



“The future is already here –  
it's just not evenly distributed.”

–William Gibson

Mar 9 Laws of Galactic Rotation

[Dynamical Regularities in Rotating Galaxies](#)

McGaugh

Mar 14 Spring Break

Mar 16 Spring Break

the future that now has passed

Mar 21 Fitting and Predicting

[Hypothesis testing with gas rich galaxies](#)

talk among yourselves

Prof. McGaugh at [IAU 379](#)

Mar 23 Data Interpretation

[Declining Rotation Curves at High Redshift?](#)

talk among yourselves

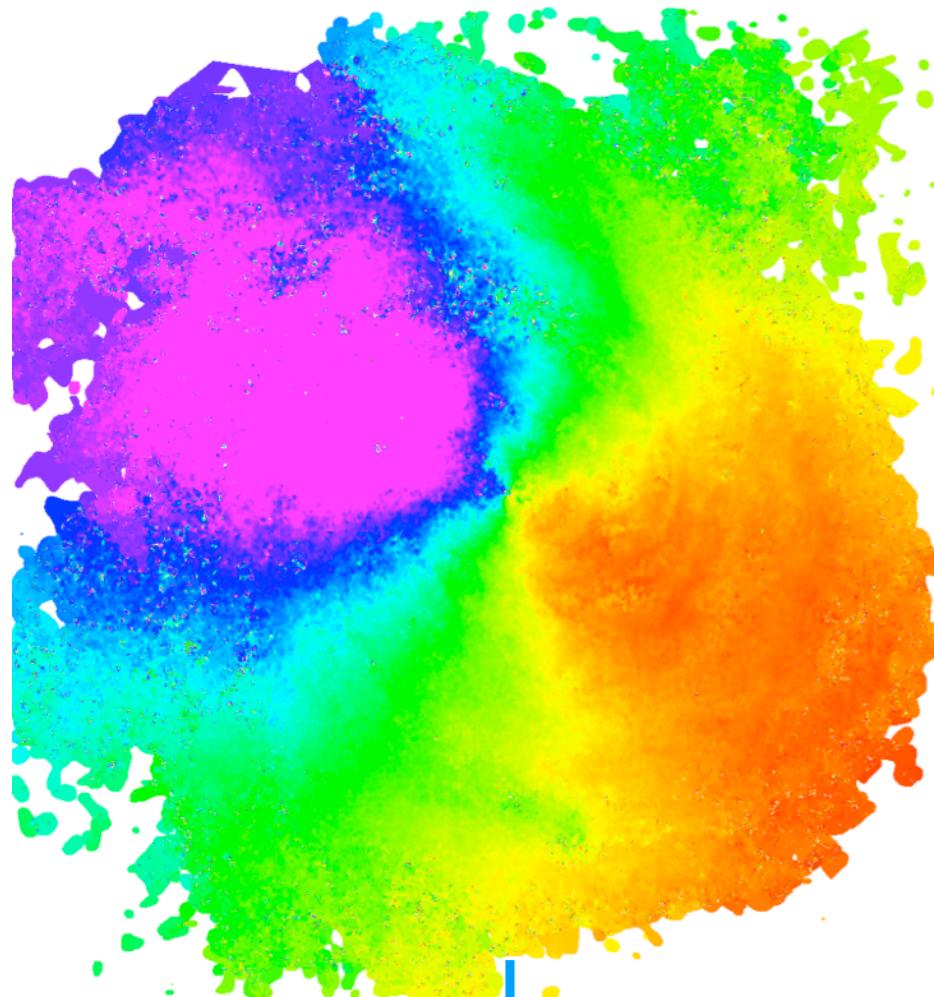
Prof. McGaugh at [IAU 379](#)

“The past is a frozen image we can  
never fully perceive.”

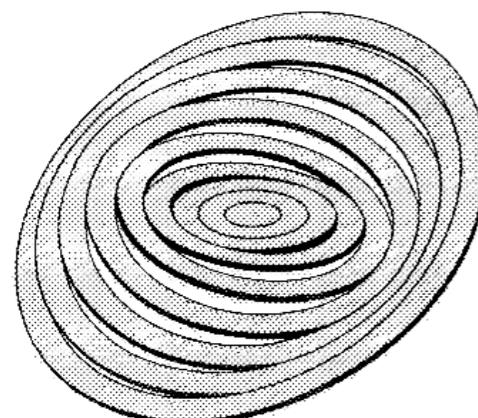
—some stuff I just made up to fill this space

# 21cm interferometric observations give atomic gas distributions and velocity fields

NGC 6946



THINGS (Walter et al. 2008; de Blok et al. 2008)



to which we make tilted ring fits

Rotation curve

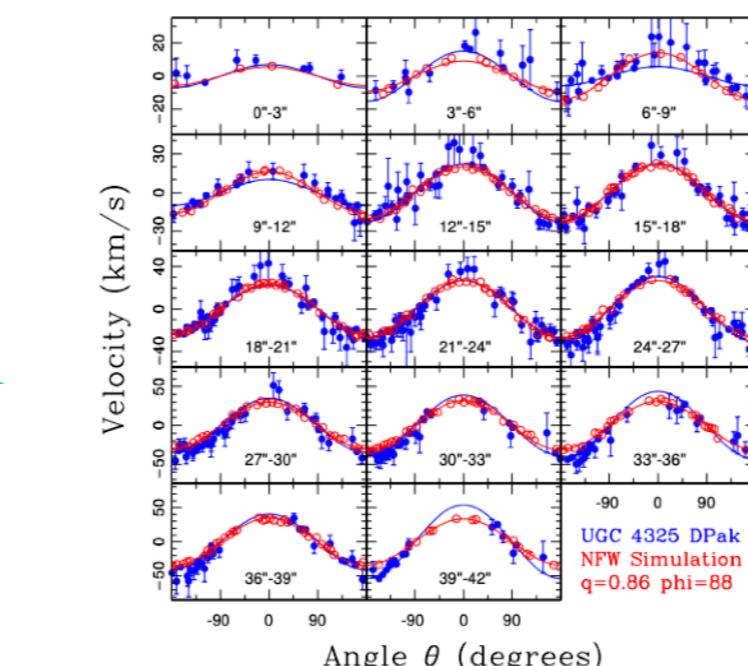
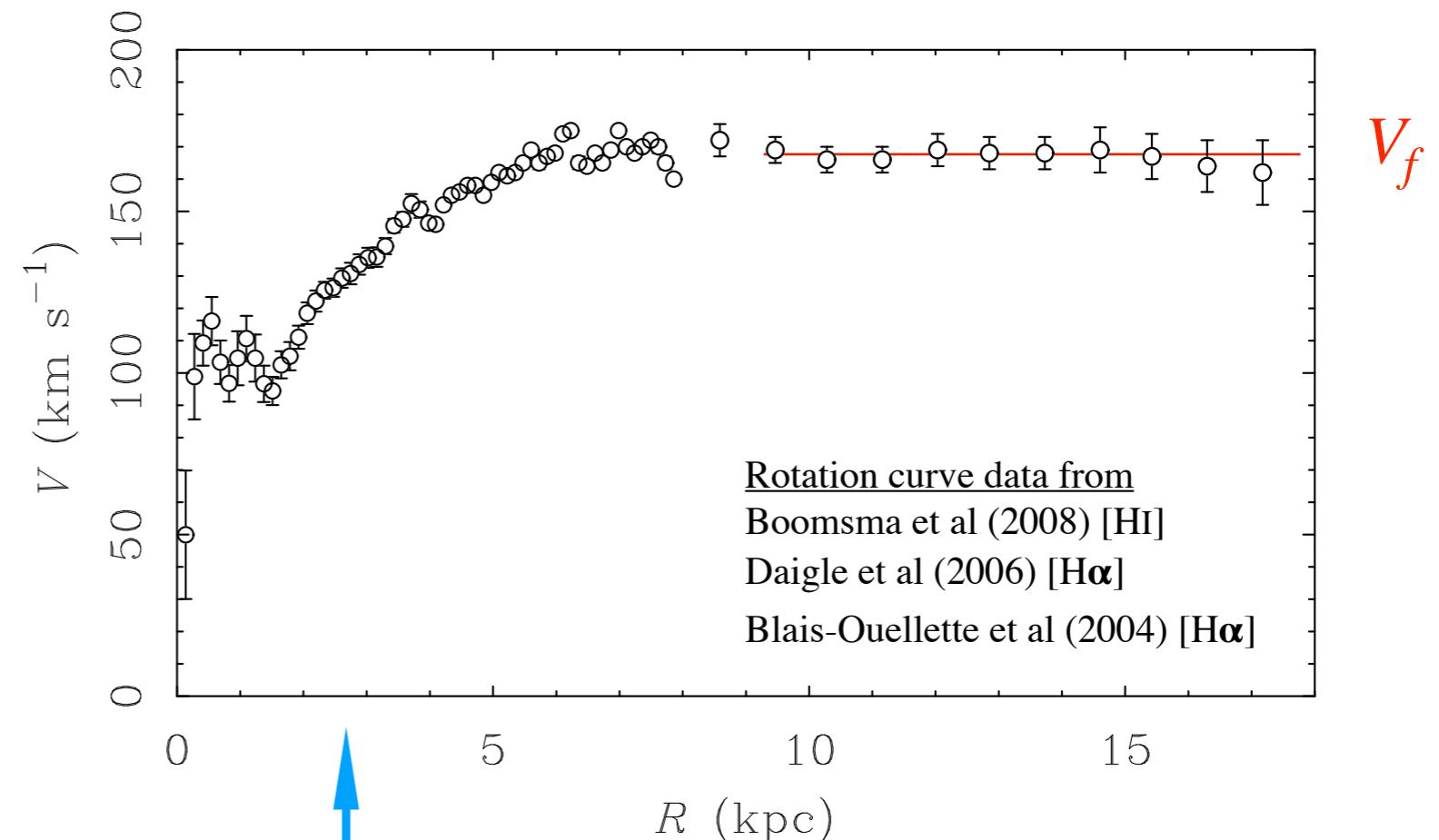
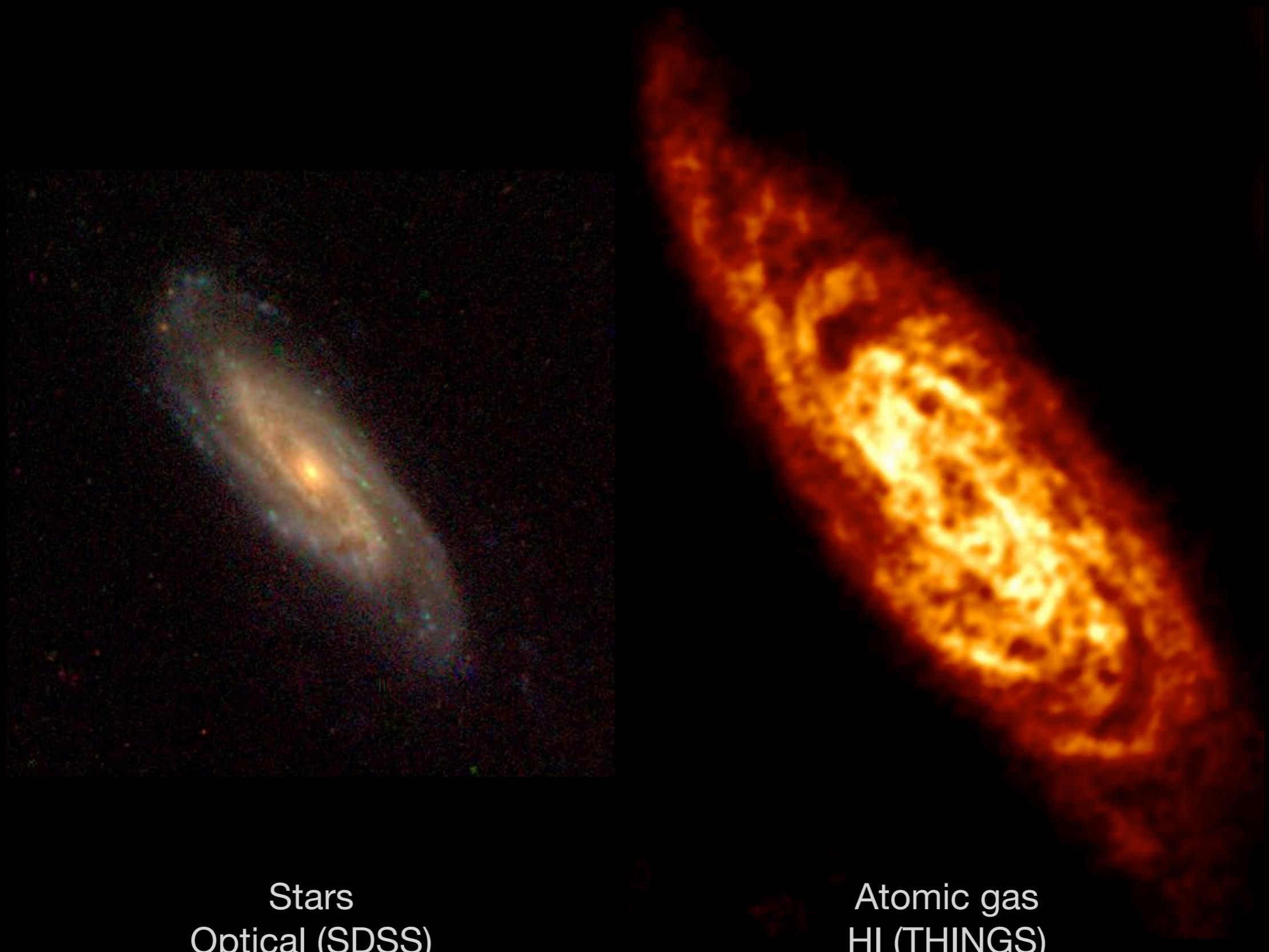


Figure 5.6: – The  $(0.86, 88^\circ)$  simulation results (red) over-plotted with the observed UGC 4325 data (blue). The simulation and data match well between  $\sim 12'' - 30''$ .

$$V \sin i = V_{\text{sys}} + V_c \cos \theta + V_r \sin \theta$$

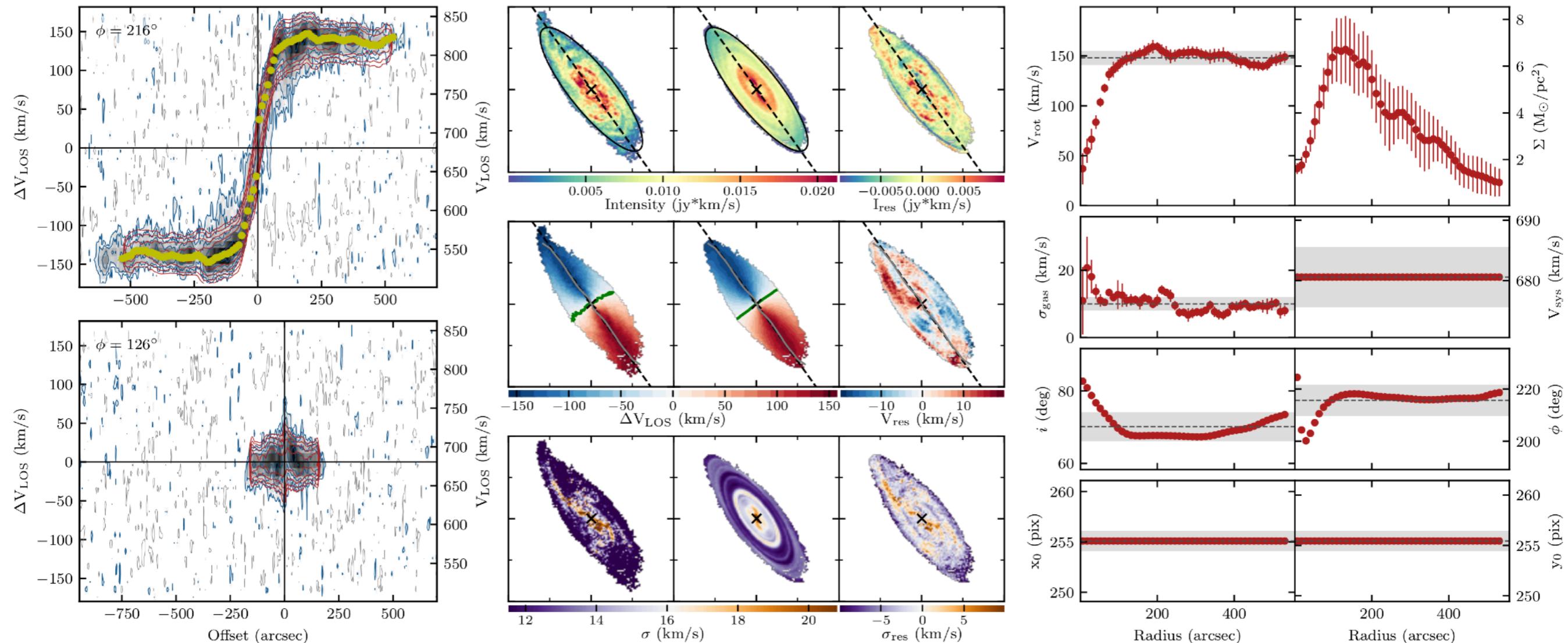
The sinusoidal variation of velocity in each ring measures the position angle, inclination, and rotation curve  $V_c(R)$ .

