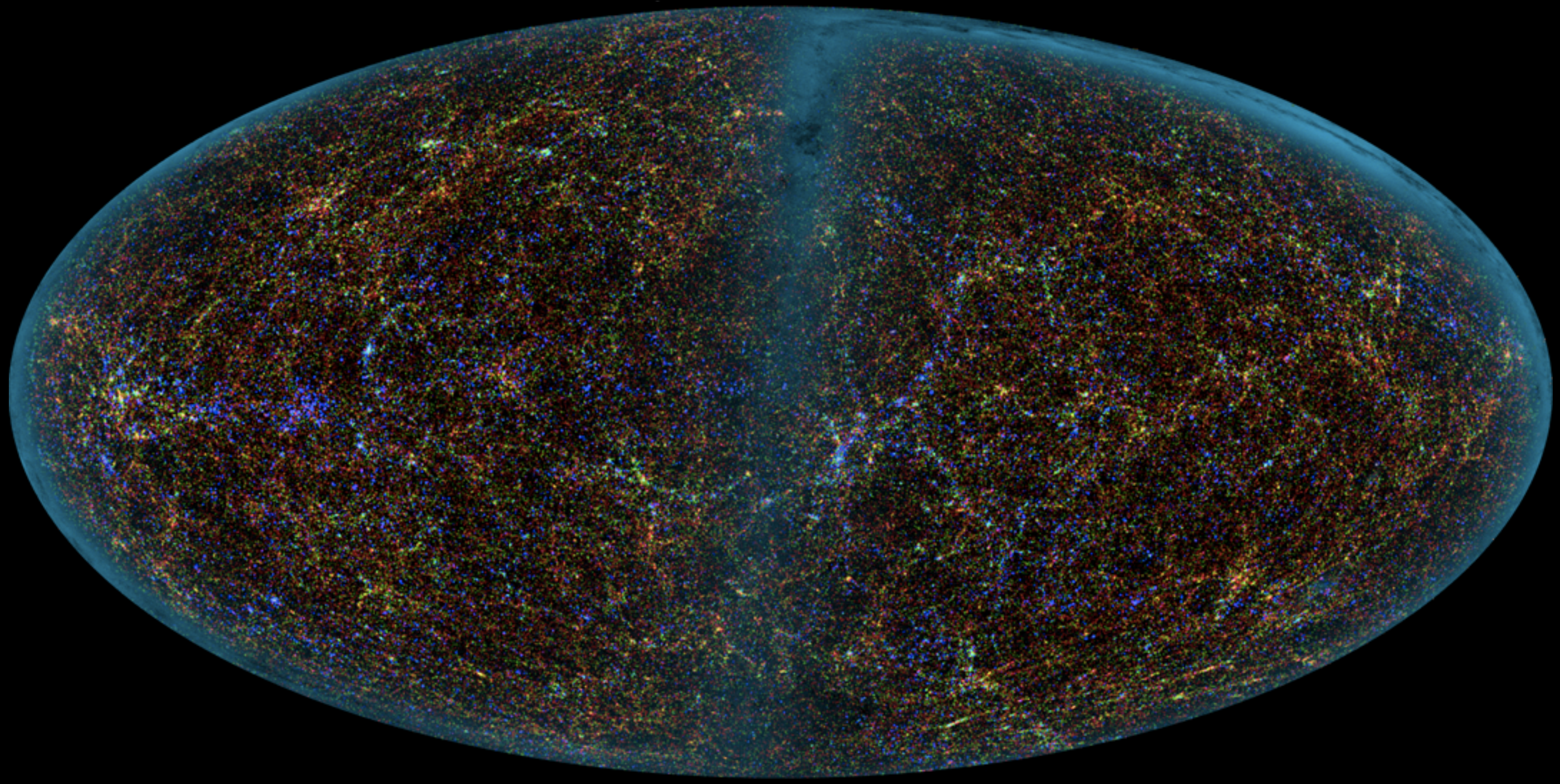
Cosmology

and Large Scale Structure



Today

Hubble Expansion

Density Parameter

Distance definitions

- proper
- comoving
- luminosity
- angular size

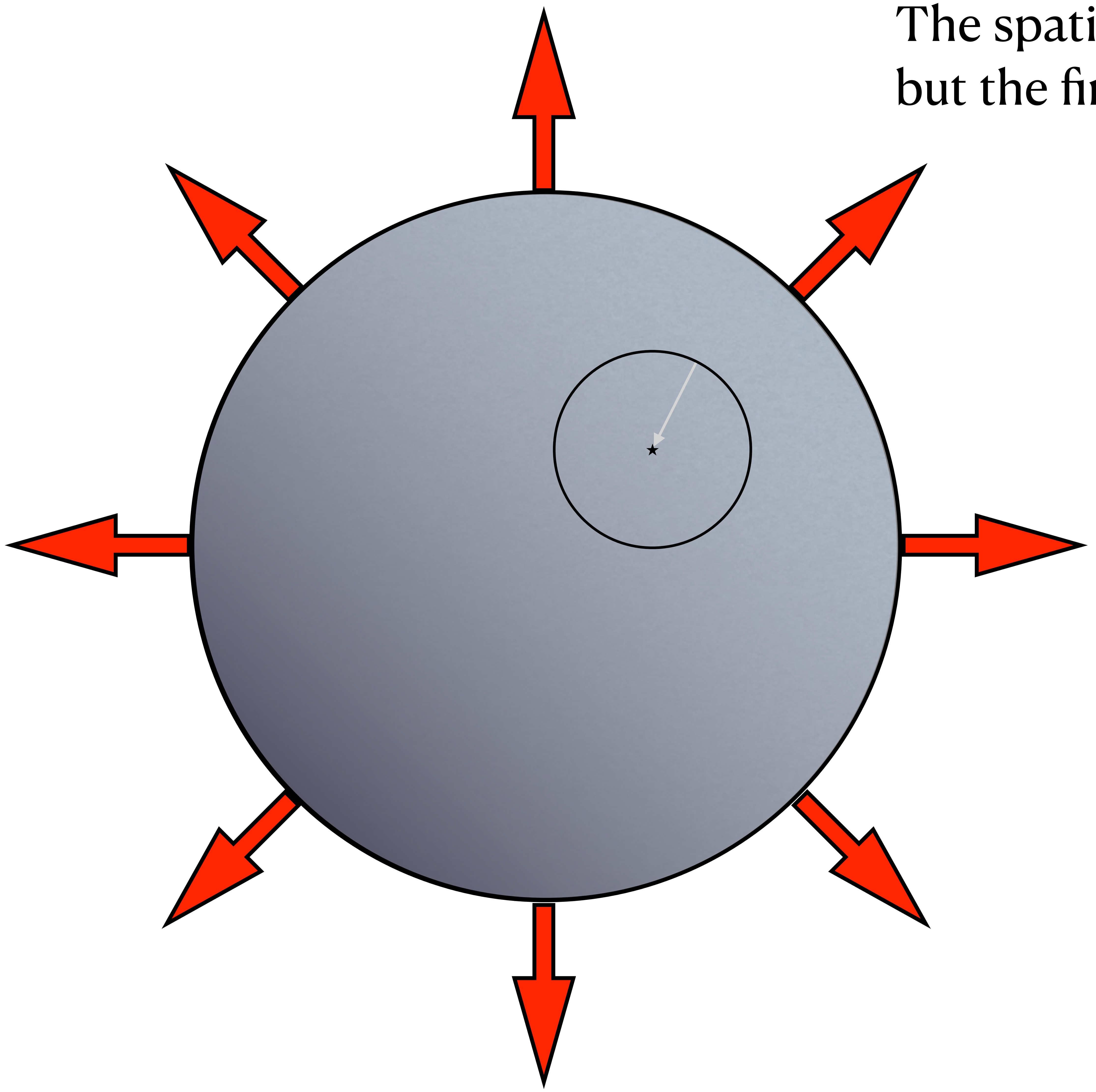
First problem set due next time
(on paper at the beginning of class)

Discuss

- What is the universe expanding into?
  - Is there a cosmic edge?
- What is meant by the primordial singularity?
  - What happened in the distant past?

Running the expansion in reverse, one comes to an initial singularity: an expanding universe has a finite age. This helps solve Olber's paradox.

The spatial extent of the universe may be infinite, but the finite speed of light imposes a *horizon*.

