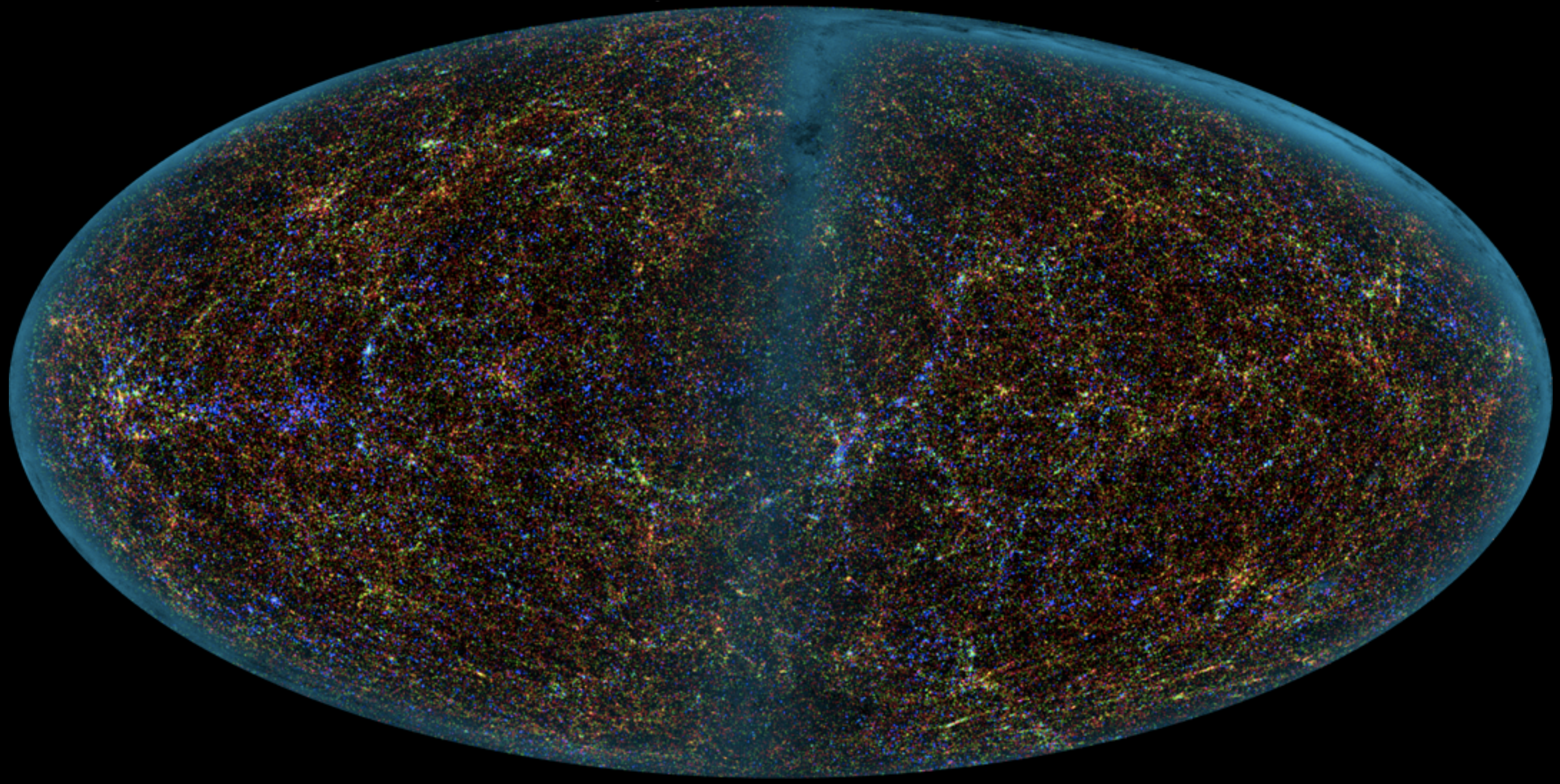
Cosmology

and Large Scale Structure



<u>Today</u>

Observational Tests

Luminosity Distance-redshift
- e.g., Type Ia SN

Angular size-redshift

Number Counts
- e.g., Galaxy N(m), N(z)

homework 2 due next time

Observational Tests

Five Classic Tests

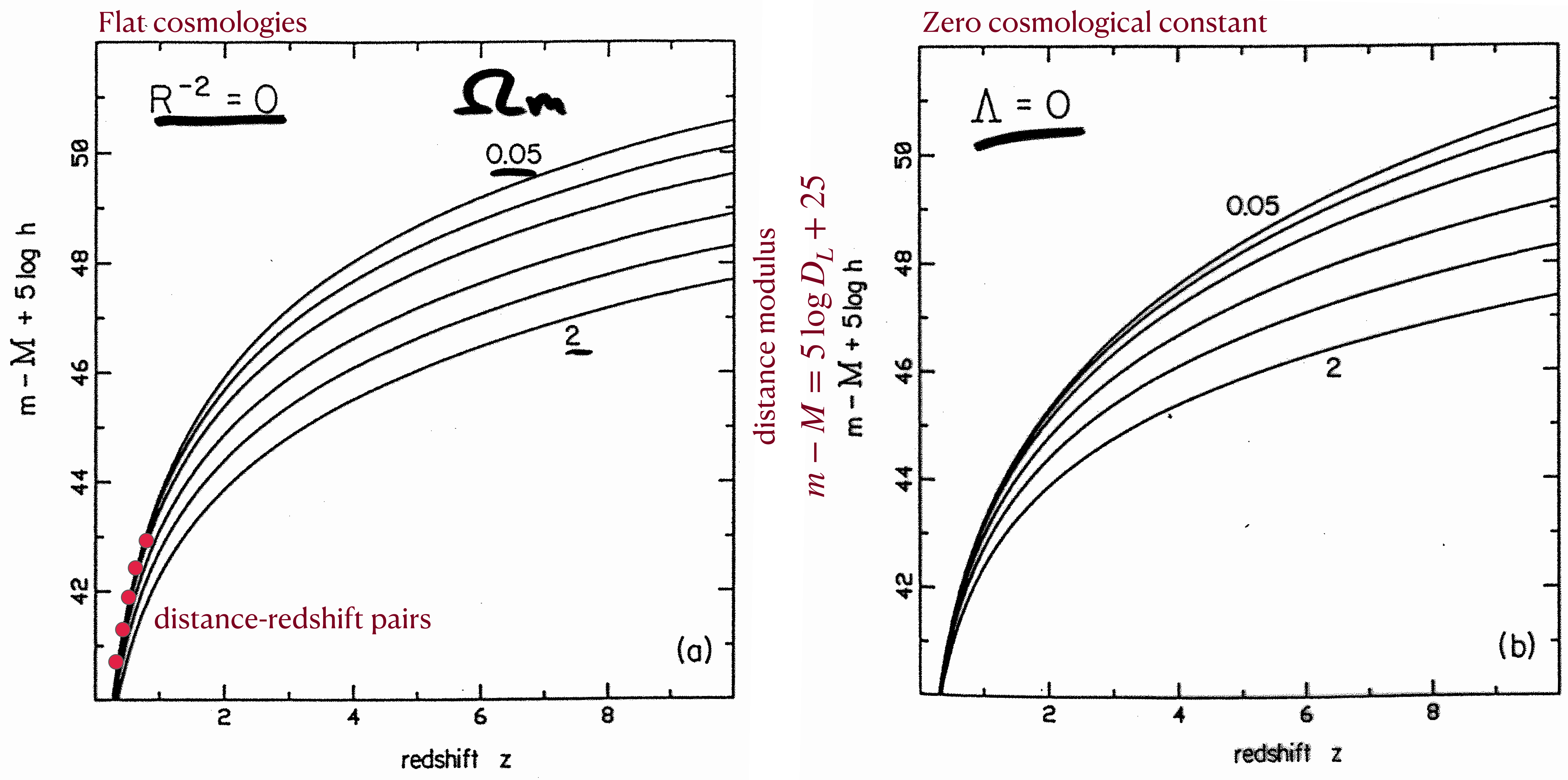
• Luminosity-redshift relation	$D_L - z$	Standard Candle
• Angular size-redshift relation	$D_A - z$	Standard Rod
• Number-redshift relation	$N(z)$	Source counts with redshift
• Number-magnitude relation	$N(m)$	Source counts with magnitude
• Tolman test	$\Sigma(z)$	Surface brightness not distance independent in Robertson-Walker geometry

Other tests are possible. E.g., one could in principle make an age-redshift test - if one could confidently measure ages of objects at cosmic distances.

- Luminosity-redshift relation

Luminosity Distance-redshift relations

Figure 13.6. Bolometric distance modulus $m - M + 5\log h$ as a function of redshift. The parameters are arranged as in figure 13.1.