# NGC 3198



# 3D modeling

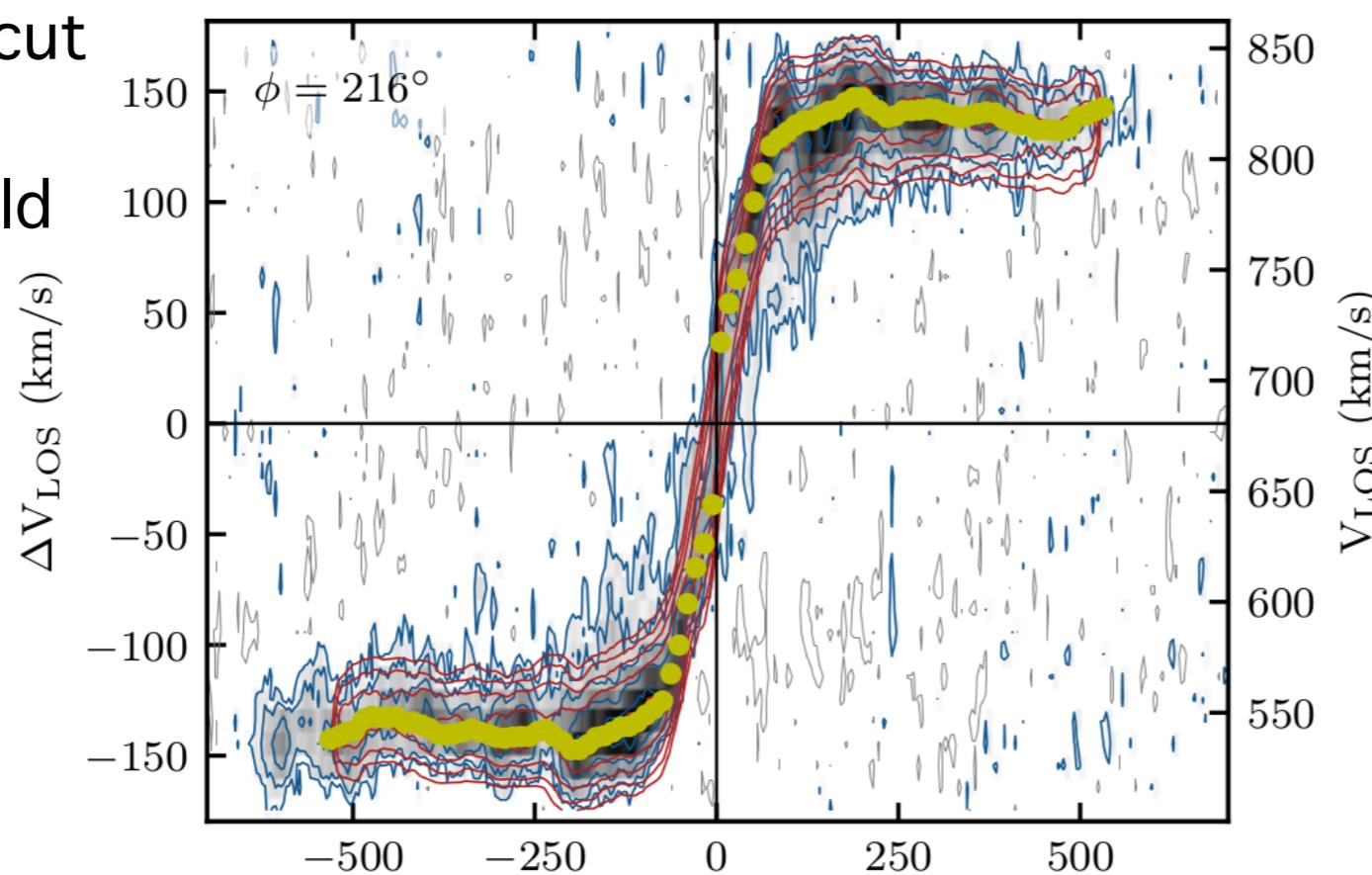
## NGC3198 - UGC05572 (THINGS)



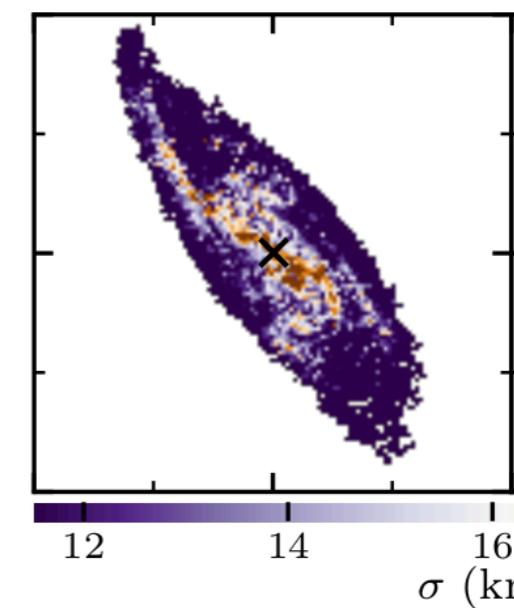
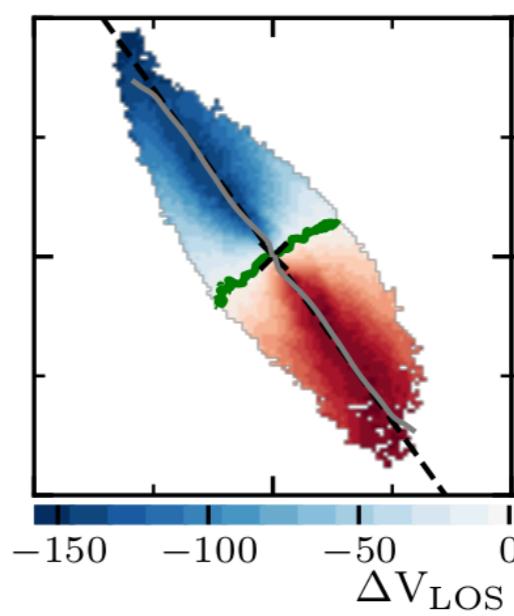
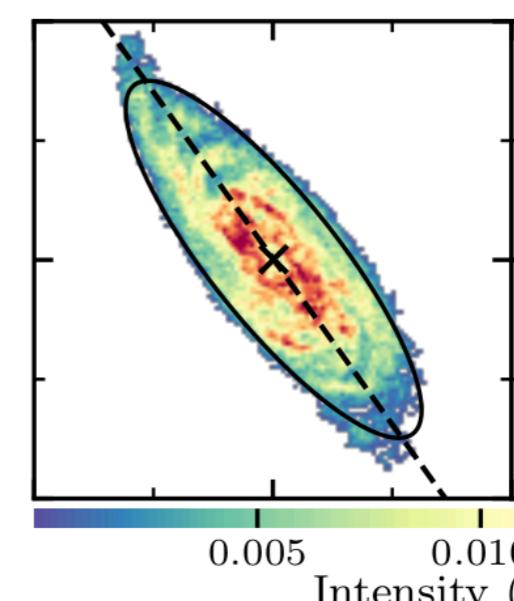
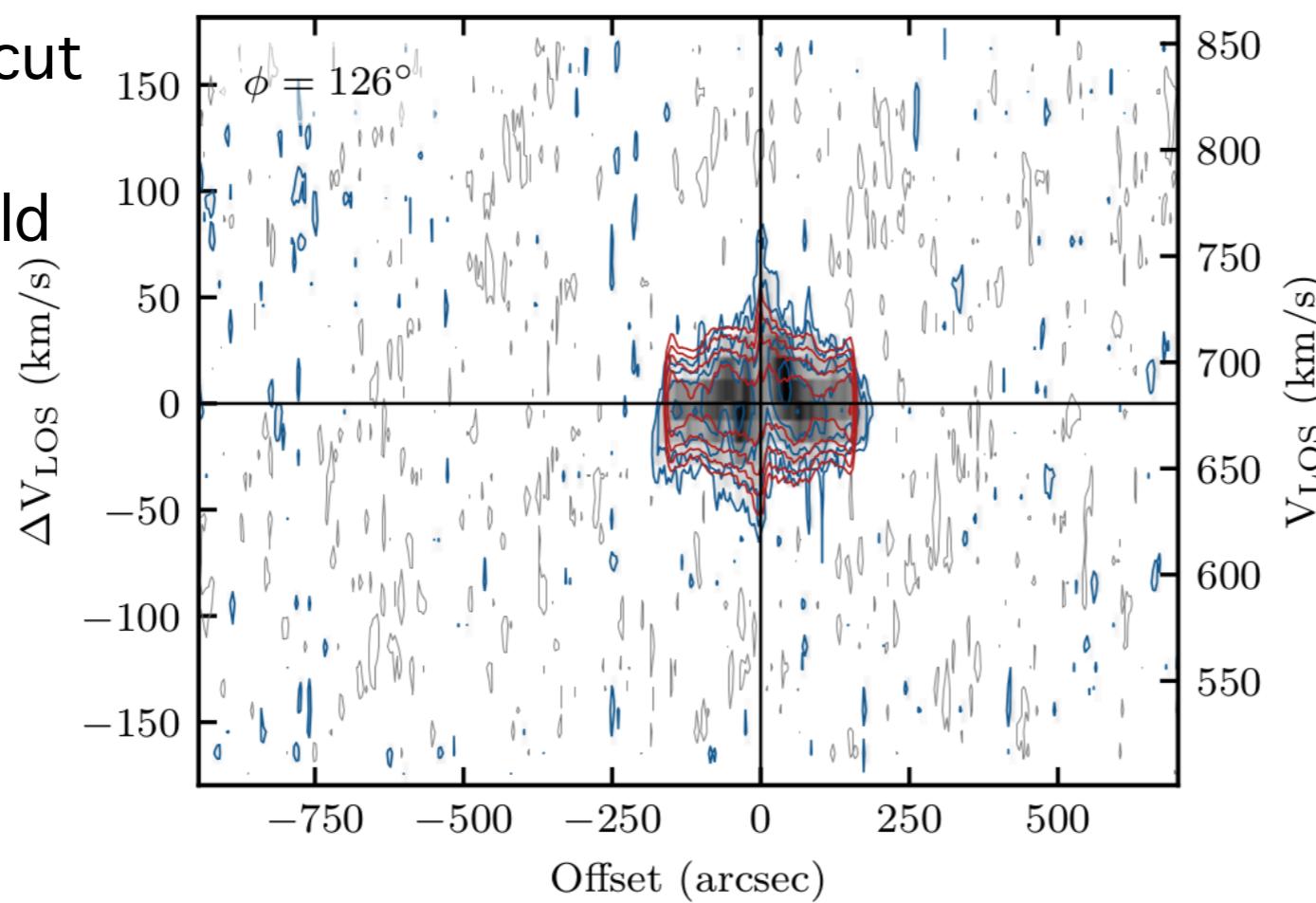
Example analysis for one galaxy. Left: position–velocity diagrams along the major (top) and minor (bottom) axis. Center: 2D HI map (top), velocity field (middle), and velocity dispersion (bottom). The left panels show the data, the center panels the fitted model, and the right panels the residuals. Right: derived radial quantities: the rotation curve (top left, with  $V_f$  noted in grey), the HI surface density (top right), followed by the velocity dispersion, system redshift, inclination, position angle, and x and y centroids.

# NGC3198 - UGC05572 (THINGS)

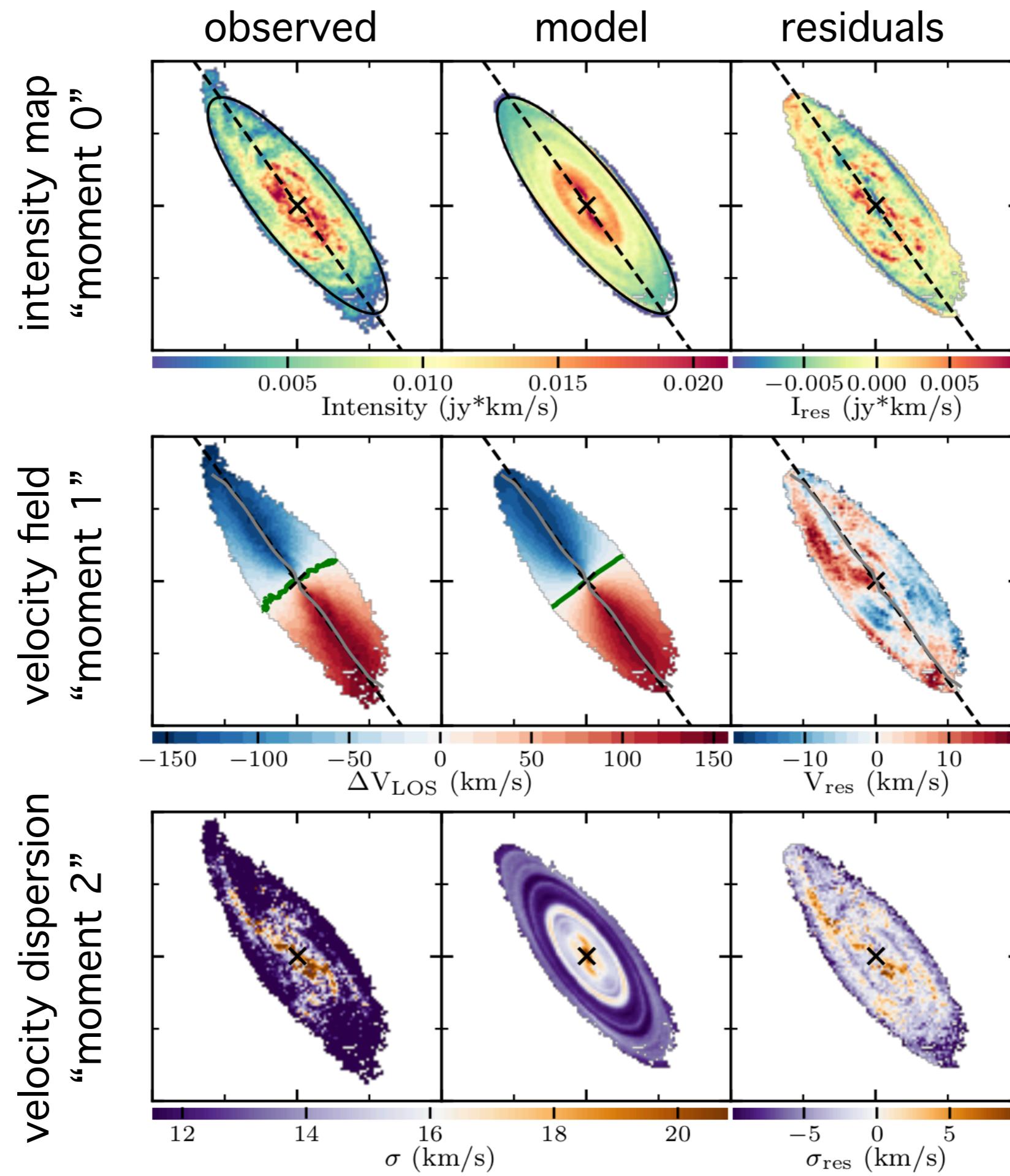
Major axis cut  
through  
velocity field



Minor axis cut  
through  
velocity field



intensity map  
“moment 0”  
velocity field  
“moment 1”  
velocity dispersion  
“moment 2”



The model is axisymmetric.

Spiral arms visible in the residual.

• Beam ( $11.4'' \times 9.4''$ )

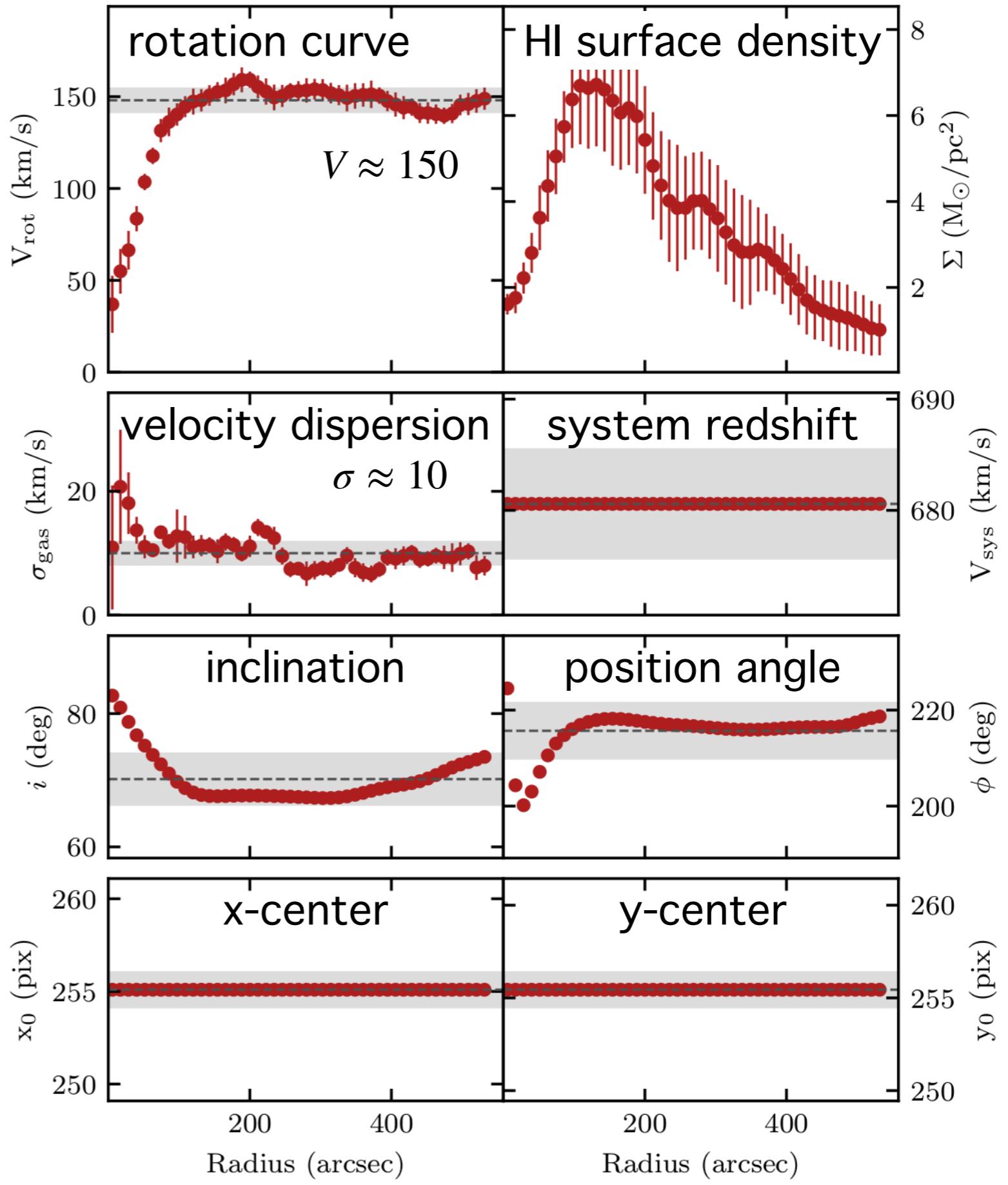
$569''$

Most spirals have large  $V/\sigma$ .  
In this case,

$$\frac{V}{\sigma} \approx 15$$

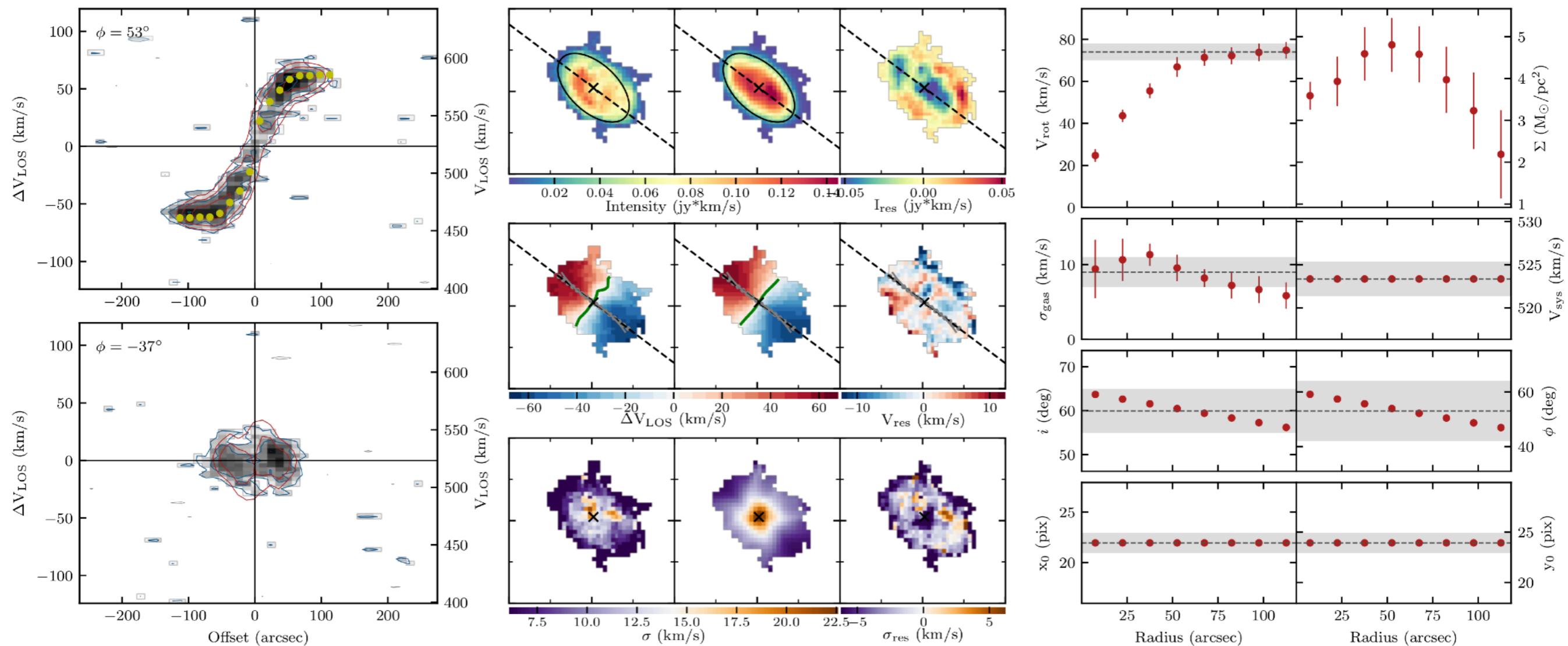
so almost all kinetic energy  
is in rotation. This is a “cold”  
disk.