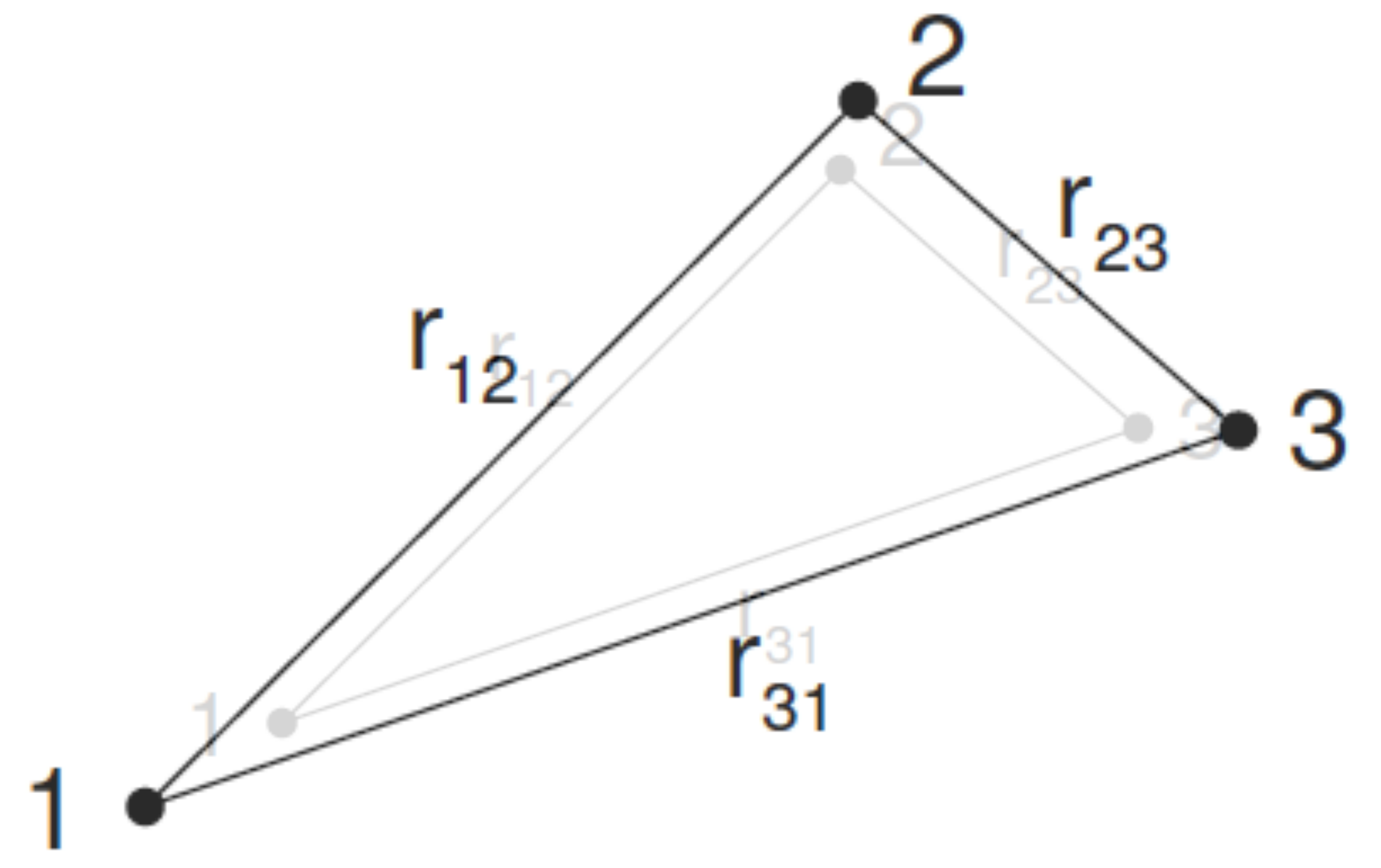


Note: in addition to the finite age of the universe, the geometry is no longer Euclidean, and light gets redshifted. Consequently, surface brightness is no longer distance independent.

The cosmic microwave background (CMB) covers the entire sky, so Olber was right - every sight line is blocked. But the intensity of the CMB is greatly reduced as its initial blazing glow (3000 K) has been redshifted to a mere 2.7 K thermal spectrum.

What does it mean?

- The universe is expanding
  - galaxies are receding form one another
  - it is *not* an explosion into some preexisting space
  - space itself is getting stretched
- The expansion has no “center”
- It does not originate from a point
  - “initial singularity” is a misnomer
  - the density is arbitrarily higher in the past, but
  - the spatial extent can still be infinite
- The age of the universe is finite



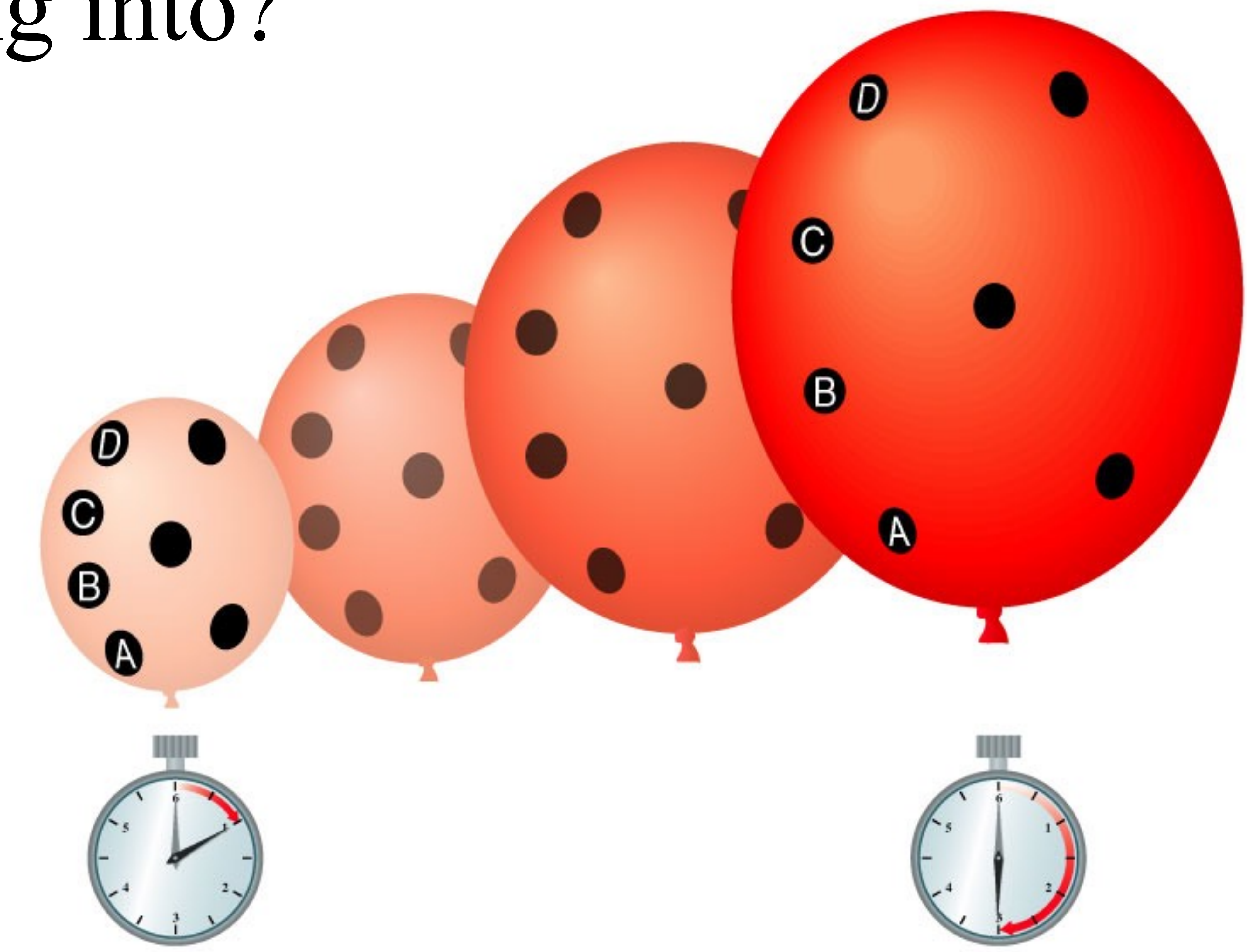
$$r_{ij}(t) = a(t)r_{ij}(t_0)$$

by convention, $a = 1$ at $t = t_0 =$ now

a was smaller in the past by $a[z(t)] = \dfrac{1}{1+z}$

- What is the universe is expanding into?

