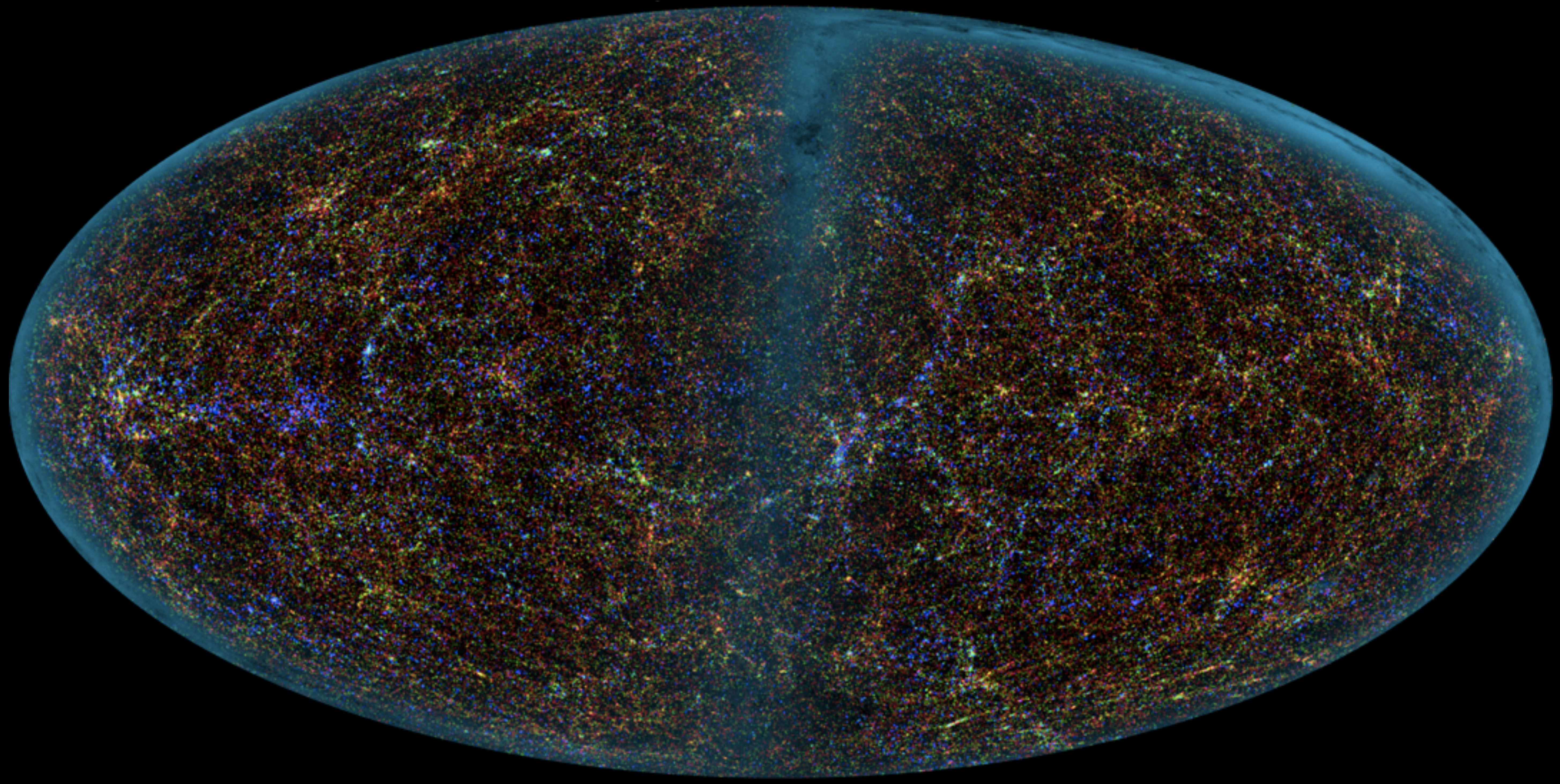
Cosmology

and Large Scale Structure



Today

Expansion dynamics

Time and Distance

13 September 2022

http://astroweb.case.edu/ssm/ASTR328/

Expansion dynamics

$$H = \frac{\dot{a}}{a}$$

$$a = (1+z)^{-1}$$

The Acceleration equation with the cosmological constant:

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{1}{3}\Lambda$$

can usually be replaced with a single variable, as $P = w\rho$ for a single medium.

The Pressure P is zero when matter dominates.
It is simply related to the energy density when radiation dominates.

You can see why the cosmological constant leads to acceleration!

$$\ddot{a} \sim \Lambda$$

$\frac{d}{dt}(\dot{a})^2 = 2\dot{a}\ddot{a}$ can be used to obtain the first order Friedmann equation

Expansion dynamics

Friedmann equation

$$H = \frac{\dot{a}}{a}$$

$$H^2 = H^2(\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda)$$

$$a = (1+z)^{-1}$$

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

Looks trivial, but H and Ω evolve. So really

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}$$

matter — radiation — curvature — cosmological constant

In general, must solve numerically.
But often we can ignore irrelevant terms -

Only one matters unless close to the redshift of matter-radiation equality $z_{\rm eq}$.

Zero in a flat universe

Zero in a sane universe

So often only two terms matter.
In the early universe, only one,
as the mass-energy dominates.

Expansion dynamics

Friedmann equation

$$H = \frac{\dot{a}}{a}$$

$$H^2 = H^2(\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda)$$

$$a = (1 + z)^{-1}$$

It is useful to consider the limit for domination by each case (matter, radiation, curvature, cosmological constant)

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

Simplifies to

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{m_0}(1 + z)^3 + \Omega_{k_0}(1 + z)^2$$

for a universe without a cosmological constant in the matter dominated era ($\Omega_m > \Omega_r$).

