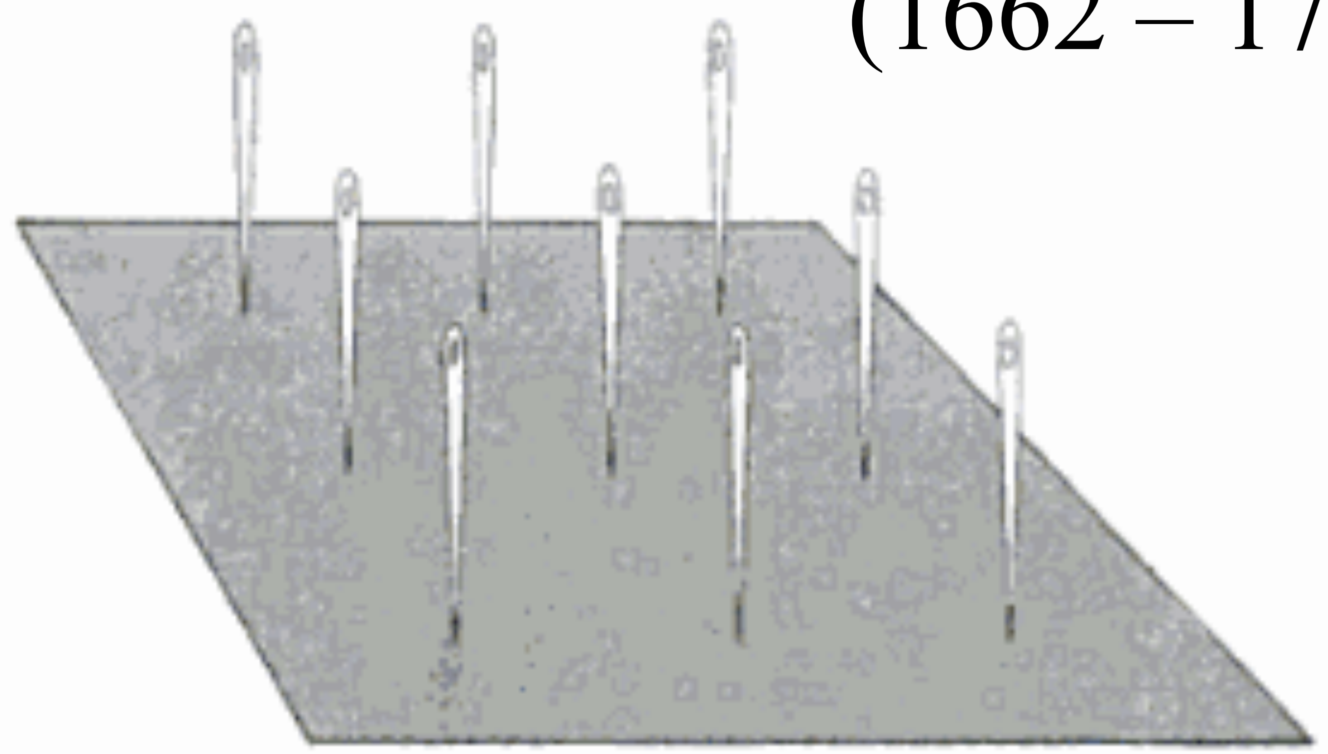


Figure 3.12. Newton agreed with Bentley that stars cannot form a finite and bounded system (as in the Stoic cosmos), for they would fall into the middle of such a system by reason of their gravitational attraction. They agreed that matter was uniformly distributed throughout infinite space, and realized that this was an unstable distribution. The particles of matter, wrote Newton, are like an array of needles standing upright on their points ready to fall one way or another, and "thus might the Sun and fixed stars be formed."

Einstein's Equation

$$\mathscr{R}_{\mu\nu} - \frac{1}{2}\mathscr{R}g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

geometry:

- $\mathscr{R}_{\mu\nu}$ Ricci curvature tensor
- $\mathscr{R}$ Ricci Scalar
- $g_{\mu\nu}$ metric tensor

$T_{\mu\nu}$ Stress-energy tensor (mass distribution, including energy, via $E = mc^2$)

$\mu\nu$ are indices of space-time (t, x, y, z)

"Mass tells space-time how to curve, and space-time tells mass how to move."
- John Wheeler

spacetime separation

$$ds^2 = g_{\mu,\nu}\, dx^\mu dx^\nu$$

Minkowski metric

$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$

4D version of Euclidean geometry

Trace of $g_{\mu,\nu} = -1, +1, +1, +1$

The Expanding Universe

$$\mathscr{R}_{\mu\nu} - \frac{1}{2}\mathscr{R}g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu}$$

Space-time is dynamic. A universe evolving according to Einstein's field equation must either expand or contract. It can not be static.

This was hard to believe, so he fudged it.

$$\mathscr{R}_{\mu\nu} - \frac{1}{2}\mathscr{R}g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu}$$

Λ is the "cosmological constant" intended to keep the universe static.

Now we believe in an expanding universe

governed by

Einstein field equation

$$\mathbf{R}_{\mu\nu} - \frac{1}{2}\mathbf{g}_{\mu\nu} = \frac{8\pi G}{c^4}\mathbf{T}_{\mu\nu} + \Lambda \mathbf{g}_{\mu\nu}$$

Roberston-Walker metric

$$ds^2 = -\,c^2\,dt^2 + a^2(t)\left(\frac{dr^2}{1-kr^2} + r^2\,d\phi^2\right)$$

Friedmann equation

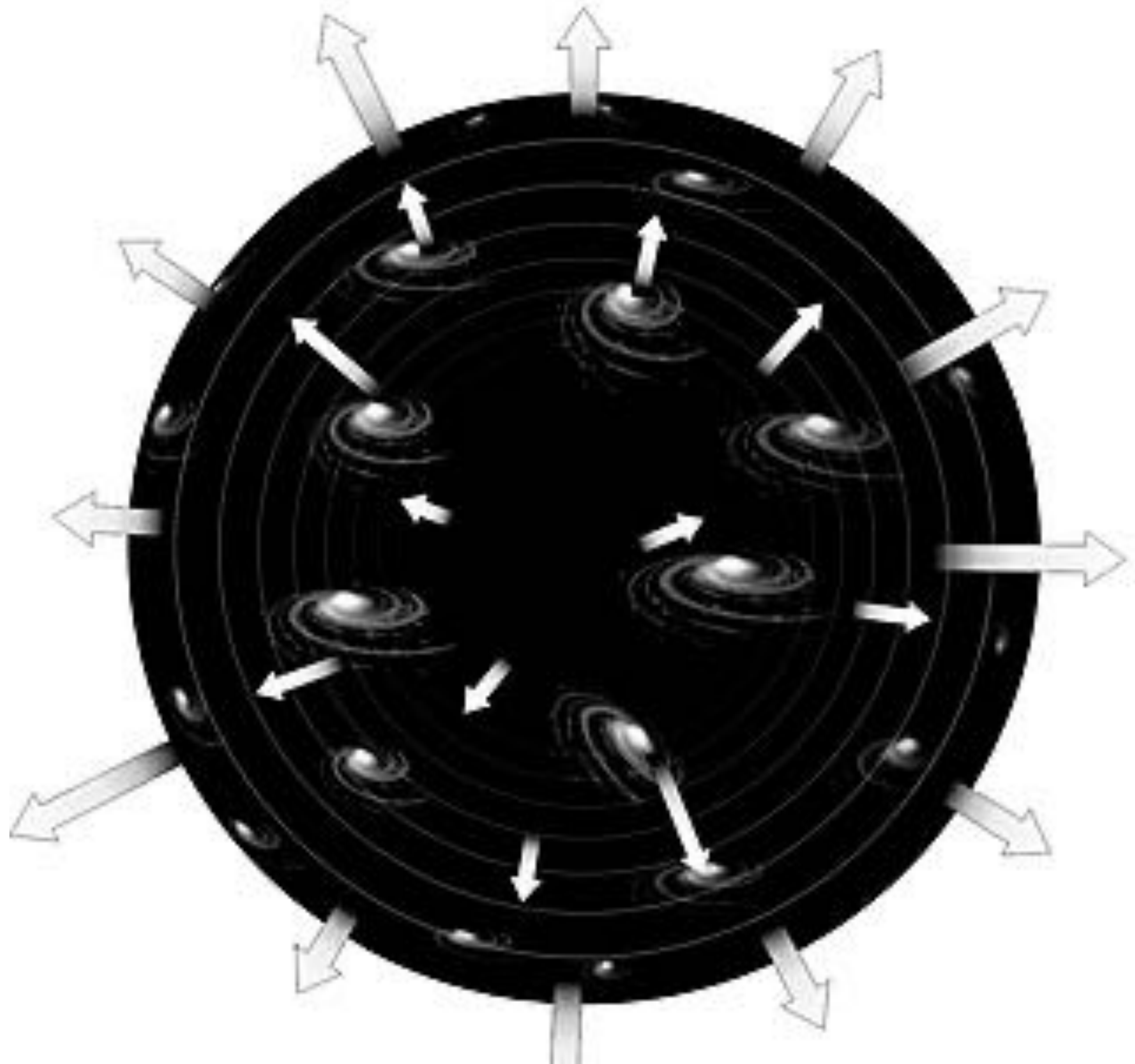
$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\rho - \frac{c^2}{a^2}k + \frac{c^2}{3}\Lambda$$