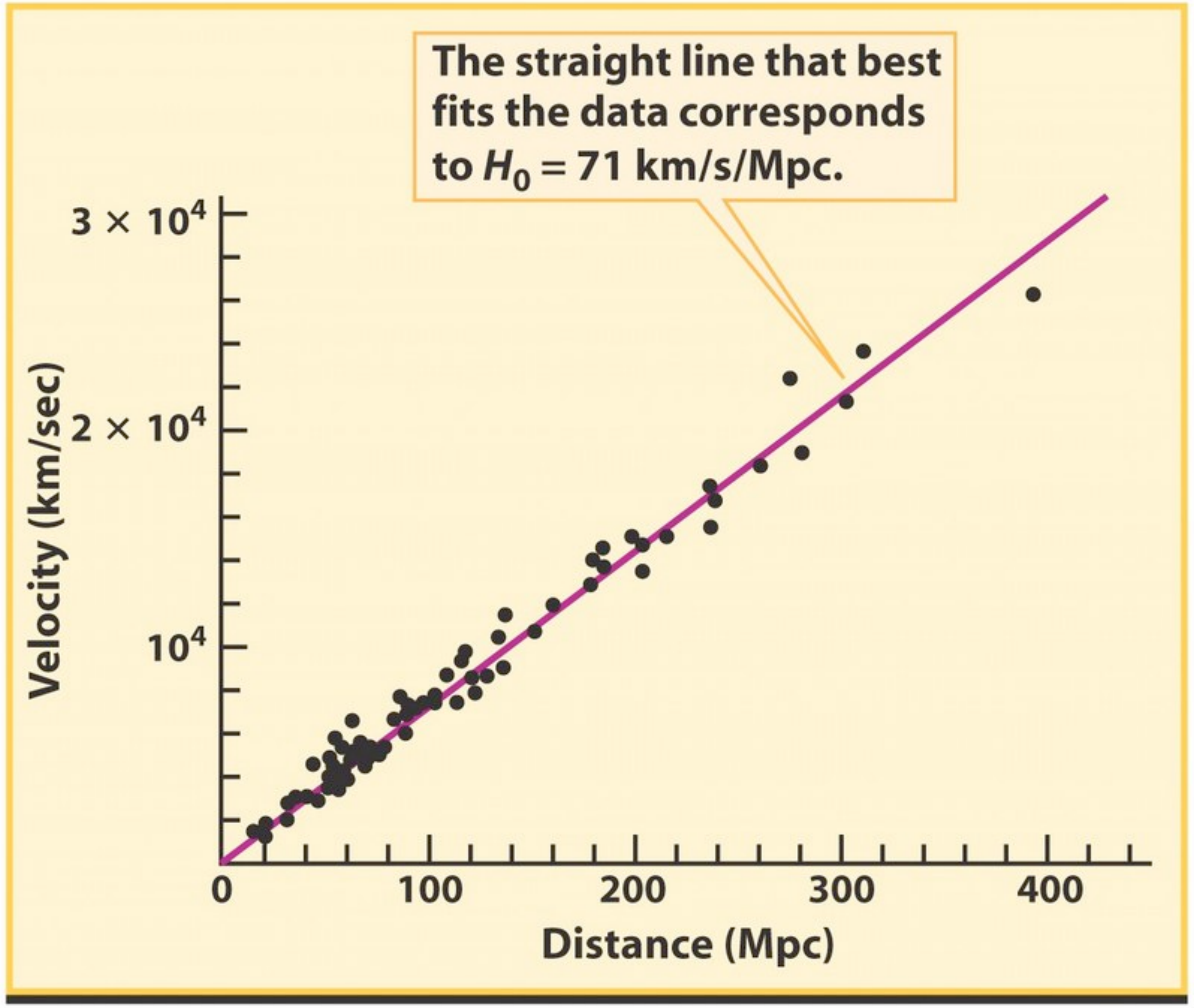
The future



One example of something that expands but has no center or edge is the surface of a balloon. The radius of the balloon represents time - the "4th dimension" so while there is no spatial center, the "center" here is the beginning of time.

This reasoning only holds as far back as theory applies, which is the Planck scale $t_P = \sqrt{\frac{G\hbar}{c^5}} \approx 5 \times 10^{-44}$ s

Expansion age of the universe



expansion factor $\longrightarrow$ zero in finite past: age $\sim \dfrac{1}{H_0}$

Crudely speaking,

$$t_H = \frac{1}{H_0} \approx 13.5 \text{ Gyr}$$ Hubble time

$$D_H \approx \frac{c}{H_0} \approx 4 \text{ Gpc}$$ Horizon distance

In detail, these quantities depend on the expansion history, which need not be linear.

Whether the expansion is steady, slows down, or speeds up depends on the contents of the stress-energy tensor. The mass and energy content of the universe determines how it self-gravitates.

The Search for Two Numbers

as cosmology was long known

- Expansion Rate
  - Sets the size scale
  - Sets the age scale

$$H_0$$

Hubble Constant = value of expansion rate measured now

- Density Parameter
  - Determines the dynamics
  - i.e., the expansion history

$$\Omega_0 = \frac{\rho_0}{\rho_c}$$

ratio of the current density
to the critical density

critical density $\rho_c = \dfrac{3H_0^2}{8\pi G}$

Modern Cosmology

- We live in an expanding universe
  - The expansion of space causes the wavelengths of photons to stretch
  - more distant objects have larger redshift (Hubble's Law: $V = H_0 d$)
- The universe may be spatially infinite
- The universe has a finite age
  - about 13 or 14 Billion years

5.1. *EVOLUTION OF ENERGY DENSITY* 81

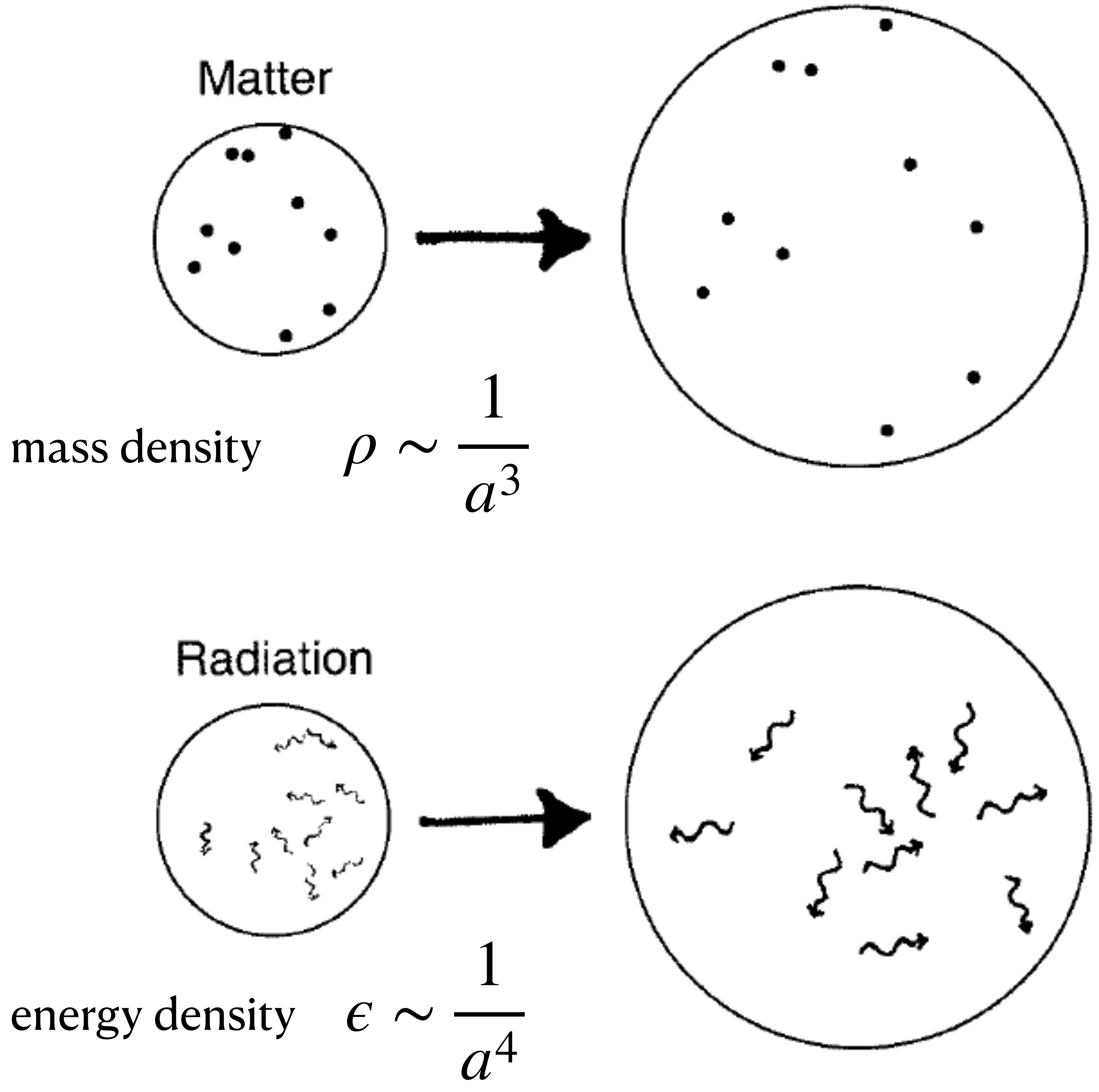
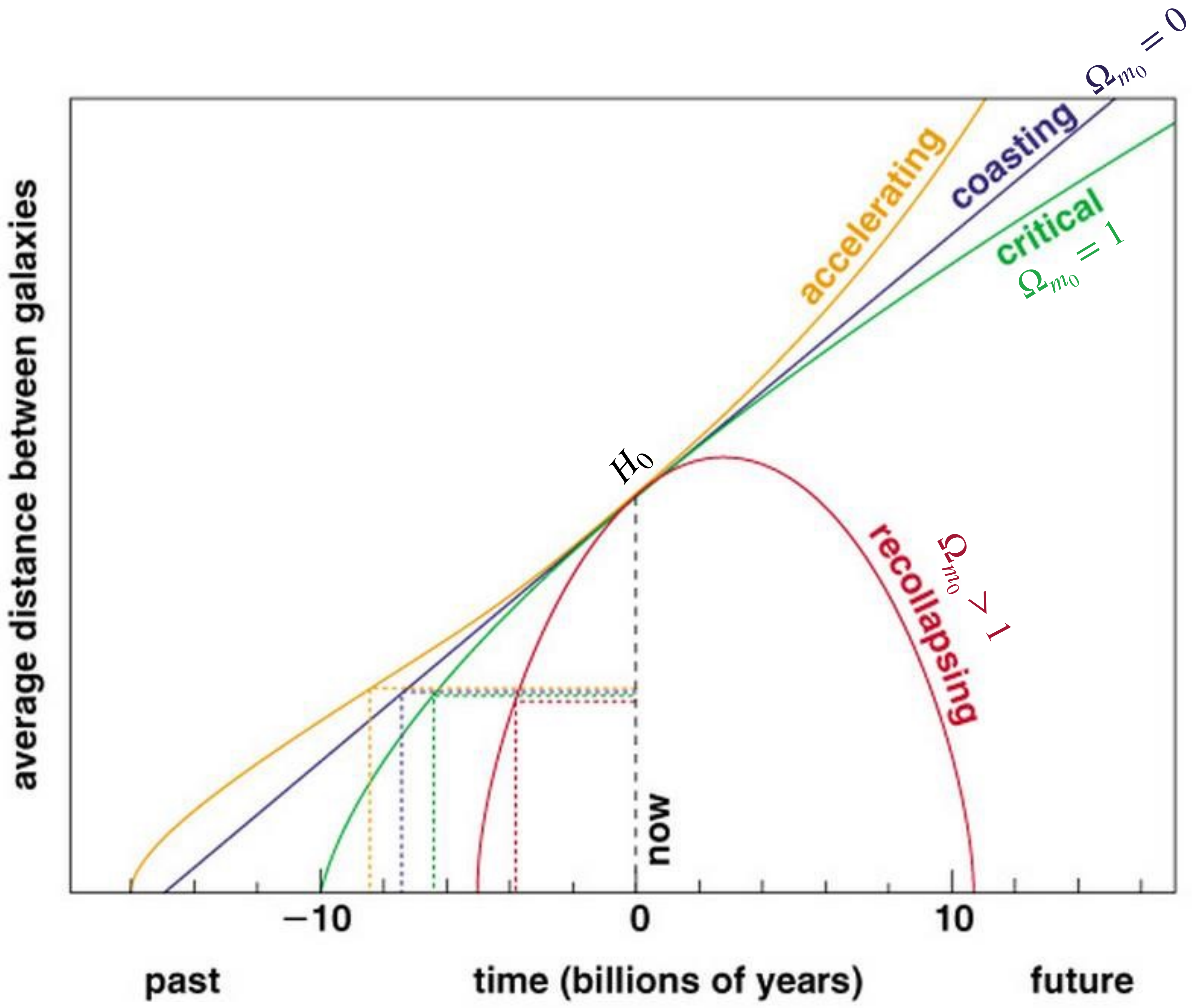