Or just

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{m_0}(1 + z)^3$$

at early times when $\Omega_m \to 1$.

Expansion dynamics

Friedmann equation

$$H^2 = H^2(\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda)$$

has some interesting limiting behaviors

$$H = \frac{\dot{a}}{a}$$

$$a = (1 + z)^{-1}$$

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

Simplifies to

$$\left(\frac{H}{H_0}\right)^2 = \Omega_{r_0}(1 + z)^4$$

for the early, radiation dominated universe when $\Omega_r \gg \Omega_m$.

Expansion history

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

Friedmann equation

$$H = \frac{\dot{a}}{a}$$

$$H^2(z) = H_0{}^2[\Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}]$$

$$a = (1+z)^{-1}$$

It is convenient to define the Expansion term

$$E^2(z) = \Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}$$

or equivalently

$$E^2(a) = \Omega_{m_0}a^{-3} + \Omega_{r_0}a^{-4} + \Omega_{k_0}a^{-2} + \Omega_{\Lambda_0}$$

So that

$$H(z) = H_0 E(z)$$

or equivalently

$$\frac{\dot{a}}{a} = H_0 E(a)$$

Generalization of the search for two numbers: now want to measure H_0, $E(z)$
where $E(z)$ contains information about the various $\mathbf{\Omega}$.

Expansion history

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

$$H = \frac{\dot{a}}{a}$$

$$a = (1+z)^{-1}$$

$$E(z) = \sqrt{\Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}}$$

Friedmann equation

$$\frac{\dot{a}}{a} = H_0 E(a)$$

If we don't know the full details of `E(a)`, we can make a Taylor expansion

$$a(t) \approx 1 + H_0(t - t_0) - \frac{1}{2} q_0 H_0^2 (t - t_0)^2 + \ldots$$

where we see the deceleration parameter as the next term after the Hubble constant

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{1}{H^2}\frac{\ddot{a}}{a}$$

so q_0 *becomes a proxy for E(z)*

Expansion history

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

If we don't know the full details of `E(a)`, we can make a Taylor expansion

$$H = \frac{\dot{a}}{a}$$

$$a(t) \approx 1 + H_0(t - t_0) - \frac{1}{2}q_0 H_0^2 (t - t_0)^2 + \dots$$

$$a = (1+z)^{-1}$$

can define higher order terms that are increasingly difficult to measure

$$E(z) = \sqrt{\Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}}$$

deceleration parameter

$$q = -\frac{a\ddot{a}}{\dot{a}^2} = -\frac{1}{H^2}\frac{\ddot{a}}{a}$$

The deceleration parameter is defined with the negative sign so it would be a positive number in a decelerating universe because we *really* expected that would be the case.

jerk

$$j = \frac{a^2\dddot{a}}{\dot{a}^3} = \frac{1}{H^3}\frac{\dddot{a}}{a}$$

snap

$$s = \frac{a^3\ddddot{a}}{\dot{a}^4} = \frac{1}{H^4}\frac{\ddddot{a}}{a}$$

crackle, pop...