expansion rate

gravitating mass

geometry

cosmological constant
a.k.a. dark energy



The Cosmological Principle

- The Universe is
  - Homogeneous (uniform in composition)
  - Isotropic (looks the same in every direction)

We need these *assumptions* in order to apply Einstein's equations to the universe as a whole.

The Perfect Cosmological Principle

- The universe looks the same from everywhere at all times.

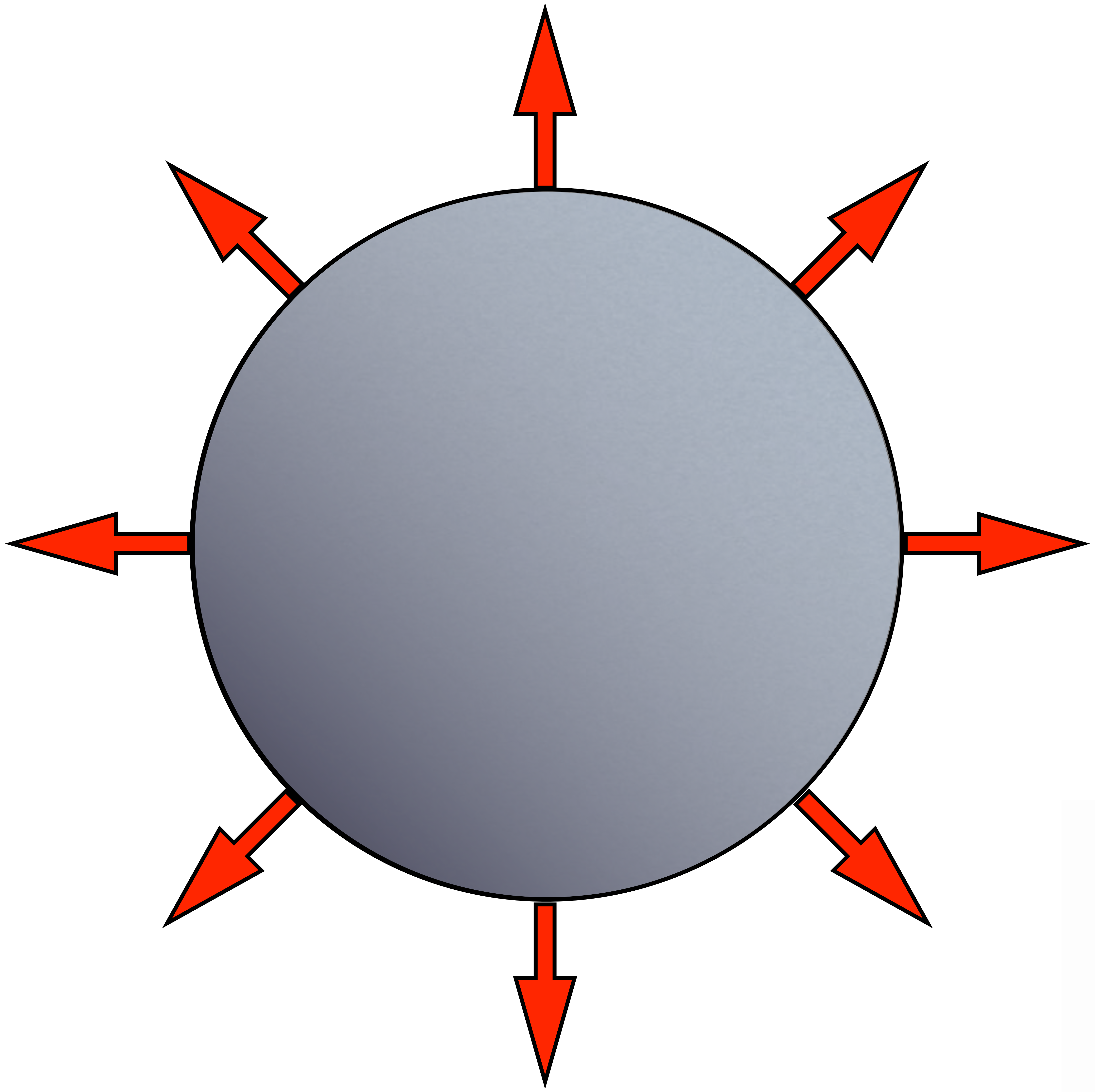
This philosophical principle led Einstein to introduce the cosmological constant.
Only works if precisely fined-tuned so that

$$\mathscr{R}_{\mu\nu} - \frac{1}{2}\mathscr{R}g_{\mu\nu} = \frac{8\pi G}{c^4}T_{\mu\nu} + \Lambda g_{\mu\nu}$$

$$\Lambda = 4\pi G\rho$$

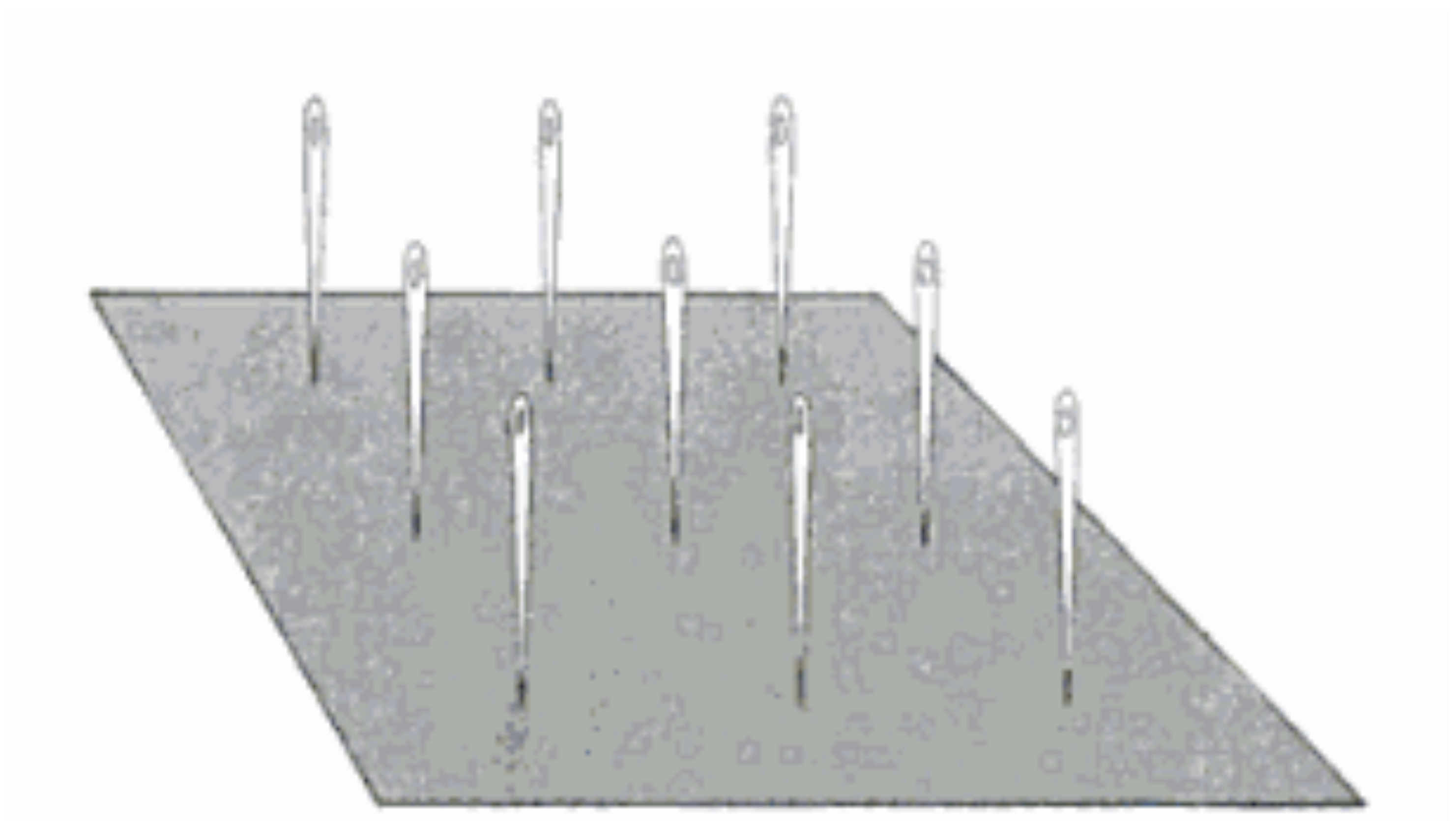
and even then it is unstable, like needles on their points.