Don’t need to worry (too  
much) about the velocity  
dispersion or flows along the  
spiral arms (both of order 10  
km/s)



Gotta do this for every galaxy, each of which has its own individual mass distribution

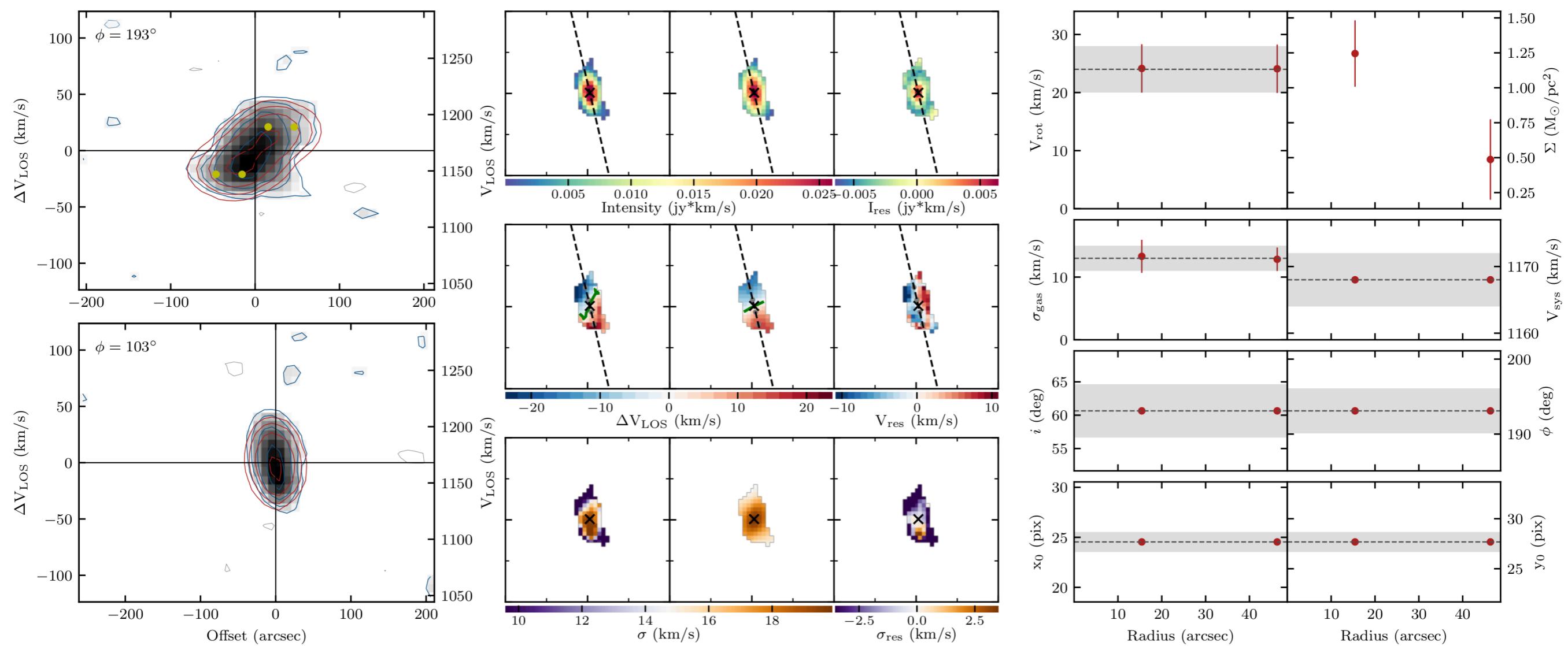
### NGC2552 – UGC04325 (WHISP)



Example analysis for one galaxy. Left: position–velocity diagrams along the major (top) and minor (bottom) axis. Center: 2D Hi map (top), velocity field (middle), and velocity dispersion (bottom). The left panels show the data, the center panels the fitted model, and the right panels the residuals. Right: derived radial quantities: the rotation curve (top left, with  $V_f$  noted in grey), the Hi surface density (top right), followed by the velocity dispersion, system redshift, inclination, position angle, and  $x$  and  $y$  centroids.

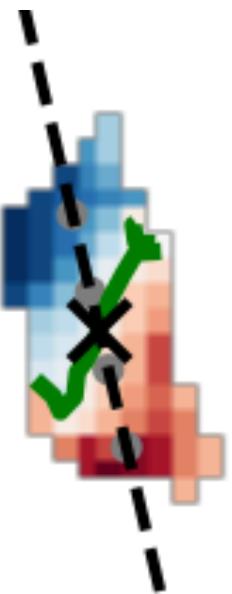
# Sometimes the data leave something to be desired

AGC740301 (ATLAS3D)



Example analysis for one galaxy. Left: position–velocity diagrams along the major (top) and minor (bottom) axis. Center: 2D  $\text{H}\alpha$  map (top), velocity field (middle), and velocity dispersion (bottom). The left panels show the data, the center panels the fitted model, and the right panels the residuals. Right: derived radial quantities: the rotation curve (top left, with  $V_f$  noted in grey), the  $\text{H}\alpha$  surface density (top right), followed by the velocity dispersion, system redshift, inclination, position angle, and  $x$  and  $y$  centroids.

Note that in this case, the kinematic minor axis is not orthogonal to the major axis.



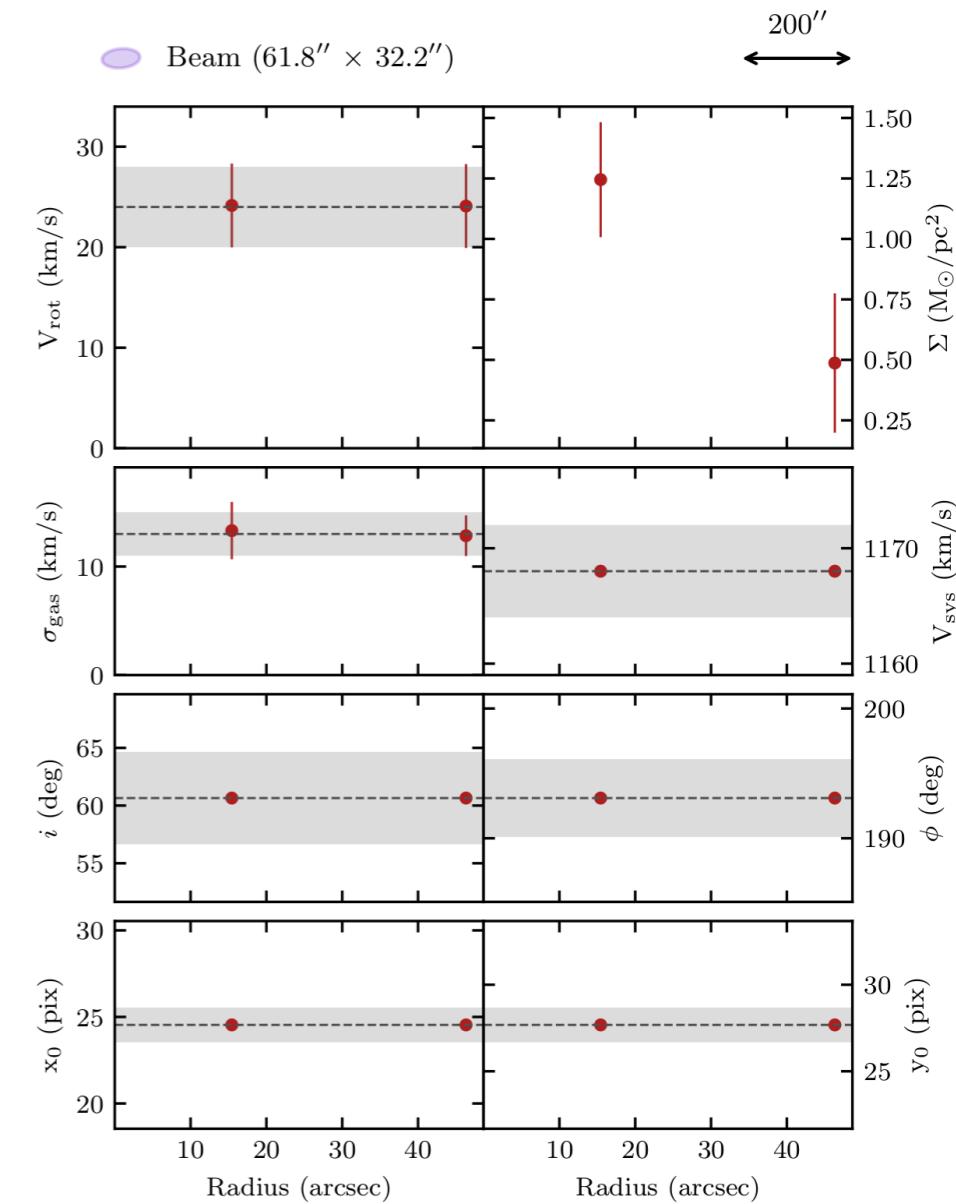
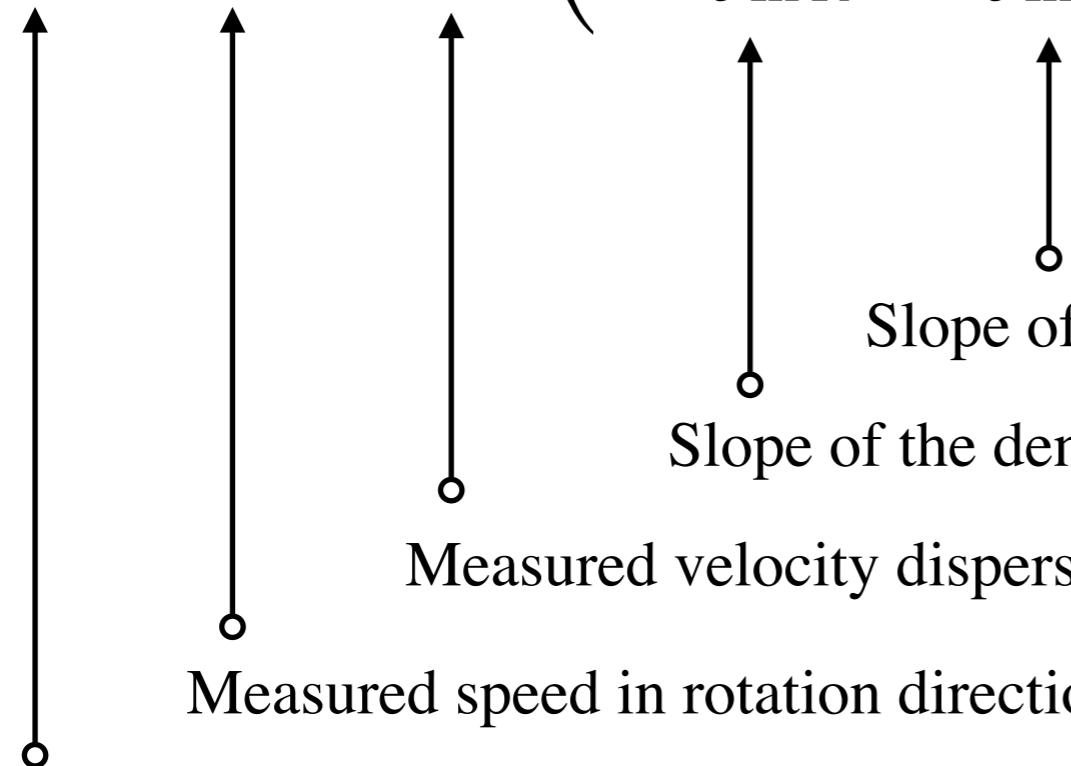
# Sometimes the data leave something to be desired

In this case, the velocity dispersion is comparable to the rotation speed,  $\sigma = 12 \text{ km/s}$  and  $V_{\text{rot}} = 23 \text{ km/s}$ . Need to account for total kinetic energy to assess the gravitational potential.

(Noncircular motion sometimes referred to as “asymmetric drift”)

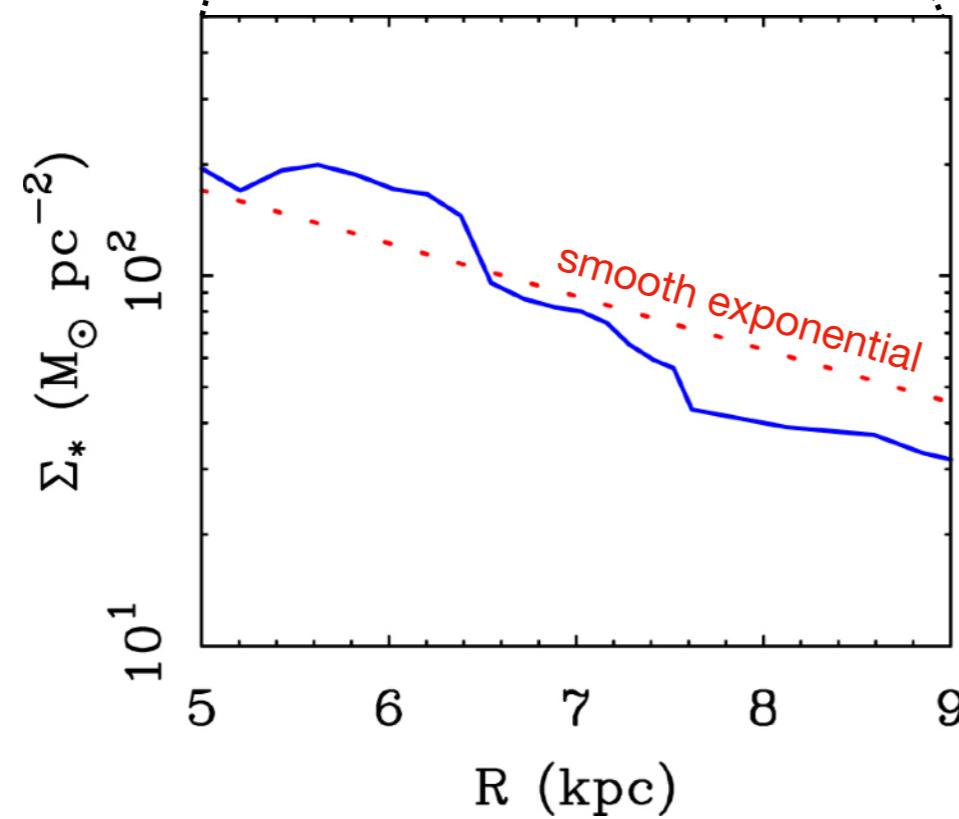
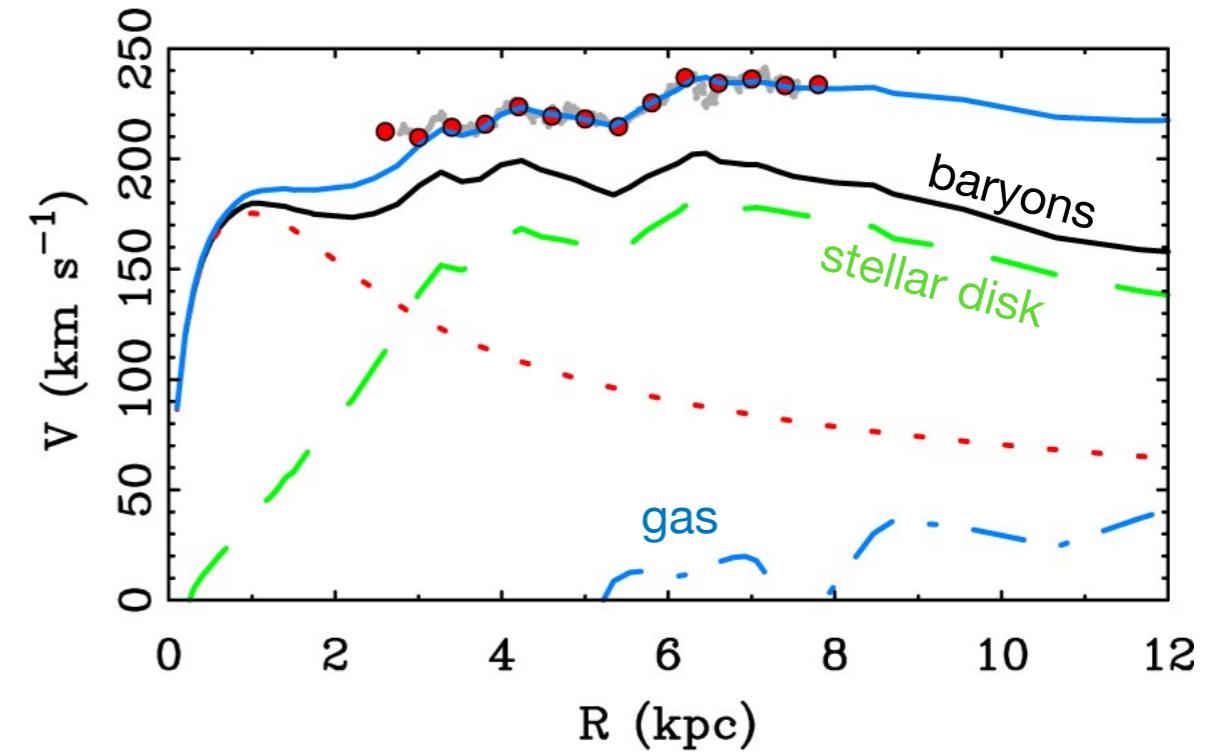
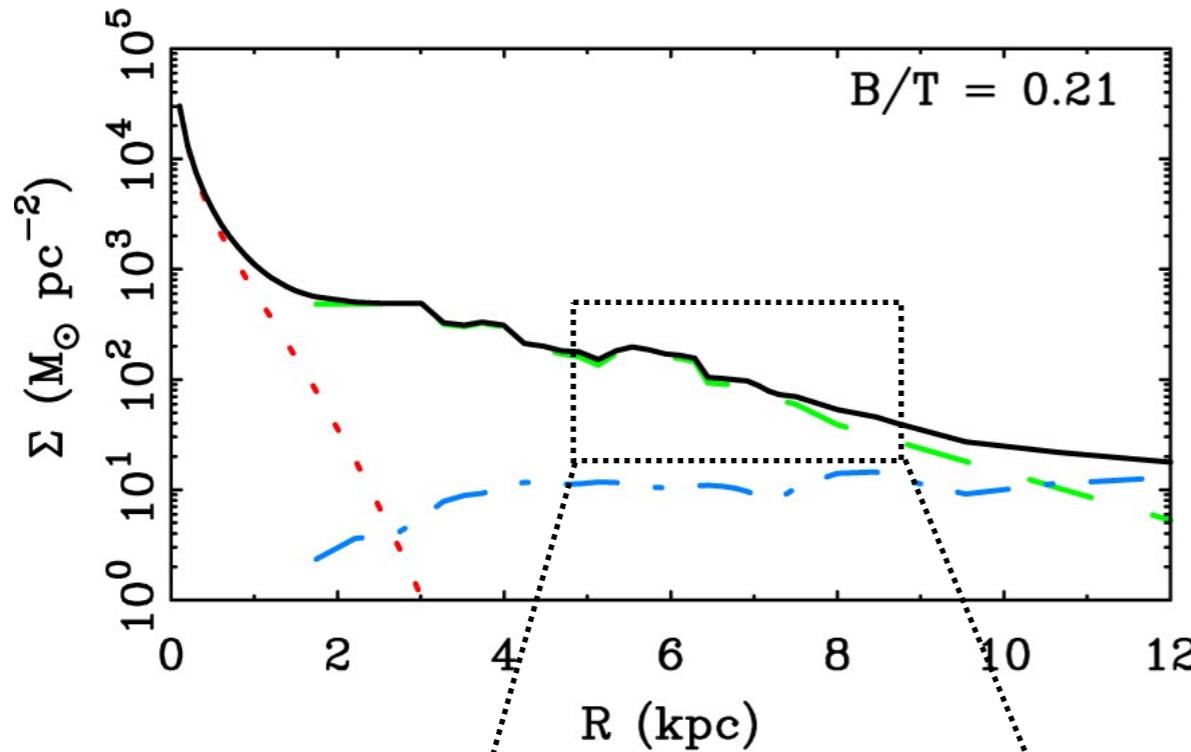
The Jeans equation is one approach to account for noncircular motion:

$$V_c^2(R) = \langle V_\phi \rangle^2 - \langle V_R \rangle^2 \left( 1 + \frac{\partial \ln \nu}{\partial \ln R} + \frac{\partial \ln \langle V_R \rangle^2}{\partial \ln R} \right)$$