Solutions from Felten & Isaacman (1986) Reviews of Modern Physics, **58**, 689

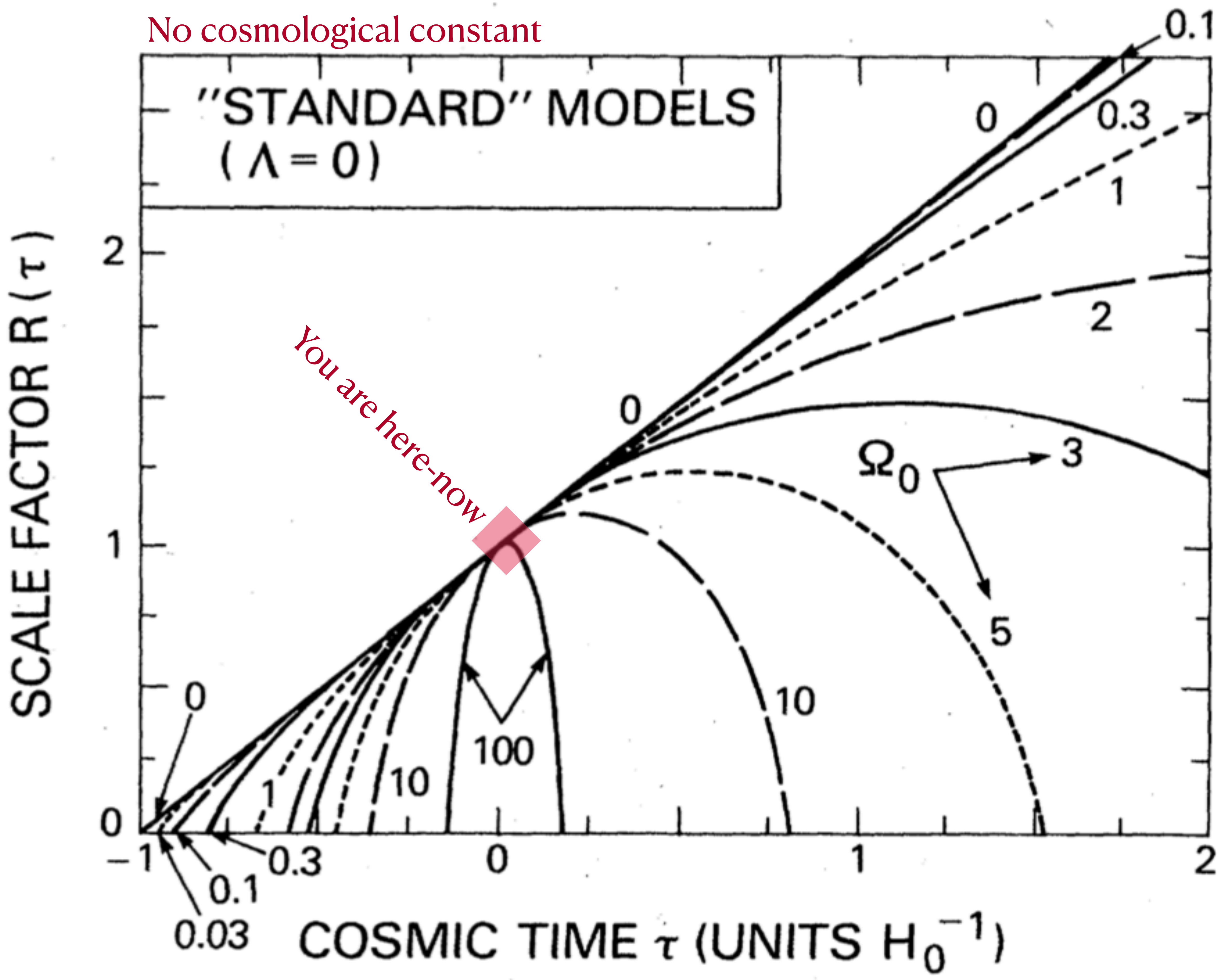


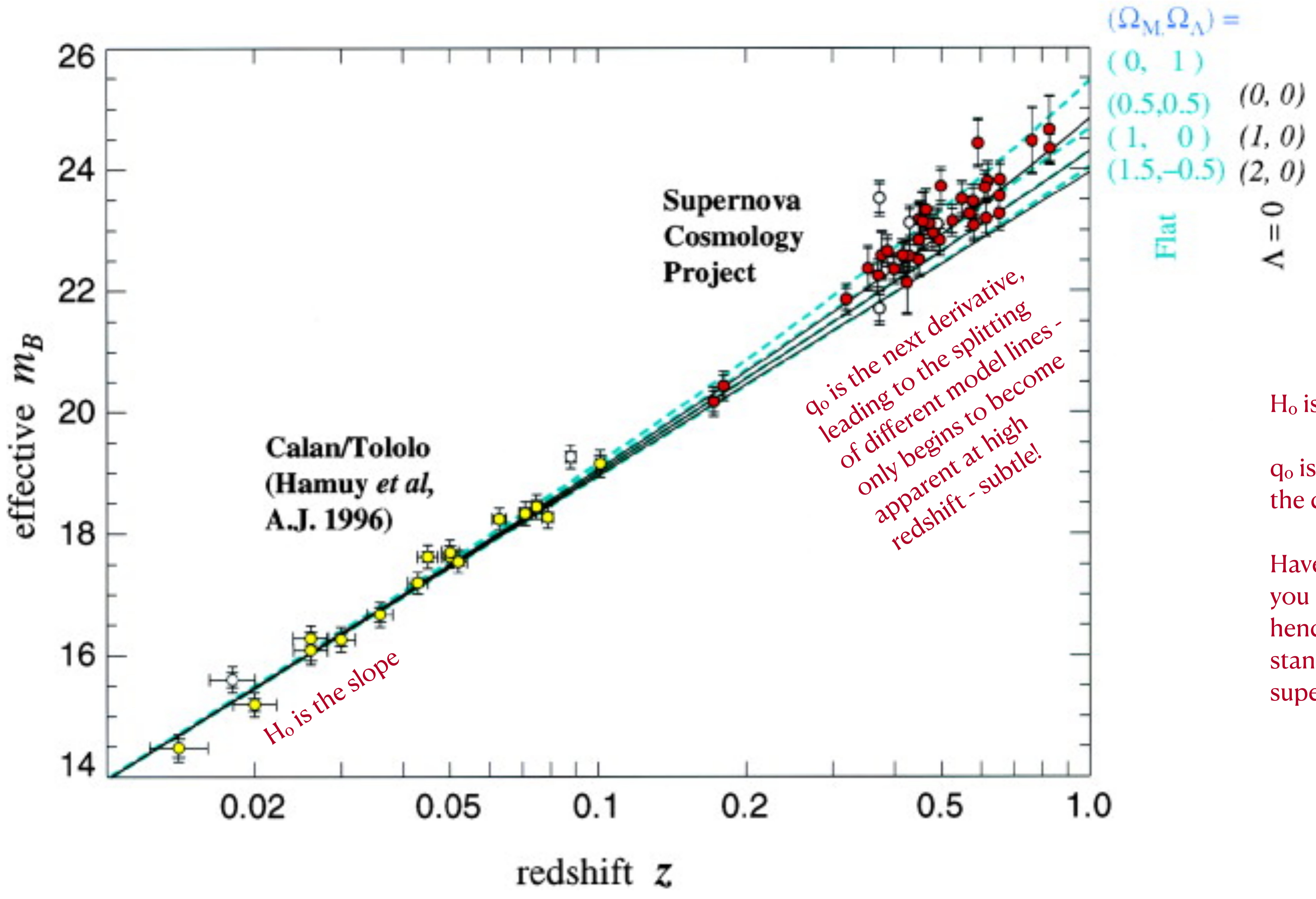
FIG. 3. "Standard" Friedmann models. The family of scale factors $R(\tau)$ for the "standard models" ($\Lambda=0$). The free parameter, shown on the curves, is Ω_0. As shown by the τ intercepts, all models have ages ≤ 1 ($\leq H_0^{-1}$ yr).

H_o is the slope

q_o is the next derivative - the change in the slope

Have to see far away before you can start to perceive q_o, hence the desire for bright standard candles like supernovae.