Peebles Fig. 13.6

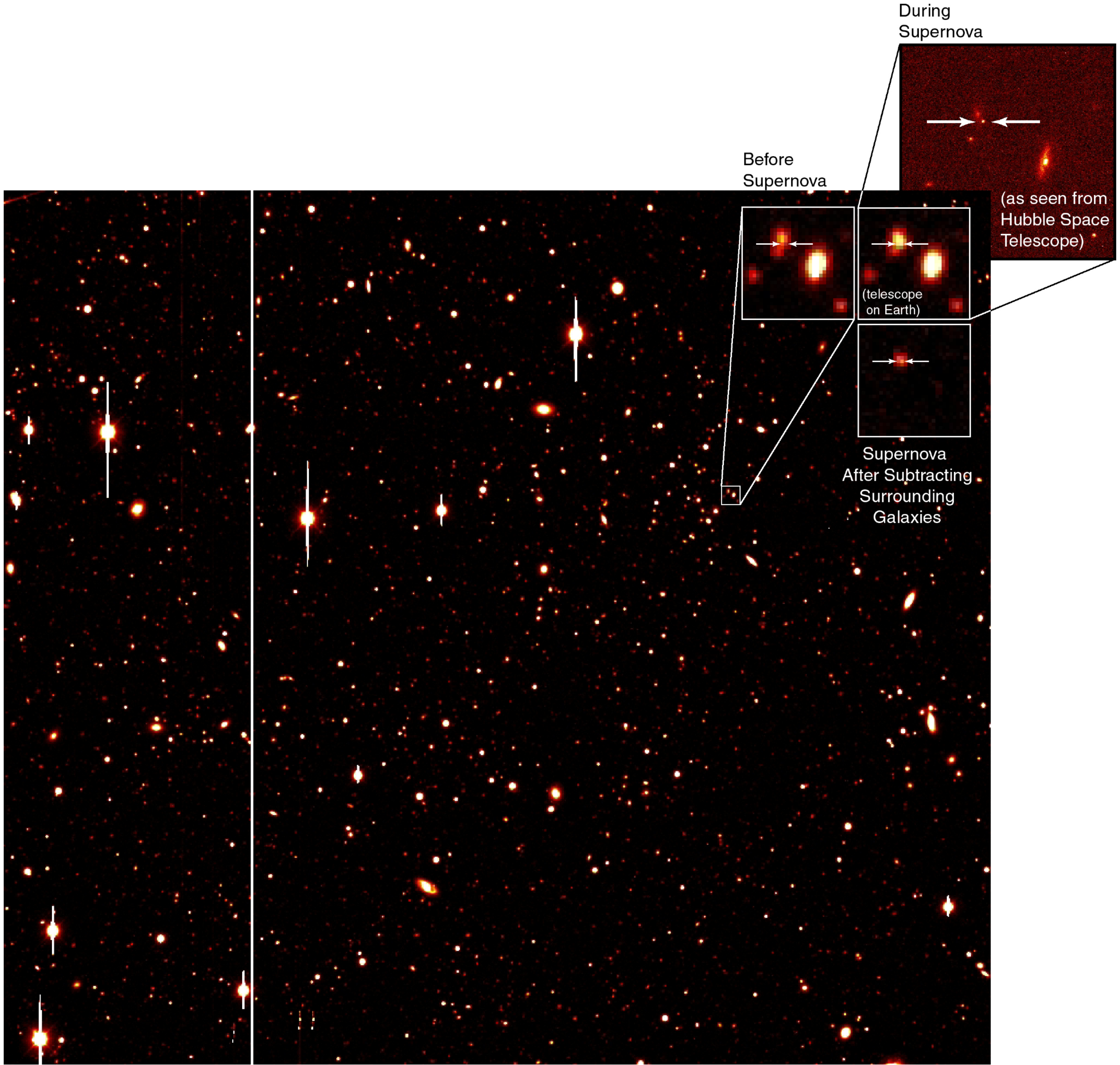
Example Standard Candle:

- Type Ia Supernovae

Survey wide swath of sky, imaging repeatedly over many nights, looking for change. If you look at enough galaxies, you'll see SN go off.

Supernova Cosmology Project

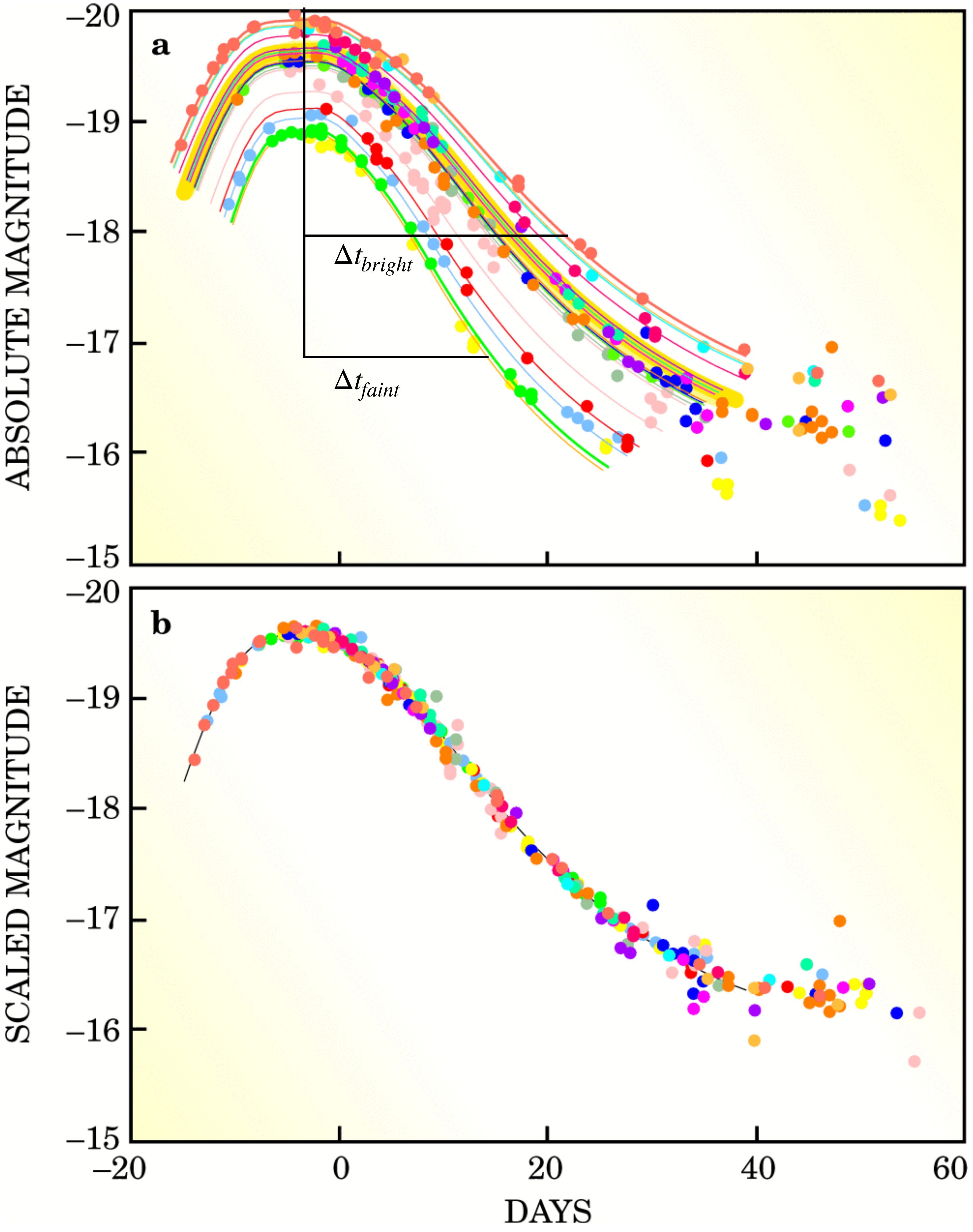
Perlmutter et al. (1998)



$$M_{max,corr} = M_{max,obs} + f(\Delta t)$$

Type Ia SN not quite standard candles, but standardizable: the peak brightness correlates with rest-frame decay time.

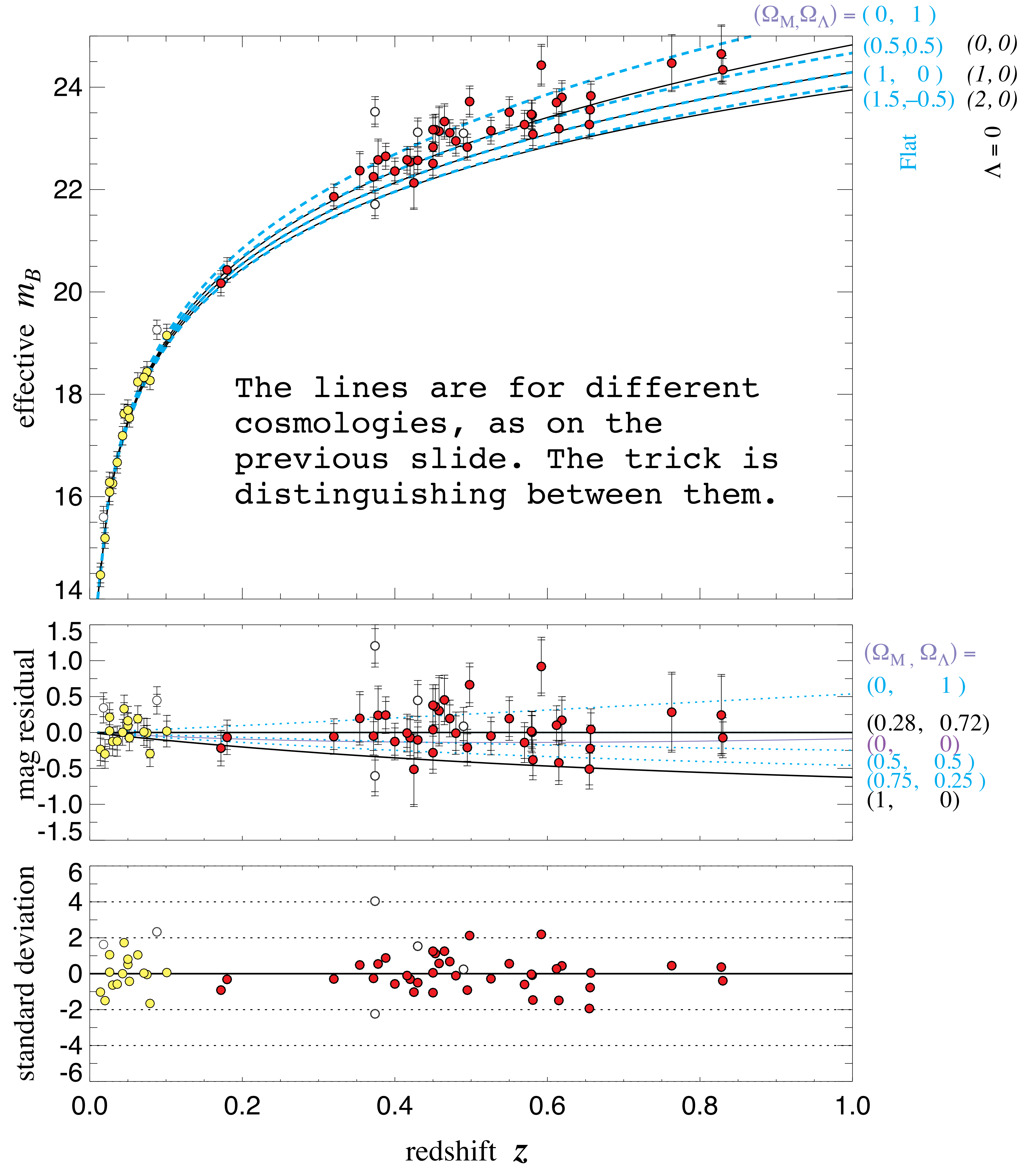
$$m_{max,corr} = M_{max,corr} - 5\log H_0 + 25 + 5\log\left(cz(1 + \frac{1-q_0}{2}z)\right)$$



