Anisotropy Essentials

ORBITAL ANISOTROPY AND THE JEANS EQUATION

In general, the orbits of particles (e.g., stars) that trace the gravitational potential are non-circular. Indeed, they can be of varied eccentricity and orientation, and the mean and distribution of these parameters can be a function of position — not just radius, but also orientation. This orbital anisotropy makes the interpretation of line-of-sight velocity measurements dependent on our viewing angle. One way to cope with this is the Jeans equation, for which the mass estimator can be written

$$G\,M(r) = r\,\sigma_r^2 \left(-\frac{d\ln n_*}{dr} - \frac{d\ln \sigma_r^2}{d\ln r} - 2\beta \right)$$

where σ_r is the velocity dispersion in the radial direction within the object, n_* is the number density of tracer particles, and β is the anisotropy parameter, which is defined as

$$\beta = 1 - \frac{\sigma_t^2}{\sigma_r^2}$$

where σ_t is the velocity dispersion in the tangential direction orthogonal to σ_r. Note that the ratio σ_t^2/σ_r^2 effectively measures a ratio of kinetic energy, quantifying the relative amount of support in rotation and pressure. In the limit of complete pressure support, $\sigma_t \ll \sigma_r$, so $\beta \to 1$. In the opposite limit of a dynamically cold, rotationally supported system, $\sigma_t \gg \sigma_r$ and $\beta \to -\infty$. Note also that the logarithmic derivative terms are pure numbers, so simpler than they look. For example, if the number density of tracers falls off as a power law like $n_* \sim r^{-3}$, then $-dn_*/dr = 3$. Similarly, the other quantities inside the parentheses are dimensionless numbers of order unity.

The velocity dispersions σ_r and σ_t are internal to the object for which the natural reference frame is at the object's center. Consequently, these quantities are not the same as the observable line-of-sight velocity dispersion σ_{los}. In the absence of information to constrain β, a common assumption is that orbits are anisotropic so that $\beta = 0$. This is convenient, but it need not be true, and examination of the mass estimator above reveals a degeneracy between the mass and anisotropy: what you get for the mass depends on what you assume about the anisotropy. It is therefore desirable to break the anisotropy when possible, e.g., by measuring the relevant quantities for two or more types of tracer particles. There are some nearby dwarf satellites where this can be done separately for young and old populations of stars.

In general the anisotropy parameter can vary as a function of radius. Consider, for example, an early type spiral with both bulge and disk. At small radii where the bulge dominate, $\beta \approx 0$ might be appropriate, but further out where the disk dominates, $\beta \ll 0$. For disk geometries where rotation dominates, it is convenient to rewrite the Jeans equation as

$$V_c^2(R) = \langle v_\phi^2 \rangle - \langle v_R^2 \rangle \left(1 + \frac{d\ln n_*}{dR} + \frac{d\ln\langle v_R^2 \rangle}{d\ln R} \right)$$

where V_c is the circular velocity of the potential, v_ϕ is the velocity in the azimuthal direction, and $\sqrt{\langle v_r \rangle^2}$ is the is the radial velocity dispersion in the plane of the disk. In this case, the azimuthal velocity v_ϕ is almost the rotation speed, but not quite: we want the true circular speed V_c but orbits usually have a little eccentricity. As a consequence, stars tend to lag the circular speed by an amount that is quantified by the second term on the right hand side. This 'asymmetric drift' is usually a small correction in disk galaxies, but is not always negligible. Within the Milky Way, the asymmetric drift is known to be a function of the age of stellar populations, ranging from $\lesssim 10\ \mathrm{km\,s^{-1}}$ for young stars to $\gtrsim 40\ \mathrm{km\,s^{-1}}$ for old stars. Stars are born on nearly circular orbits ($v_\phi \approx 220\ \mathrm{km\,s^{-1}}$) but some mechanism scatter them over time, diverting some of their azimuthal motion into radial motion, thus causing their orbital eccentricity to grow so they lag progressively behind the circular speed of the potential.