# Final Review - Cosmology

3 Empirical Pillars of the Hot Big Bang

- Hubble Expansion
- Big Bang Nucleosynthesis
- Relic Radiation field (CMB)

## Basic parameters

Expansion Rate $H_0$ Hubble parameter $H(z)$

Mass Density $\Omega_m = \frac{\rho_m}{\rho_{crit}}$ $\rho_{crit} = \frac{3H_0^2}{8\pi G}$

Baryon Density $\Omega_b$ $\Omega_m = \Omega_b + \Omega_{DM}$

Need dark matter because $\Omega_m > \Omega_b$

Cosmological constant $\Lambda$ $\Lambda$ does not vary
but $\Omega_\Lambda$ does because $\rho_{crit}$ does

Power spectrum $\sigma_8$ normalization of amplitude
$n$ tilt $(n \approx 1)$

Deceleration parameter $q = \frac{1}{2}\Omega_m - \Omega_\Lambda$

# Governing Equations

Robertson-Walker metric

$$ds^2 = c^2 dt^2 - a^2(t)\left[\frac{dr^2}{1-kr^2} + r^2 d\Omega^2\right]$$

Acceleration equation

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3P) + \frac{1}{3}\Lambda$$

Friedmann equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G}{3}\left(\rho_m + \frac{\varepsilon_r}{c^2}\right) - \frac{kc^2}{(aR_0)^2} + \frac{c^2}{3}\Lambda$$

OR

$$\left(\frac{H}{H_0}\right)^2 = \Omega_m a^{-3} + \Omega_r a^{-4} + \Omega_k a^{-2} + \Omega_\Lambda$$

The expansion factor is related to the redshift by

$$a = \frac{1}{1+z} \qquad \text{with } a = 1 \text{ at } z = 0 \quad \text{(now)}$$

also,

$$\sum \Omega_i = 1 \qquad \text{i.e.} \qquad \Omega_m + \Omega_r + \Omega_k + \Omega_\Lambda = 1$$

If the universe if geometrically flat, $\Omega_k = 0$

and $\Omega_m + \Omega_\Lambda = 1$ at late times when $\Omega_r \to 0$.

At early times, $\Omega_m \to 1$ until $\Omega_m = \Omega_r$ at matter-radiation equality

# Radiation Domination

$\rho_m \sim a^{-3}$ while $\varepsilon_r \sim a^{-4}$

so at some point (now long ago), $\rho_m = \dfrac{\varepsilon_r}{c^2}$

Prior to this, radiation dominates - $\Omega_r$ is the only term that matters in the Friedmann equation

During this time, $a \sim t^{1/2}$

As the universe expands, the radiation field cools

$T_r \sim a^{-1} \sim (1+z)$ $\qquad\qquad$ $\varepsilon_r = \alpha T_r^4$ radiation constant

Consequently $T_r \sim t^{-1/2}$

such that $T^2 t$ = constant

In detail $\quad T_r = K t^{-1/2}$ with $K = \left(\dfrac{32\pi\alpha G}{3c^2}\right)^{-1/4} \simeq 10^{10}\ \mathrm{K\,s}^{1/2}$

In addition to the radiation background, there should also be a neutrino background with

$T_\nu = \left(\dfrac{4}{11}\right)^{1/3} T_r$ $\qquad$ so $T_{\nu_0} = 1.95$ K

for $T_{r_0} = 2.75$ K

The prefactor arise because photons are bosons but neutrinos are fermions

# Big Bang Nucleosynthesis

The isotopes of the light elements (H, He, Li) formed in the first few minutes when the whole universe was dense and hot enough to be one big fusion reactor.

Neutrons fuse with protons to make deuterium ($^2H$ or D), tritium, $^3He$, $^4He$, and $^{6\,\&\,7}Li$.

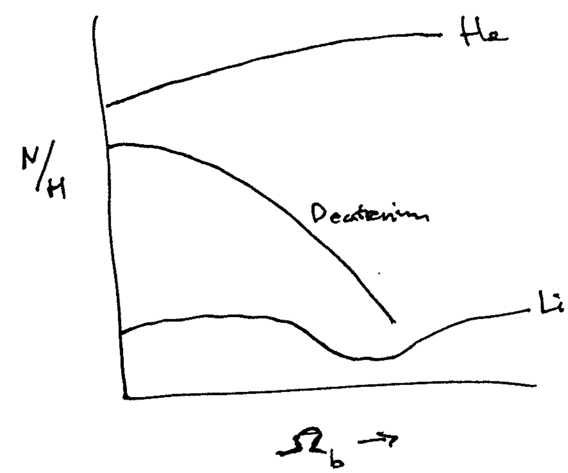
The abundance ratios $D/H$, $He/H$, $Li/H$

depend on the neutron-to-proton ratio, which in turn depends on the freeze-out abundance of neutrons and the decay lifetime of free neutrons (~ 8 min)

The baryon density $\Omega_b$ can be inferred with observations of the primordial abundances of the light isotopes.