An expanding universe solves the stability problem that Newton & Bentley corresponded about.



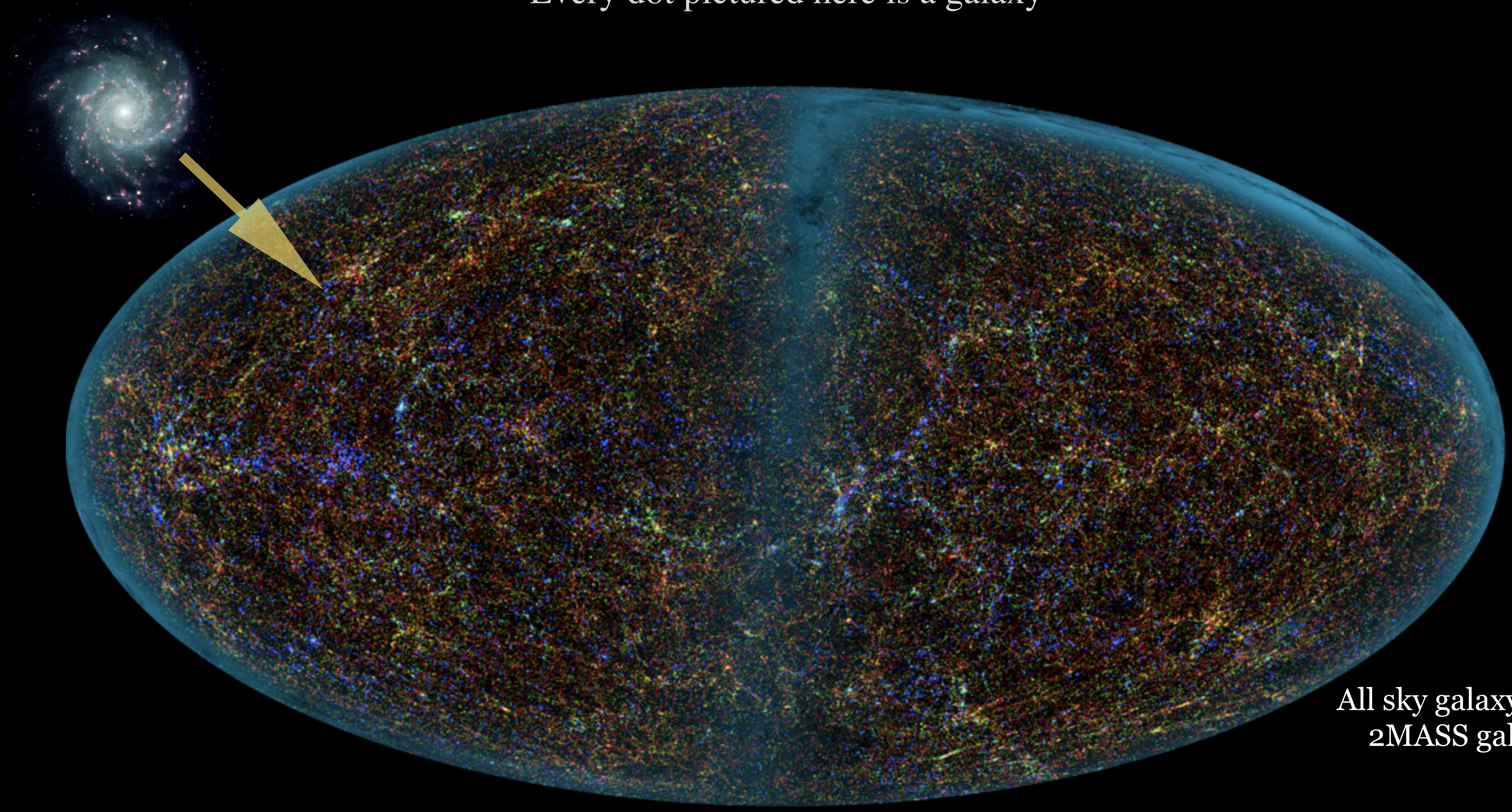
Fundamentally, this was the same problem Bentley & Newton had encountered. A static universe will collapse to a point under the mutual gravitational attraction of all its constituent parts.

Einstein's cosmological constant suffers the same instability problem: it only holds the universe constant for one very specific, fine-tuned value. Any perturbation, and it drives accelerated expansion. General Relativity cannot satisfy the "Perfect" Cosmological Principle.



Does this look homogeneous and isotropic?