• Luminosity-redshift relation

<u>Hubble diagram</u>
apparent magnitude vs. redshift
equivalent to distance modulus for standard candle
(M constant)

This example for Type Ia SN from the
Supernova Cosmology Project
(won the Nobel Prize in Physics in 2011)



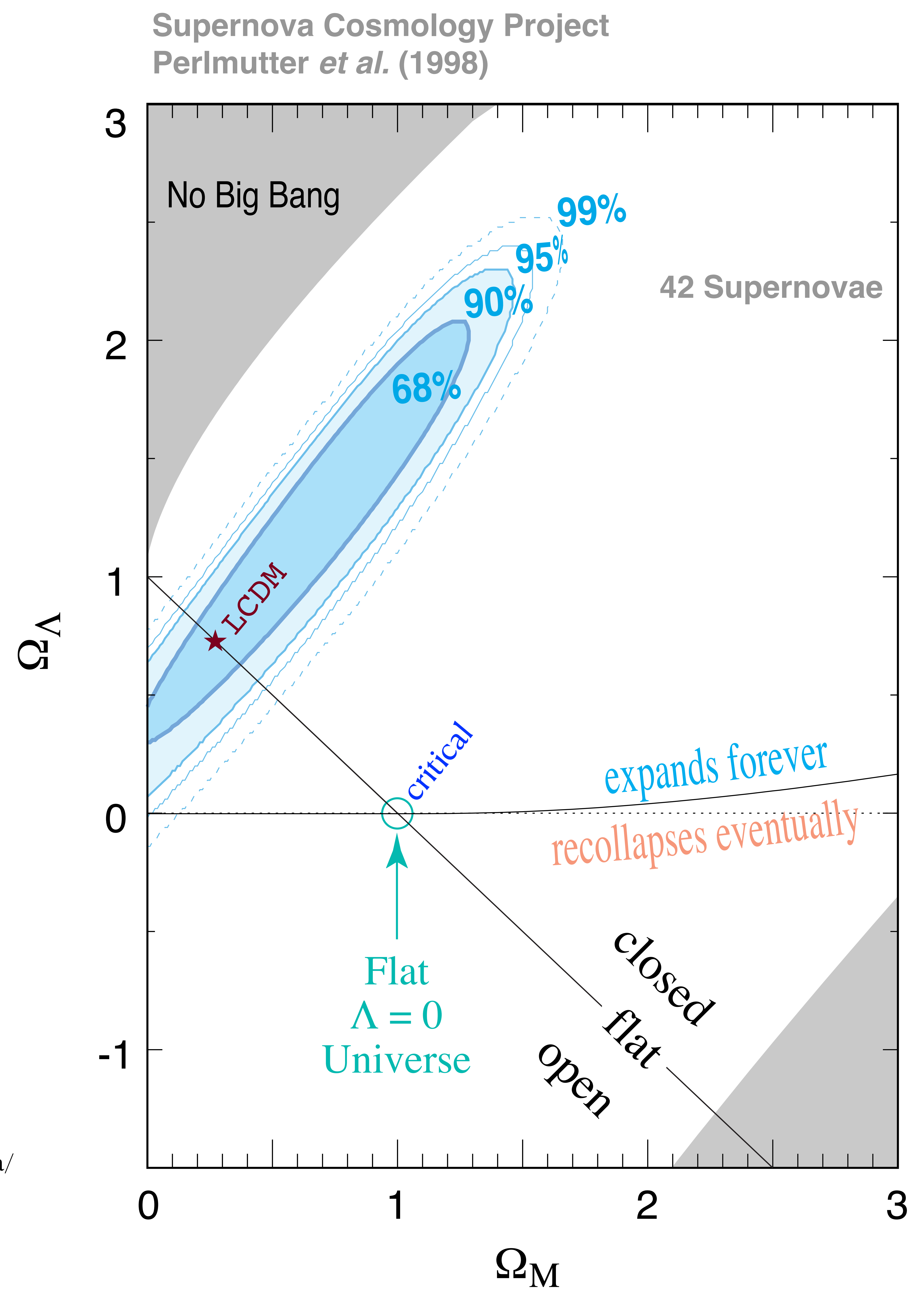
Perlmutter, *et al.* (1998)

- Luminosity-redshift relation

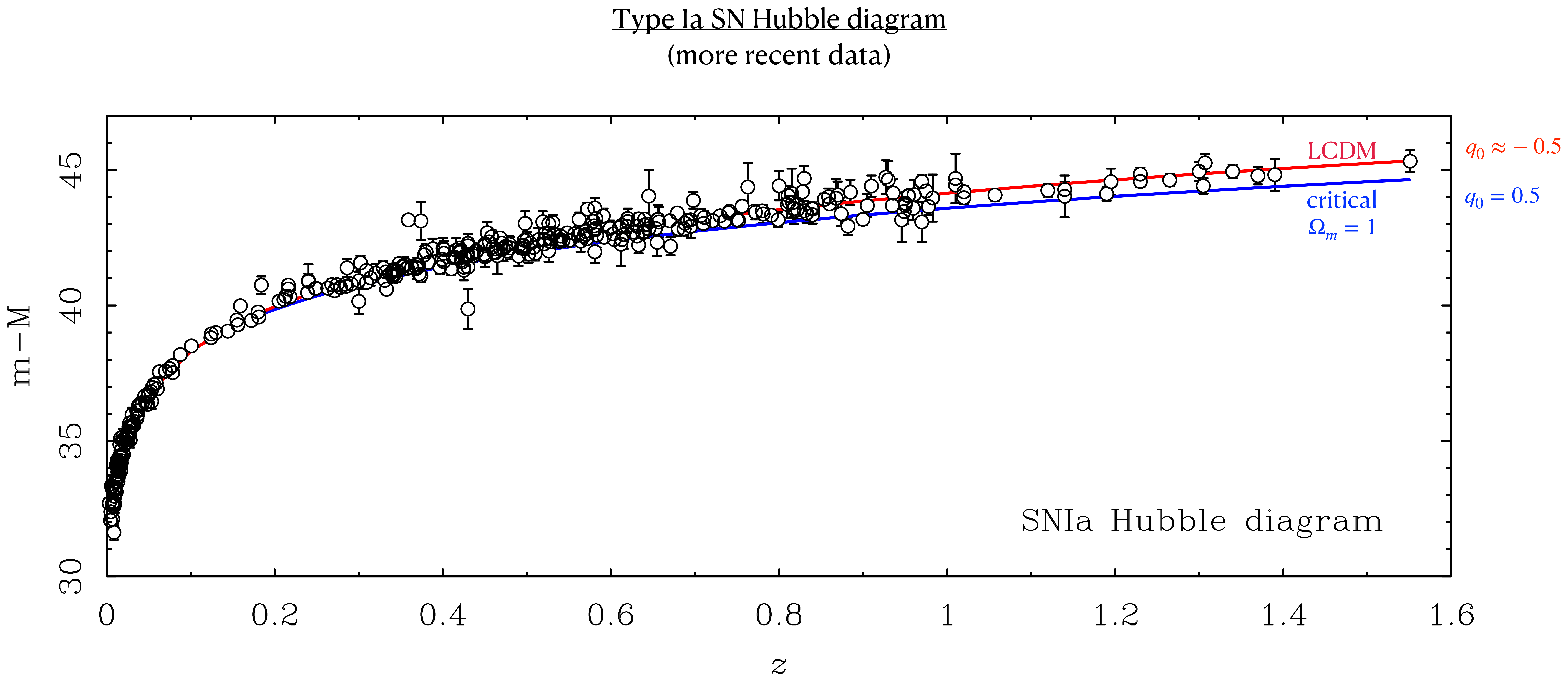
The Type Ia SN constraint practically excluded non-zero cosmological constant, but it did not provide a strong constraint on Ω_m and Ω_Λ individually.

LCDM depended on the inclusion of other information, like independent measures of the mass density and the assumption of a flat geometry.

There is a LOT more to the history of this subject; see https://tritonstation.com/2019/01/28/a-personal-recollection-of-how-we-learned-to-stop-worrying-and-love-the-lambda/



- Luminosity-redshift relation



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https://tritonstation.com/2019/01/28/a-personal-recollection-of-how-we-learned-to-stop-worrying-and-love-the-lambda/