As $\Omega_b$ increases,

- $D/H$ quickly declines
- He gradually increases
- Li wiggles




Baryon density precisely known

$0.04 < \Omega_b < 0.05$

with most of the uncertainty in $H_0$

[ $\Omega_b h^2 = 0.0225$ with small formal uncertainty ]

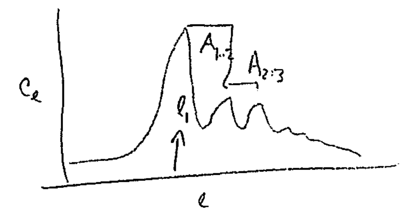
# Cosmic Microwave Background (CMB)

A fundamental ~~tenat~~ of the Hot Big Bang cosmology is that the universe should be full of a relic radiation field. This was discovered in the '60s and is now known as the CMB.
It has a perfect thermal spectrum with $T_{\gamma 0} = 2.75$ K.
The surface of last scattering at $z = 1090$ marks the transition from an opaque plasma to a transparent atomic gas after recombination forms free electrons and protons into hydrogen.

The CMB temperature is very uniform on the sky – it is very nearly the same in every direction we look: the early universe obeys the cosmological principle, being both homogeneous and isotropic when $t = 3.8 \times 10^5$ yr.

There is a dipole moment on the sky due to our motion wrt the CMB of $\sim 600$ km $s^{-1}$ (discovered in '70s)
There are fluctuations on smaller scales at the level $\frac{\Delta T}{T} \approx 10^{-5}$
(discovered in '92)
These fluctuations represent early seeds for the formation of large scale structure, which were predicted to have a larger amplitude ($\frac{\Delta T}{T} \sim 10^{-2}$). That this is not the case is one line of evidence for non-baryonic cold dark matter, which is needed to grow structure in the allotted time.
(The other primary line of evidence for CDM is $\Omega_m > \Omega_b$.)

The pattern of fluctuations in the CMB, quantified as the acoustic power spectrum, is a sensitive probe of cosmic parameters




- The location of the first peak, $\ell_1$, is primarily sensitive to the geometry of the universe. Its observed location indicates that the universe is flat $|\Omega_k| < 0.005$.

- The amplitudes of the peaks and their ratios $A_{1:2}$, $A_{2:3}$, ... are sensitive to the matter content of the universe. Alternate compression and rarefaction peaks are a competition between baryonic drag and the net driving term provided by non-baryonic dark matter.

## Large Scale Structure

LSS starts from the small fluctuations seen in the CMB: $\delta = \frac{1}{3}\frac{\Delta T}{T}$

From this initial condition, structure grows as $\delta \sim a(t)$.

The dark matter forms structure first. The baryons subsequently fall into the halos formed by CDM after they fully decouple from the photons (not till $z \sim 200$).

Fundamental Observations

Mass Density

There are <u>many</u> ways to estimate the gravitating mass density of the universe - the over-all $\Omega_m = \frac{\rho_m}{\rho_{crit}}$ $\left( \rho_{crit} = \frac{3H_0^2}{8\pi G} \right)$

To give just a few examples:

- Cluster $M/L$

  measure mass of cluster (e.g., virial $M \approx \frac{1.5 R_e \sigma^2}{G}$.

  measure luminosity, combine to get $M/L$ (assume to be global)

  measure average luminosity density of the universe $j$ by integrating luminosity function of galaxies

  combine to get density: $\rho_m = \frac{M}{L} j$

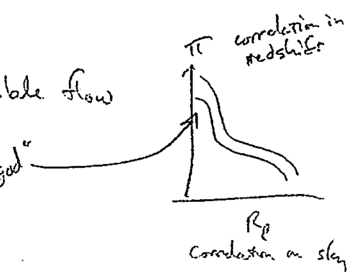
- weak lensing

  measure lensing shear over large scales

  $\Omega_m \approx 0.2$ by this method so far

- peculiar velocity field

  masses like clusters distort the Hubble flow




  amount of distortion depends on mass

$$\frac{\delta V}{V} \approx \frac{d \ln H}{d \ln \rho} \frac{\delta \rho}{\rho} \approx -\frac{1}{3} \frac{\Omega_m^{0.6}}{b} \frac{\delta N}{N}$$

  $b$ is bias factor relating mass $\rho$ and galaxy number density $N$

  $\Omega_m \approx 0.25$ by this method

- LOTS of others

Fundamental Observations

Age Scale

- Oldest Stars

  $t_{GC} = 13.3 \pm 0.1$ Gyr (recent)

  consistently in the range

  Globular clusters $t_{GC} = 12 - 15$ Gyr

  calibration of stellar evolution depends on distance scale

- White dwarf

  maximum ages along cooling function

  measure drop off in luminosity function

  (no WDs fainter than $M \approx 16$)

  Account for preceding stellar evolution

  $$t_{WD}^{max} = 12.5^{+1.4}_{-3.5} \text{ Gyr}$$

- Radio isotope chronometers  eg. Th/Eu

  $t_{TH} = 12.8 \pm 3$ Gyr  oldest star

  measure abundances of r-process radioactive elements in ancient, metal poor stars relative to stable elements

- Interstellar dust grains

  measure abundances of radio isotopes in meteoritic grains account for evolution of parent star, evolution preceding it, and age of solar system (each 4 ~ 5 Gyr):

  $t_{grain} \simeq 13.7 \pm 1.3$ Gyr