Figure 5.1: The dilution of non-relativistic particles ("matter") and relativistic particles ("radiation") as the universe expands.

Possible expansion histories - depends on mass density Ω_m

$a(t)$



t

The Hubble “constant” is the slope measured now

$$H = \frac{\dot{a}}{a} = \frac{1}{a}\frac{da}{dt}$$

Much effort in cosmology is directed towards determining *a(t)*.

Notional distances:

Hubble's "constant" is the current expansion rate $H = \frac{\dot{a}}{a} = \frac{1}{a}\frac{da}{dt}$

Proper distance: real separation in km or Mpc or furlongs.
The proper distance between galaxies increases as the universe expands.

$$D_{\mathrm{proper}} = a(t)\, d_{\mathrm{comoving}}$$

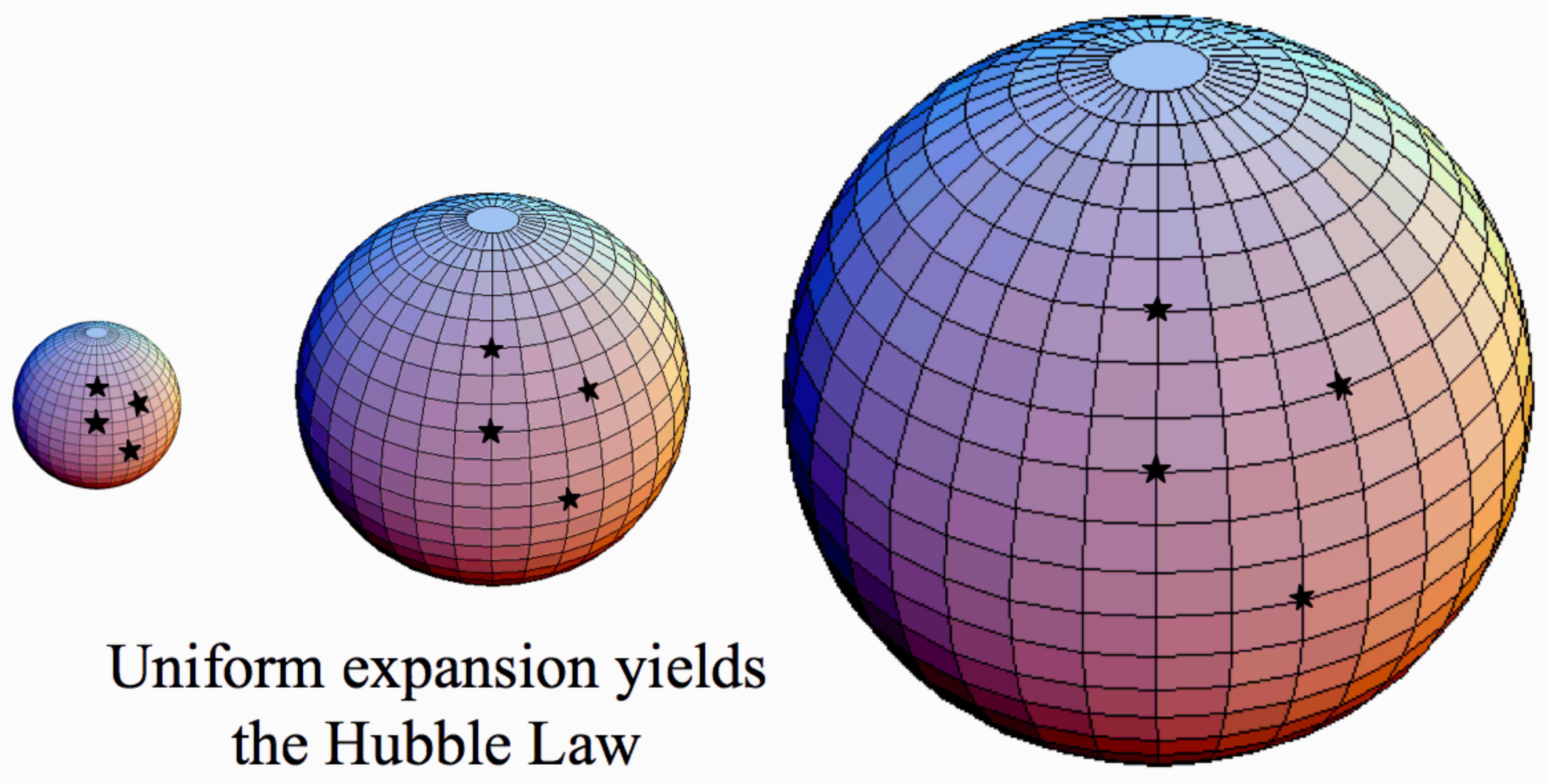
Comoving distance: separation on a grid of coordinates that expands along with the universe.
The comoving separation remains fixed as the universe expands.

