 Galactic Kinematics

The Local Standard of Rest (LSR)
Solar motion
disk velocity dispersion
Galactic Rotation
The Local Standard of Rest

Let's define a coordinate system:

**Position**: \((R, \theta, z)\)
- \(R\) = galactocentric distance
- \(\theta\) = azimuthal coordinate
- \(z\) = height above/below the plane

**Velocity**: \((\Pi, \Theta, Z)\)
- \(\Pi\) = velocity in/out from center
- \(\Theta\) = tangential velocity
- \(Z\) = velocity up and down
Define a point in space that is moving on a perfectly circular orbit around the center of the galaxy at the Sun's galactocentric distance. We measure all velocities of stars relative to this point, which is known as the Local Standard of Rest.

The velocity of the Local Standard of Rest (LSR) is then given by

\[
\begin{align*}
\Pi_{LSR} &= 0 \\
\Theta_{LSR} &= \Theta_0 \\
Z_{LSR} &= 0
\end{align*}
\]

More generally, if the Galactic potential is not axis-symmetric (e.g., because of the Galactic bar), then the LSR orbit is oval.
Now we define the velocity of stars relative to the LSR. For example, look at three hypothetical orbits:

- Star A lags the LSR (negative Theta velocity)
- Star B leads the LSR (positive Theta velocity)
- Star C has Theta=0

*Note: The LSR is not the orbit of the Sun!!!*
What is the Sun's motion relative to the LSR?

Look at all the disk stars around us, and measure their radial velocities (v_r) and proper motions (μ). Do this for lots of stars, and take the average along different lines of sight.

- If the sun wasn't moving, what would you expect to see?
Solar motion: sun moving “upwards” at 7 km/s wrt other stars

Fig 2.9 'Galaxies in the Universe' Sparke/Gallagher CUP 2007
The residual solar motion wrt the average of local stars is

\[ U_\odot = -10 \text{ km s}^{-1} \]
\[ V_\odot = 5 \text{ km s}^{-1} \]
\[ W_\odot = 7 \text{ km s}^{-1} \]

Some say \( V = 15 \text{ km/s} \)!

The Sun is moving

- a bit towards the galactic center
- faster than the LSR
- northward out of the galactic plane

Currently we are near the mid-plane

(Remember this doesn't account for the rotation of the disk!)
The Velocity Distribution of Stars

Make a histogram of the $Z$ (up/down) velocities of stars of different spectral type:

- A stars ("A")
- K giants ("gK")
- M dwarfs ("dM")

*(what is different about these groups of stars?*)

The spread in velocities -- called the **velocity dispersion** and calculated as the standard deviation of the distribution -- is different for each group:

<table>
<thead>
<tr>
<th>Stars</th>
<th>Dispersion (km/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>9</td>
</tr>
<tr>
<td>gK</td>
<td>17</td>
</tr>
<tr>
<td>dM</td>
<td>18</td>
</tr>
<tr>
<td>white dwarfs</td>
<td>25</td>
</tr>
</tbody>
</table>

\[
\sigma_z = \sqrt{\sum W_i^2}
\]
Remember also that different groups of stars had different disk thicknesses:

<table>
<thead>
<tr>
<th>Stars</th>
<th>Dispersion (km/s)</th>
<th>Scale height (pc)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B</td>
<td>6</td>
<td>60</td>
</tr>
<tr>
<td>A</td>
<td>9</td>
<td>120</td>
</tr>
<tr>
<td>gK</td>
<td>17</td>
<td>270</td>
</tr>
<tr>
<td>dM</td>
<td>18</td>
<td>350</td>
</tr>
<tr>
<td>white dwarfs</td>
<td>25</td>
<td>500</td>
</tr>
</tbody>
</table>

**Question #1:** Why does dispersion increase with spectral type?

**Question #2:** Why do dispersion and scale height increase together?
Oort limit - imagine the disk as a plane parallel slab

