Mass components

Stars in galaxies

- Spiral & Irregular galaxies – Exponential disks

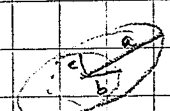
$$\Sigma(R) = \Sigma_0 e^{-R/R_d}$$

is a decent first approximation to the azimuthally averaged face-on surface brightness.

Can measure light profile, assume a mass-to-light ratio, and solve the Poisson equation to obtain the gravitational potential of the observed stars.

- Elliptical galaxies 3D ellipsoids seen in projection

spherical $a = b = c$



can be oblate, prolate, or triaxial
$a = b > c$; $a > b = c$; $a > b > c$
(all can be a fcn of radius; position angle can twist)

Classical "$r^{1/4}$" or de Vaucouleurs profile

$$\Sigma(r) = \Sigma_e \, e^{-7.67\left[\left(\frac{r}{R_e}\right)^{1/4} - 1\right]}$$

R_e = effective radius (contains $\frac{1}{2}$ light)

$\Sigma_e = \Sigma(R_e)$

Sersic profile: generalized hybrid

$$\Sigma(r) = \Sigma_e \, 10^{-b_n\left[\left(\frac{r}{R_e}\right)^{\frac{1}{n}} - 1\right]}$$

adds shape parameter n

$n = 1 \rightarrow$ exponential disk
$n = 4 \rightarrow$ de Vaucoulers profile

Galaxies with both bulge & disk can be fit as separate components (e.g. exponential disk + "classical" $r^{1/4}$ bulge) or as a single Sersic profile (with n managing the transition from bulge to disk).

The ISM

The Interstellar Medium contains gas in all phases — molecular, atomic, and ionized (plasma) as well as dust.

In the Milky Way, all of these are largely confined to the plane of the disk, though there probably exists a tenuous quasi-spherical corona of hot, ionized gas at radii far beyond the edge of the stellar disk, ~~[illegible]~~ extending to ~300 kpc (very roughly).
This corona contains relatively little mass, though it might integrate up to $\sim 10^{10} M_\odot$ all the way out.

IN the disk of the Milky Way, in order of mass:

Atomic gas	HI	~60-70% of ISM mass
Molecular gas	H_2	~30% " " "
Ionized gas	HII	< 10% " " "
Dust		≪ 1%

Total gas mass in MW $\sim 10^{10} M_\odot$
dust mass maybe $\sim 10^{8} M_\odot$

HI extends to ~20 kpc
(The stellar $R_d \approx 2.2$ kpc, so almost all stellar mass is interior to 10 kpc)

$M_* \simeq 5$ or $6 \times 10^{10} M_\odot$

so the Milky Way has a gas fraction $f_g \lesssim 20\%$ which is typical for bright spirals.

HI Atomic gas

detected by spin-flip transition of H

$\overset{\uparrow}{\oplus}\ \overset{\uparrow}{\ominus} \longrightarrow \uparrow\oplus\ \ominus\downarrow$

transition frequency very low energy

$$\nu_{10} = \frac{8}{3} g_I \left(\frac{m_e}{m_p}\right) \alpha^2 \underbrace{(cR_M)} = 1420.4 \text{ MHz}$$

the 21 cm line

nuclear gaunt factor $g_I = 5.6$ for proton

fine structure constant $\alpha = \frac{1}{137}$

Rydberg frequency

Emission coefficient

$$A_{ul} = \frac{64\pi^4}{3hc^3} \nu_{ul}^3 \left|\mu_{10}^*\right|^2$$

Bohr magneton (dipole moment of e^-)

numerically

$$A_{10} = 2.85 \times 10^{-15}\ s^{-1}$$

$$\left|\mu_{10}^*\right| = \frac{\hbar}{2} \frac{e}{m_e c}$$

radiative half-life $\tau_{1/2} = \frac{1}{A_{10}} \approx 11 \times 10^6$ yr

$(3.5 \times 10^{14}$ s $= 11$ Myr$) = \frac{1}{\tau_{1/2}}$

Very low critical density $(\ll 1\ cm^{-3})$

so almost always in LTE through collisional excitation

Can also define "spin temperature"