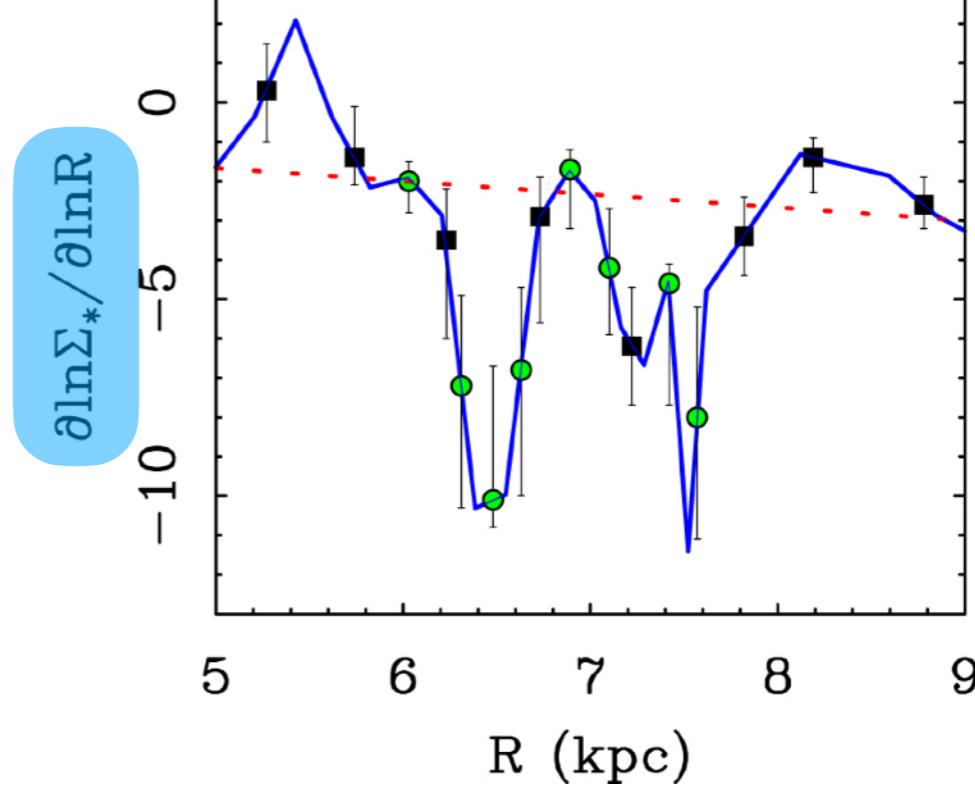


Velocity of a test particle on a circular orbit (a representation of the potential)

Example: model Milky Way



$$V_c^2(R) = \langle V_\phi \rangle^2 - \langle V_R \rangle^2 \left( 1 + \frac{\partial \ln \nu}{\partial \ln R} + \frac{\partial \ln \langle V_R \rangle^2}{\partial \ln R} \right)$$



Now have all that we need to describe the mass distribution,  
the corresponding gravitational potential, and  
the resulting equilibrium kinematics

- Mass distribution
  - stellar disk
  - central bulge
  - atomic gas
  - molecular gas
  - etc.



- Gravitational Potential
  - Each component distinct
  - Add linearly in Newton
- Kinematics
  - rotation
  - pressure (velocity dispersion)

## Empirical Laws of Galactic Rotation

- Flat rotation curves (Rubin-Bosma Law)

Rotation curves tend asymptotically towards a constant rotation velocity that persists to indefinitely large radii:  $V(R \rightarrow \infty) \rightarrow V_f$

- Tully-Fisher relation (Luminous, Stellar Mass, and Baryonic TF relations)

The baryonic mass of galaxies scales as the fourth power of the flat rotation velocity:  $M_b = A V_f^4$

- Central density relation (lower surface brightness galaxies exhibit larger mass discrepancies)

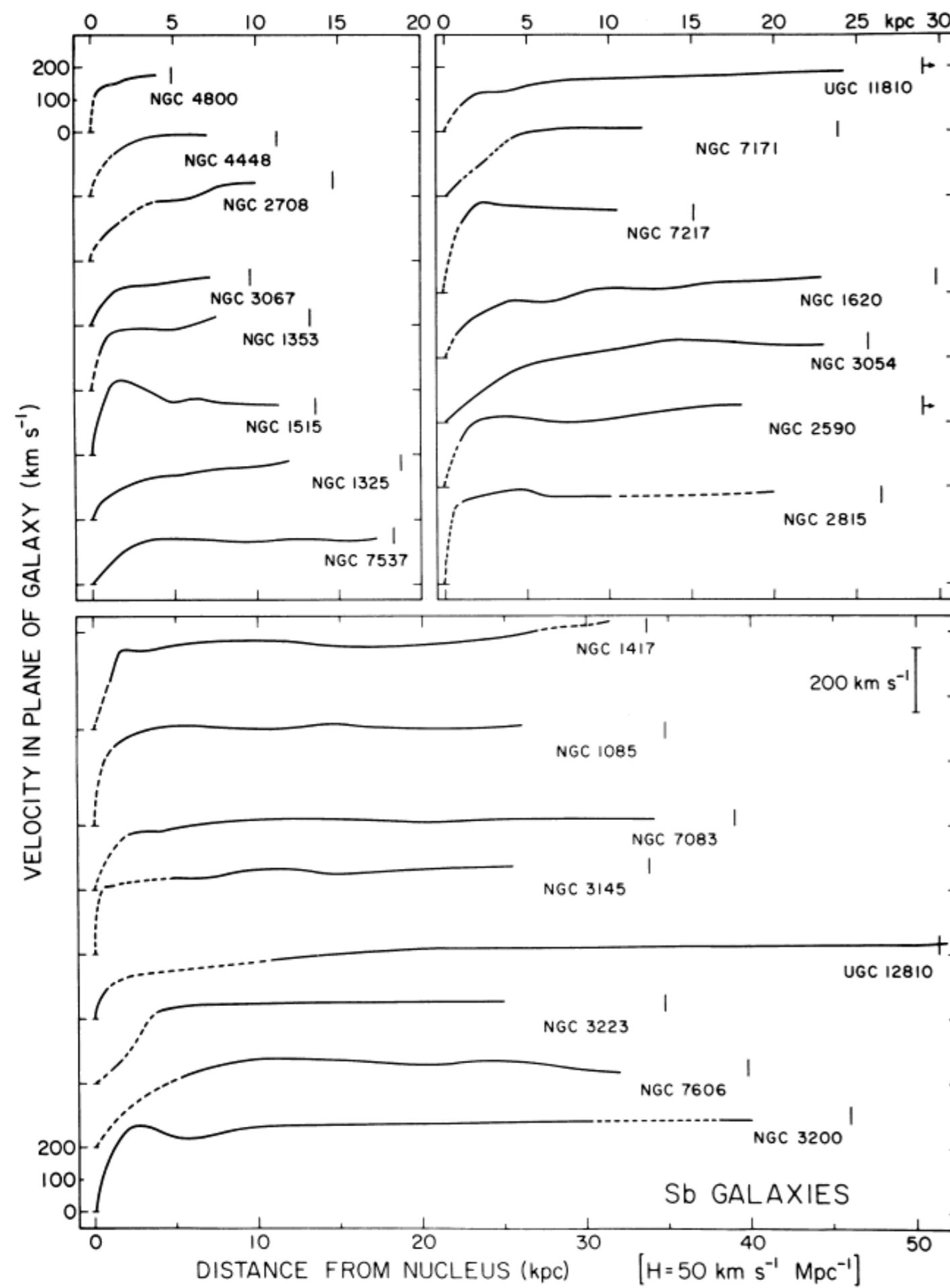
The central dynamical surface densities of galaxies is related to their central surface brightnesses:  $\Sigma_{dyn}(R \rightarrow 0) = f[\Sigma_*(R \rightarrow 0)]$

- Renzo's rule (Sancisi's Law)

“For any feature in the luminosity profile there is a corresponding feature in the rotation curve and vice versa.” (Sancisi 2004).

- Radial acceleration relation

The observed centripetal acceleration is related to that predicted by the observed distribution of baryons:  $g_{obs} = \mathcal{F}(g_{bar})$



Rotation curves tend to become flat at large radii

$$V \propto \text{const}$$

$$M \propto R$$

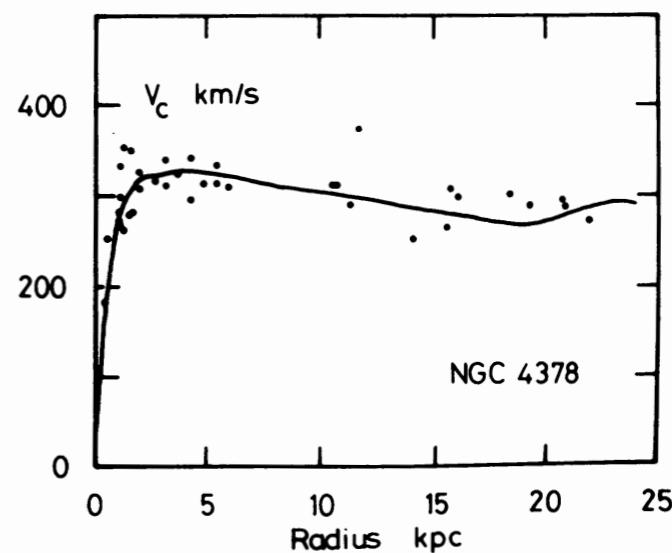
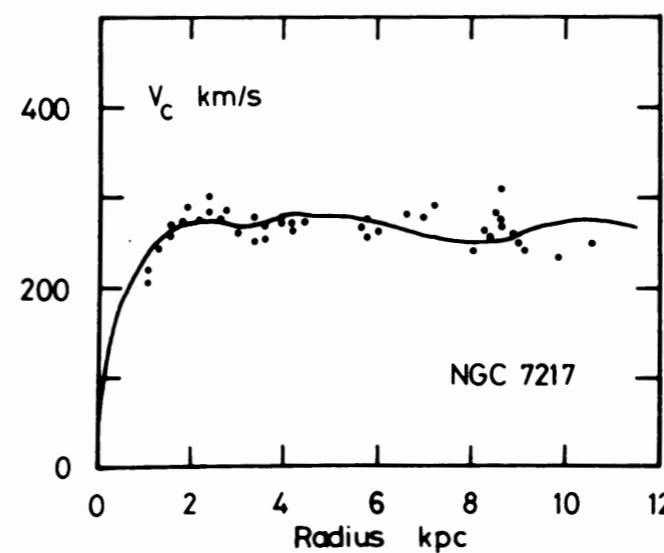
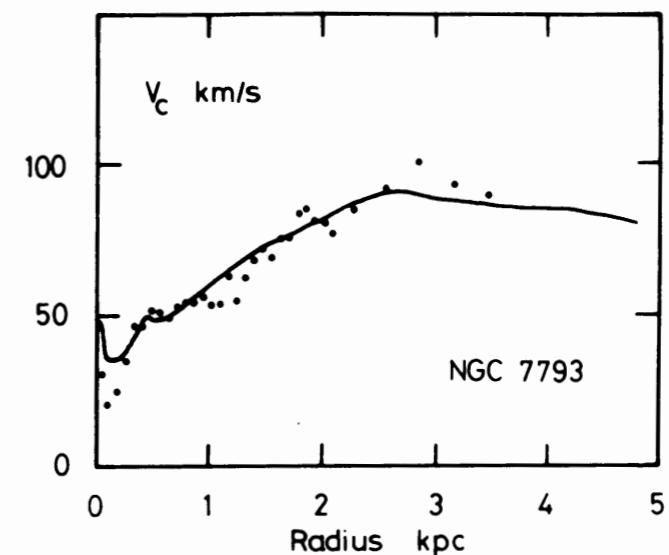
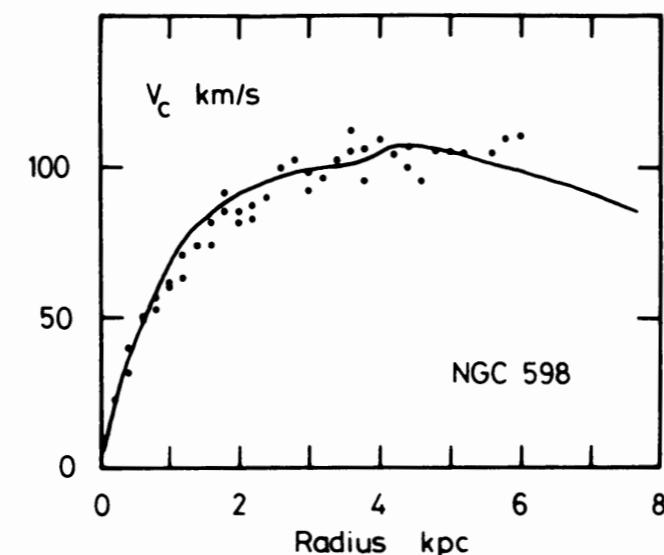
$$\rho \propto R^{-2}$$

Optical data from  
Rubin, Thonnard, & Ford 1978, *ApJ*, 225, L107

# Kalnajs IAU 100 (1982)

KALNAJS : The customary approach of deducing mass distribution from rotation curves involves an implicit or explicit extrapolation of the velocity data, and the often reported rise of M/L usually begins where the observed information runs out. I would like to show you a slide depicting four rotation curves computed from photometric data which has been converted into mass distributions by assuming that M/L is constant within a galaxy. The photometry extends to faint enough limits to completely determine the rotation curves. For NGC 4378 it was necessary to decompose the light into a bulge and a disk. For the others the decomposition gave essentially the same curves as would have been obtained from pure disks.

The rotation curves agree well with the observed velocity points, and thus demonstrate that the flat rotation curves of NGC 7217 and NGC 4378 need not lead one to conclude that there is dark matter in the outer parts of these galaxies.



Rotation curves computed from photometry assuming a constant M/L within each galaxy. The dots are the measured velocities. The values of M/L used are 5.0, 2.9, 4.2 and 6.5.

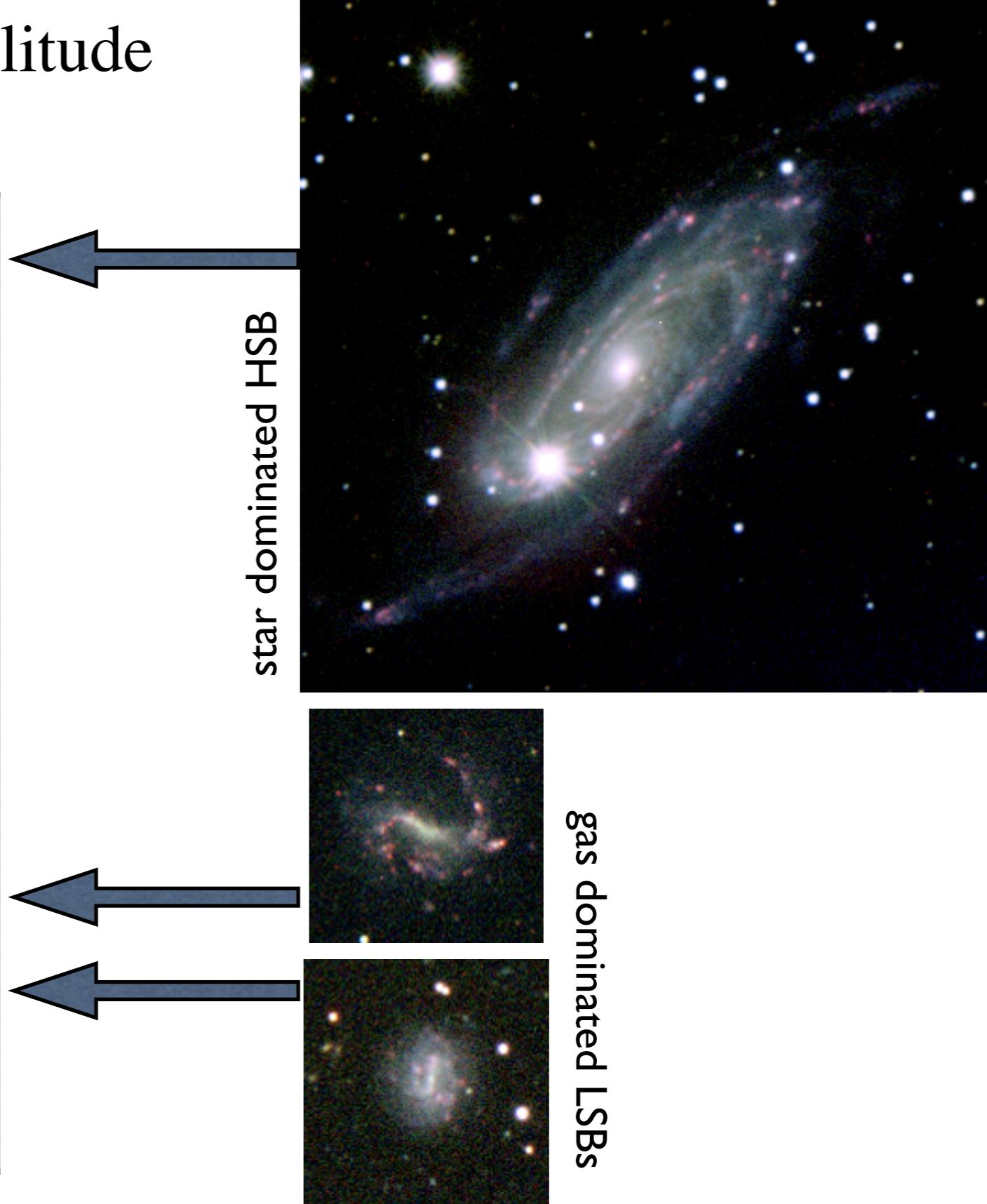
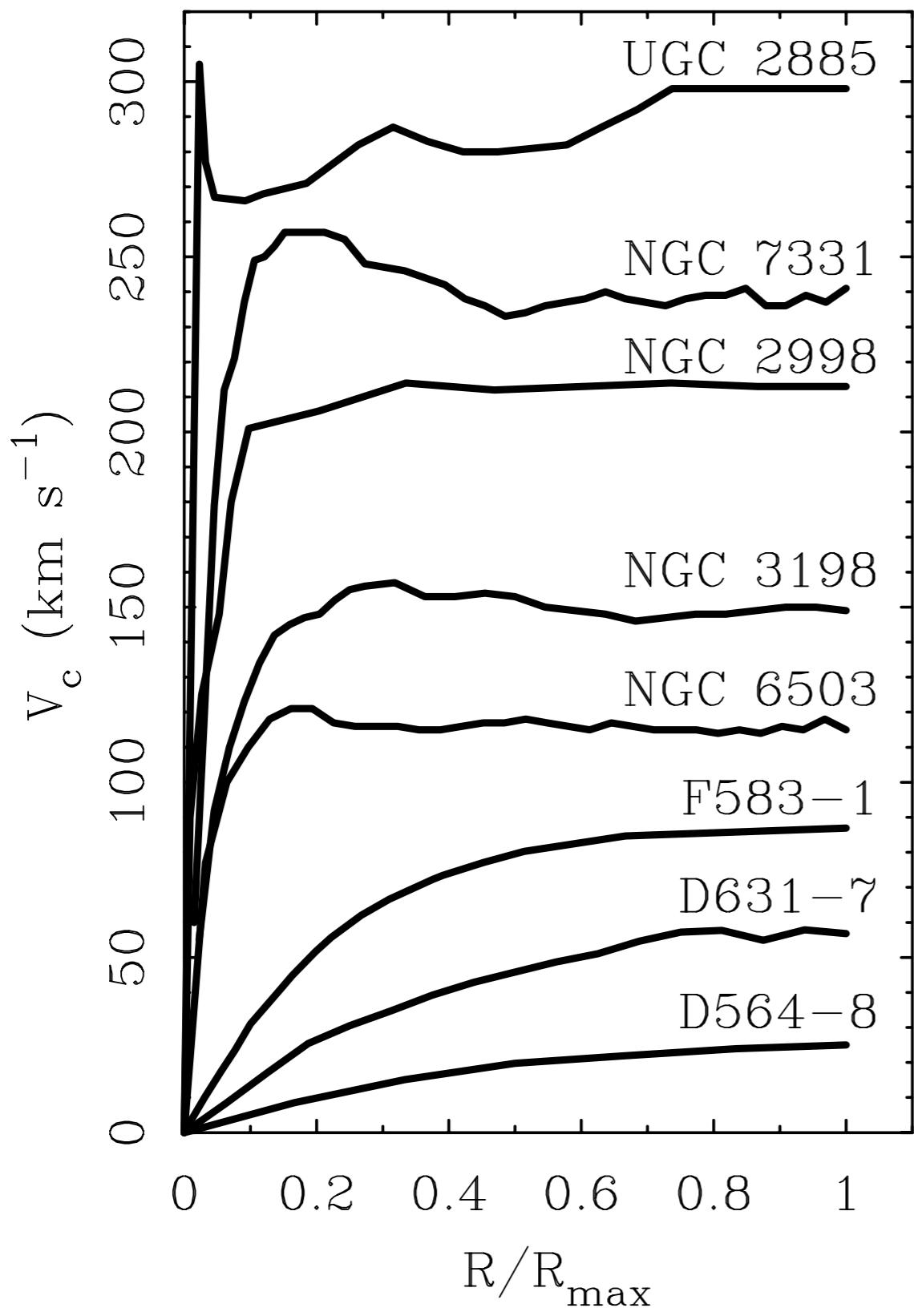
..... :  
..... : ?  
..... : !!!!!  
somebody : HA, HA, HA.  
..... : \*\*\*, ???, !!!