First, think of balancing KE with PE for a small mass m orbiting a big mass M:
\[ \frac{1}{2}mv^2 \sim \frac{GMm}{r} \]

So we can solve for the big mass M:
\[ v^2 \sim \frac{2GM}{r} \]

Now, instead of a big mass M, think of a circular patch of radius r and surface density Sigma (in M_{sun}/pc^2). It has a total mass:
\[ M \sim \Sigma_0 \pi r^2 \]

So plug that in and get:
\[ v^2 \sim 2\pi G \Sigma_0 r \]
\[ \sigma_z^2 \sim 2\pi G \Sigma_0 z_0 \]

So if we measure velocity dispersions and scale heights for groups of stars, we can measure the mass density of the Galaxy's disk. This was first done in the early 1960s by Jan Oort and is called the Oort limit. A recent (and more sophisticated) analysis gives \( \sim 70 \text{ M}_{\text{sun}}/\text{pc}^2 \).

Now let's just add up all the mass we see:

<table>
<thead>
<tr>
<th></th>
<th>( \text{M}_{\text{sun}}/\text{pc}^2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Stars</strong></td>
<td>25</td>
</tr>
<tr>
<td><strong>Stellar remnants (mostly WDs)</strong></td>
<td>20</td>
</tr>
<tr>
<td><strong>Gas (HI+H2)</strong></td>
<td>5</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>50</td>
</tr>
</tbody>
</table>

Are we happy with these sums?
The Rotation Curve of the Milky Way

Now, what about measuring how fast the disk is rotating? How could you measure that? Observations show that $\Theta_0(R_0) \sim 220$ km/s

So we can use this to derive...

...the orbital period of the Sun...

$$P = \frac{2\pi R_0}{\Theta_0} \sim 230 \text{ million years}$$

...and the mass of the Galaxy.

$$M(R) = \frac{\Theta_0^2 R_0}{G} \sim 9 \times 10^{10} M_\odot$$

How might we get $R_0$?

This is similar to (but a bit more than) the observed mass in stars and gas. So everything is fine! *Or is it??*

This analysis is only for the Sun's galactocentric radius ($R_0$). Can we do this throughout the Galaxy and get $v(R)$?
The Tangent-Point Method

Look at gas clouds in the Milky Way. Using 21-cm radio emission, we can get their radial velocity via the doppler shift.

Imagine looking at some line of sight through the galaxy and observing the gas clouds:

So \( v(C) = v(R_{\text{min}}) = v(R_0\sin(l)) \).
So $v(C) = v(R_{\text{min}}) = v(R_0 \sin(l))$.

We can do this mapping for all $R < R_0$. At $R > R_0$, the geometry becomes ambiguous, and we need to actually know the true distance to whatever object we are measuring the velocity of. This is harder, although still possible.

From this, we construct the rotation curve of the Milky Way:

*Figure 22.27* The rotation curve of the Milky Way Galaxy. The IAU standard values of $R_0 = 8.5$ kpc and $\Theta_0 = 220$ km s$^{-1}$ have been assumed. (Figure from Clemens, *Ap. J.*, 295, 422, 1985.)
Fig 2.20 (D. Hartmann) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007
terminal velocity curve

$v_{term}$ (km s$^{-1}$)

\[ l \]

-20 \quad -30 \quad -40 \quad -50 \quad -60 \quad -70
Fig 2.21 (Burton, Honma) 'Galaxies in the Universe' Sparke/Gallagher CUP 2007
bumps & wiggles in rotation curves

$\Sigma$ (M$_{\odot}$ pc$^{-2}$)

$R$ (kpc)

$V_c$ (km s$^{-1}$)

$V_t$ (km s$^{-1}$)

$R$ (kpc)

$\text{terminal velocity curve}$

$\text{stars}$

$\text{gas}$
Renzo’s rule: When you see a feature in the light, you see a corresponding feature in the rotation curve.
Fitting the bumps & wiggles at small radii successful in predicting the rotation curve at large radii.