- Angular size-redshift relation

Ideal case:
a **Standard Rod**
an object of constant, known size ℓ

angular extent & size

$$\theta = \frac{\ell}{D_A}$$

Angular size distance

$$D_A = \frac{D_p}{(1+z)}$$

Note that

$$D_A = \frac{D_L}{(1+z)^2}$$

ANGULAR-DIAMETER DISTANCE 137

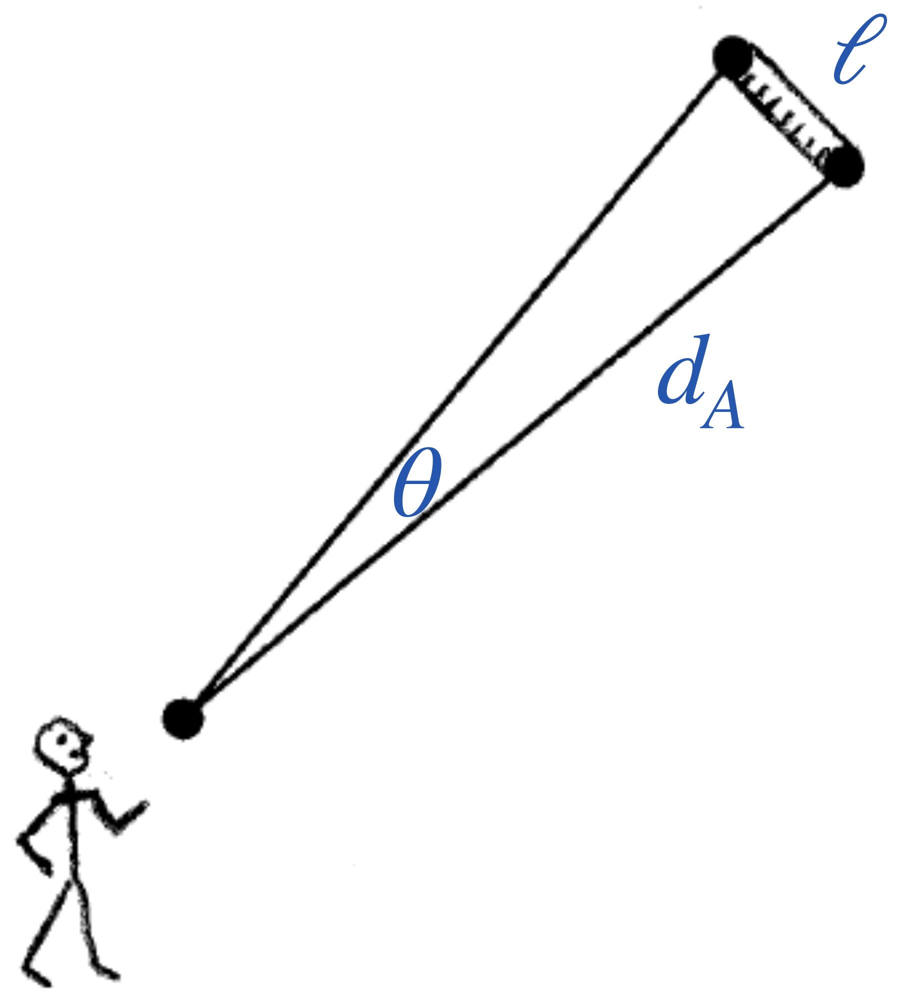


Figure 7.3: An observer at the origin observes a standard yardstick, of known proper length ℓ, at comoving coordinate distance r.

• Angular size-redshift relation

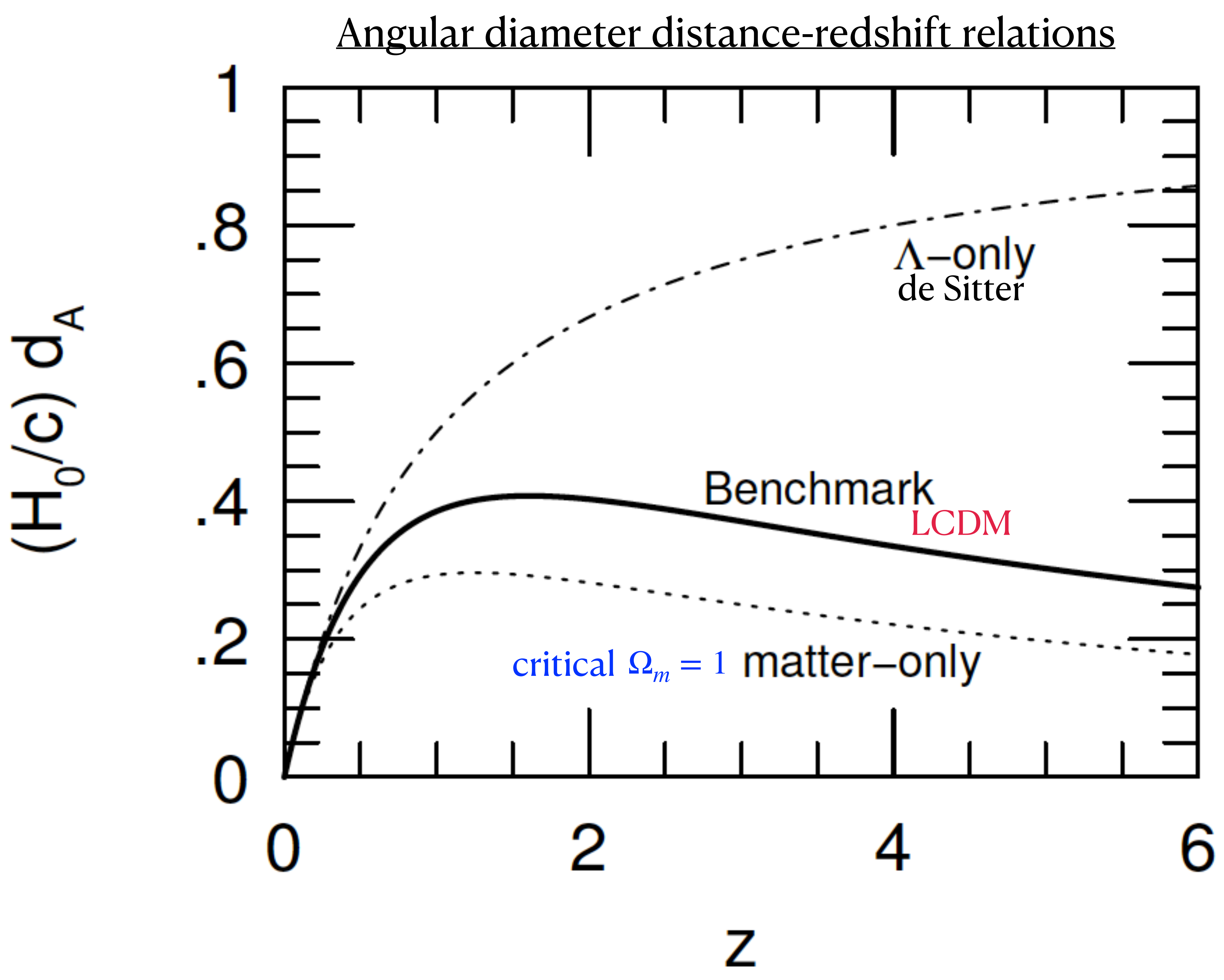


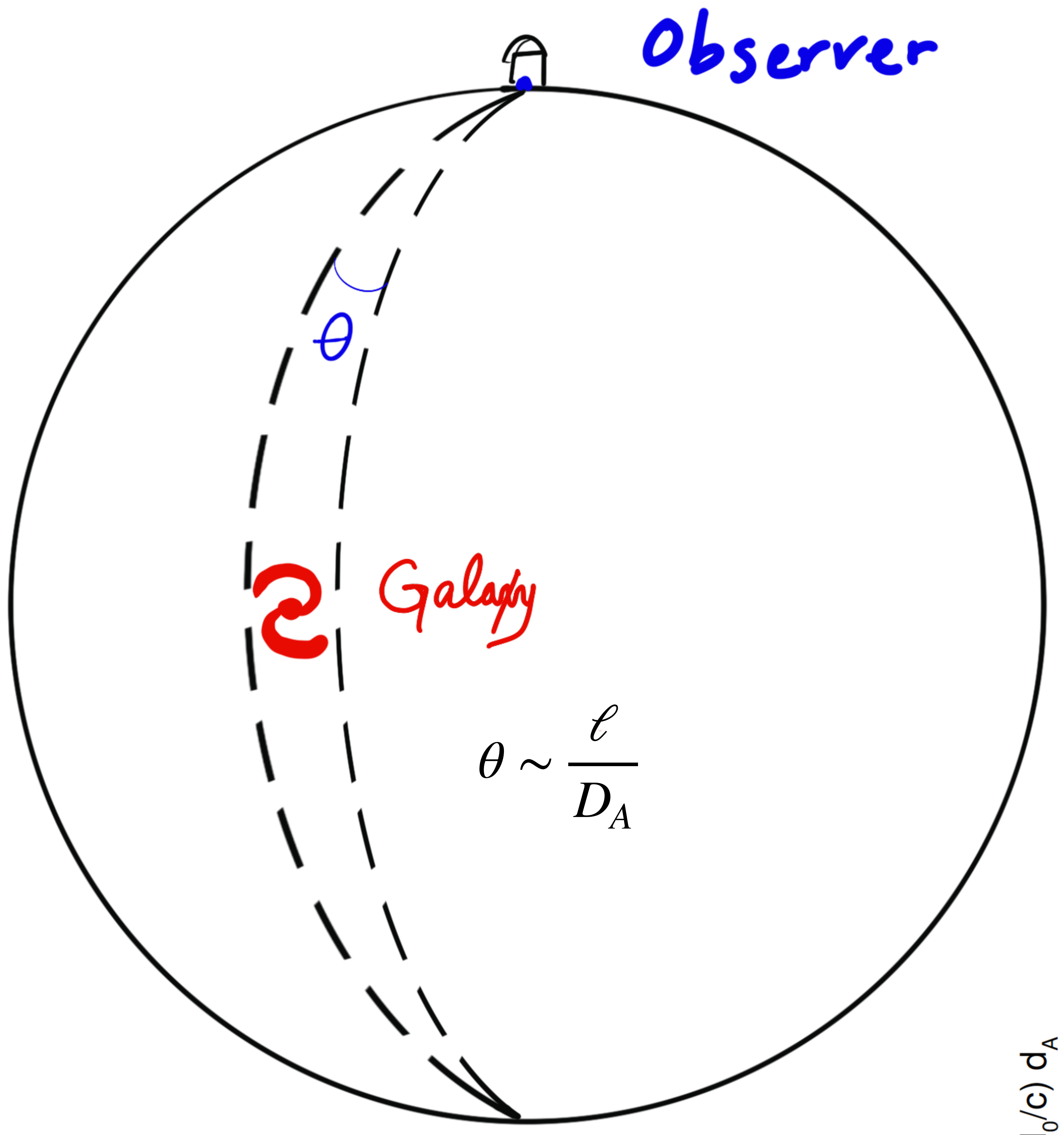
Figure 7.4: The angular-diameter distance for a standard yardstick with observed redshift z. The bold solid line gives the result for the Benchmark Model, the dot-dash line for a flat, lambda-only universe, and the dotted line for a flat, matter-only universe.

Ryden Fig. 6.4
(2nd ed)

Note that the angular diameter distance never exceeds the Hubble length.