(The audience becomes restive and the massive halo enthusiasts slowly regain their composure).

HAUD : This is very interesting, but note the limited extent of the rotation curves of these four galaxies. Three of them extend to radii less than 12 kpc and the fourth one reaches 25 kpc. Usually the M/L starts to increase rapidly only outside roughly 30 kpc, and only giant galaxies have coronas.

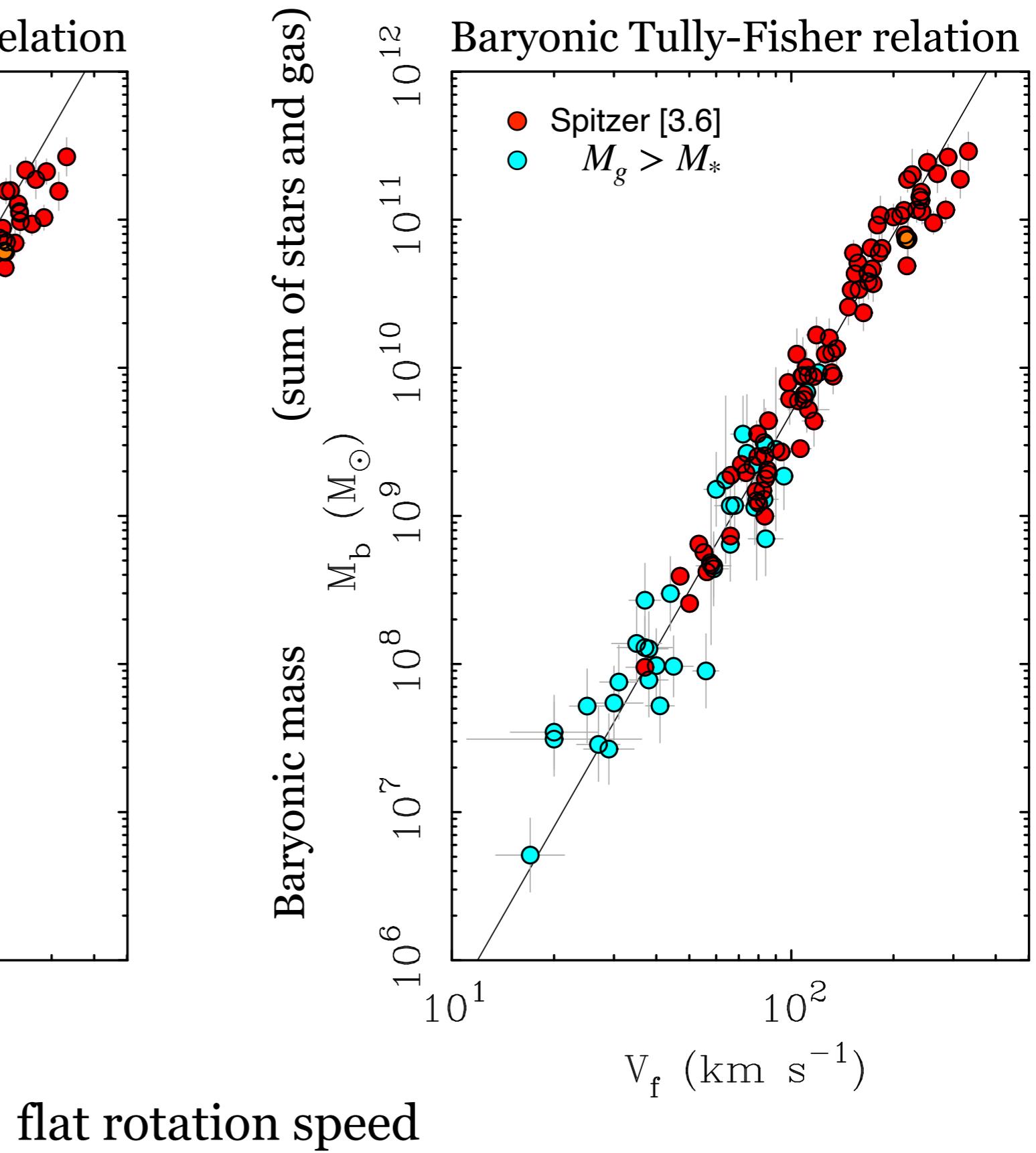
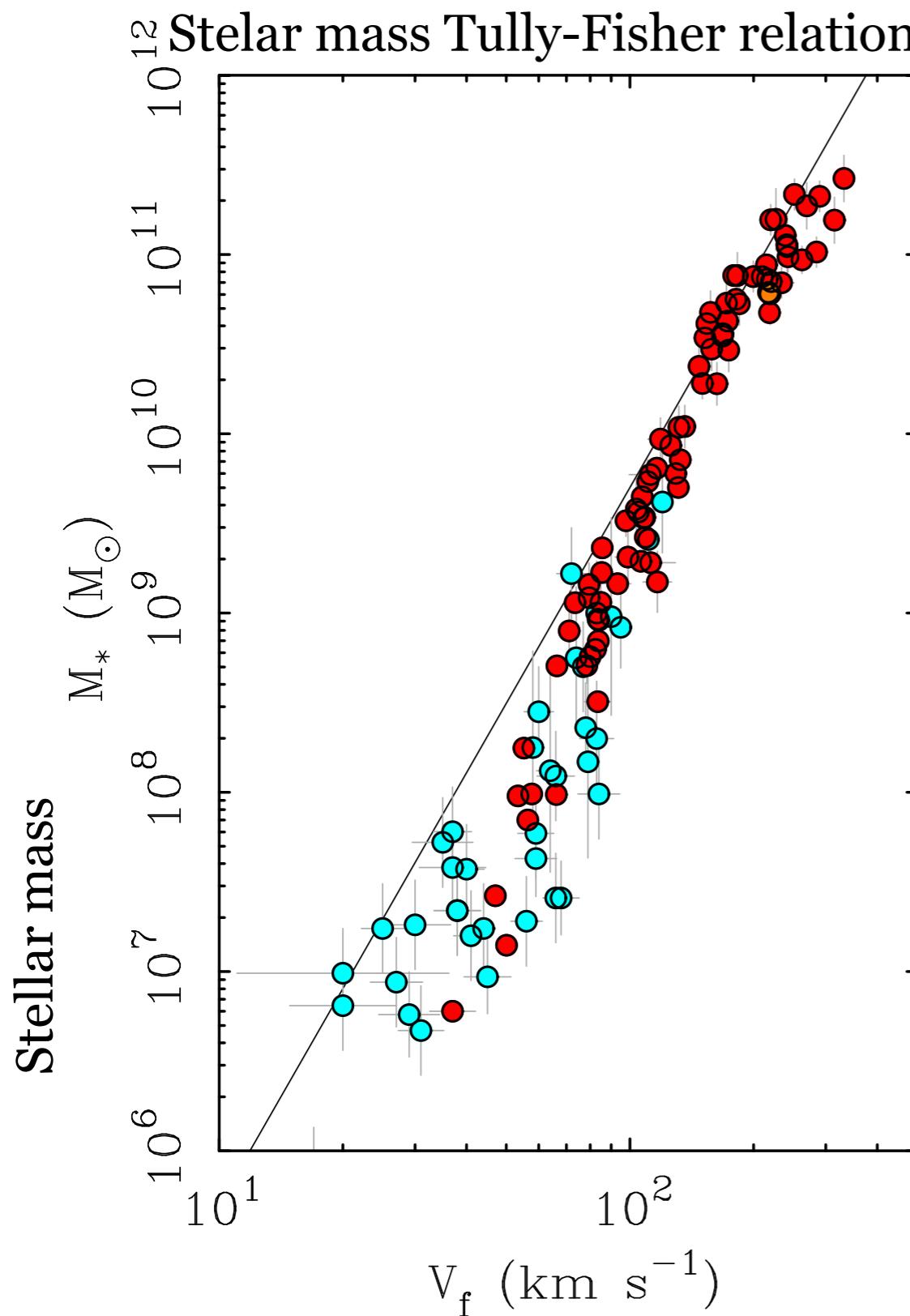
RUBIN (to Kalnajs) : It is true that the analysis of the rotation curves presents the mass interior to any R, but not the distribution of the mass. Thus, while the mass could be in a disk, there are other reasons, stability especially, that suggest a halo. The velocities you show for NGC 4378 and 7217 come from our data, and both rotation curves are fairly exceptional in that the velocities fall slightly with increasing R. I suspect you would have more difficulty in fitting with constant M/L a flat or slightly rising rotation curve which extends to very large radii. In any case, it seems to me, you must be saying that the surface brightness of these galaxies falls slower than exponentially with increasing R.

# Tully-Fisher: Rotation curve amplitude correlates with observed mass:

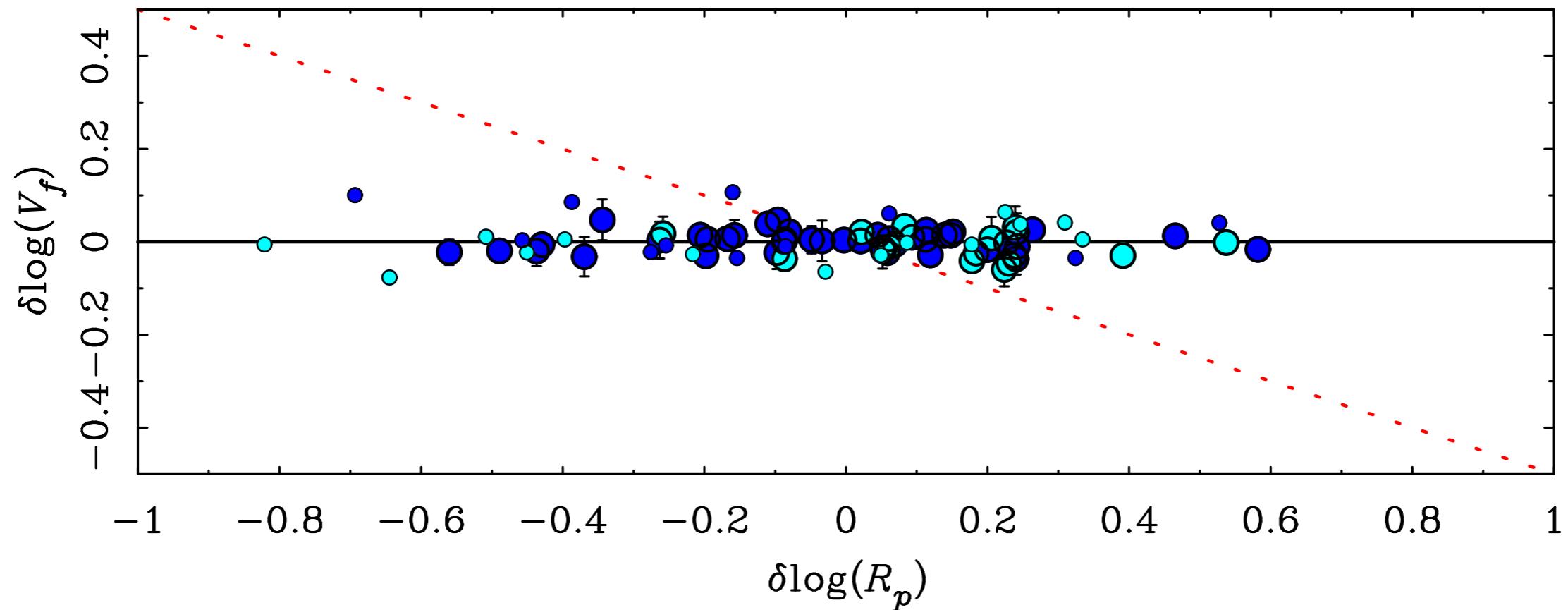


Flat rotation curves continue to occur in quite small systems (e.g., Leo P with  $V_f \sim 15$  km/s)

# Tully-Fisher relations: amplitude of flat rotation speed correlates with mass

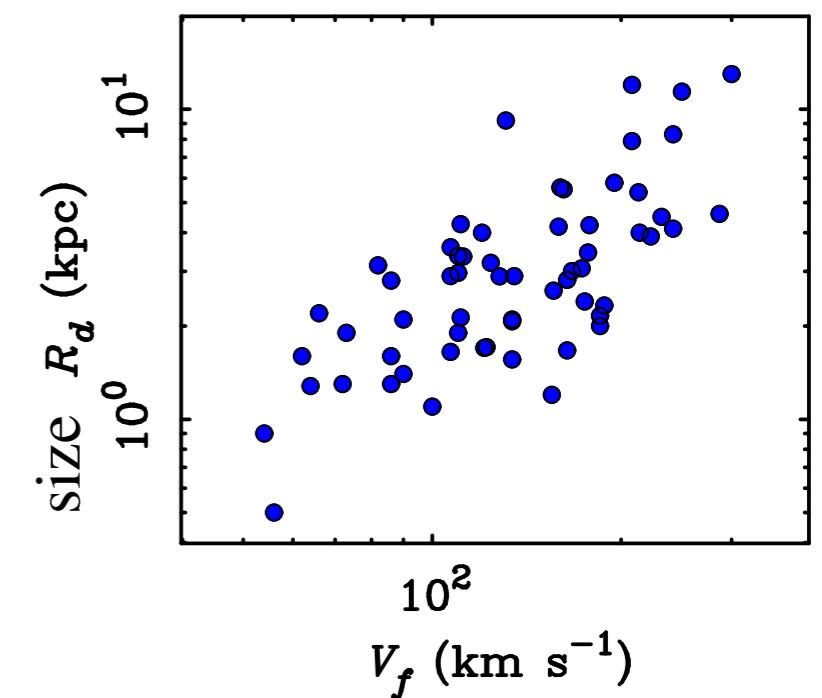


# No residuals from BTFR with size or surface density for disks

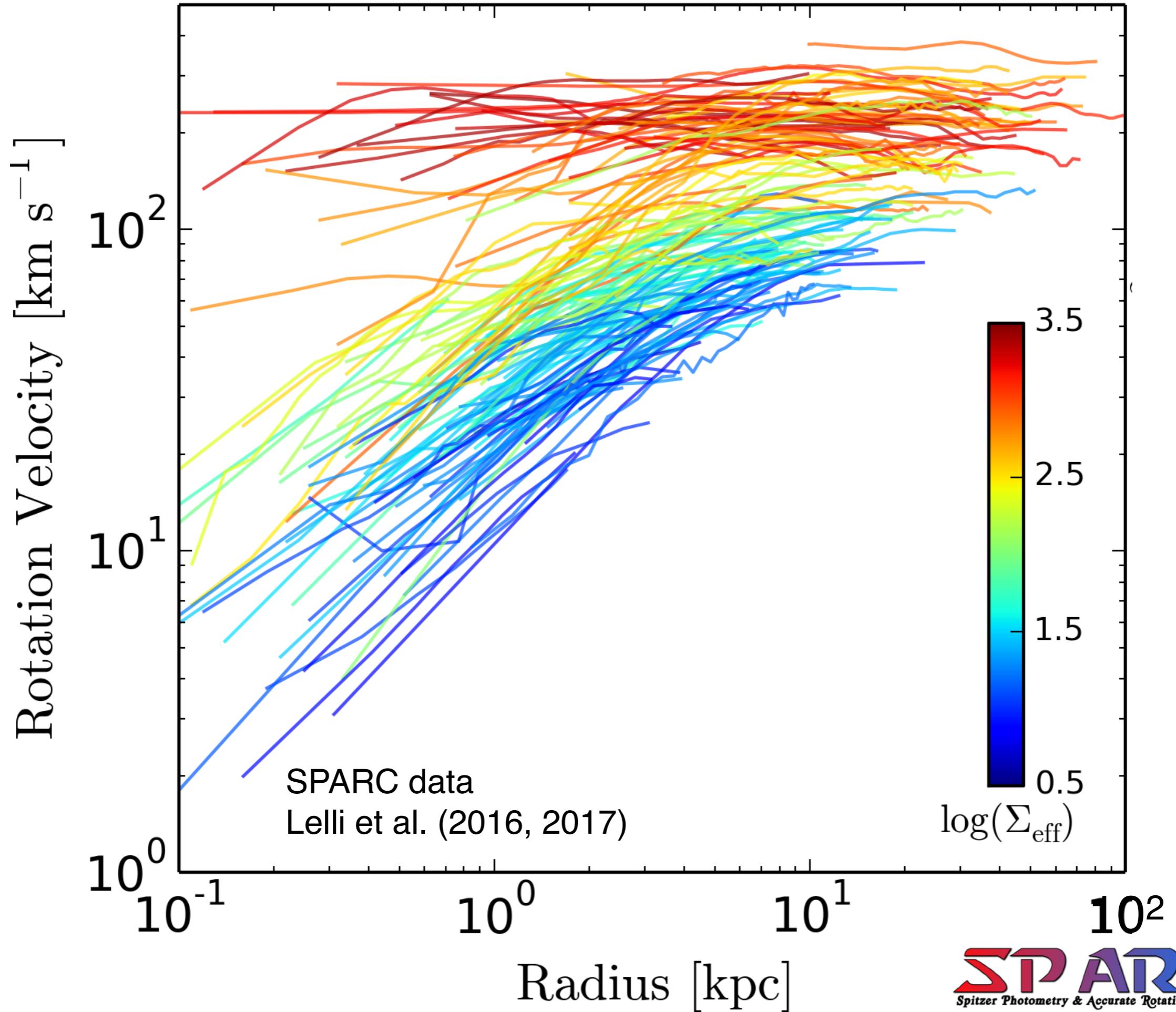


$$V^2 = \frac{GM}{R} \rightarrow \frac{\delta \log(V)}{\delta \log(R)} = -\frac{1}{2} \text{ expected slope (dotted line)}$$