Luminosity distance: equivalent to inverse-square distance in a Euclidean geometry.

$$d_{\mathrm{L}} = (1+z)D_{\mathrm{proper}}$$

Angular-diameter distance: equivalent angular scale to that in a Euclidean geometry.

$$d_{\mathrm{A}} = \frac{D_{\mathrm{proper}}}{(1+z)}$$



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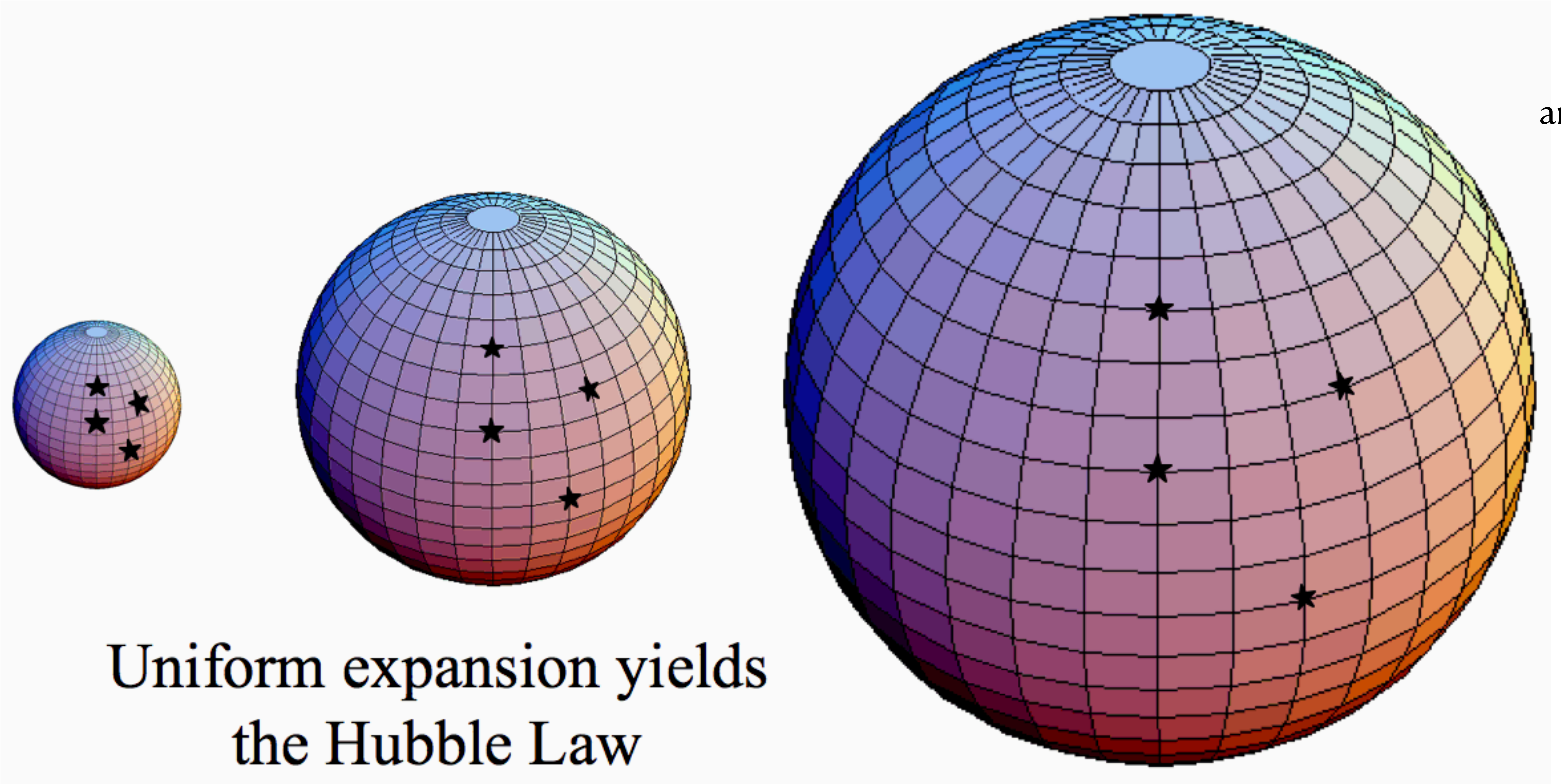
Angular-diameter distance: equivalent angular scale to that in a Euclidean geometry.

angular size $$\theta = \frac{\ell}{d_A}$$ intrinsic size



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ANGULAR-DIAMETER DISTANCE 137

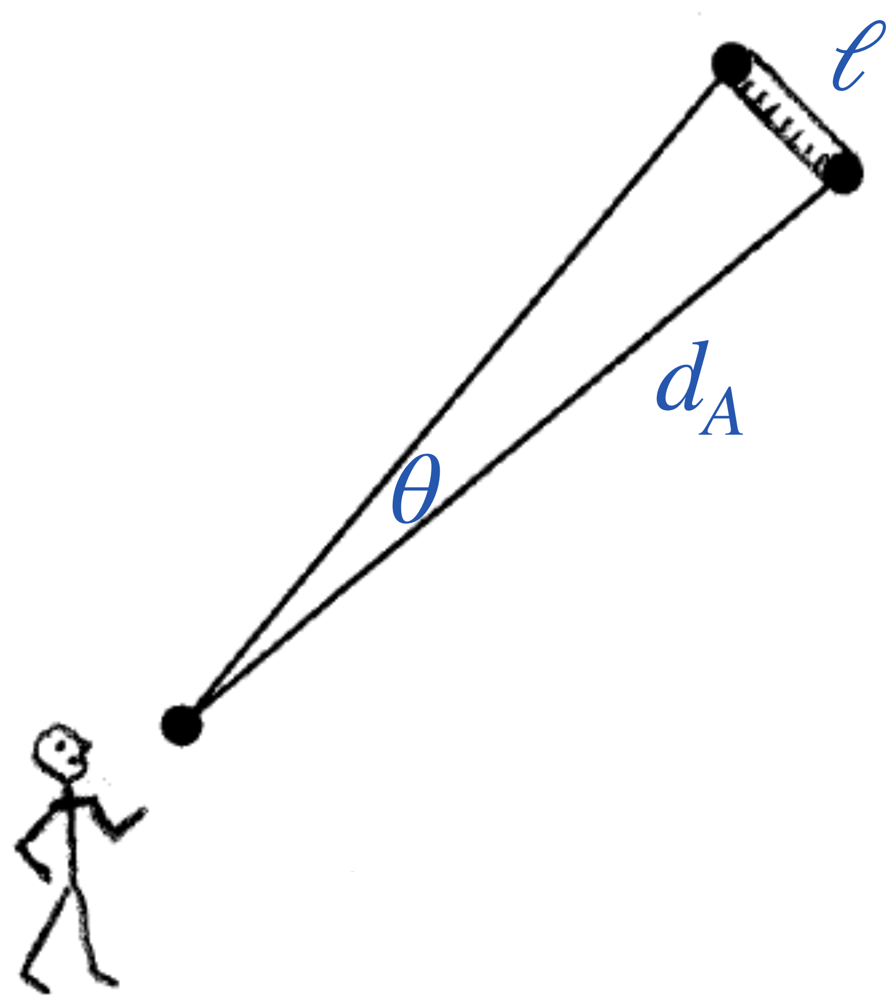


Figure 7.3: An observer at the origin observes a standard yardstick, of known proper length ℓ, at comoving coordinate distance r.

The luminosity distance and the angular diameter distance are identical to the proper distance in a Euclidean geometry.

We define luminosity and angular size distances for the convenience of our Euclidean intuitions when employing the Robertson-Walker geometry of a homogeneous and isotropic universe in General Relativity.

Magnitude system

http://astroweb.case.edu/ssm/ASTR620/mags.html

measured quantities

apparent magnitude & flux

The apparent magnitude is how bright an object appears to us. The flux is its measured brightness in physical units (e.g., photons/s/m²).

$$m = -2.5 \log(f/f_0) = -2.5 \log f + \xi$$

ξ zero point

Vega system: $m_{Vega} = 0$

A0 stars have zero color: B-V =0, U-B =0, etc.
commonly used for broad band filters

AB system: $\xi = -48.6$

$$m = -2.5 \log(f_\nu) - 48.6$$

monochromatic flux at frequency ν

intrinsic quantities

absolute magnitude & luminosity

absolute magnitude is the apparent magnitude an object would have if 10 pc distant.

$$M - M_\odot = -2.5 \log\left(\frac{L}{L_\odot}\right)$$

normalized here to the sun.

Luminosity can also be expressed in physical units like Watts or ergs/s or photons/s

distance modulus (m-M)
(neglecting interstellar extinction)

$$m - M = 5 \log\left(\frac{d_L}{10\ \mathrm{pc}}\right) = 5 \log d_L - 5$$

the absolute magnitude is defined as the apparent magnitude an object would have if 10 pc distant.