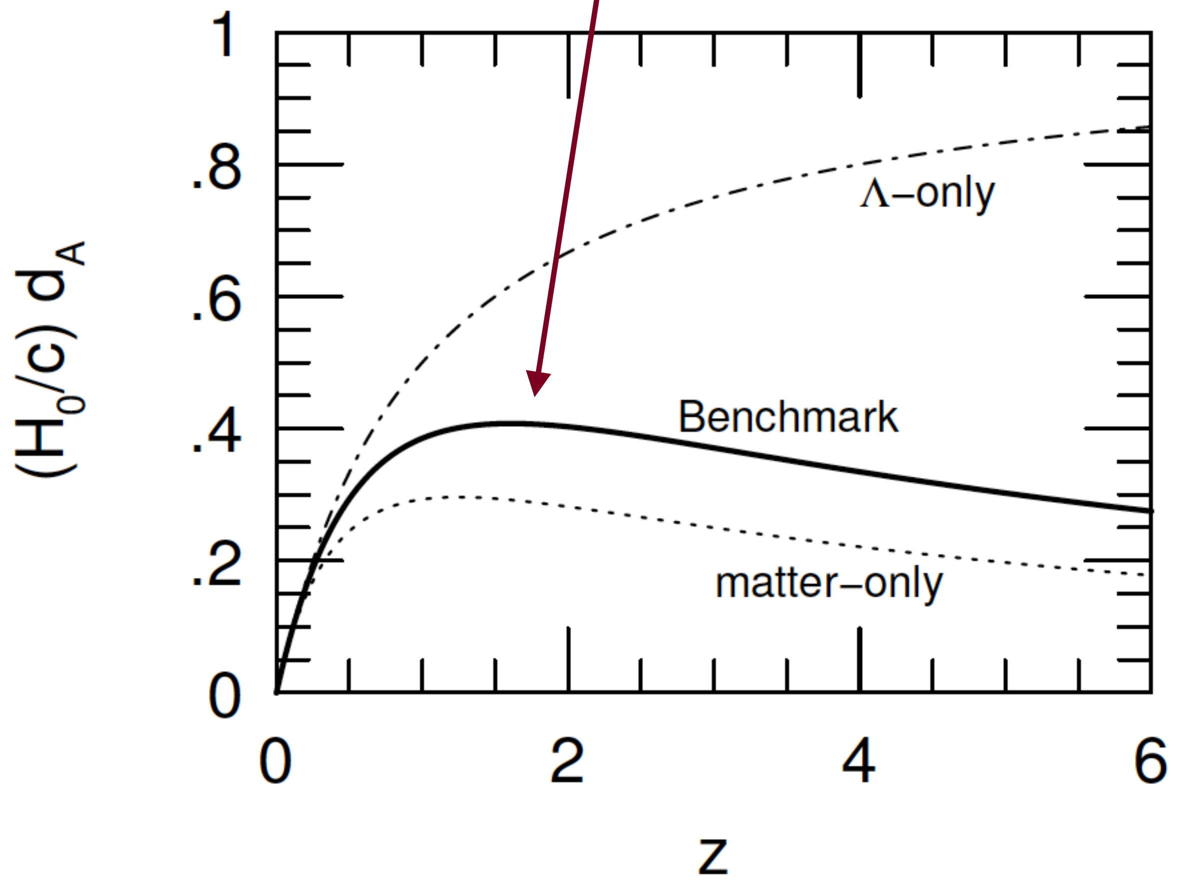
Sometimes has a maximum!

Angular size can have a minimum in non-Euclidean geometries because of the divergence of light rays. Beyond the distance corresponding to this minimum size, objects start to look bigger again!



$$\theta \sim \frac{\ell}{D_A}$$

For LCDM, the minimum angular size occurs around $z \approx 1.6$ at $D_A \approx 1.75$ Gpc

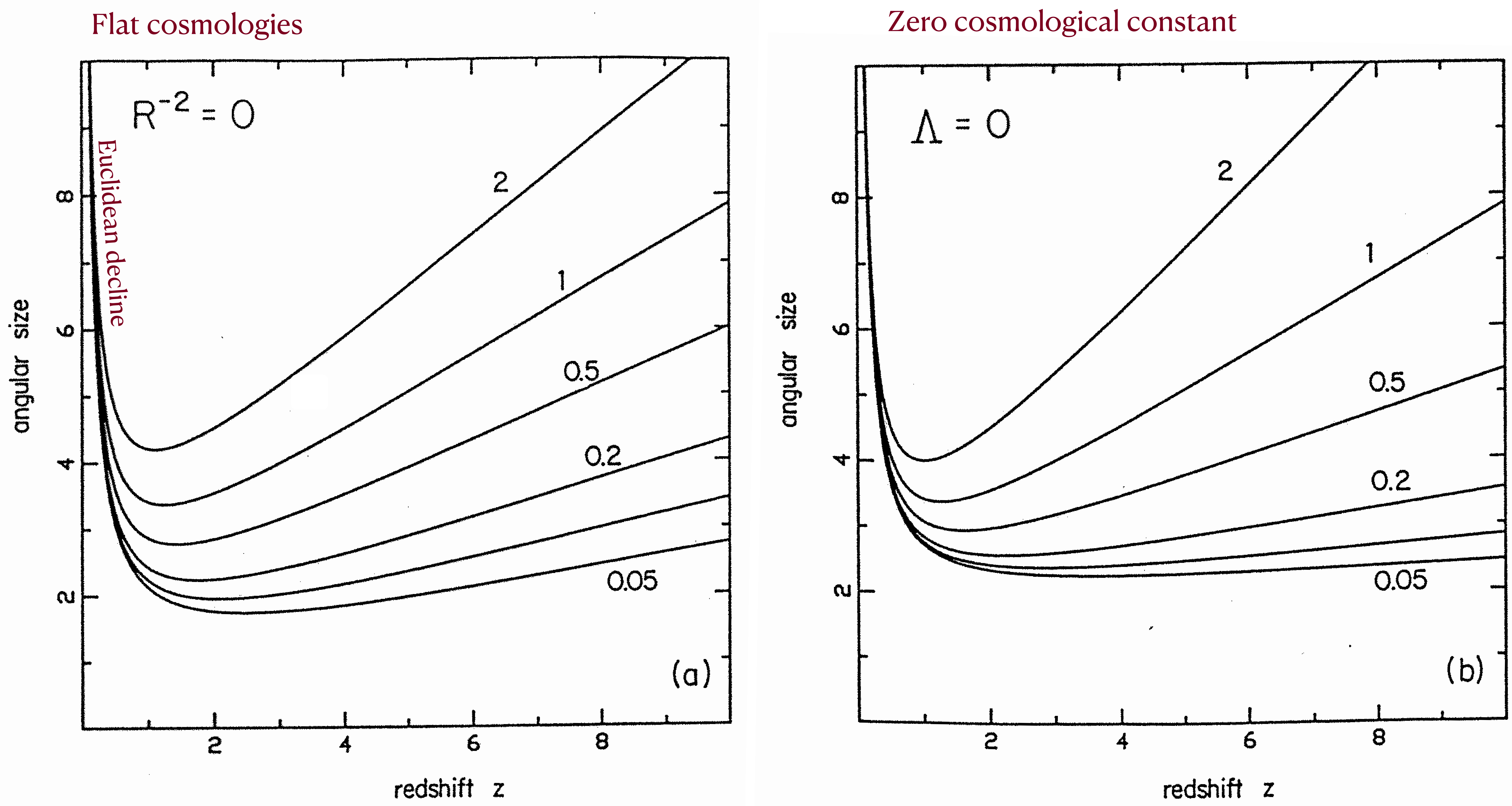


• Angular size-redshift relation

Peebles Fig. 13.5

Angular Size-redshift relations

Figure 13.5. Angular size as a function of redshift. The vertical axis is the factor $F_\theta = (1+z)/H_o a_o r(z)$ in equation (13.47). The parameters are arranged as in figure 13.1.



Initially objects decline in angular size with increasing distance, but this trend reverses at high redshift in the Robertson-Walker geometry!