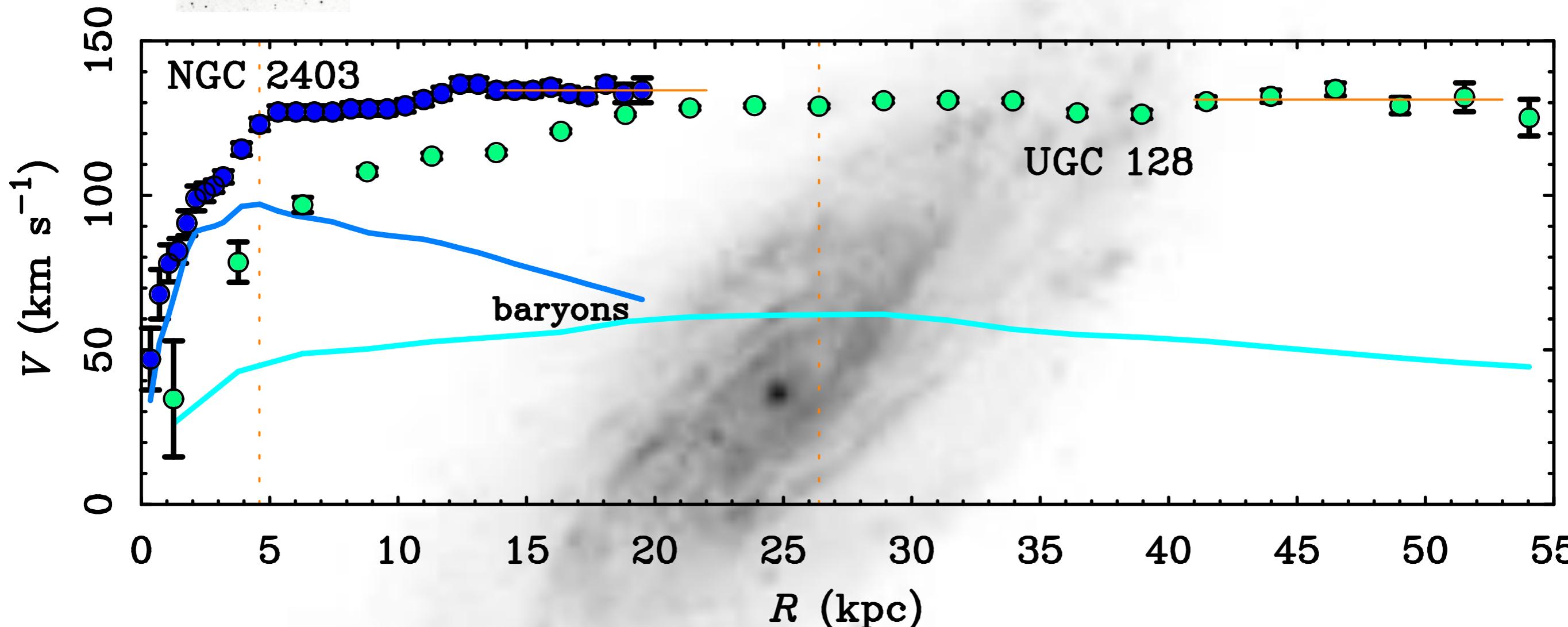
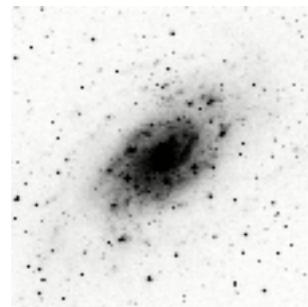
Note: large range in size at a given mass or velocity



# Rotation curve shape correlates with baryonic surface density



TF pair



Radius in physical units (kpc)

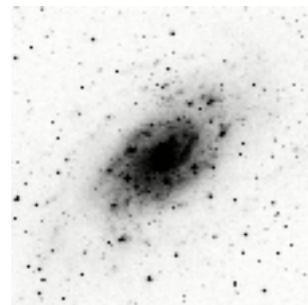
Persic & Salucci 1996

de Blok & McGaugh 1996

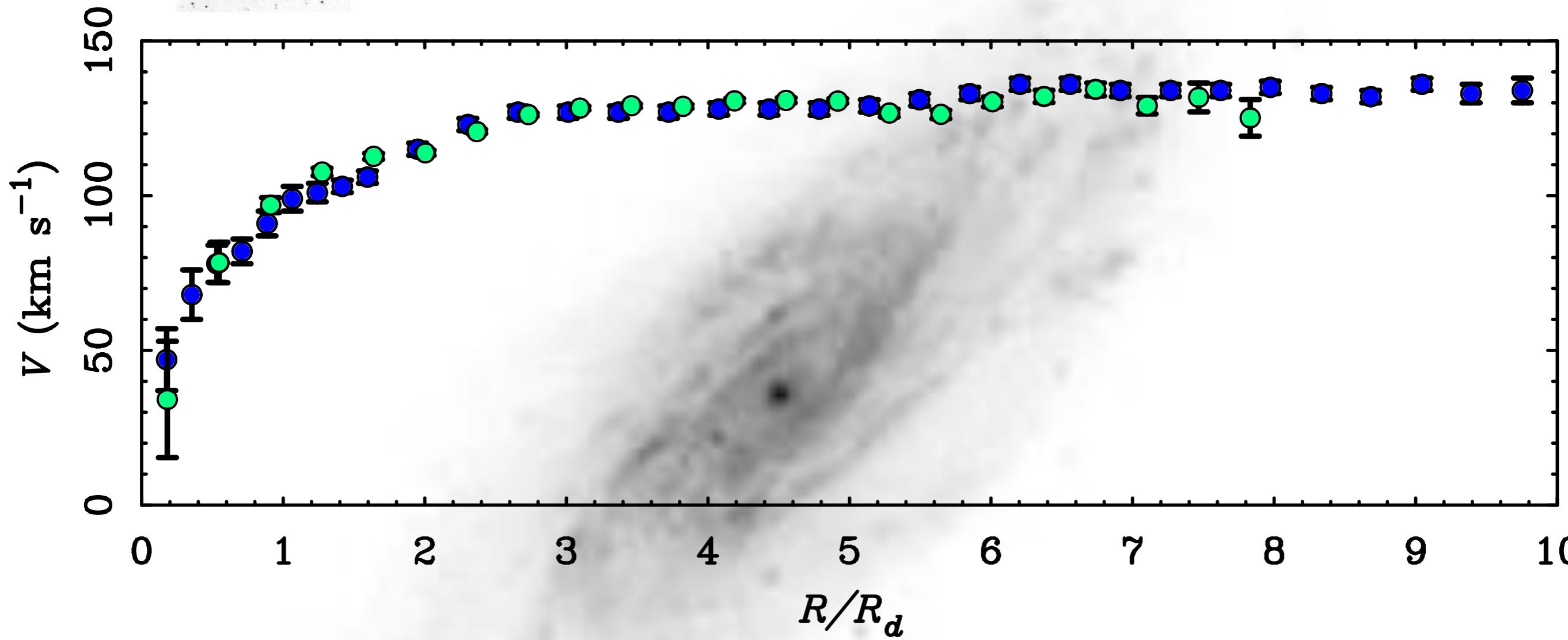
Tully & Verheijen (1998)

Nordermeer & Verheijen (2007) [URC nor quite right formulation]

Swaters et al. (2009)



The dynamics knows about the distribution of baryons, not just their total mass



Radius normalized by size of disk.

Persic & Salucci 1996

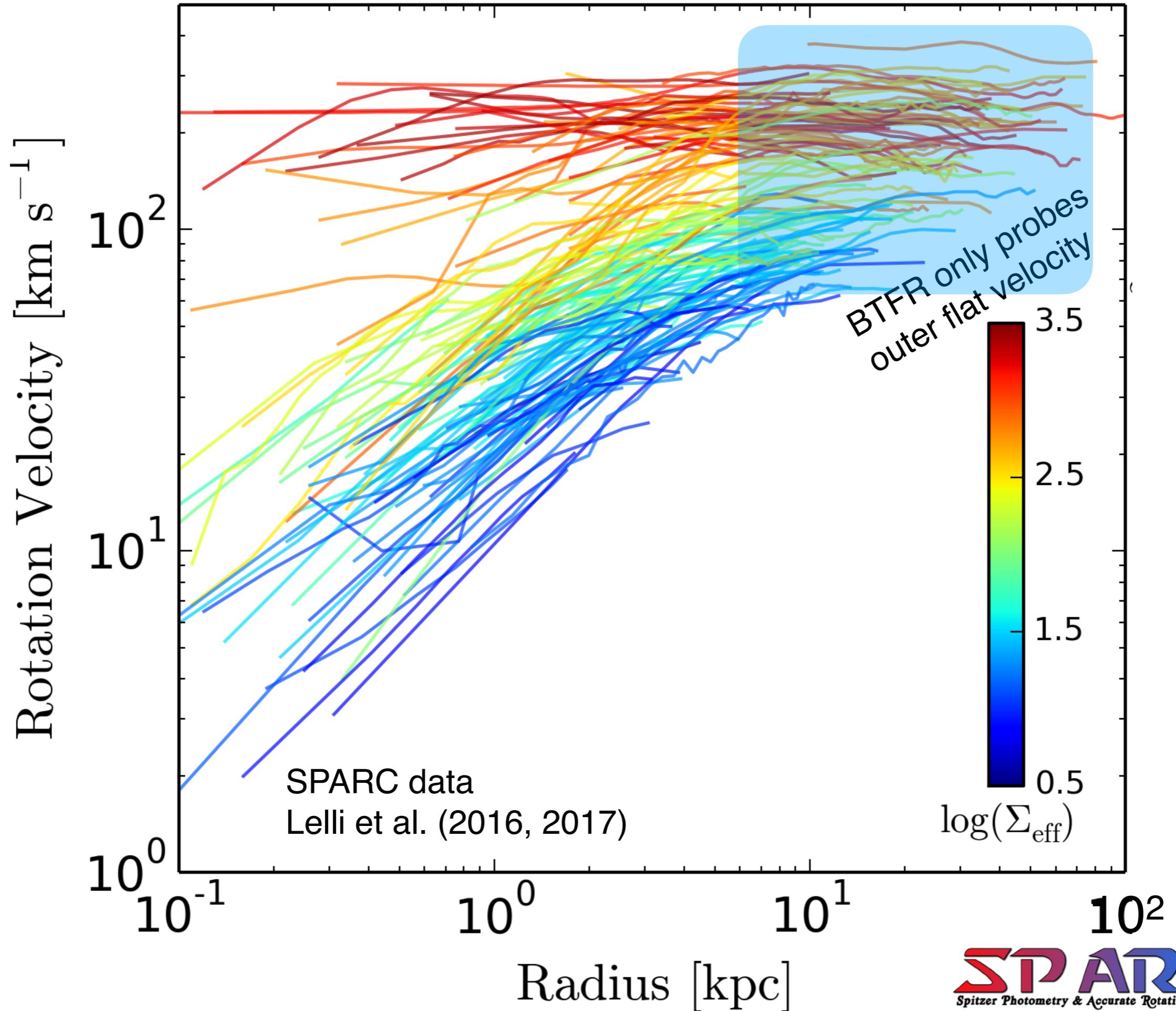
de Blok & McGaugh 1996

Tully & Verheijen (1998)

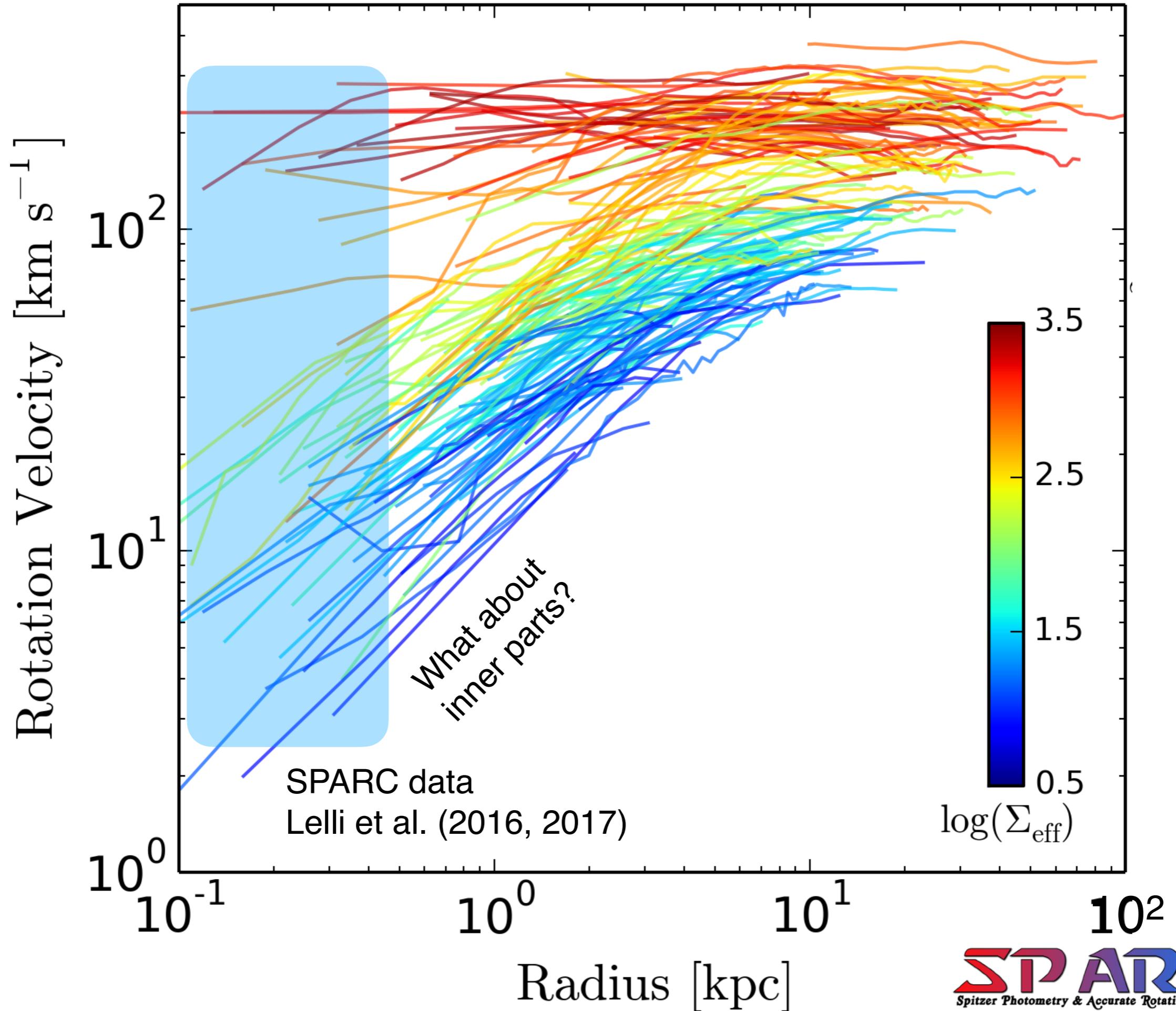
Nordermeer & Verheijen (2007) [URC nor quite right formulation]

Swaters et al. (2009)

# Rotation curve shape correlates with baryonic surface density



# Rotation curve shape correlates with baryonic surface density



# Central Density Relation

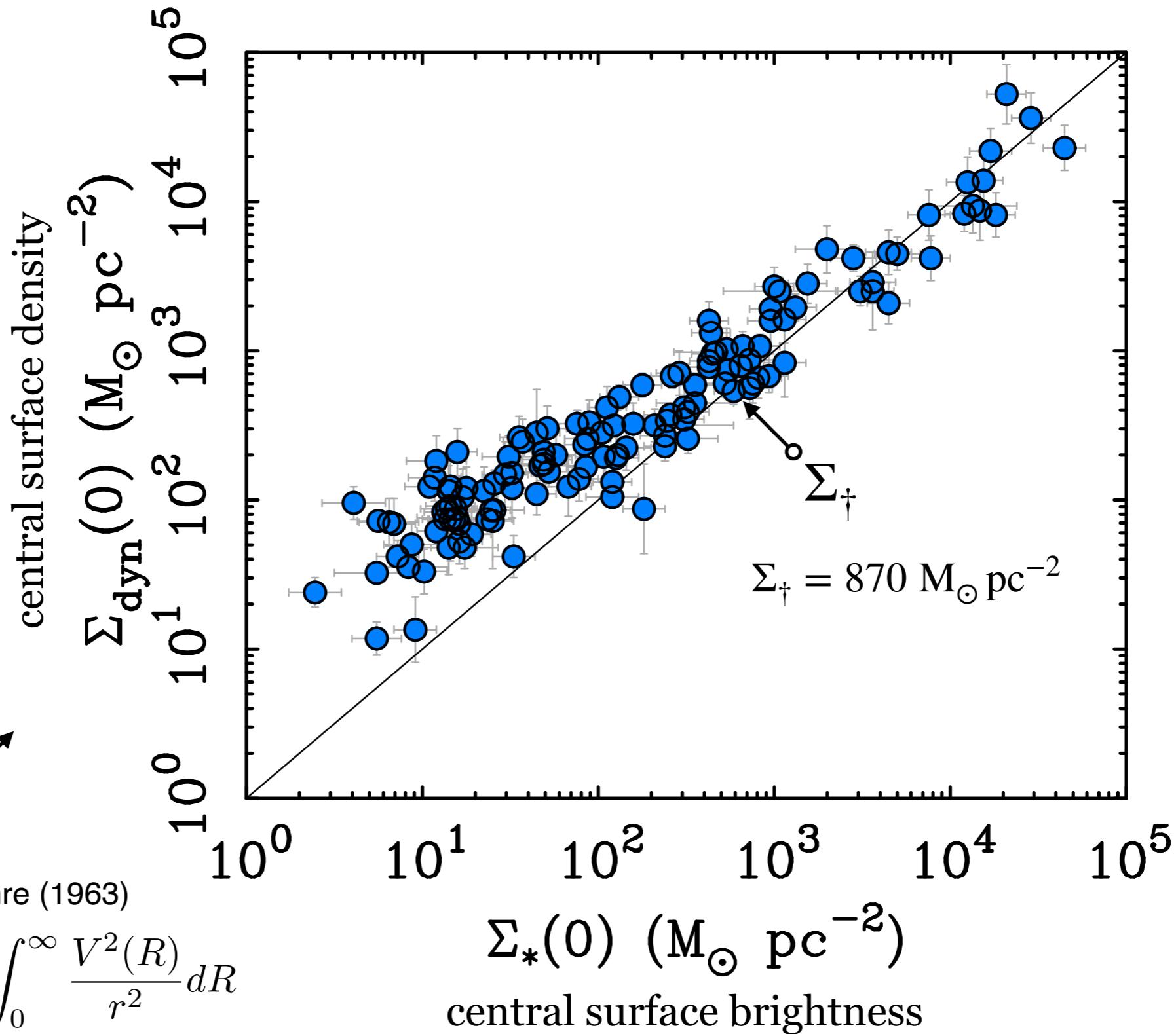
Lelli et al. (2016)

The *dynamical* central mass surface density correlates with the central surface brightness of stars in galaxies.