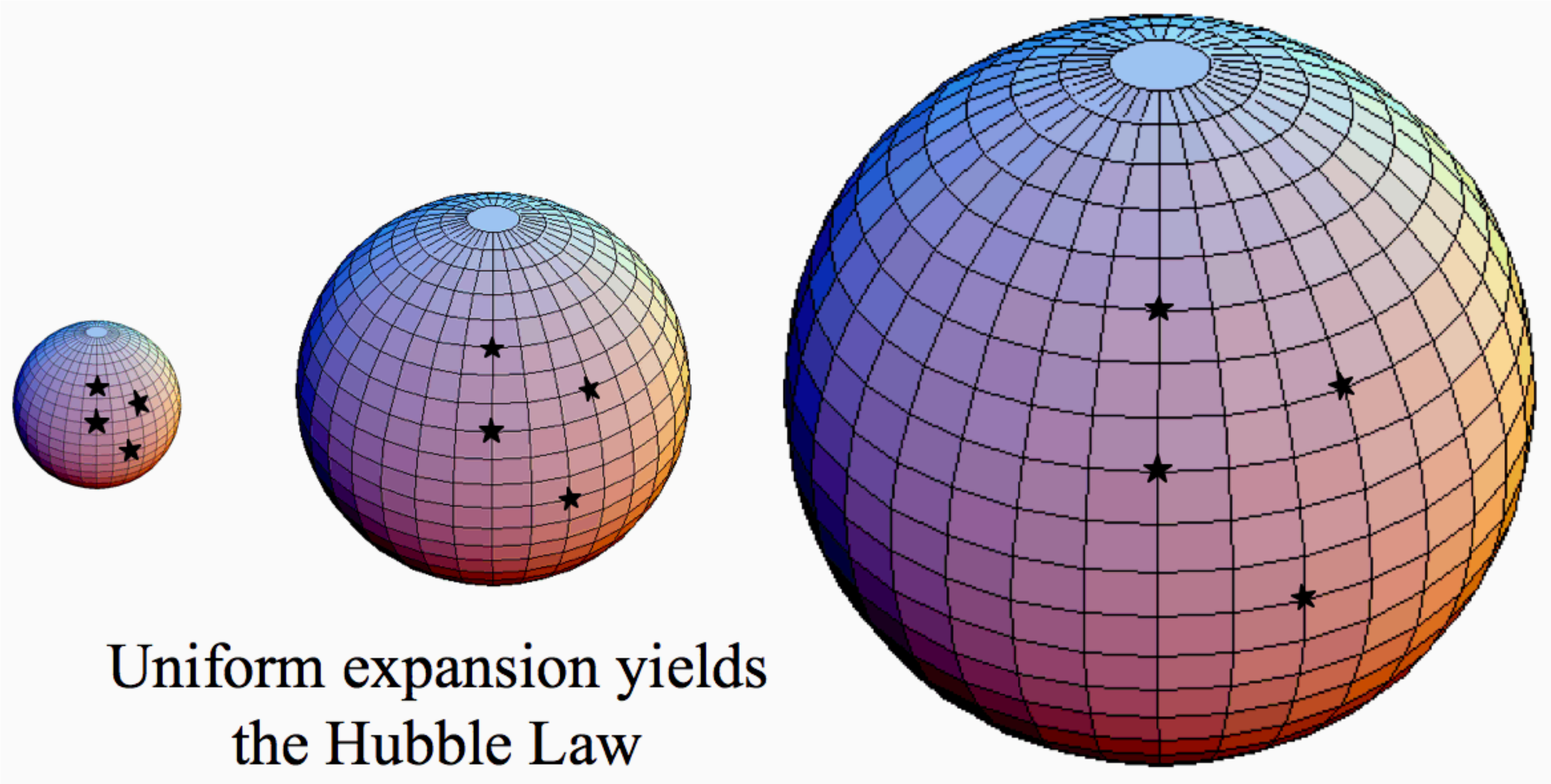
$$m - M = 5 \log d_L + 25 \quad \text{in Mpc}$$

The -5 turns into +25 if distance is measured in Mpc.
Note that this is a **luminosity distance**

NOTE: All logarithms are base ten unless otherwise specified. Natural logs will be written $\ln(x)$

Metrics

geometry in an expanding space-time



$$ds = a(t)dr$$

geometry in an expanding space-time

3D Euclidean geometry

For Cartesian coordinates (x,y,z) the separation between points is

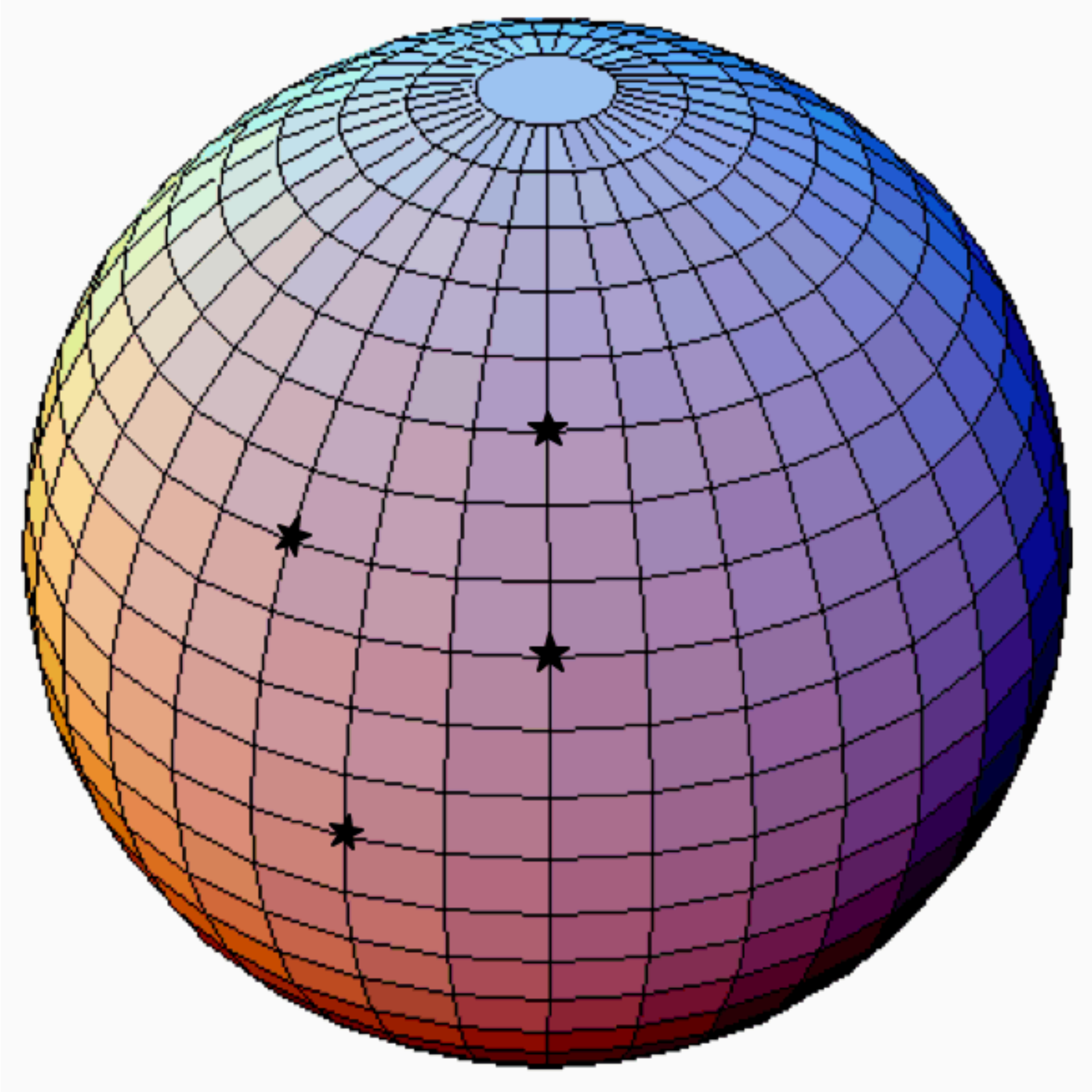
$$d\ell^2 = dx^2 + dy^2 + dz^2$$

For Spherical coordinates (r, θ, ϕ) the separation between points is

$$d\ell^2 = dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2)$$

$$d\ell^2 = dr^2 + r^2 d\Omega^2$$

$\sin\theta$ appears because E-W distances are smaller at high latitudes than at the equator.