- Angular size-redshift relation

L.I. Gurvits et al.: The "angular size – redshift" relation f

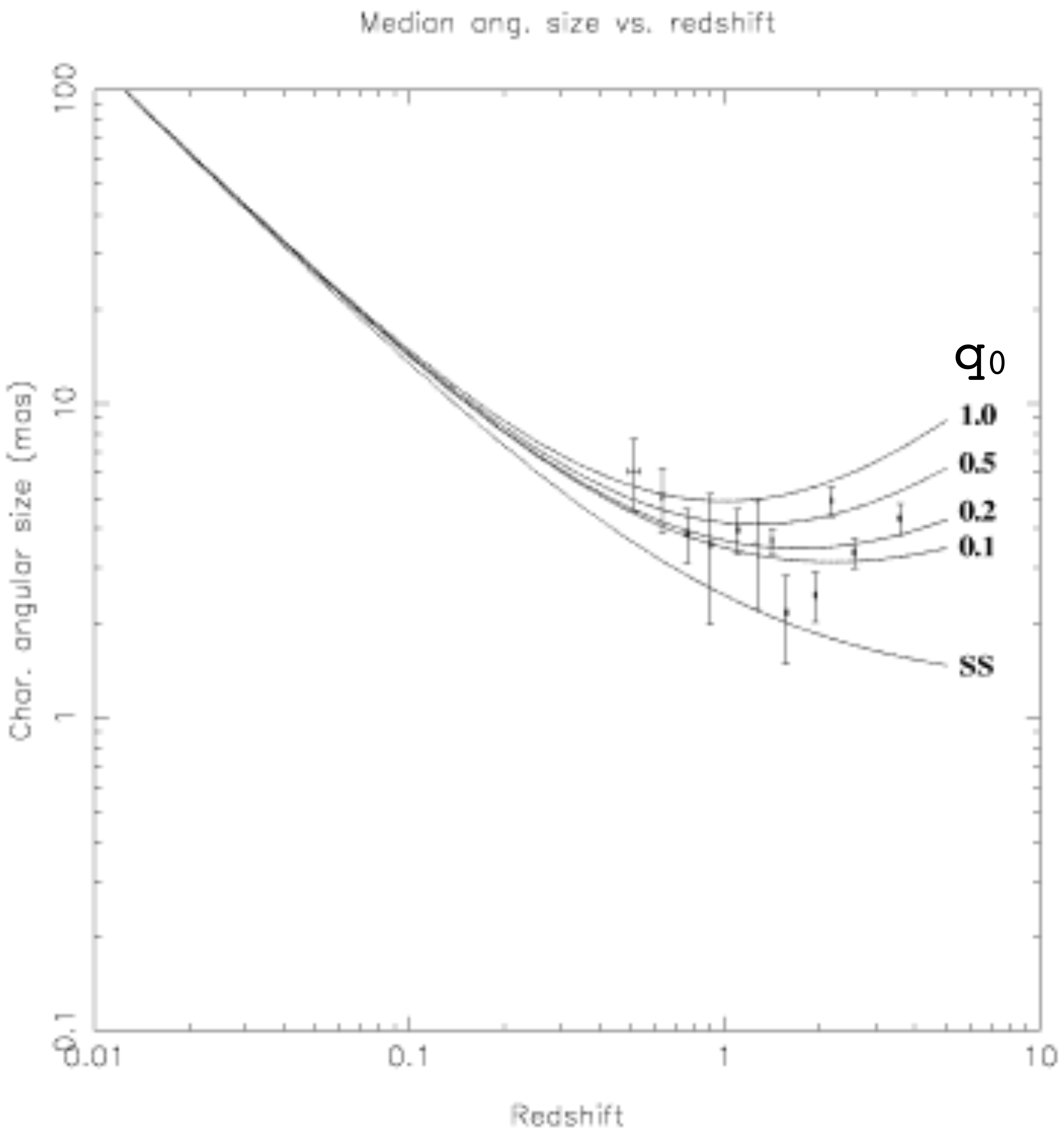


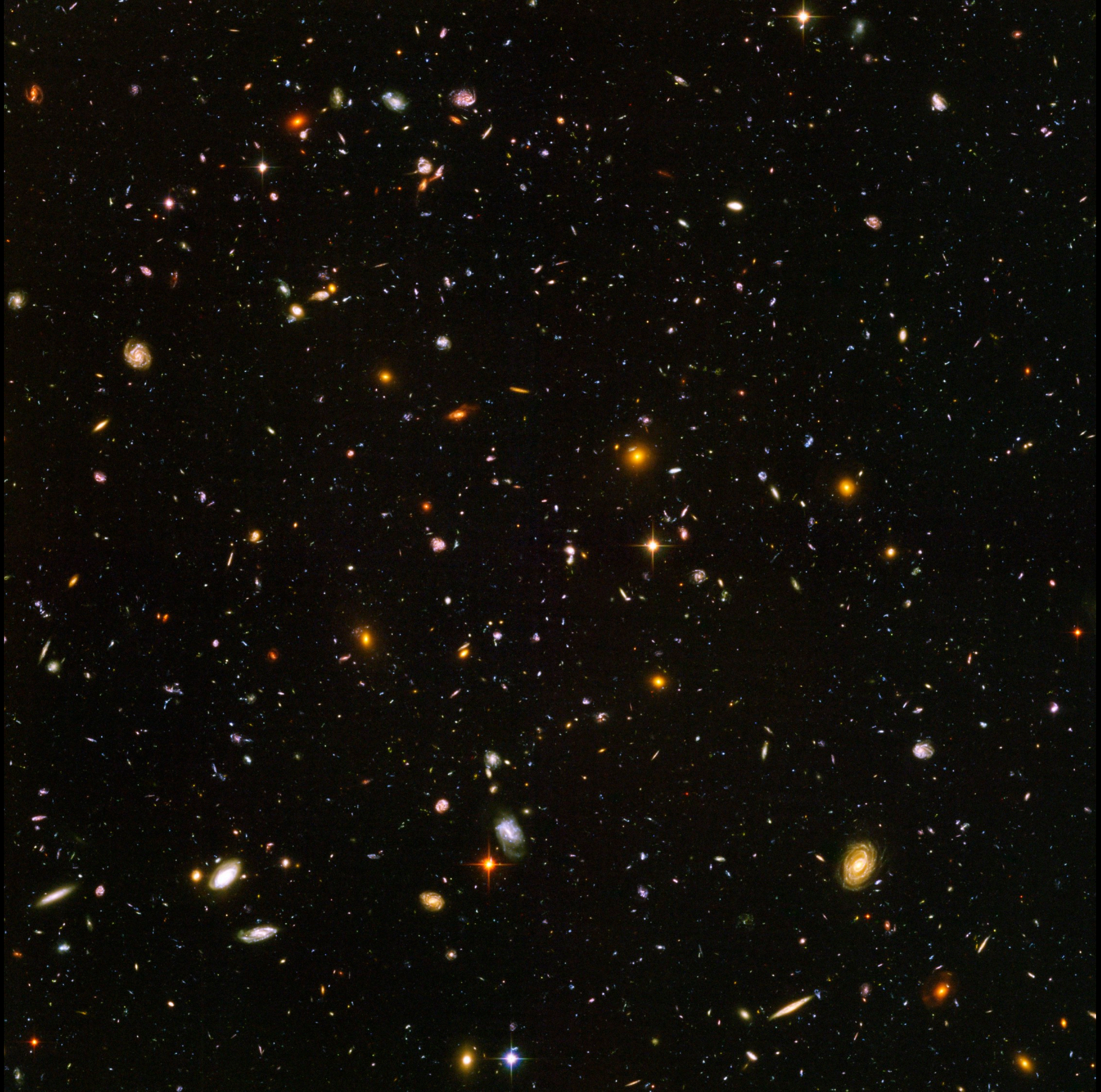
Fig. 10. Median angular size versus redshift for 145 sources (binned into 12 bins, 12–13 sources per bin) with $-0.38 \le \alpha \le 0.18$ and $L \ge 10^{26}$ W/Hz. The solid lines correspond to the linear size parameter $lh = 22.7$ pc, the Steady-state model (SS) and models of a homogeneous, isotropic Universe with $\Lambda = 0$ and various shown values of $q_\circ$. None of the solid lines represents the best fit.

Gurvits et al. (1999)
Fig. 10

angular sizes of compact radio sources

- Number-magnitude relation

Hubble Ultra Deep Field



Cosmic Volume

Robertson-Walker metric

$$ds^2 = -c^2dt^2 + a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2(d\theta^2 + \sin^2\theta d\phi^2)\right]$$

Volume

$$V = a^3\int_0^r \frac{r^2dr}{\sqrt{1-kr^2}}\int_0^\pi \sin\theta d\theta\int_0^{2\pi} d\phi$$

An open universe ($\Omega_m < 1$) is “bigger” than a closed universe ($\Omega_m > 1$)

integrates to

$$V = \frac{4\pi}{3}(ar)^3 f(r) \qquad f(r) = \begin{cases} \frac{3}{2}\left[\frac{\sin^{-1} r}{r^3} - \frac{\sqrt{1-r^2}}{r^2}\right] \\ 1 \\ \frac{3}{2}\left[\frac{\sqrt{1+r^2}}{r^2} - \frac{\sinh^{-1} r}{r^3}\right] \end{cases} \quad \text{for} \quad k = \begin{cases} 1 \\ 0 \\ -1 \end{cases}$$

Cosmic Volume

in terms of the proper distance

$$D_p = a(t) \int_0^r \frac{dr}{\sqrt{1 - kr^2}}$$

we can make the Taylor expansion

Sandage (1988, ARA&A, 26, 561)

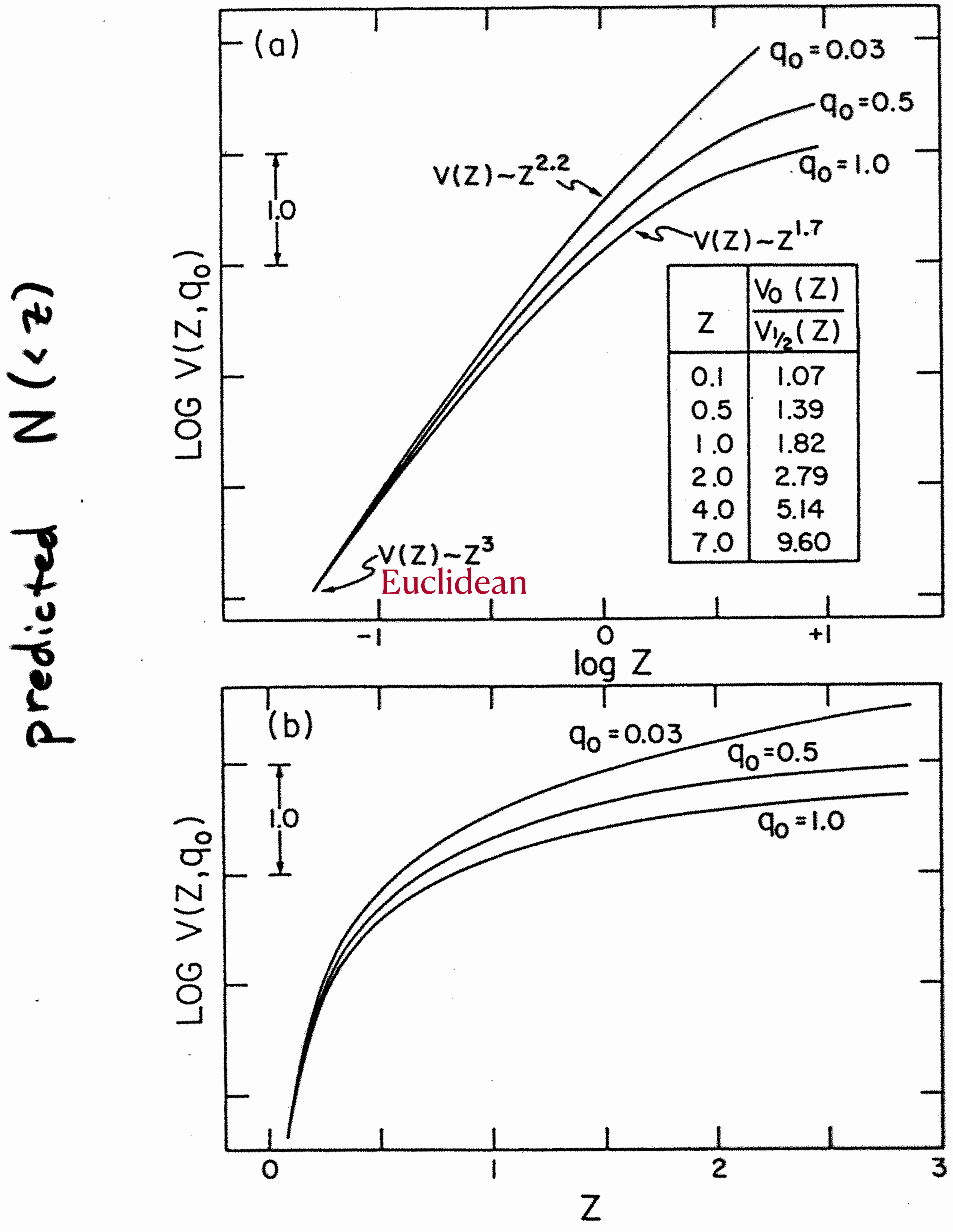
$$V = \frac{4\pi}{3} D_p^3 \left[1 - \frac{k}{5} \left(\frac{D_p}{R} \right)^2 + \mathcal{O} \left(\frac{D_p}{R} \right)^4 \right]$$