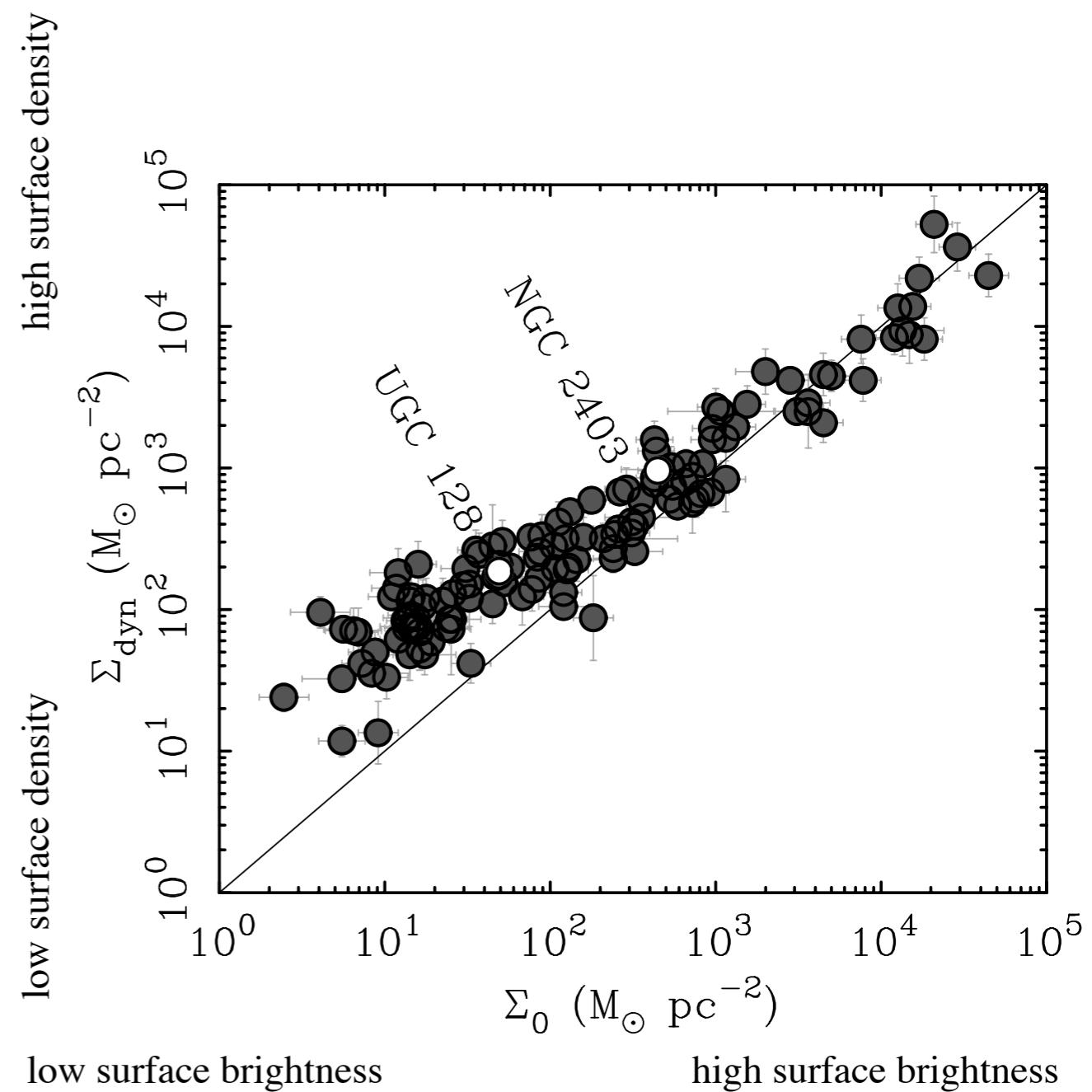
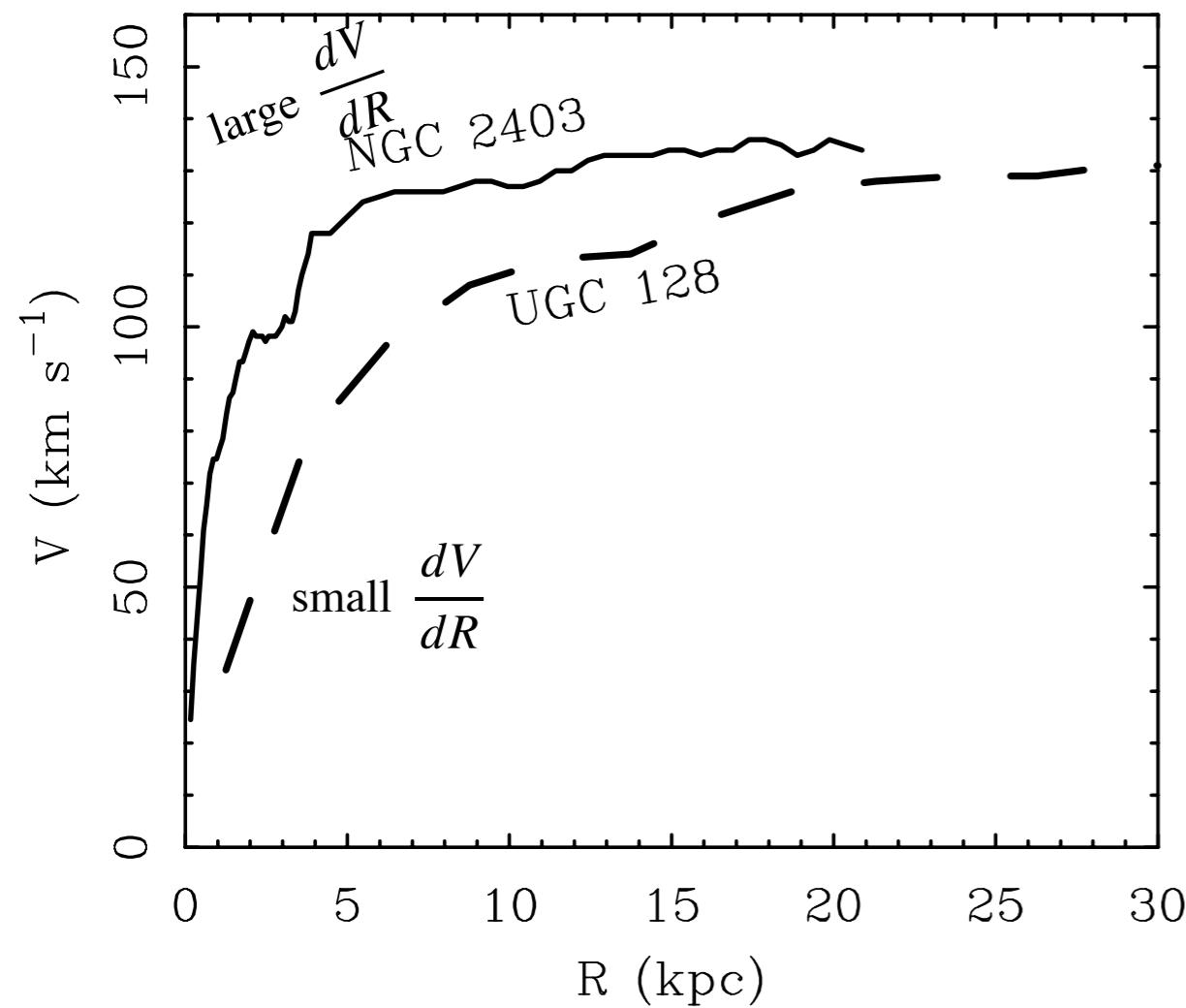
Mass surface  
density  
indicated by  
kinematics

$$\Sigma_{dyn}(0) = \frac{1}{2\pi G} \int_0^\infty \frac{V^2(R)}{r^2} dR$$

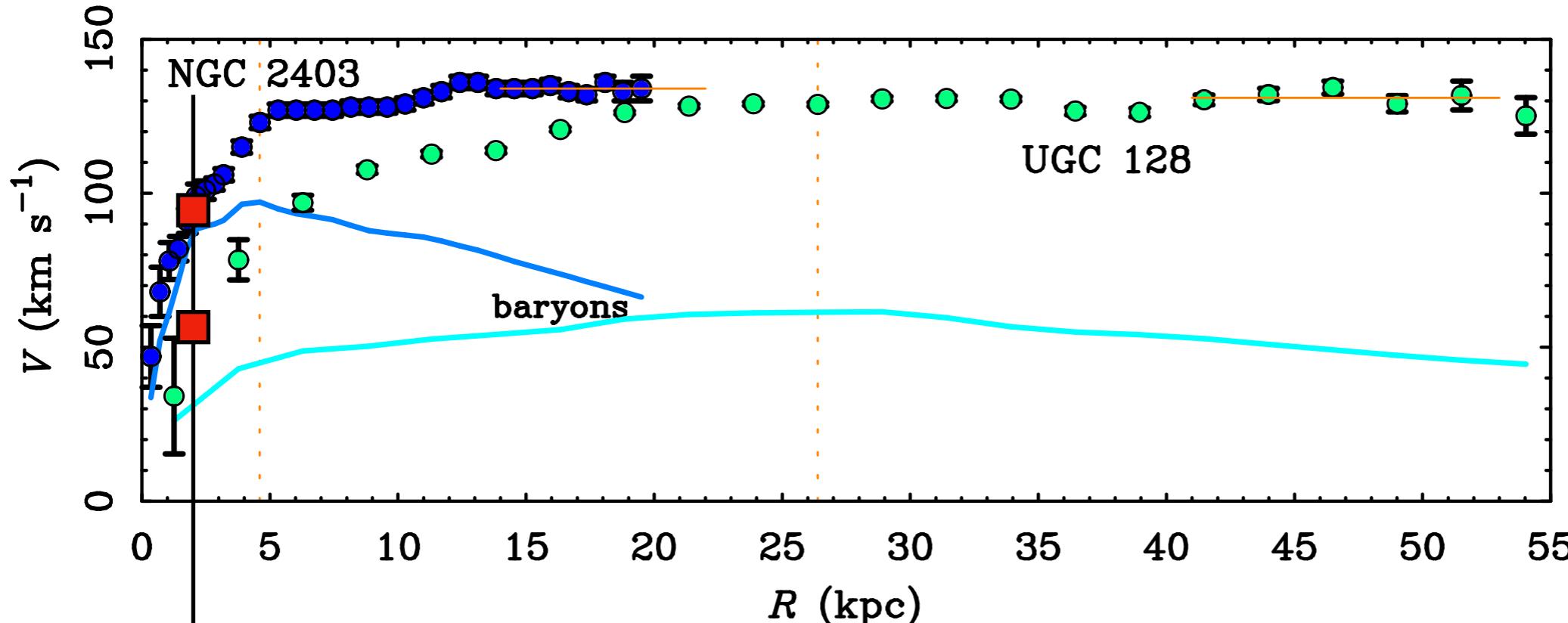
Toomre (1963)



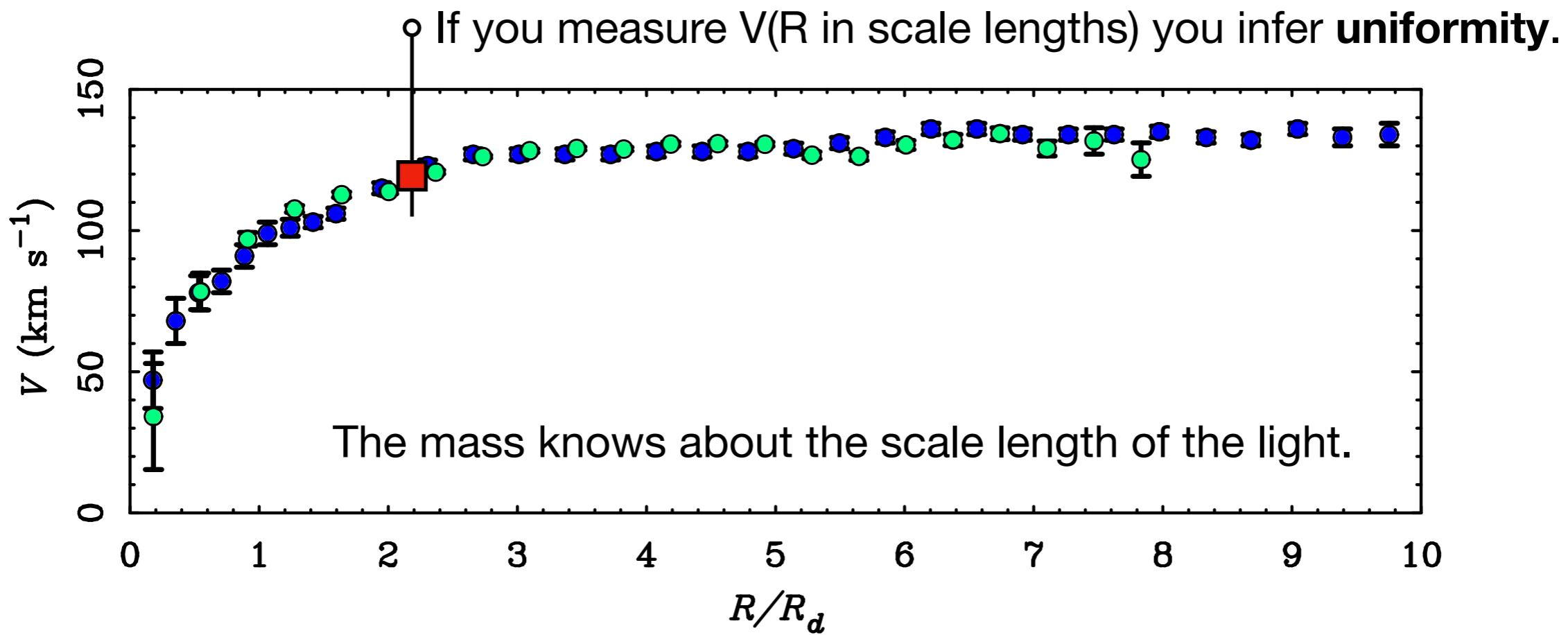
# Central Density Relation



What you get depends on how you look at it: what you assume & what you choose to measure:



○ If you measure  $V(R$  in kpc) you infer **diversity**.



- Renzo's Rule: (2004 IAU; 1995 private communication)  
*“When you see a feature in the light, you see a corresponding feature in the rotation curve.”*

NGC 6946

stellar disk



optical

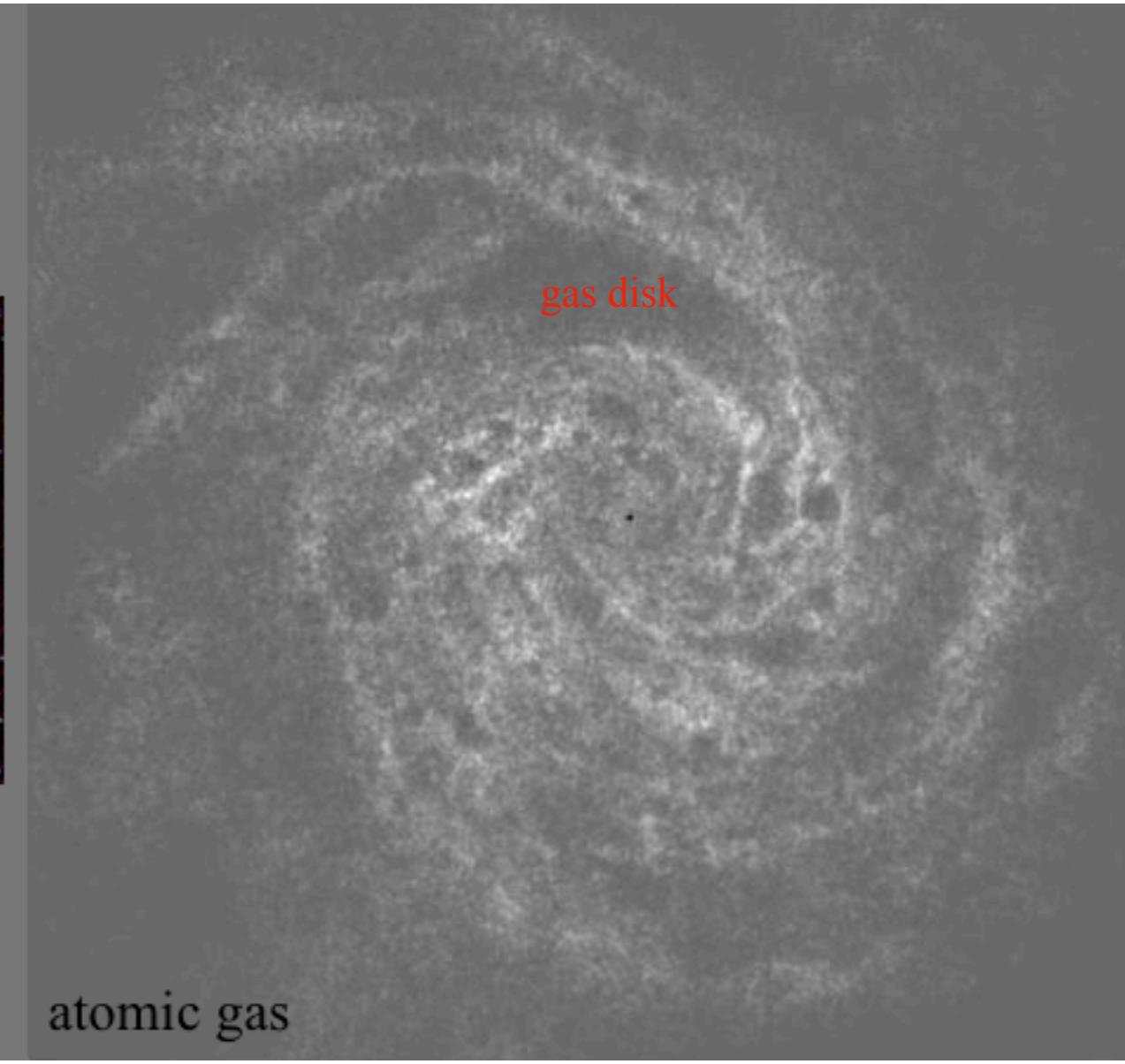
central bulge



near infrared

gas disk

atomic gas

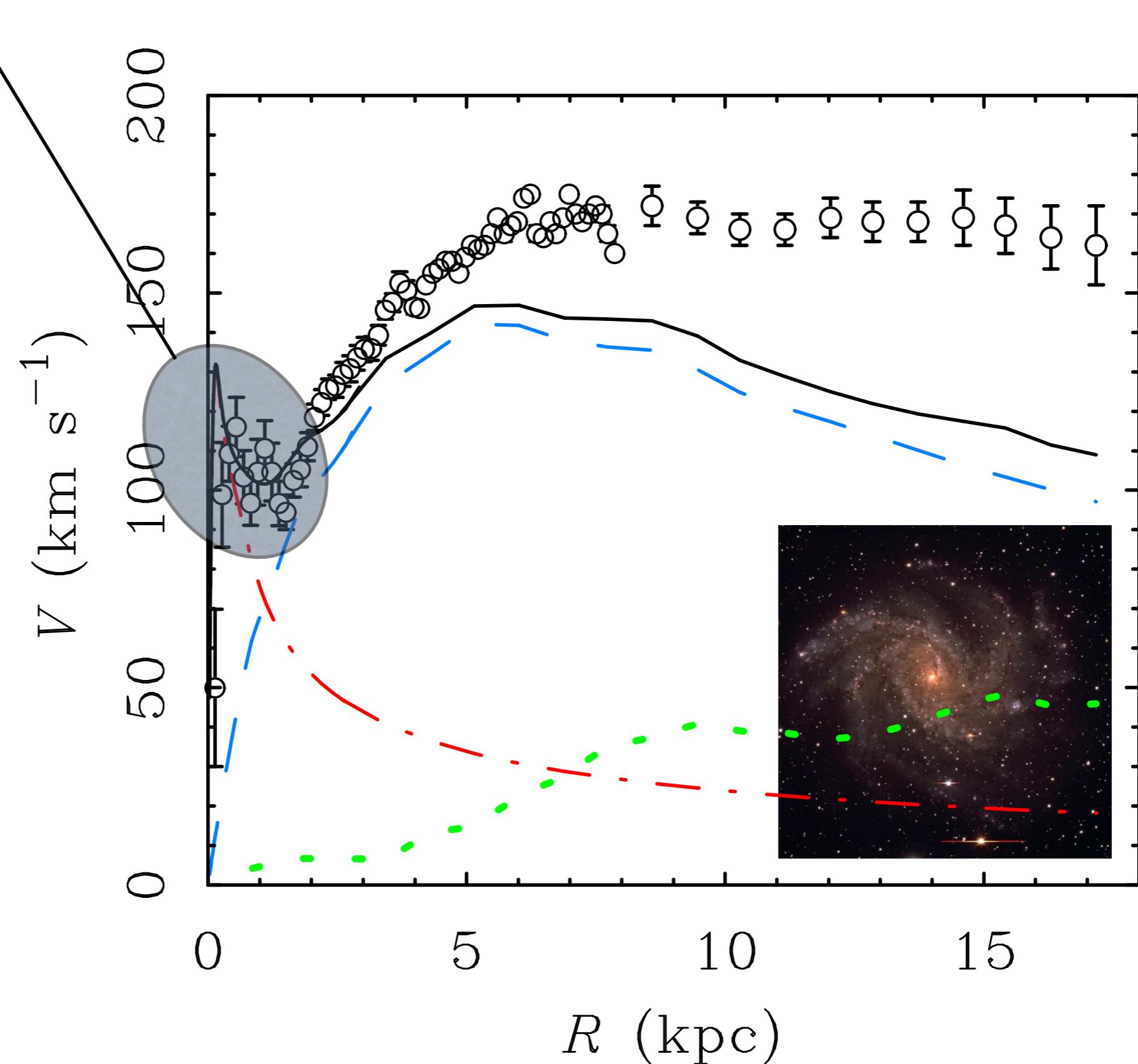


The central bulge component of NGC 6946 is only  $\sim 5\%$  of the total light, but it has a perceptible effect on the kinematics.