Every dot pictured here is a galaxy



All sky galaxy distribution
2MASS galaxy survey

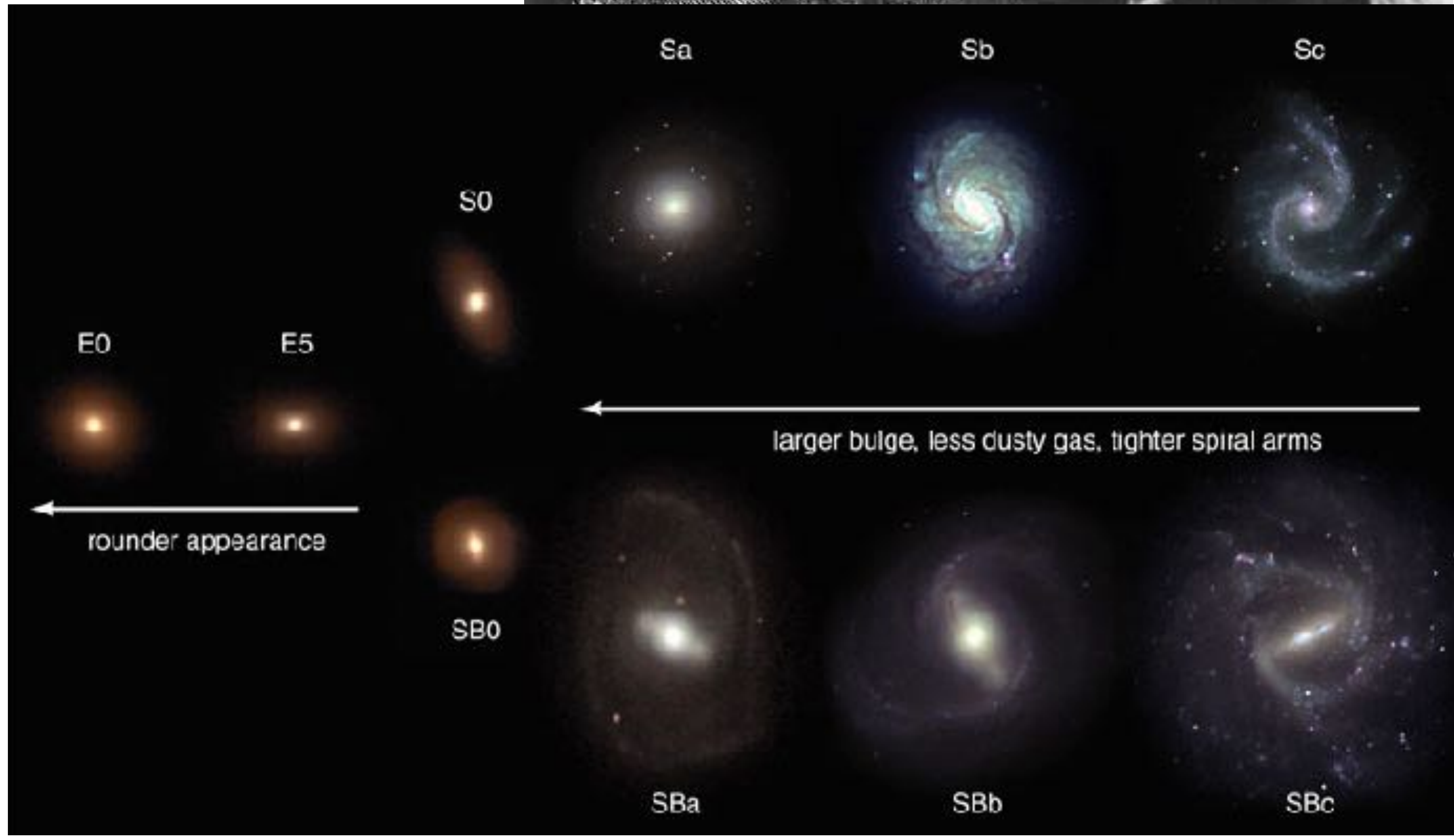
The color-coding corresponds to redshift: redder galaxies are more distant.
The distribution of galaxies is structured into enormous filaments and walls surrounding giant voids.

Hubble

- Showed that galaxies were distant systems, comparable in size to the Milky Way
  - settled the Great Debate after ten years.

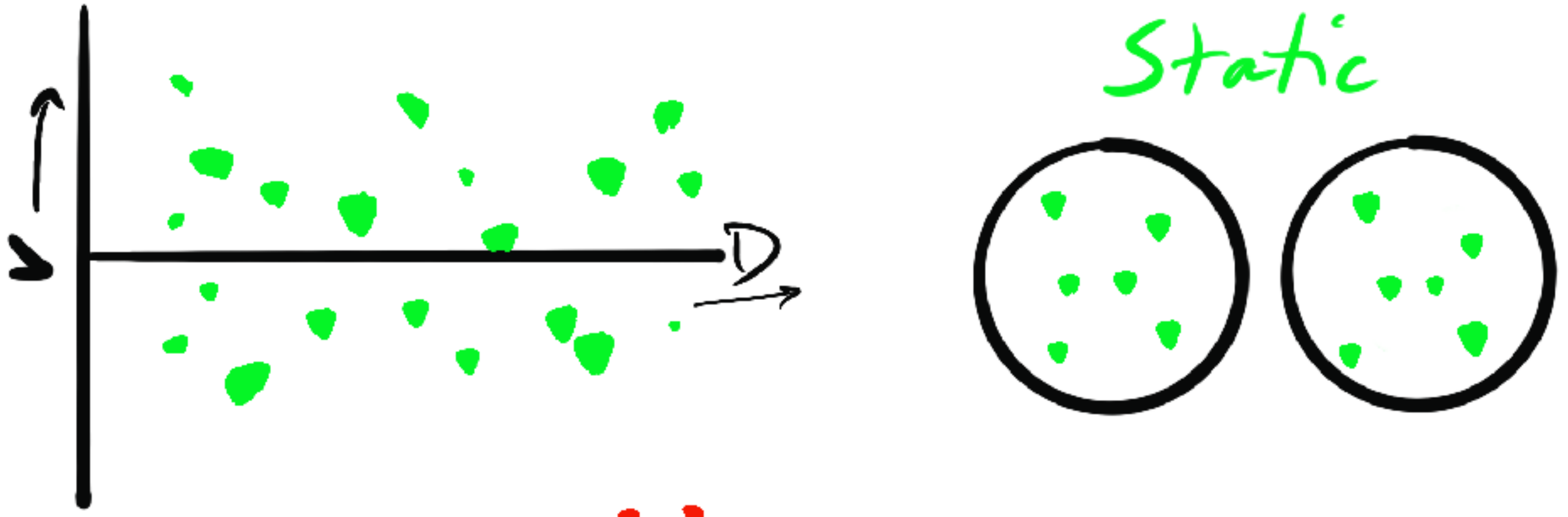
- Classified galaxy morphology (Hubble types)
  - tuning fork diagram; spiral & elliptical galaxies

- Demonstrated the expansion of the Universe.