Hubble Diagram



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Expansion history

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

$$H = \frac{\dot{a}}{a}$$

$$a = (1+z)^{-1}$$

$$E(z) = \sqrt{\Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}}$$

There is an analytic solution for matter domination - can parameterize the expansion as a cycloid

$$a = \frac{\Omega_m}{2(1-\Omega_m)}(\cosh\eta - 1)$$

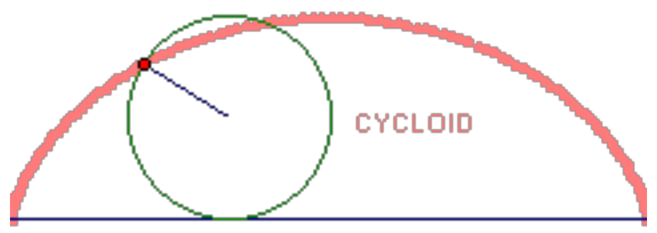
Peebles 13.10

$$H_0 t = \frac{\Omega_m}{2(1-\Omega_m)^{3/2}}(\sinh\eta - \eta)$$

where η is the development parameter - related to the conformal time

The current value of the development parameter is

$$\cosh\eta_0 = \frac{2}{\Omega_{m_0}} - 1$$



This is of no use now because of Lambda, BUT it does become useful for the growth of structure in the early universe, when every protogalaxy can be considered its own little island universe with $\Omega_m \gtrsim 1$

comoving coordinates constant

Robertson-Walker metric

$$ds^2 = -c^2dt^2 + a^2(t)[dr^2 + S_k^2(r)d\Omega^2]$$

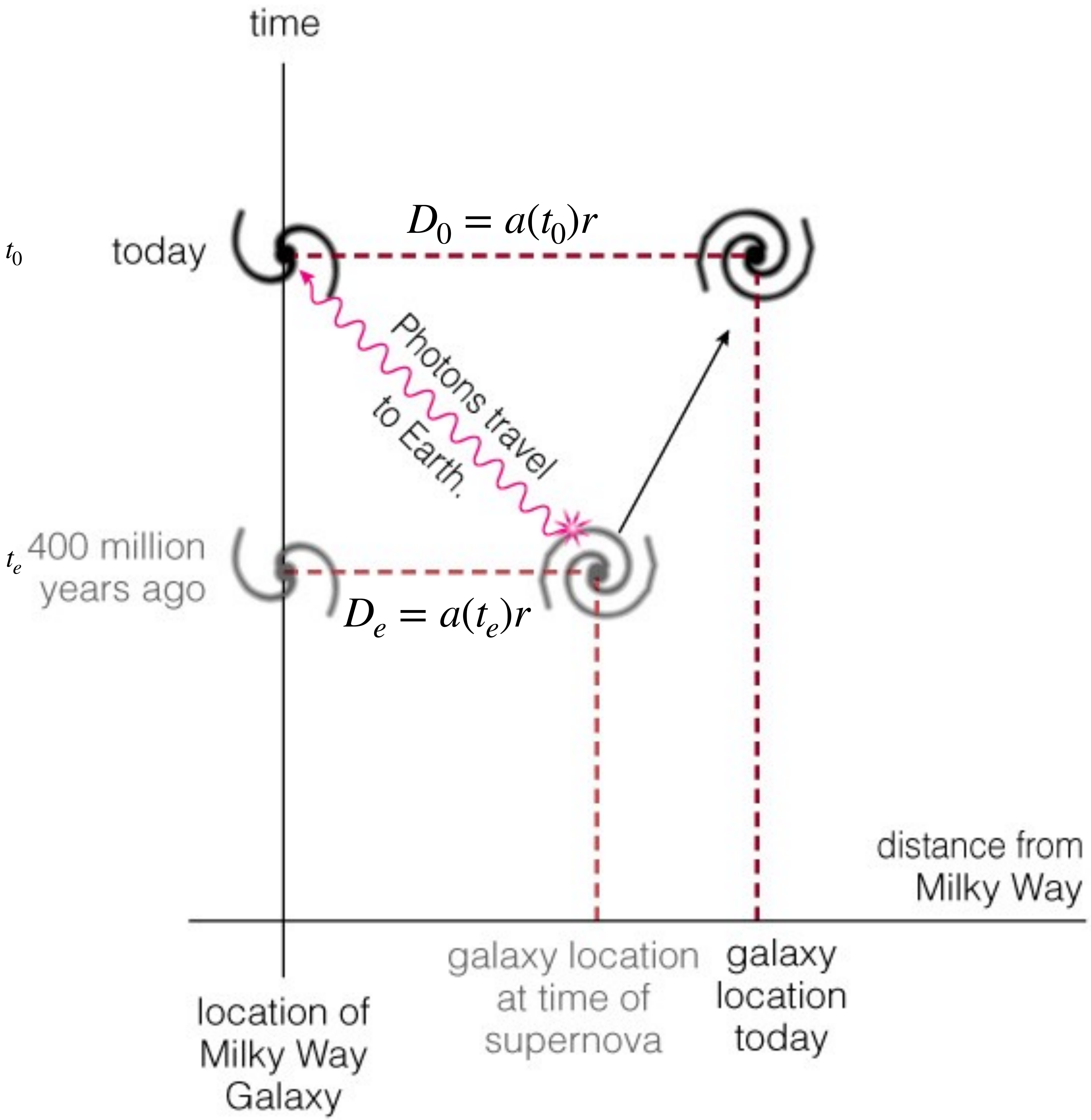
for photons, $ds = 0$ so this becomes

$$cdt = a(t)dr$$

Once we know (or assume) what kind of universe we live in, we specify the expansion history *a(t)*.

we know the expansion factor from the redshift

$$\frac{a(t_0)}{a(t_e)} = 1 + z$$



comoving coordinates constant

Once we know (or assume) what kind of universe we live in, we specify the expansion history *a(t)*.