$$\frac{N_1}{N_0} = \frac{g_1}{g_0} e^{-\frac{\nu h_{10}}{kT_S}}$$

$g_1 = 3$ upper states

$g_0 = 1$ ground state

Note $\frac{h\nu}{k} \ll 1$K very small, so $e^{-\frac{\nu h_{10}}{kT}} = 1$ if $T_S \approx T$ anywhere near equilibrium

so $\frac{N_1}{N_0} = \frac{3}{1}$ & $N_H = N_0 + N_1 = 4N_0$ good for counting atoms counting 21 cm photons → counting HI atoms

$$M_{HI} = \frac{16\pi m_H}{3 A_{ul} h c} D^2 \underbrace{\int F_\nu \, dv}_{\text{flux integral in Jy}\cdot\text{km/s}}$$

$1 \text{ Jansky} = 10^{-26} \text{ W m}^{-2} \text{ Hz}^{-1}$

Numerically

$$M_{HI} = \left(2.34 \times 10^5 M_\odot\right) D^2 \int F_\nu \, dv$$

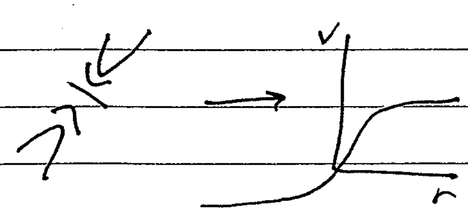
D in Mpc (luminosity distance $(1+z)\times D_L$ in cosmology) because it's a line

$M_{\text{atomic gas}} = \frac{1}{X} M_{HI}$ — Hydrogen mass fraction

Can measure HI masses accurately to a few % IF care is taken.

HI emission ALSO gives velocity field from Doppler effect – important tracer of the gravitational potential

Fit velocity field with tilted ring model to obtain rotation curve



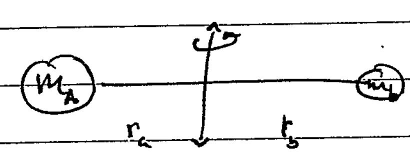
Molecular ISM

cold ~30K
"dense" ~100 cm^{-3}

very clumpy; need to be to self-shield against interstellar radiation field (mild UV can photodissociate many molecules)
most mass in GMCs: Giant Molecular Clouds ~$10^6 M_\odot$
... this is where stars are born

Diatomic molecules boring - or at least hard to excite
↳ H_2, O_2, N_2 because symmetry prevent them from having a dipole moment.
Hence no radiation if the rotate. (Vibrational states much higher energy)

Polar molecules (like CO) have permanent dipole moment thanks to asymmetry
leading to rich rotational spectrum (often at mm & cm wavelengths)



Angular momentum

$$L = \left(\frac{m_a m_b}{m_a + m_b}\right)(r_a + r_b)\,\omega$$ is quantized

rotational energy

$$E = \frac{J(J+1)\hbar^2}{2I}$$

moment of inertia

$$I = m_a r_a^2 + m_b r_b^2$$

need I to be large to generate detectable emission in cold molecular gas quantum mechanics requires $\Delta J = \pm 1$ so

$$\nu = \frac{hJ}{4\pi m_e r_e^2} \qquad m_e = \frac{m_a m_b}{m_a + m_b}$$

$$r_e = r_a + r_b$$

Rotational transitions lead to ladder spectrum

gets more complicated for multi-atom molecules

Can play same tricks as with HI:

$$M_{H_2} = 1.1 \times 10^4 D^2 F_{co} \qquad X_{co} = 2.8 \times 10^{20} \text{ cm}^{-2} (\text{K km/s})^{-1}$$