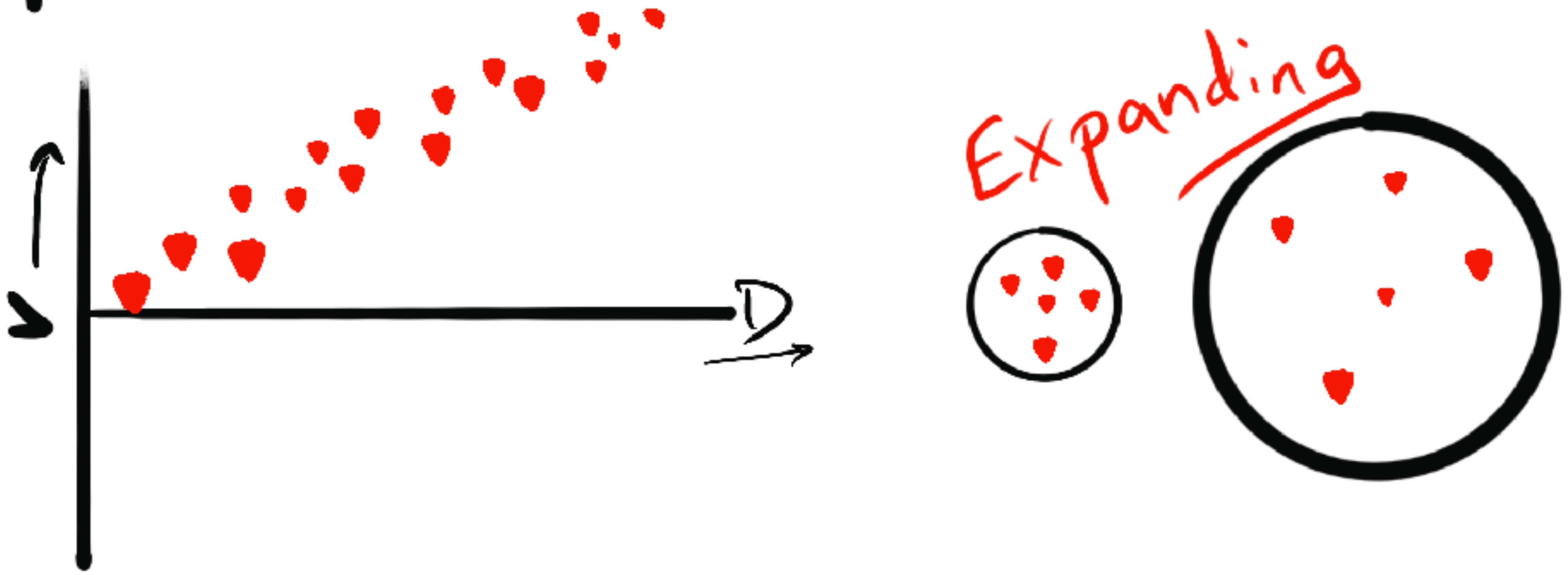


1920s & 30s

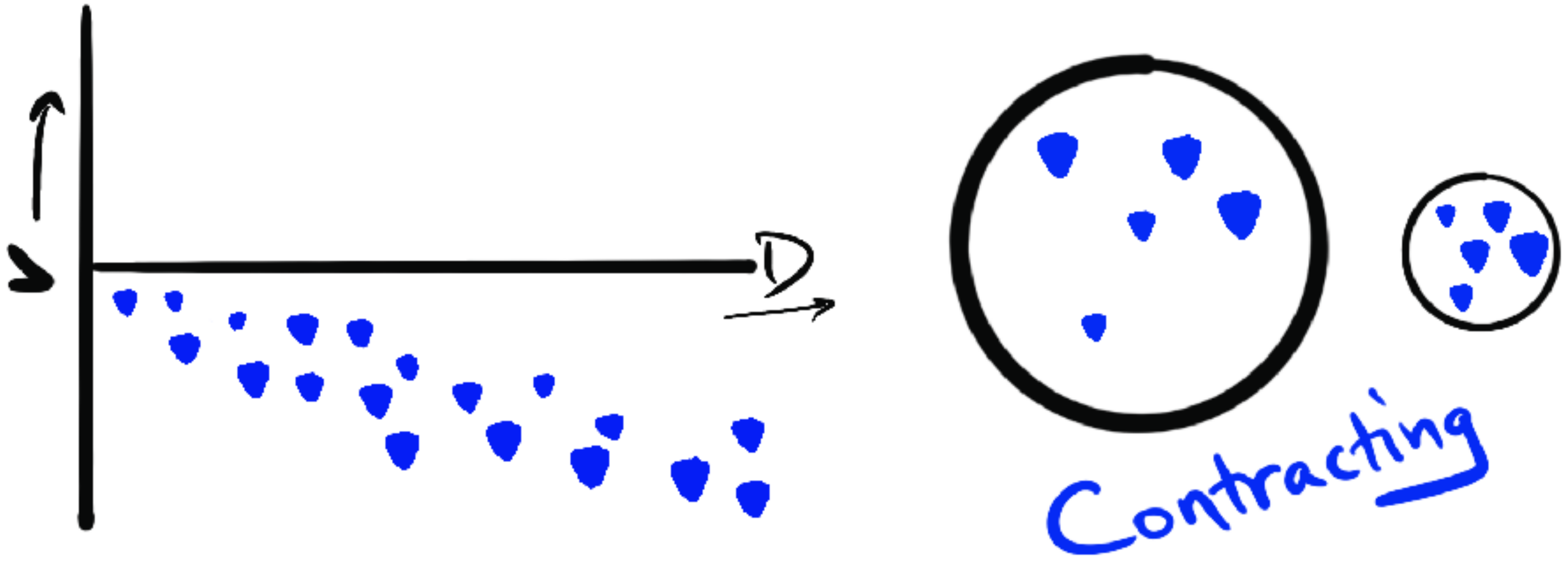
Why expansion? What should we expect to see?



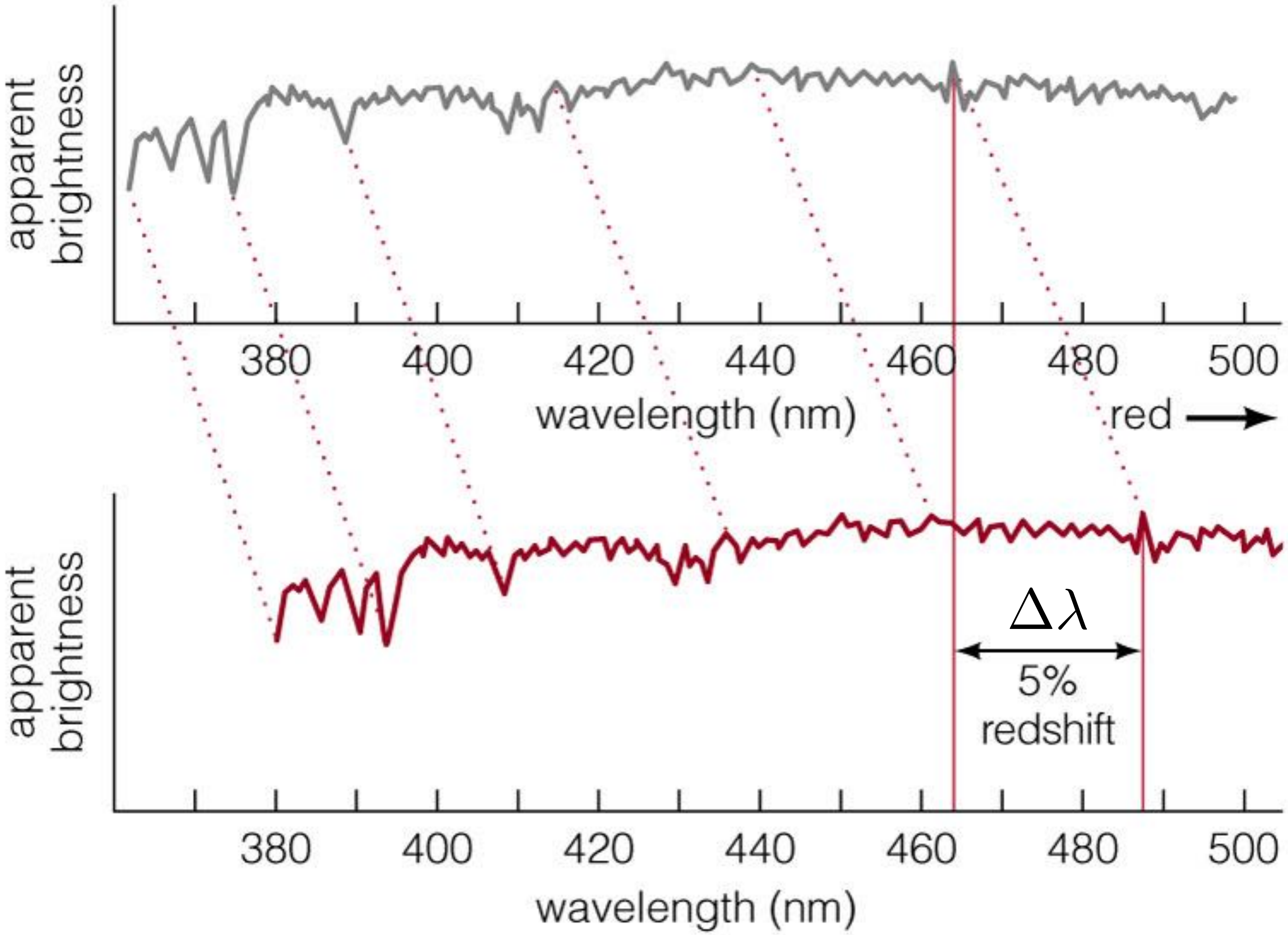
In a static universe, galaxies might move around, with some moving towards us and some away, but there should be no systematic trend one way or the other.



In an expanding universe, galaxies will appear to be moving systematic away from us in proportion to their distance. The more remote a galaxy, the more stretching of the space in between. This is the origin of Hubble's expansion law $V = H_0 D$.



In a contracting universe, galaxies will appear to be moving systematic towards us (blueshifted instead of redshifted).



$$z = \frac{\Delta\lambda}{\lambda}$$

$$z = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}$$

Spectra get stretched and redshifted as the universe expands

Hubble knew from Slipher's work that the spectral features of virtually all galaxies are ***redshifted*** $\Rightarrow$ they're all moving away from us.