we know the expansion factor from the redshift $\frac{a(t_0)}{a(t_e)} = 1 + z$

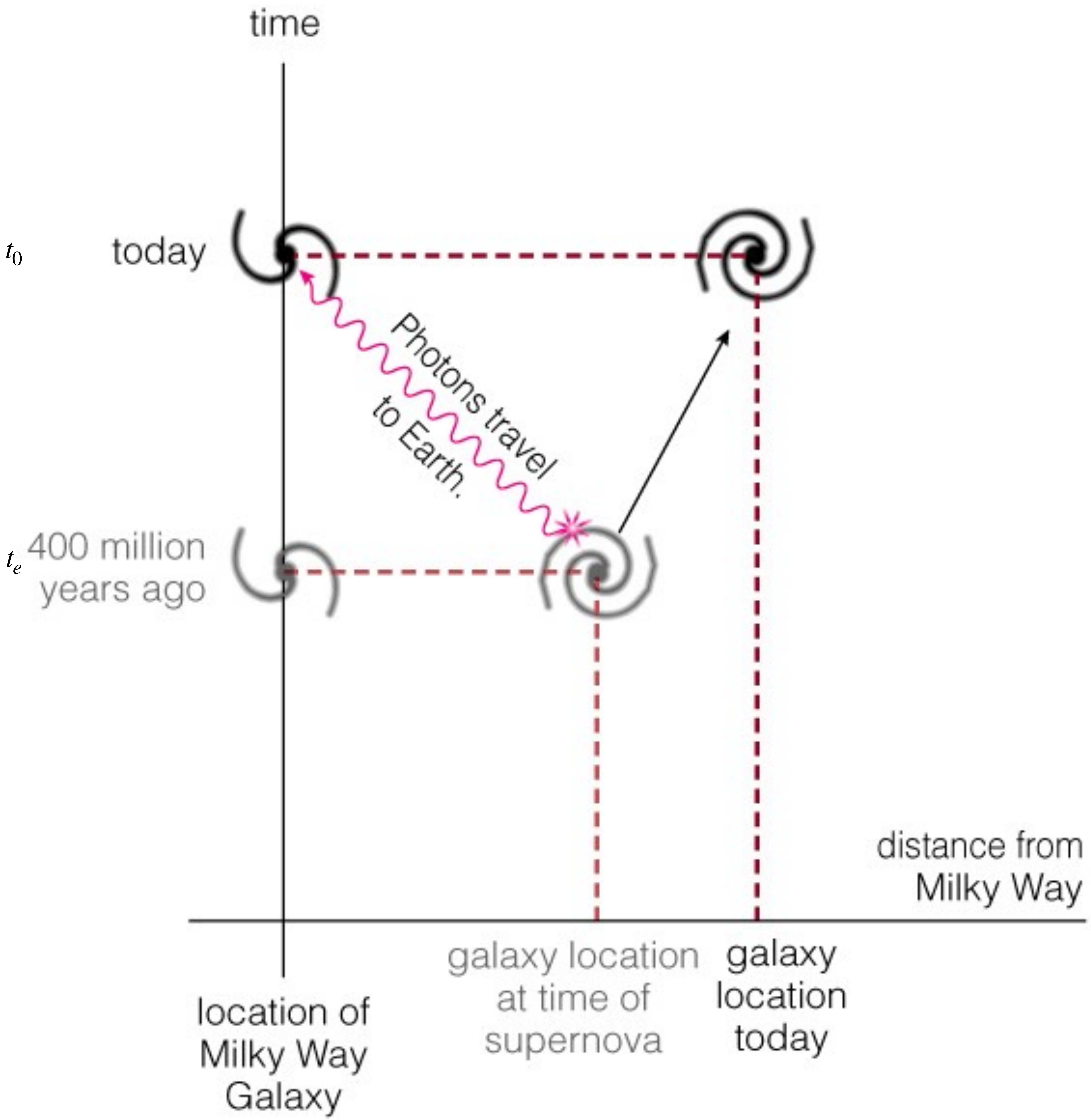
a photon propagating through the expanding universe traverses a distance element

$$d\ell = cdt = a(t)dr$$

The comoving separation between two points is fixed, so

$$r = \int_0^r dr = c\int_{t_e}^{t_0} \frac{dt}{a(t)}$$

Relates observed redshift to the time of photon emission (400 Myr ago in the example at right).



comoving coordinates constant

Once we know (or assume) what kind of universe we live in, we specify the expansion history *a(t)*.

