Cosmological Overdensities & Enclosed Mass: Relating R$_\triangle$, M$_\triangle$, and V$_\triangle$

It is conventional in cosmology to refer to structures by the density contrast they represent with respect to the critical density of the universe. The mass enclosed within a radius encompassing the over-density Δ is

$$M_\Delta = \frac{4\pi}{3}\Delta\rho_{crit}R_\Delta^3.$$

With the definition of critical density

$$\rho_{crit} = \frac{3H_0^2}{8\pi G},$$

this becomes

$$M_\Delta = \frac{\Delta}{2G}H_0^2 R_\Delta^3.$$

By the same token, the circular velocity of a tracer particle at R_Δ is

$$V_\Delta^2 = \frac{GM_\Delta}{R_\Delta}.$$

Consequently,

$$M_\Delta = (\Delta/2)^{-1/2}(GH_0)^{-1}\ V_\Delta^3.$$

The choice of reference Δ is somewhat arbitrary. For the current best-fit ΛCDM cosmology, the "virial" radius within which the mass has had time to settle down occurs around $\Delta \approx 100$. (The exact virial value of Δ depends weakly on the the cosmological parameters. The definition of virial radius, while formally meaningful, is nevertheless somewhat arbitrary as the mass profiles of dark matter halos merge smoothly into their surroundings.) For a Hubble constant $H_0 = 72\ \mathrm{km\,s^{-1}\,Mpc^{-1}}$,

$$M_{100} = (4.6\times 10^5\ \mathrm{km^{-3}\,s^3\,M_\odot})V_{100}^3.$$