## Baryonic Mass Components of Galaxies

$$
M_{b}=M_{*}+M_{g}=\Upsilon_{*} L+\frac{1}{X}\left(M_{H I}+M_{H_{2}}\right)
$$

$$
X \approx 0.73 \text { (hydrogen fraction) }
$$

- Stars $\quad M_{*}=\Upsilon_{*}^{i} L_{i} \quad L_{i}=4 \pi D^{2} F_{i}$
- $\Upsilon_{*}^{i}$ is the stellar mass-to-light ratio in photometric band $i$
- Cold Gas
- Atomic gas - H I
- $M_{H I}=2.36 \times 10^{5} D^{2} F_{H I}$ negligible within the optical radius
- Molecular gas - $\mathrm{H}_{2}$
- $M_{H_{2}}=1.1 \times 10^{4} D^{2} F_{C O} \quad$ using carbon monoxide as a proxy also scales with stellar mass
at least for late type gaxies $M_{H_{2}} \approx 0.07 M_{*}$ at least for late type galaxies


## Stellar mass-to-light ratios from stellar population models


mean galaxy colors with
CSP models (lines)

galaxies color coded by Hubble type

## Stellar population models

## Typically, redder colors mean higher mass-to-light ratios






Can use multiple colors, but most of the information is in the first one.

## Baryonic Mass of Galaxies

$$
\begin{array}{r}
M_{b}=M_{*}+M_{g}=\Upsilon_{*} L+X^{-1}\left(M_{H I}+M_{H_{2}}\right) \\
X^{-1} \approx 1.33-1.42 \\
\text { hydrogen fraction }
\end{array}
$$

- Stars $\quad M_{*}=\Upsilon_{*}^{i} L_{i} \quad L_{i}=4 \pi D^{2} F_{i}$
- $\Upsilon_{*}^{i}$ is the stellar mass-to-light ratio in photometric band $i$

To a surprisingly good approximation ( $\sim 20 \%$ ), for star forming (late type) galaxies

$$
M_{*} \approx 0.5 L_{[3.6]} \quad \approx 0.63 L_{K}
$$

For early type galaxies and the bulge component of spirals

$$
M_{*} \approx 0.9 L_{[3.6]} \quad \approx 1.1 L_{K}
$$

That gets us the total mass. We also need to know its distribution.
Fit ellipses
Get surface brightness profile


For analytic approximation to the mass profile like exponential disk $\quad \Sigma(R)=\Sigma_{0} e^{-R / R_{d}}$ There can be a formula for the corresponding rotation curve $\quad M_{*}(R)=2 \pi \int_{0}^{R} \Sigma\left(R^{\prime}\right) R^{\prime} d R^{\prime}$ $V_{*}^{2}(R)=\frac{2 G M_{*}}{R_{d}}\left(\frac{R}{2 R_{d}}\right)^{2}\left[I_{0}\left(\frac{R}{2 R_{d}}\right) K_{0}\left(\frac{R}{2 R_{d}}\right)-I_{1}\left(\frac{R}{2 R_{d}}\right) K_{1}\left(\frac{R}{2 R_{d}}\right)\right] \quad M_{*}=2 \pi R_{d}^{2} \Sigma_{0}$

Examples for the size and mass of NGC 6946

Progressive approximations in mass modeling

- Point Mass
- "spherical" disk
- thin exponential disk
- thick exponential disk
- surface density $\Sigma(R)$
- 2D $\Sigma(R, \phi)$ [e.g., bars]
- 3D $\rho(R, \phi, z)$
- 3D + non-equilibrium

We numerically solve the Poisson equation to obtain the gravitational potential $\Phi_{*}$ from the observed surface density $\Sigma_{*}(R)$
$I$ and $K$ are modified Bessel functions



$$
\begin{array}{r}
\text { ikik }=\left[I_{0}(y) K_{0}(y)-I_{1}(y) K_{1}(y)\right] \\
y=\frac{R}{2 R_{d}}
\end{array}
$$

$$
\begin{aligned}
& M_{*}=3.3 \times 10^{10} \mathrm{M}_{\odot} \\
& R_{d}=2.44 \mathrm{kpc}
\end{aligned}
$$





Surface density profile $\Sigma_{b}(R)$


Numerically solve the Poisson equation

$$
\begin{aligned}
& \nabla^{2} \Phi_{*}=4 \pi G \rho_{*}(R, \theta, Z) \\
& \mathrm{g}_{*}(R)=\frac{V_{*}^{2}}{R}=-\frac{\partial \Phi_{*}}{\partial R}=2 \pi G \Sigma_{*}(R)
\end{aligned}
$$

for each observed component. Velocities add in quadrature:

$$
V_{b}^{2}(R)=V_{*}^{2}(R)+V_{g}^{2}(R)
$$



## Stellar orbits in galaxies

M105<br>Elliptical Galaxy

NGC 628
Spiral Galaxy

Pressure Supported
Eccentric radial orbits Random orientations

Rotationally Supported
Nearly circular orbits Same direction, same plane

## M33 velocity field



Rotation curves extracted using "tilted ring" fits

Fit ellipses that most closely match the circular velocity at a given radius. In principle, get ellipse center, position angle, axis ratio, inclination, and rotation velocity. In practice, usually have to fix some of these parameters.
tilted ring model


NGC 6822 (Weldrake \& de Blok 2003)


## 21 cm interferometric observations give atomic gas distributions and velocity fields

## NGC 6946


tilted ring model

to which we make tilted ring fits

Rotation curve


The sinusoidal variation of velocity in each ring measures the position angle, inclination, and rotation curve $\mathrm{V}_{\mathrm{c}}(\mathrm{R})$.

$$
V \sin i=V_{s y s}+V_{c} \cos \theta+V_{r} \sin \theta
$$



Example analysis for one galaxy. Left: position-velocity diagrams along the major (top) and minor (bottom) axis. Center: 2D Hi map (top), velocity field (middle), and velocity dispersion (bottom). The left panels show the data, the center panels the fitted model, and the right panels the residuals. Right: derived radial quantities: the rotation curve (top left, with Vf noted in grey), the Hi surface density (top right), followed by the velocity dispersion, system redshift, inclination, position angle, and x and y centroids.