we know the expansion factor from the redshift

a photon propagating through the expanding universe traverses a distance element

$$d\ell = c\,dt = a(t)dr$$

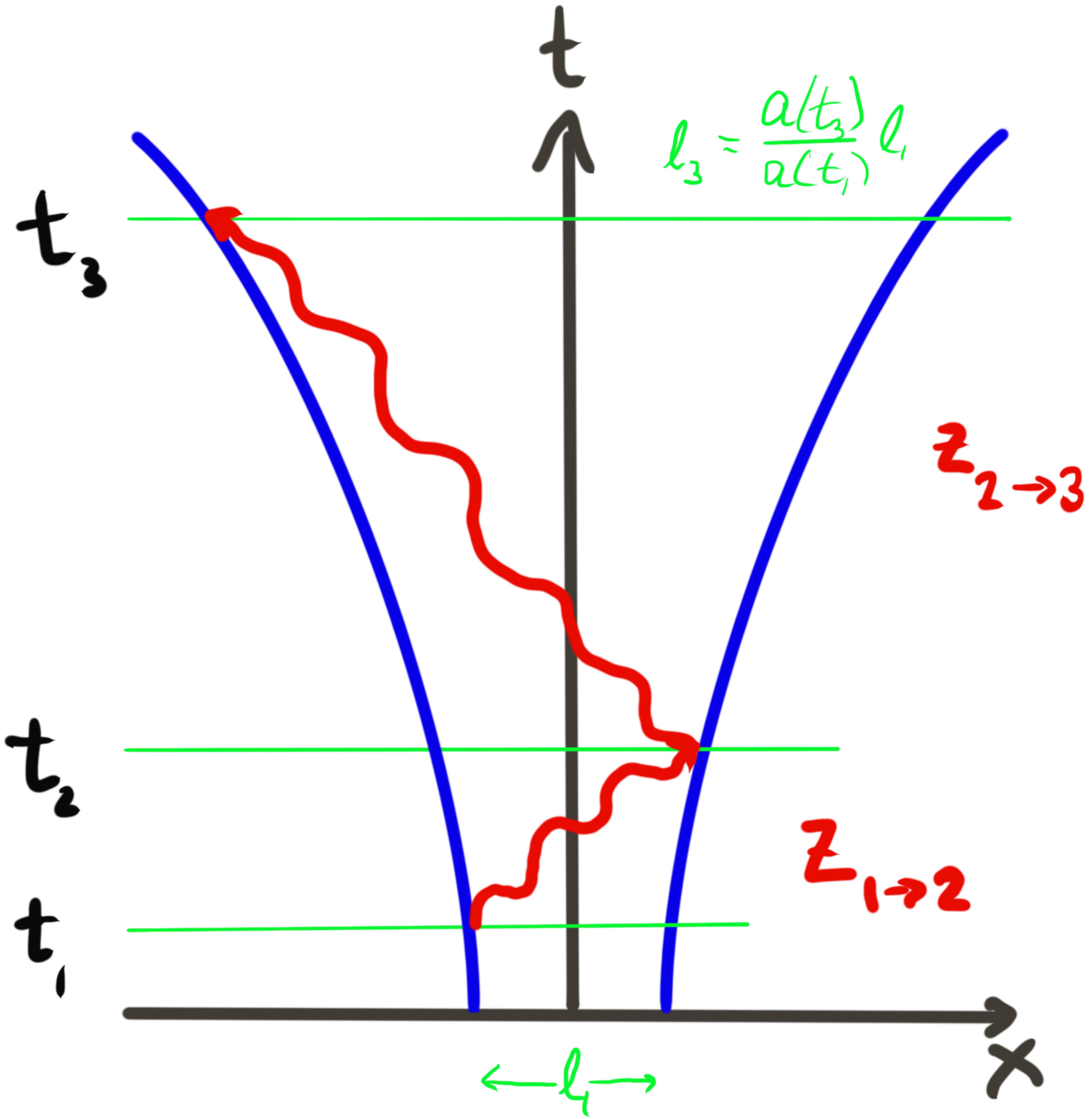
The comoving separation between two points is fixed, so

$$r = c\int_{t_1}^{t_2} \frac{dt}{a(t)} = c\int_{t_2}^{t_3} \frac{dt}{a(t)}$$

and

$$\frac{a(t_2)}{a(t_1)} = 1 + z_{1\to 2} \qquad \frac{a(t_3)}{a(t_2)} = 1 + z_{2\to 3}$$

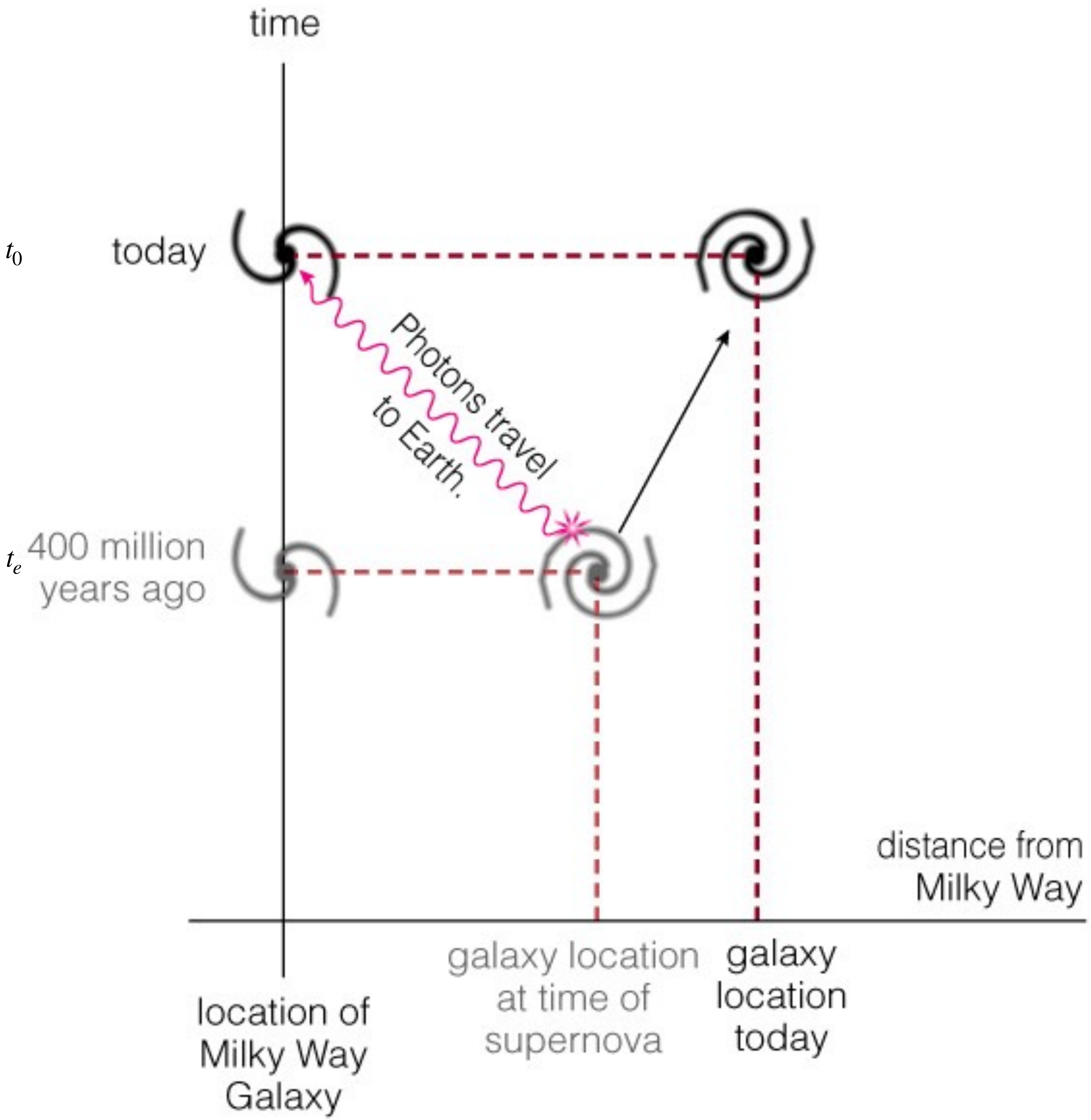
relates the redshift to the expansion factor