$\frac{k}{R^2}$ is the curvature of space

Note that the volume increases as the curvature becomes more negative,
so a closed universe is “small”
and an open universe is “big”

Cosmic Volume

Sandage 1988, ARAA, 26, 561



Z	$V_0(Z) / V_{1/2}(Z)$
0.1	1.07
0.5	1.39
1.0	1.82
2.0	2.79
4.0	5.14
7.0	9.60

Figure 1 Theoretical $N(z, q_0)$ relations for three values of q_0. Plotted is the integral count, i.e. the total number of galaxies in a complete (volume-limited) sample that have redshifts smaller than z. Parts (*a*) and (*b*) are the same function but plotted as log z (*a*) and z (*b*).

Note: decelerating cosmologies have "less" volume than Euclidean. The long(N)-m diagram should have a slope < 0.6.

as $q_0 \downarrow$, $V \uparrow$

Note: in 1988, we didn't consider $q < 0$ to be a physical possibility

- Number-redshift and Number-magnitude relations

Since the volume depends on curvature, source counts N(m) provide a test

For sources of luminosities L and constant comoving number density $\Phi(L)$,

$\underbrace{\text{comoving number density}}_{\text{homogeneity, no evolution}}$

Number-redshift:

$$N(<z) = \frac{4\pi}{3H_0^2} z^3 \int_0^\infty \Phi(L) \left[1 + \frac{3}{2} z (1 + q_0)\right] dL$$

Number-magnitude:

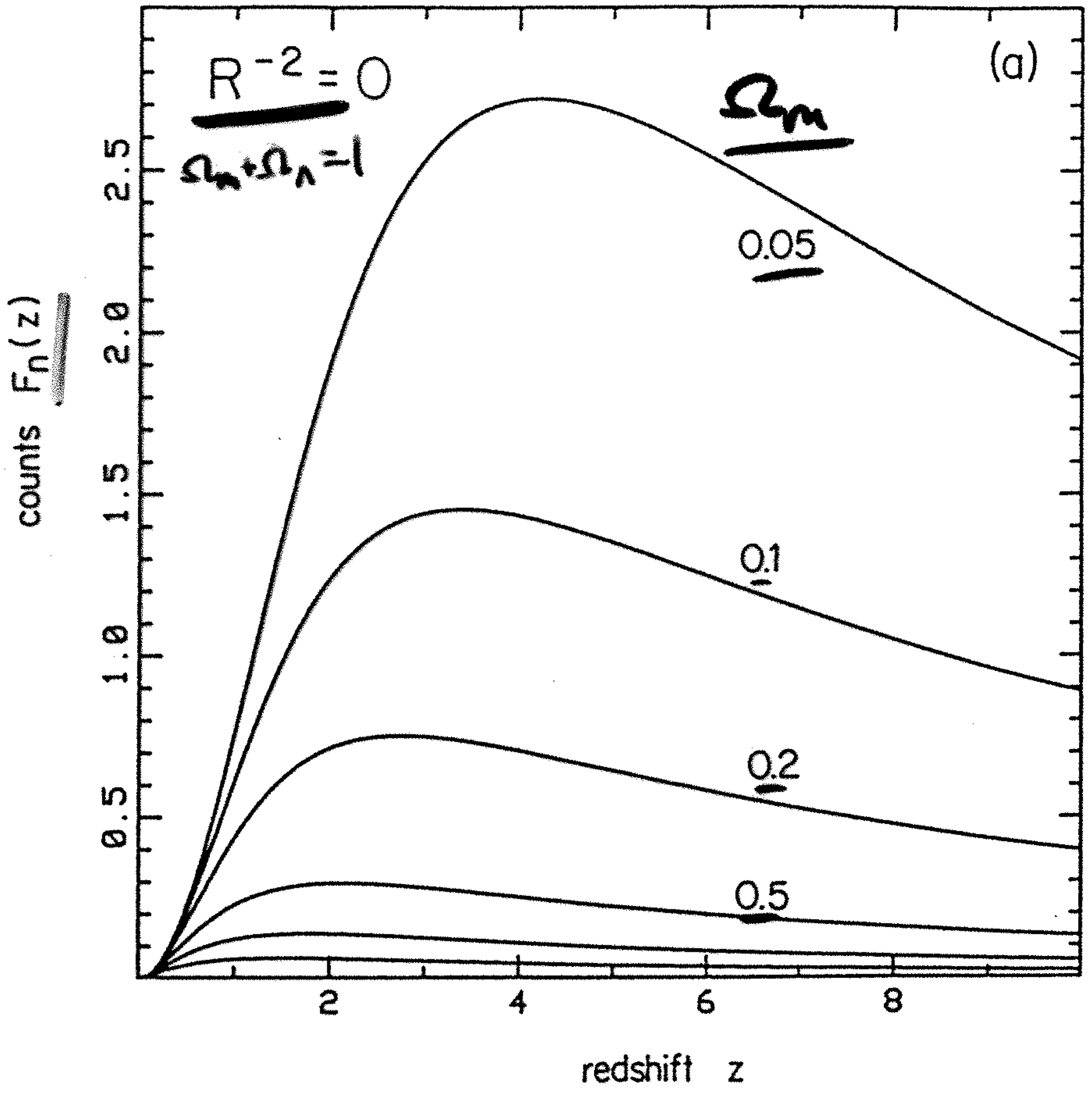
$$N(<f) = \frac{4\pi}{3} (4\pi f)^{-3/2} \int_0^\infty \Phi(L) \left[1 - 3H_0 \left(\frac{L}{4\pi f}\right)^{1/2}\right] L^{3/2} dL$$

Historically, radio source counts in the 1960s played an important role in excluding the Steady State cosmology.

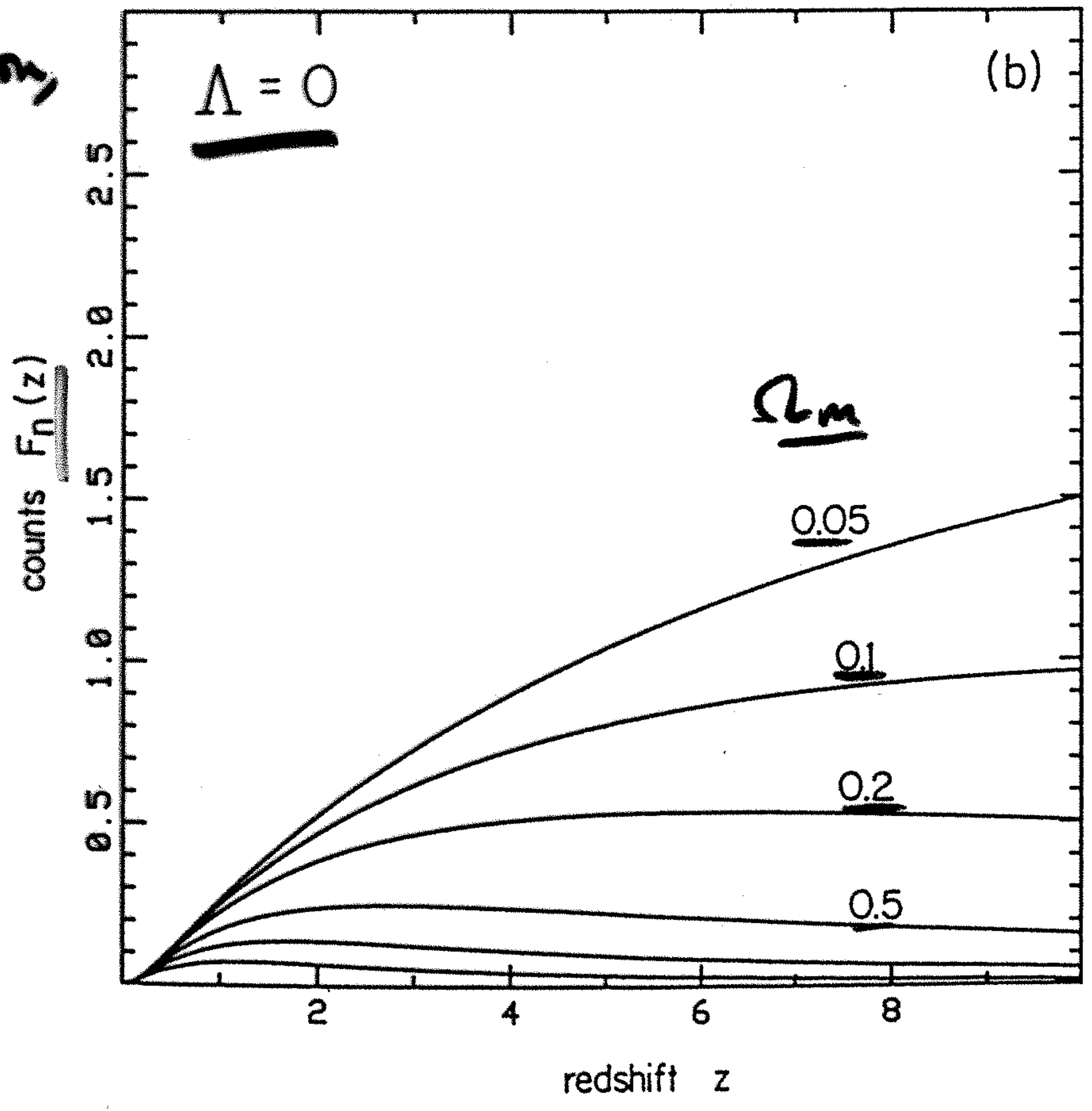
Peebles 13.8

Differential counts $\frac{dN}{dz}$

Flat



No cosmological constant



Peebles

Figure 13.8. Counts as a function of redshift. The vertical axis is the dimensionless function $F_n(z)$ in dN/dz in equation (13.61). The parameters are arranged as in figure 13.1.