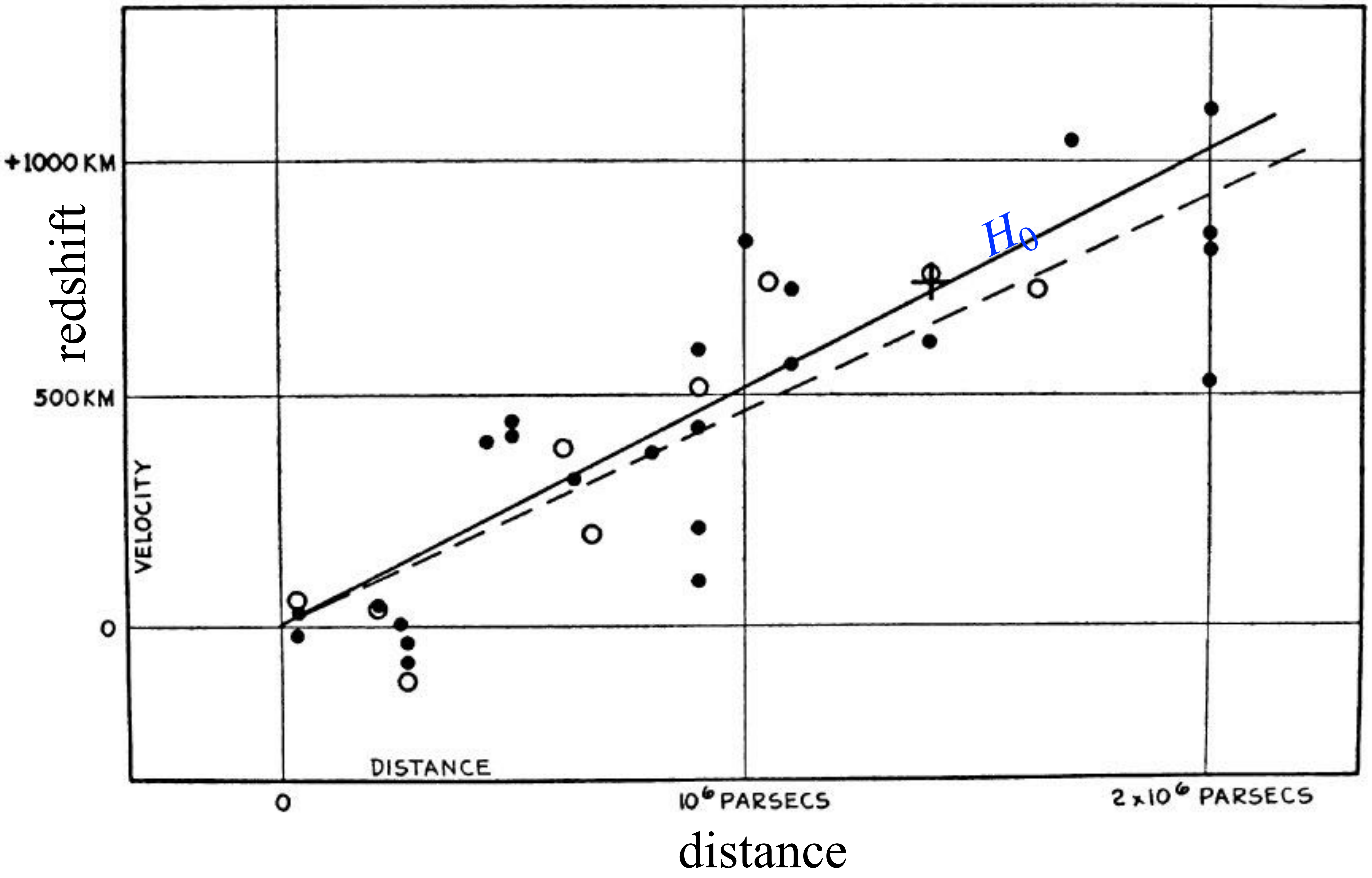
The slope of this relation is the expansion rate of the universe. Hubble's constant quantifies the size and age of the universe.