The proper distance

$$D_p(t_0) = c\int_{t_e}^{t_0} \frac{dt}{a(t)}$$

$$a(t) \approx 1 + H_0(t - t_0) - \frac{1}{2}q_0H_0^2(t - t_0)^2 + \ldots$$



In terms of redshift,

$$D_p(z_e) = \frac{c}{H_0}\int_0^{z_e} \frac{dz}{E(z)}$$

For **zero** cosmological constant, there is an exact solution known as Mattig's equation:

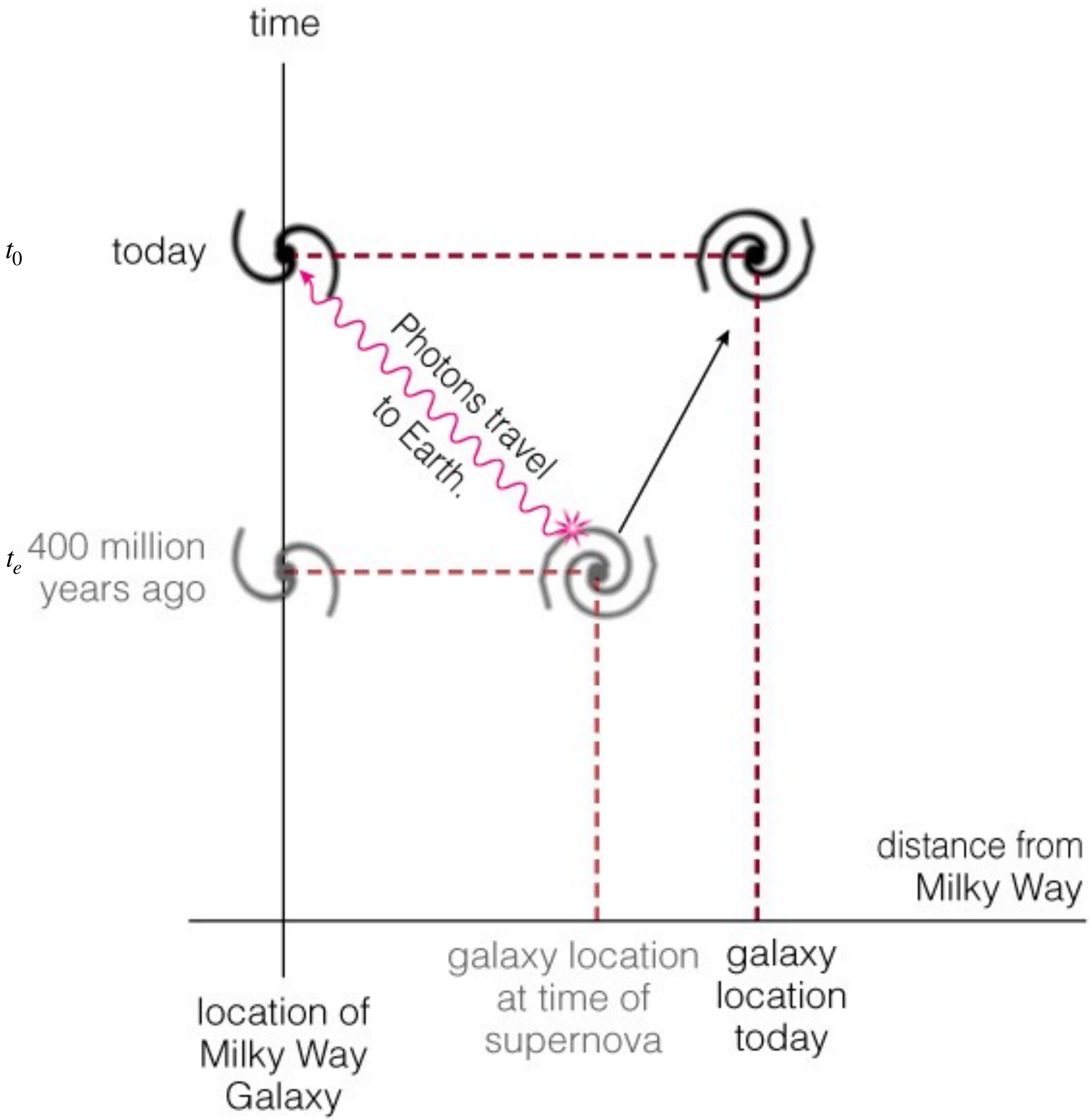
$$D_p(z) = \frac{2c}{H_0}\frac{[z\Omega_m + (\Omega_m - 2)(\sqrt{1 + z\Omega_m} - 1])}{\Omega_m^2(1+z)}$$