Metrics

geometry in an expanding space-time

4D Minkowski Spacetime

For Cartesian coordinates (t,x,y,z) the separation between points is

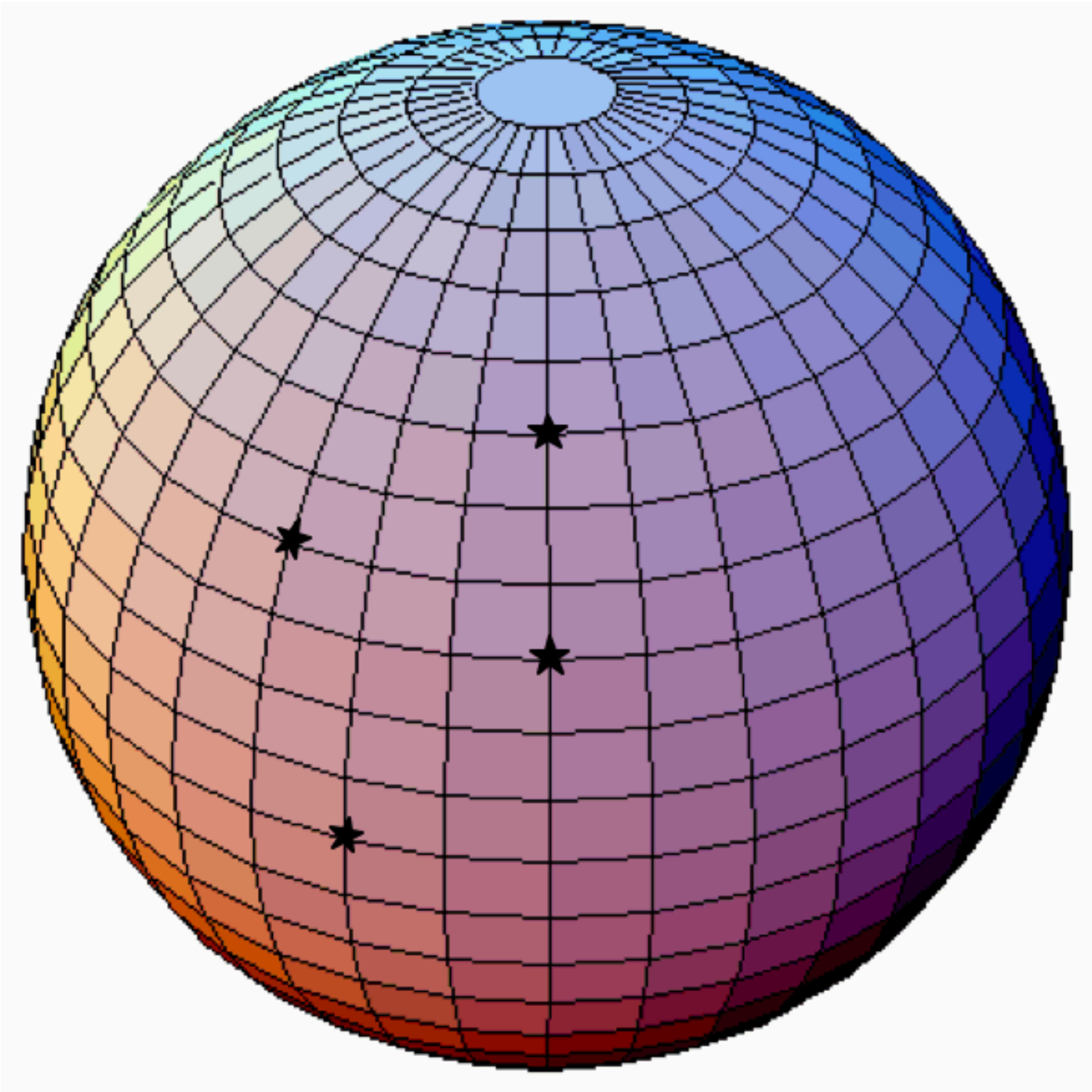
$$ds^2 = -c^2dt^2 + dx^2 + dy^2 + dz^2$$
$$ds^2 = -c^2dt^2 + d\ell^2$$

For Spherical coordinates (t, r, θ, ϕ) the separation between points is

$$ds^2 = -c^2dt^2 + dr^2 + r^2d\Omega^2$$

For a photon, $ds = 0$

$d\Omega$ is a convenient placeholder for the angular terms that we can usually ignore since we assume isotropy.



Metrics

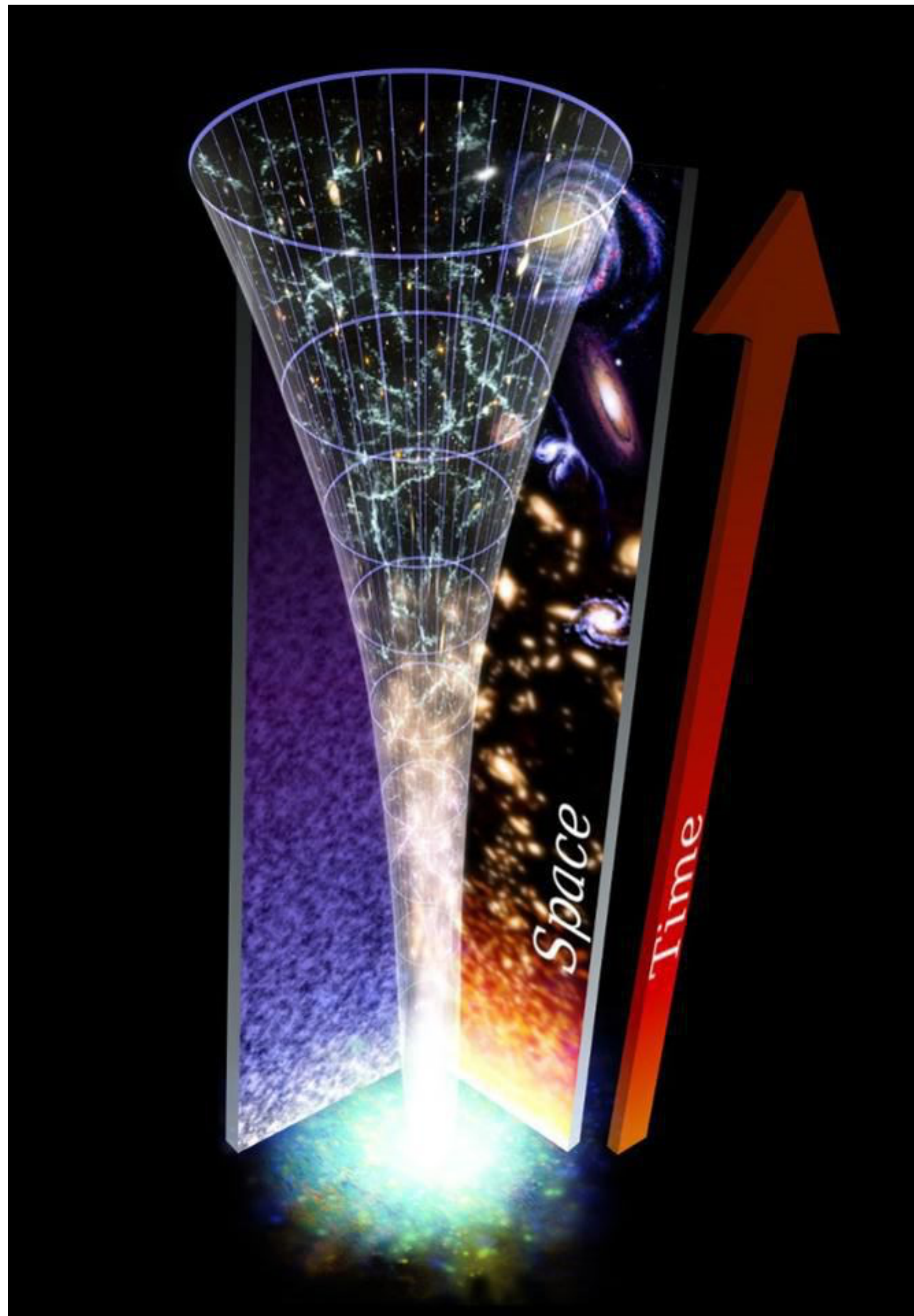
geometry in an expanding space-time

4D Robertson-Walker Spacetime

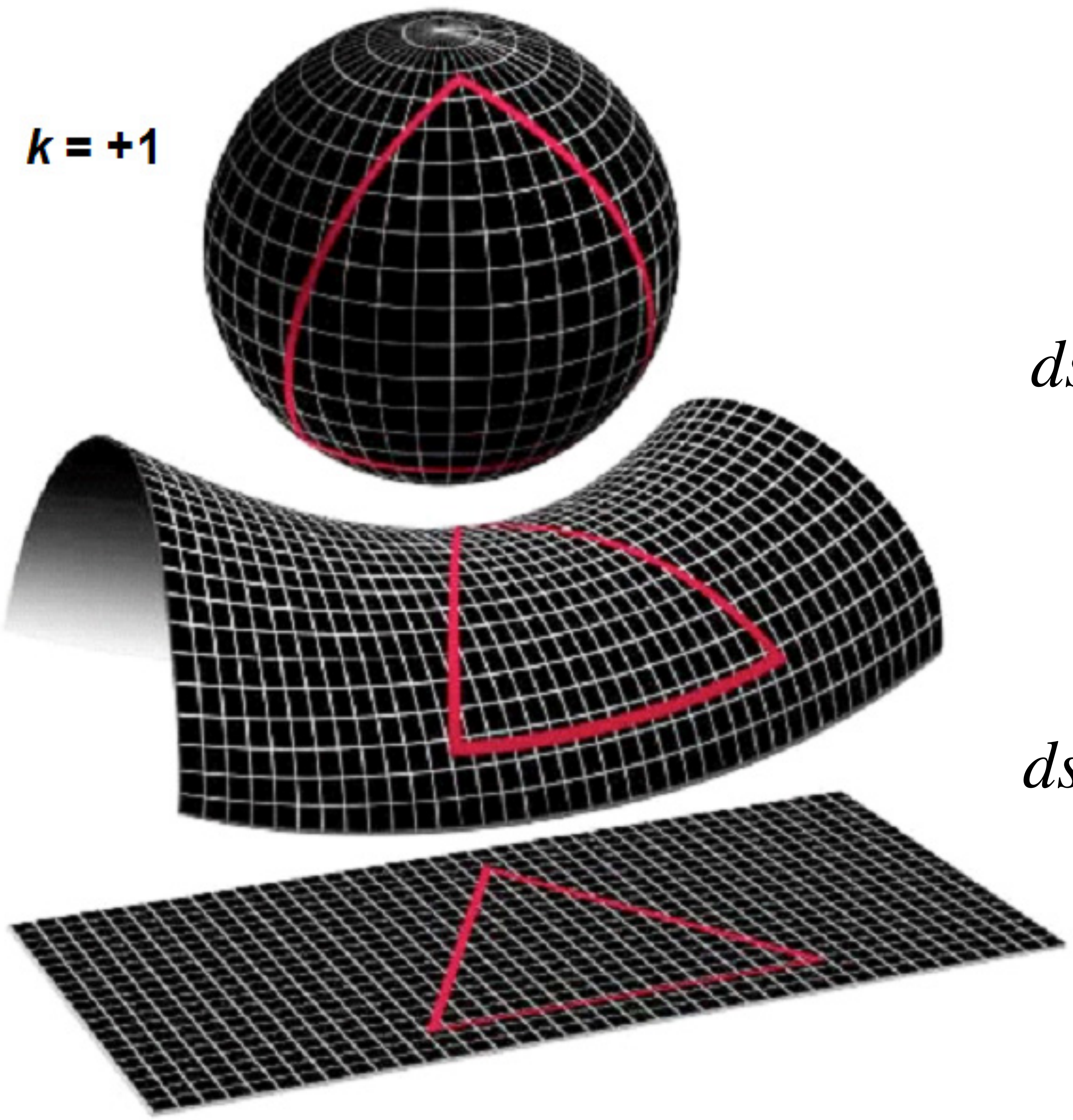
Derived from General Relativity
assuming homogeneity and isotropy