Hubble's law

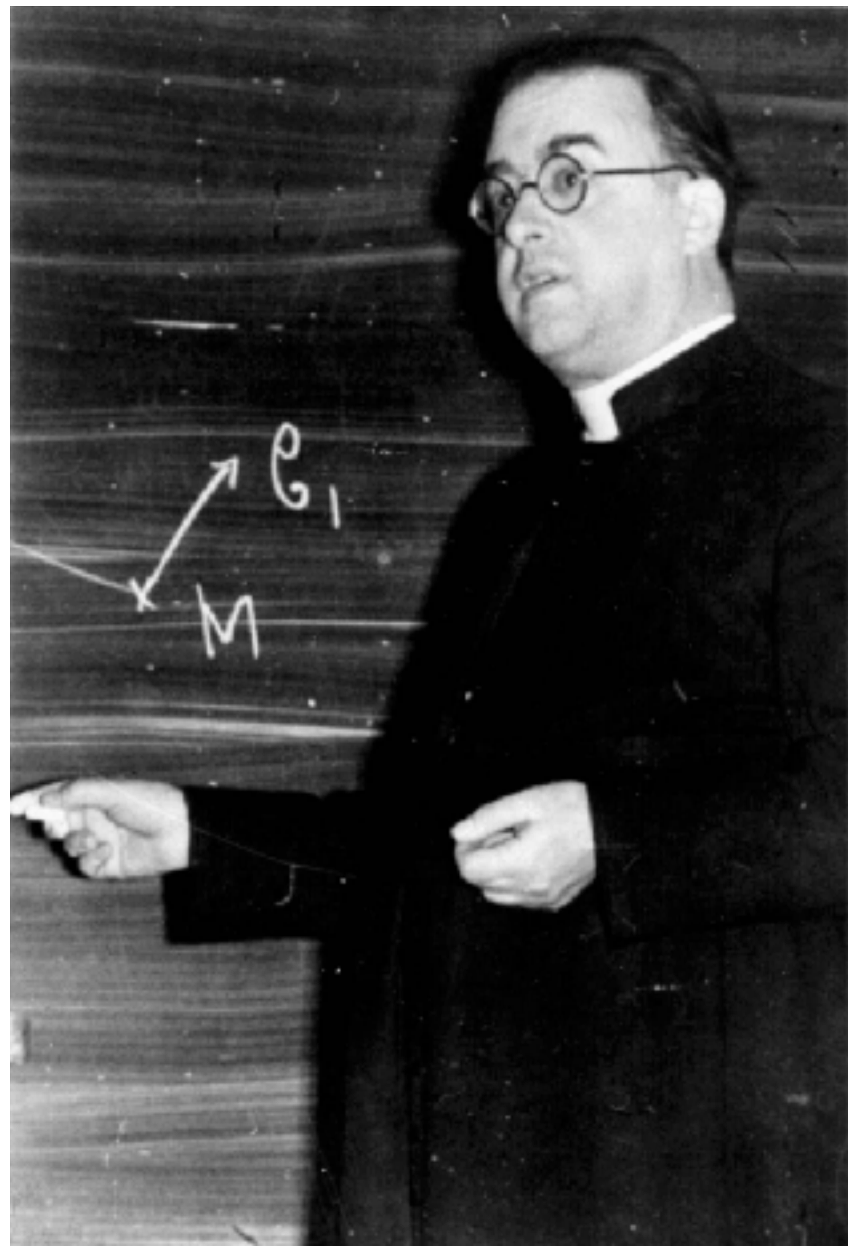
$$V = H_0 D$$

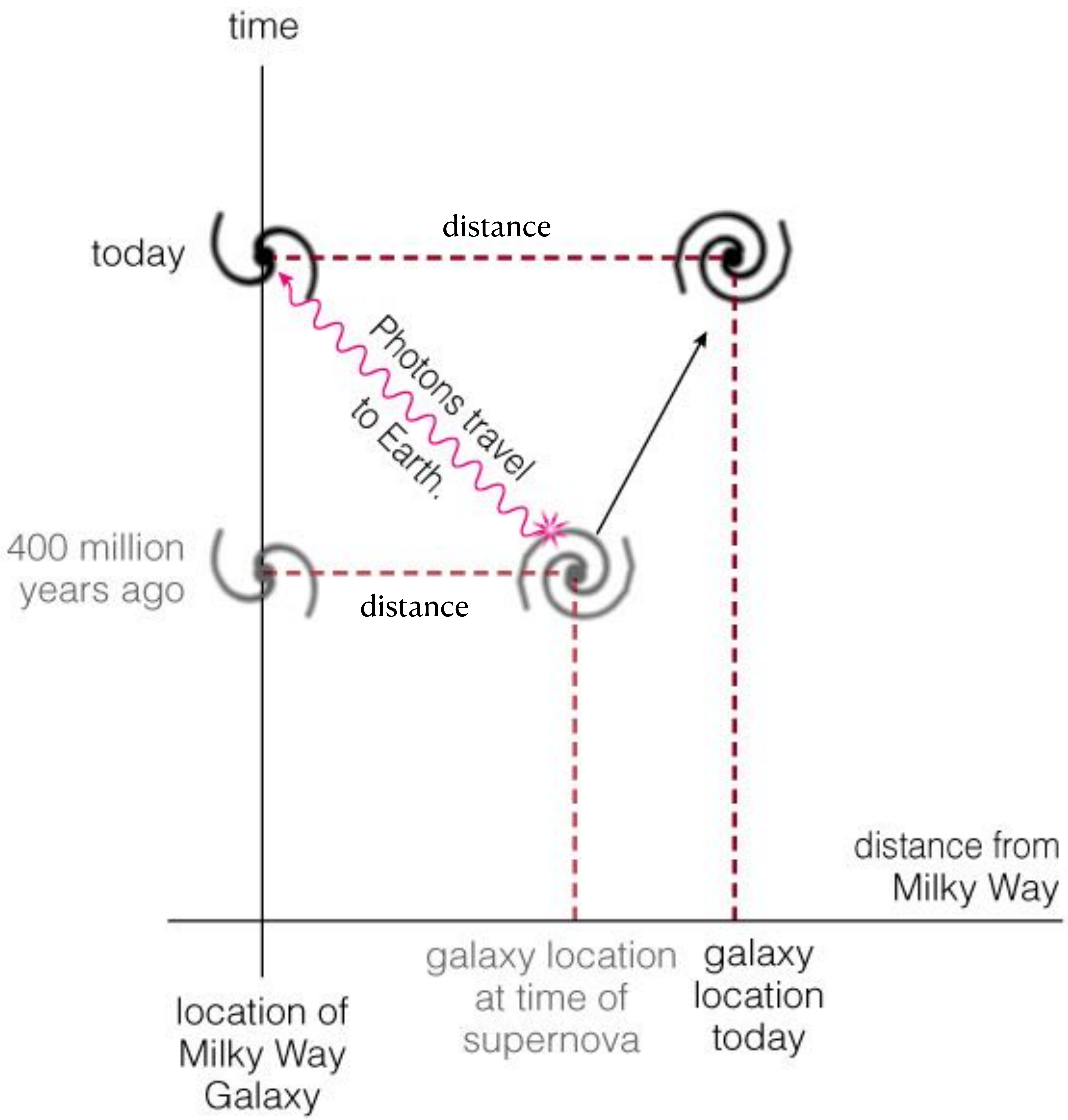
Slipher measured redshifts

Hubble measured distances



Lemaître put them together

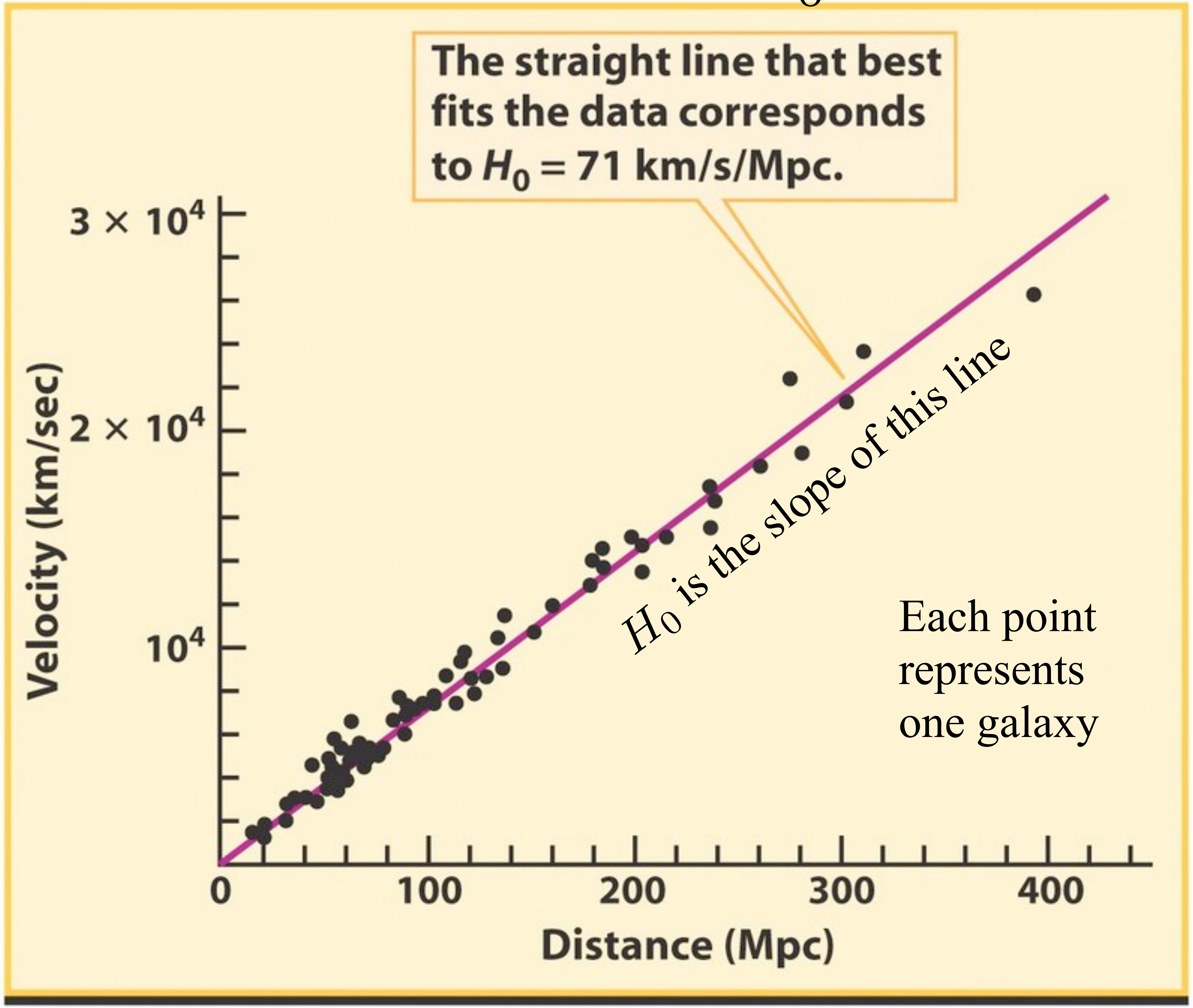




The universe expands while photons are traveling between distant galaxies.

Looking further away is equivalent to looking back in time.

Hubble's law: $V = H_0 d$



Hubble's earliest estimate was about 500 km/s/Mpc

Historically, there was a long-running debate between 50 (Sandage) and 100 (de Vaucouleurs) km/s/Mpc

Modern estimates range from 67 - 75 km/s/Mpc

Currently, there is a tension between 67 (Planck) and 73 (traditional measurements) km/s/Mpc

$H_0 = 67.4 \pm 0.5\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ (Planck 2020)

$H_0 = 73.2 \pm 0.9\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$ (arXiv:2408.11770)