In general, there is no analytic solution, but can approximate with the Taylor expansion:

$$D_p(z) = \frac{c}{H_0}\left[z - \frac{1}{2}(1+q_0)z^2\right]$$

Where the leading term is Hubble's Law

$$D_p(z) = \frac{cz}{H_0}$$



For time rather than distance

Friedmann equation

$$\frac{\dot{a}}{a} = H_0 E(a)$$

$$H_0 \int_{t_e}^{t_o} dt = \int_{a_e}^{1} \frac{da}{aE(a)} = \int_0^{z_e} \frac{dz}{(1+z)E(z)}$$

$$H_0(t_0 - t_e) = \int_0^{z_e} \frac{dz}{(1+z)E(z)}$$

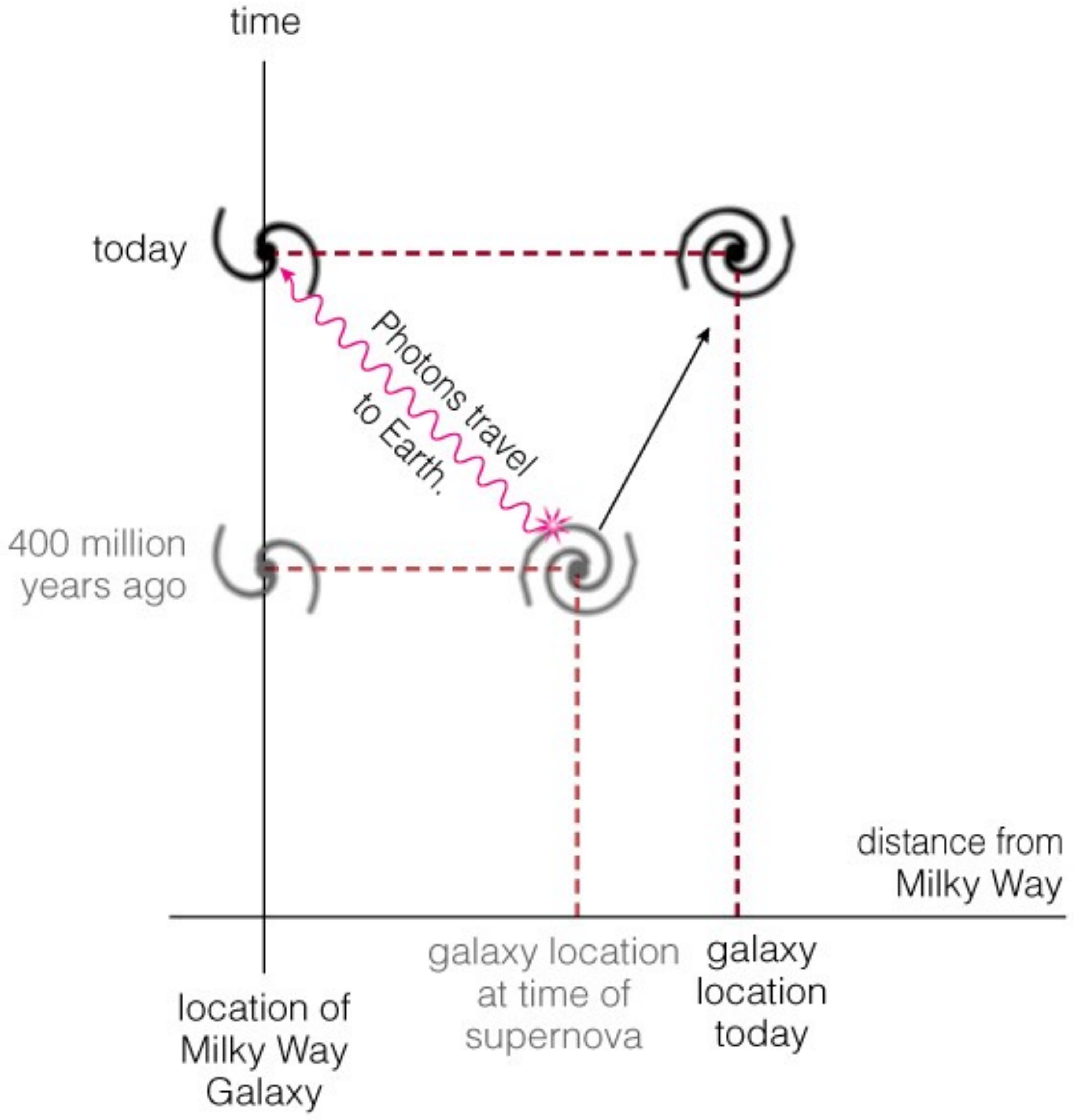
$(t_0 - t_e)$ is the **Lookback time** - the time since the photon was emitted.

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

$$H = \frac{\dot{a}}{a}$$

$$a = (1+z)^{-1}$$

$$E(z) = \sqrt{\Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}}$$



The age of the universe is obtained by setting

$$t_e = 0\ ;\ z \to \infty$$

$$H_0 t_0 = \int_0^\infty \frac{dz}{(1+z)E(z)}$$

which can be approximated by

$$t_0 \approx \left(\frac{2}{3H_0}\right)(0.7\Omega_{m_0} + 0.3 - 0.3\Omega_{\Lambda_0})^{-0.3}$$

There is no deep theory in this last formula.
It is just a fitting formula that approximates the answer to a few %.

Similarly, the redshift-age of a matter dominated universe can be approximated as

$$\frac{1}{t(z)} \approx H(z)[1 + \frac{1}{2}\Omega_m^{0.6}(z)]$$ Peacock (3.46)

$$\Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

$$H = \frac{\dot{a}}{a}$$

$$a = (1+z)^{-1}$$

$$E(z) = \sqrt{\Omega_{m_0}(1+z)^3 + \Omega_{r_0}(1+z)^4 + \Omega_{k_0}(1+z)^2 + \Omega_{\Lambda_0}}$$