for number conservation,

$$n = n_0 \left(\frac{R_0}{R}\right)^3$$

$$\frac{dN}{dz} = n_0 \frac{F_n(z)}{H_0^3}$$

$$F_n(z) = \frac{[H_0 R_0 r(z)]^2}{E(z)}$$

Differential counts $\frac{dN}{dz}$

Yoshii & Peterson (1995)

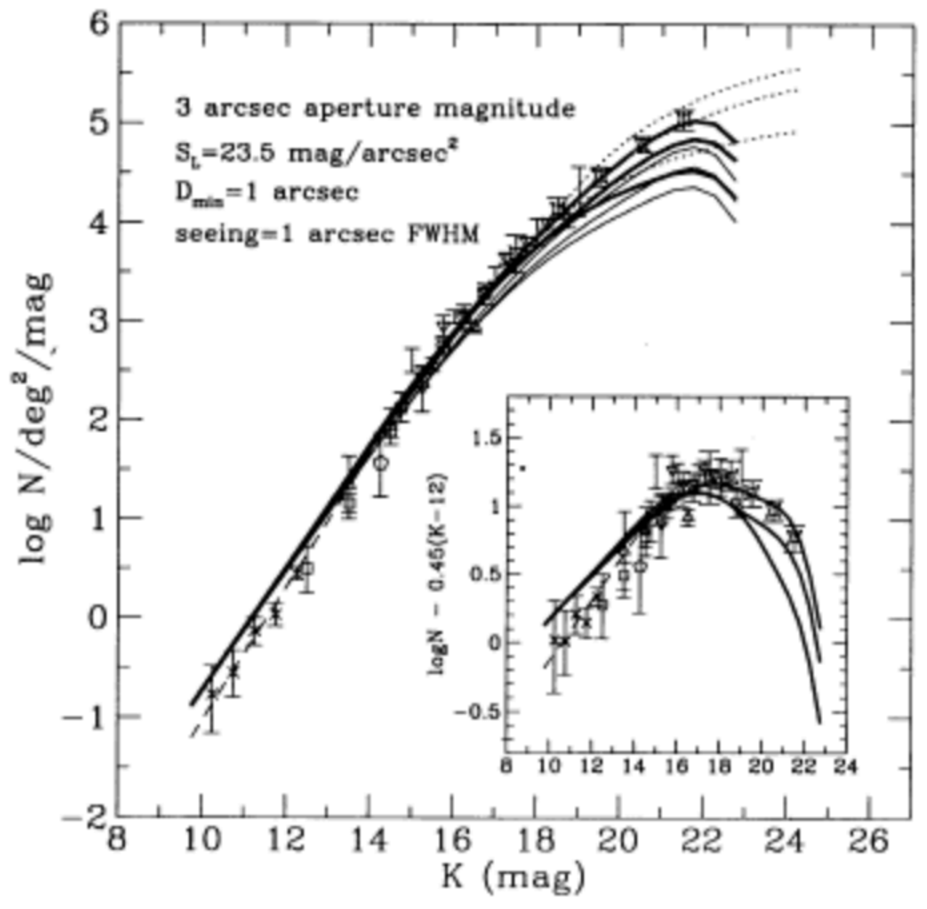


FIG. 2*a*

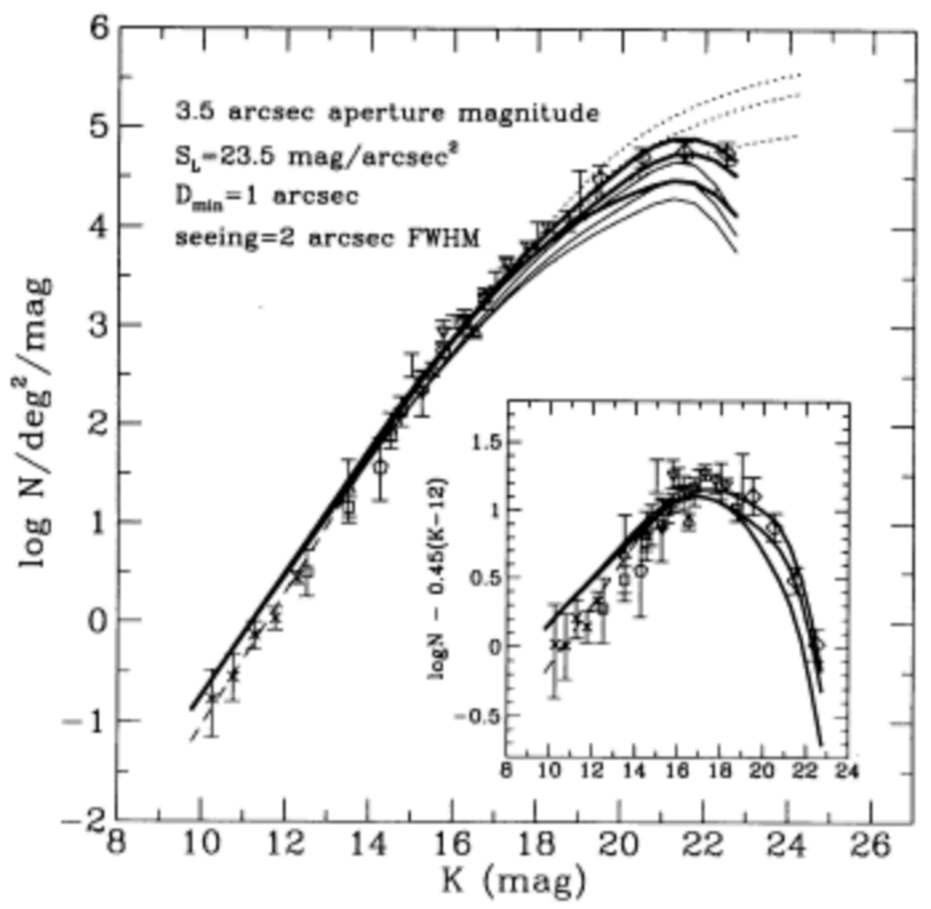


FIG. 2*b*

TABLE 1

PREDICTED NUMBER COUNT SLOPE AND REDSHIFT DISTRIBUTION OF BLUE-SELECTED FAINT GALAXIES[a]

PARAMETERS			COUNT SLOPE γ (B_J = 17–24)		MEDIAN z_{med} (B_J = 23–24)		MAXIMUM z_{max} (B_J = 23–24)	
Ω_0	λ_0	h	No Evolution	Evolution	No Evolution	Evolution	No Evolution	Evolution
Model Predictions with No Isophotal Selection Effects								
1	0	0.5......	$0.38^{+0.01}_{-0.02}$	$0.43^{+0.02}_{-0.02}$	$0.49^{+0.00}_{-0.16}$	$0.85^{+0.24}_{-0.32}$	$1.76^{+0.00}_{-0.36}$	$2.98^{+1.20}_{-0.49}$
0.2	0	0.6......	$0.39^{+0.01}_{-0.02}$	$0.43^{+0.01}_{-0.01}$	$0.47^{+0.00}_{-0.13}$	$0.69^{+0.06}_{-0.19}$	$1.39^{+0.00}_{-0.22}$	$2.56^{+0.26}_{-0.87}$
0.2	0.8	0.8......	$0.40^{+0.01}_{-0.02}$	$0.44^{+0.01}_{-0.01}$	$0.45^{+0.00}_{-0.12}$	$0.62^{+0.44}_{-0.13}$	$1.25^{+0.00}_{-0.19}$	$2.34^{+0.32}_{-0.74}$
Model Predictions with the Isophotal Selection Effects[b]								
1	0	0.5......	$0.36^{+0.02}_{-0.02}$	$0.40^{+0.02}_{-0.02}$	$0.41^{+0.00}_{-0.11}$	$0.62^{+0.09}_{-0.19}$	$1.15^{+0.00}_{-0.16}$	$2.41^{+0.41}_{-1.16}$
0.2	0	0.6......	$0.37^{+0.01}_{-0.01}$	$0.40^{+0.02}_{-0.01}$	$0.40^{+0.00}_{-0.10}$	$0.54^{+0.03}_{-0.12}$	$1.04^{+0.00}_{-0.14}$	$1.53^{+0.12}_{-0.51}$
0.2	0.8	0.8......	$0.39^{+0.01}_{-0.02}$	$0.42^{+0.01}_{-0.00}$	$0.39^{+0.00}_{-0.09}$	$0.50^{+0.02}_{-0.08}$	$0.97^{+0.00}_{-0.13}$	$1.25^{+0.09}_{-0.18}$
Observational Results[c]								
			0.42 ± 0.01		0.47 ± 0.05		1–1.5	

[a] The range of uncertainty in predicted quantities is set by separate sum of (+) and (−) changes of each quantity due to the changes of input parameters from our standard choice in the model, i.e., the redshift of galaxy formation $z_F = 5 \rightarrow 10$, the UV strength parameter $x = 0.2 \rightarrow 0$, the UV SED N3379 → N4649, the Pence's galaxy mix → the Tinsley's mix, the luminosity function slope index $\alpha = -1.1 \rightarrow -1.3$, and the Hubble constant $h \rightarrow 1$.

[b] The isophotal magnitude scheme and the observational conditions of 1".5 FWHM seeing, $S_L = 27.5$ mag arcsec^{-2}, and $D_{min} = 2''$ are adopted for the purpose of comparison with Colless et al.'s 1993 LDSS-2 redshift measurements of galaxies sampled from a photometric catalog of Jones et al. 1991.

[c] γ is taken from Jones et al. 1991 after their estimate on the total magnitude scheme is converted into that on the isophotal magnitude scheme using γ(total) − γ(isophotal) ≈ 0.02. z_{med} and z_{max} are taken from Colless et al. 1993.

"we find that these number count data favor a flat, low-density $\Omega_m \sim 0.2$ universe with a nonzero cosmological constant."