The error bars do not overlap - the discrepancy is $\sim 5\sigma$

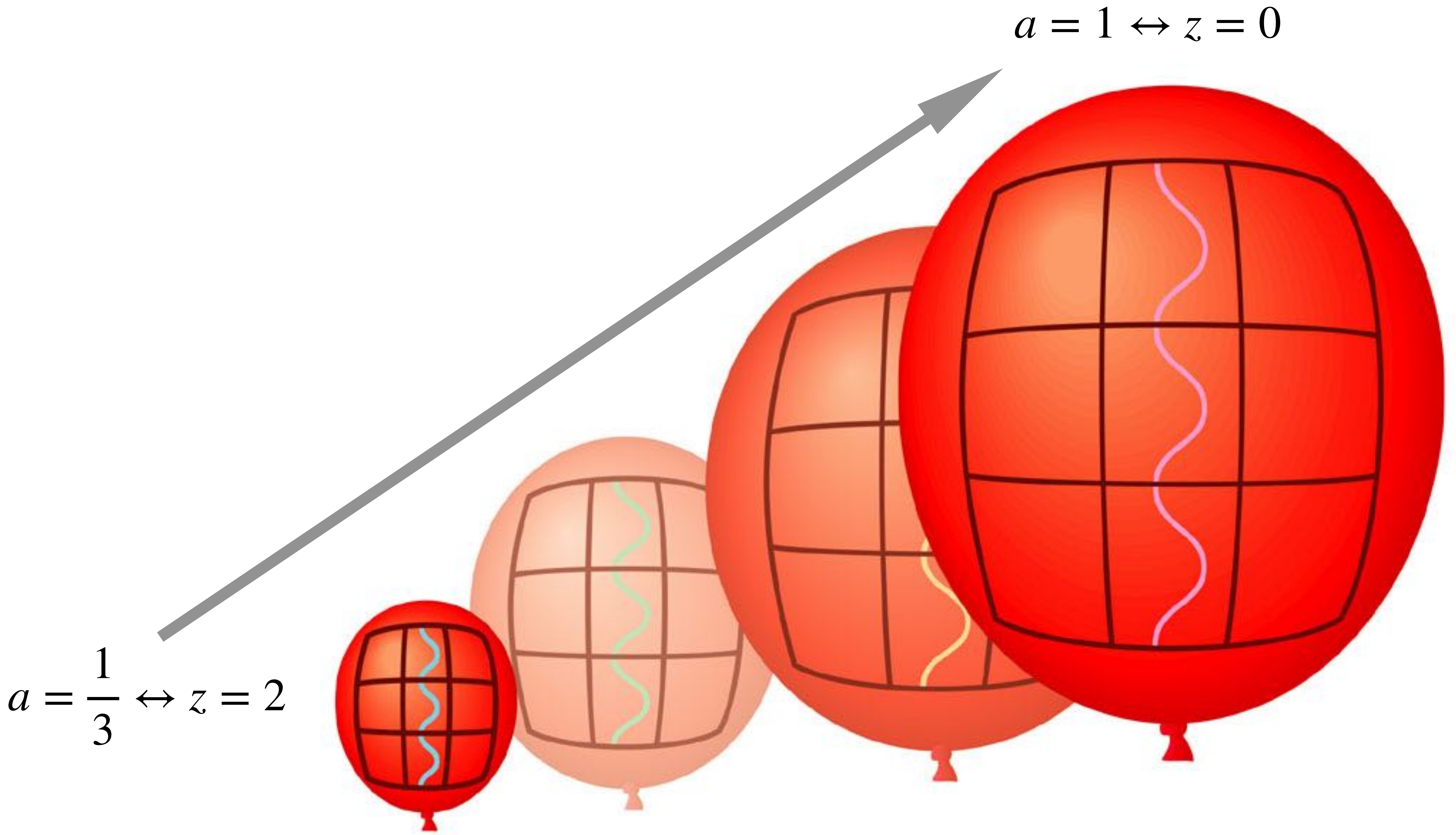
Expansion stretches photon wavelengths causing the ***cosmological redshift***: stretching of space, *not* an explosion.

$$V = H_0 d$$ is linear in the Proper Distance

redshift $$z = \frac{\Delta\lambda}{\lambda} = \frac{\lambda_{obs} - \lambda_{em}}{\lambda_{em}}$$ related to **expansion factor** a by $$a = \frac{1}{1+z}$$ by convention, right now $a = 1$.

$$z = \frac{\Delta\lambda}{\lambda} \approx \frac{V}{c}$$ in the non-relativistic limit

but more generally $$1 + z = \sqrt{\frac{1 + V/c}{1 - V/c}}$$

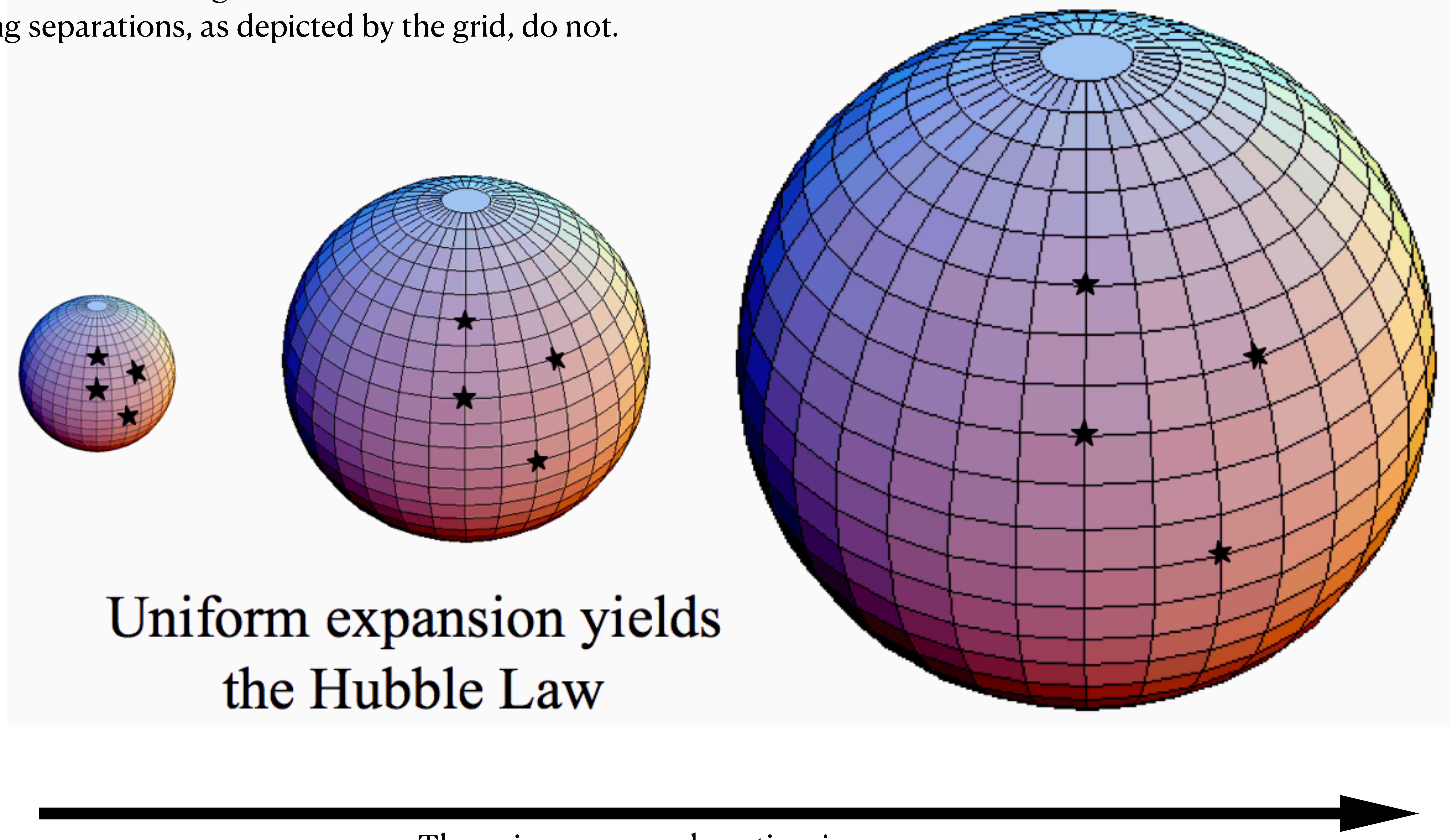


$$D_{\text{proper}} = a(t)\, d_{\text{comoving}}$$

$$H = \frac{\dot{a}}{a} = \frac{1}{a}\frac{da}{dt}$$

Hubble's "constant" is the current expansion rate

The proper distance between galaxies increases with time.
Their comoving separations, as depicted by the grid, do not.



The universe expands as time increases

Expansion stretches photon wavelengths causing the ***cosmological redshift***: stretching of space, *not* an explosion.