$$\underbrace{ds^2}_{\text{Event separation}} = -c^2dt^2 + \underbrace{a^2}_{\text{Expansion factor}}(t)[dr^2 + \underbrace{S_k^2}_{\text{Geometric factor}}(r)d\Omega^2]$$

For a photon, $ds = 0$



Geometric factor



$$S_k(r) = \begin{cases} R\sin\left(\frac{r}{R}\right) & \text{for } k = +1 \quad \text{Positively curved} \\ r & \text{for } k = 0 \quad \text{Flat} \\ R\sinh\left(\frac{r}{R}\right) & \text{for } k = -1 \quad \text{Negatively curved} \end{cases}$$

$$ds^2 = -c^2dt^2 + a^2(t)[dr^2 + S_k^2(r)d\Omega^2]$$

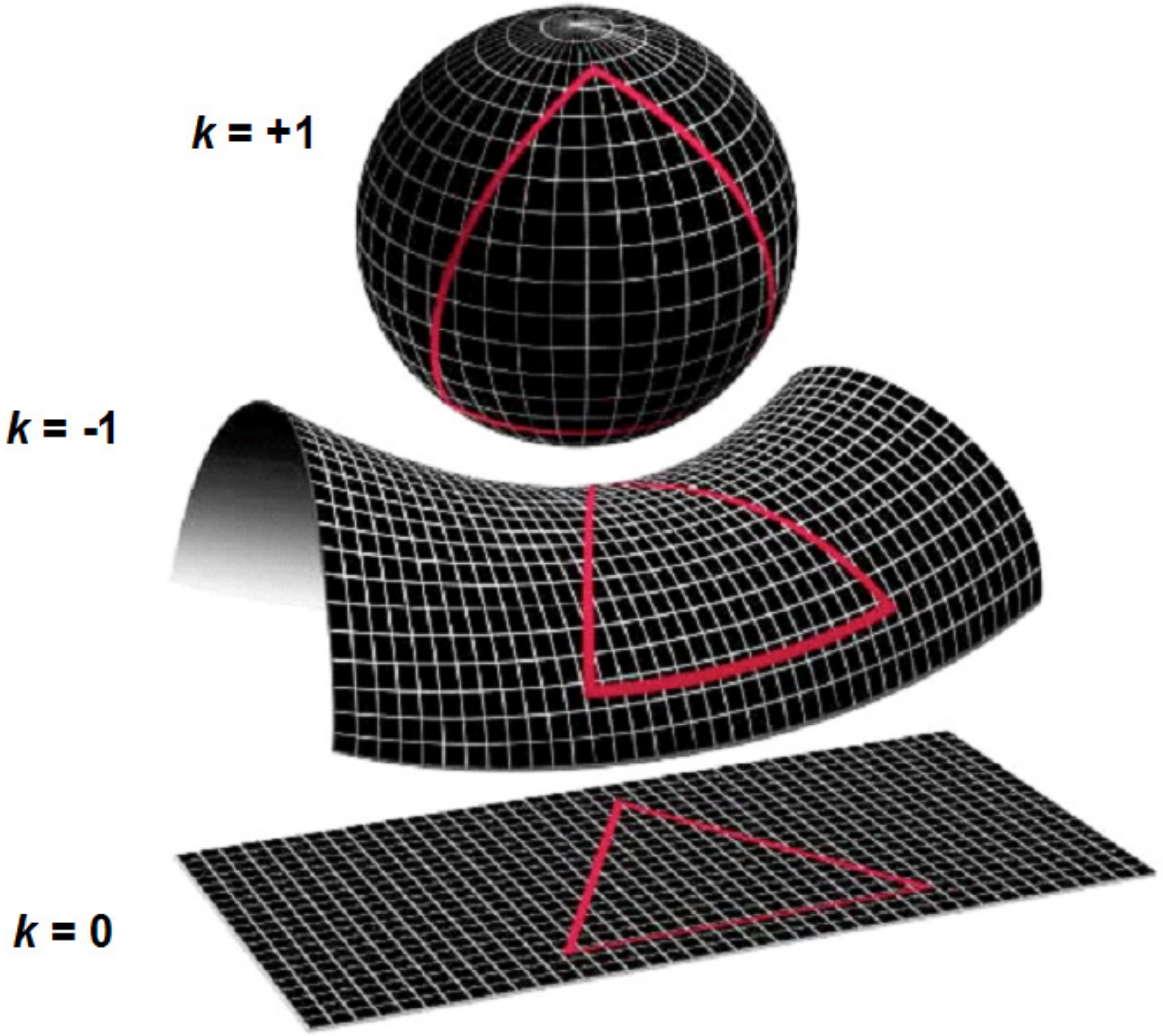
Ryden's notation

$$ds^2 = c^2dt^2 - a^2(t)\left[\frac{dr^2}{1 - kr^2} + r^2d\Omega^2\right]$$

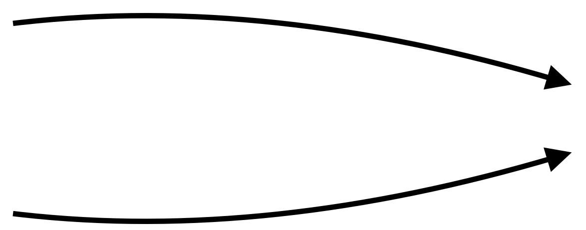
Another common notation swaps (- + + +) convention for (+ - - -) convention; absorbs difference in the definition of the comoving coordinate

r proper distance

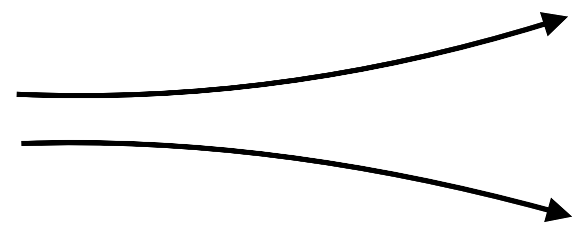
R curvature scale (no curvature in flat geometry)



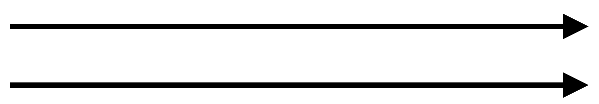
❖ **Positively** curved geometry
Finite total volume
Parallel rays converge
[exceeds critical density; eventually re-collapses]



❖ **Negatively** curved geometry
Infinite volume
Parallel rays diverge
[below critical density; expands forever]



❖ **Flat** geometry
Infinite volume
Parallel rays remain parallel
[exactly critical density; expands forever - just barely]



To get the proper distance to a galaxy we observe, we need to integrate over the expansion since the time of photon emission:

$$c\int_{t_e}^{t_0} \frac{dt}{a(t)} = \int_0^r dr = r$$

where we make use of the fact that for photons, $ds = 0$

so that

$$ds^2 = -c^2dt^2 + a^2(t)[dr^2 + S_k^2(r)d\Omega^2]$$

becomes

$$cdt = a(t)[dr^2 + S_k^2(r)d\Omega^2]^{1/2}$$

we know the expansion factor from the redshift

$$\frac{a(t_0)}{a(t_e)} = \frac{1}{a(t_e)} = 1 + z$$

but we need to know what kind of universe we live in to specify $S_k(r)$