Note the up-down-up morphology - this requires a maximal bulge; can't explain that with a dark matter halo.

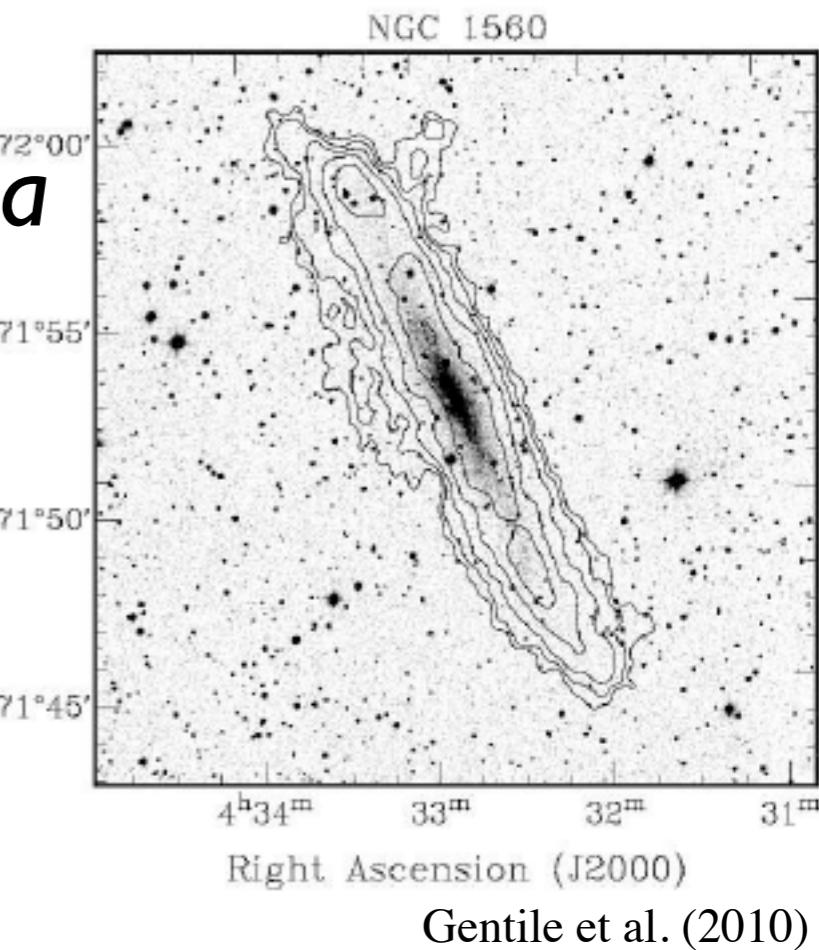
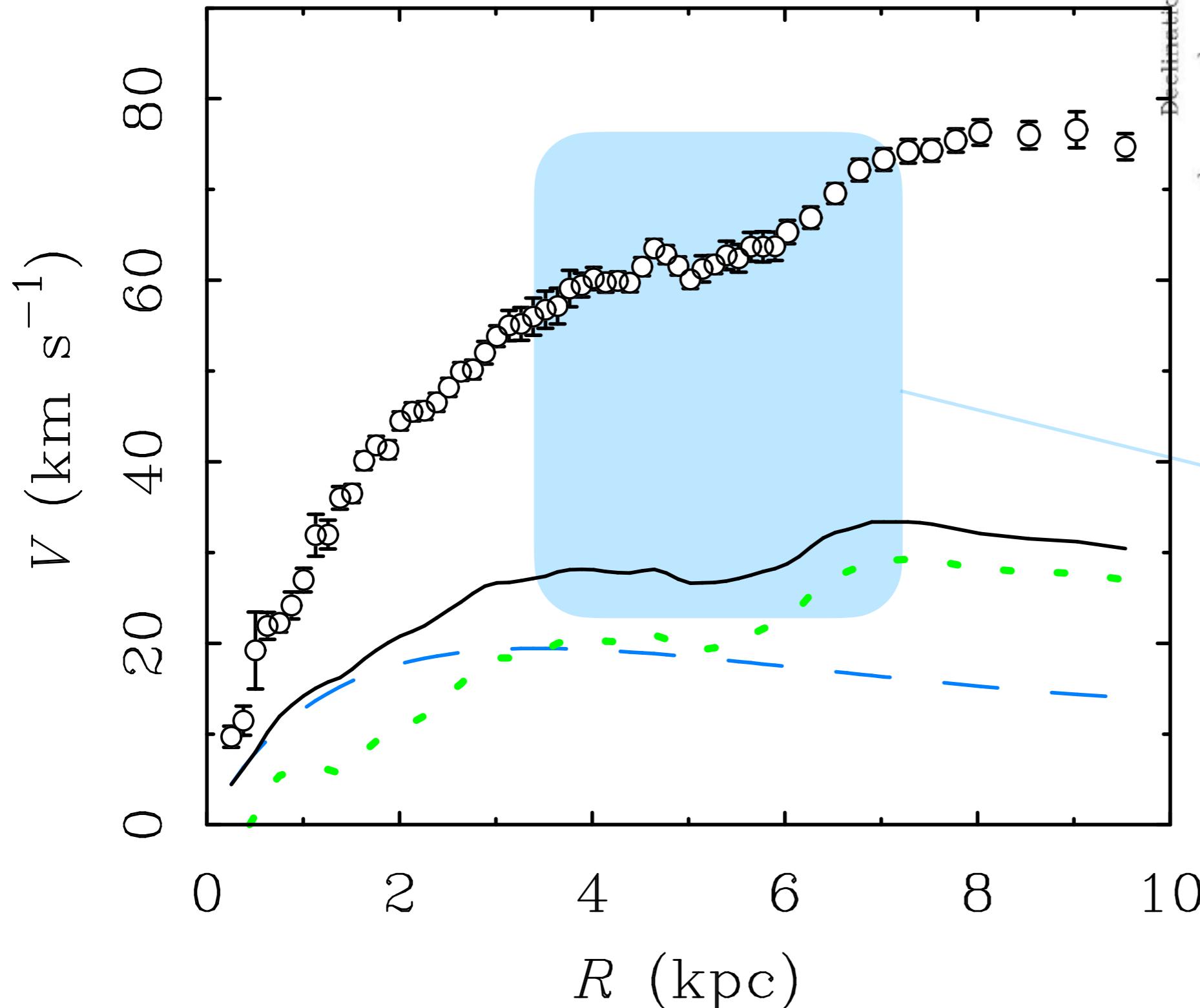
$$V^2 = GM/R$$

**M** is small  
but so is **R**



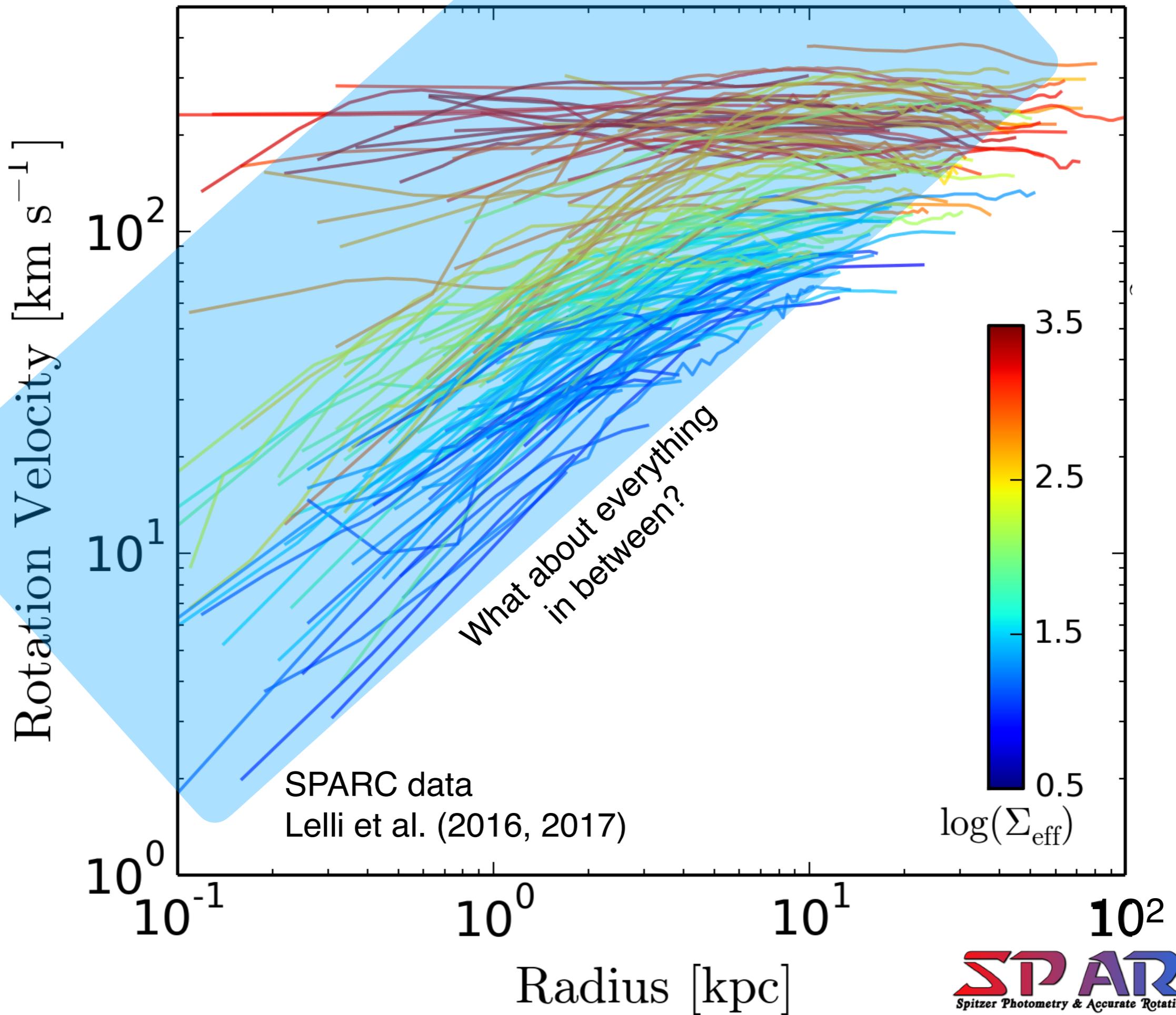
# Renzo's Rule:

“When you see a feature *in the light*, you see a corresponding feature *in the rotation curve*.”

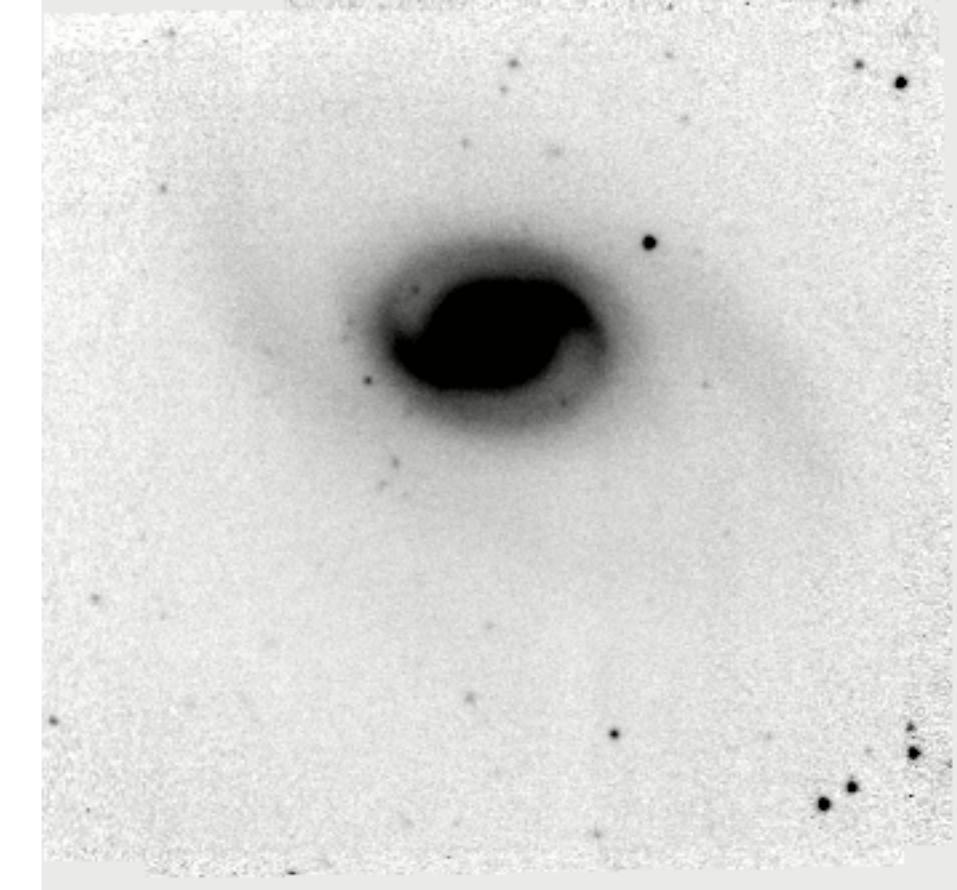
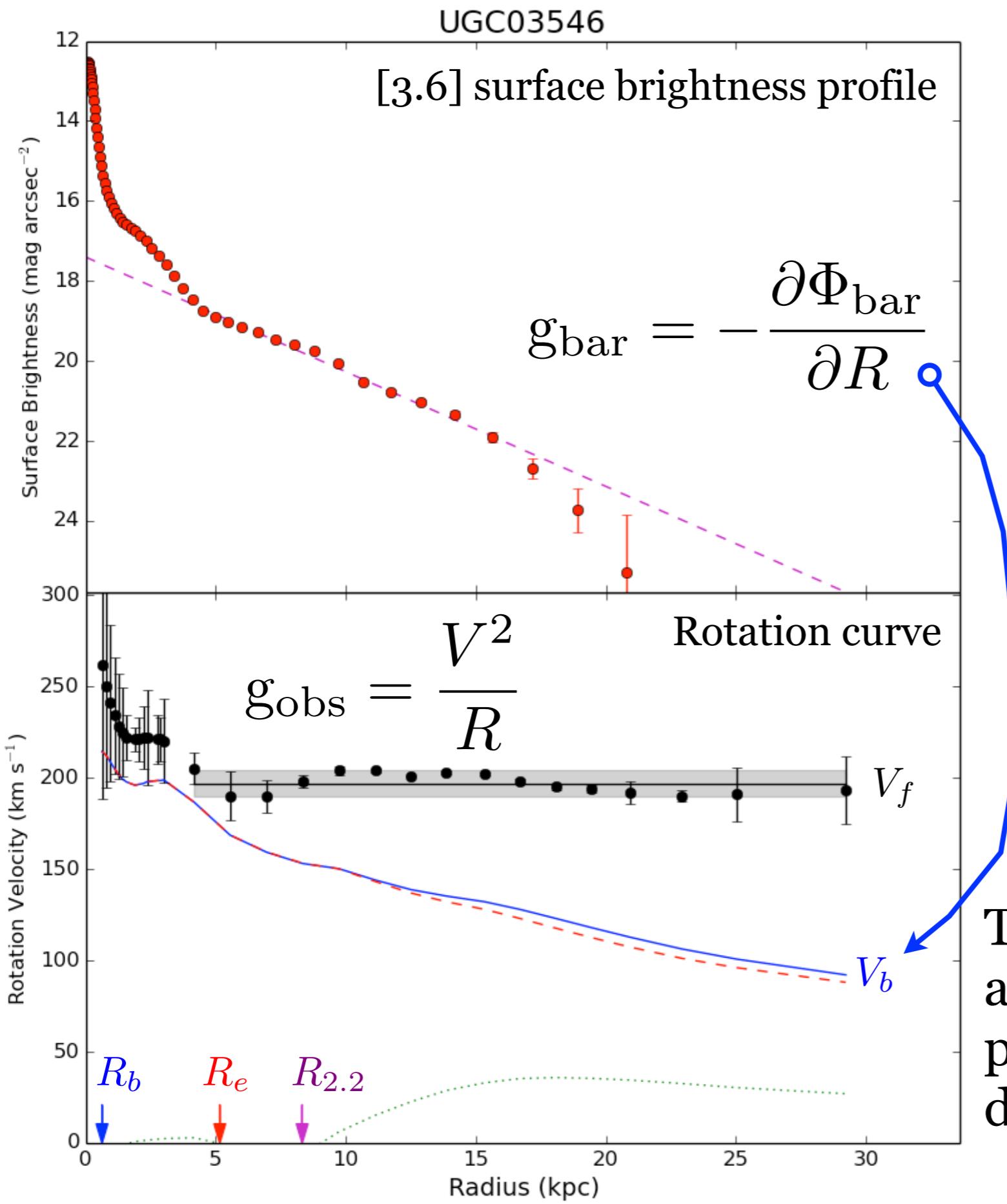


In NGC 1560, a marked feature in the gas is reflected in the kinematics, even though it accounts for little of the dynamical mass.

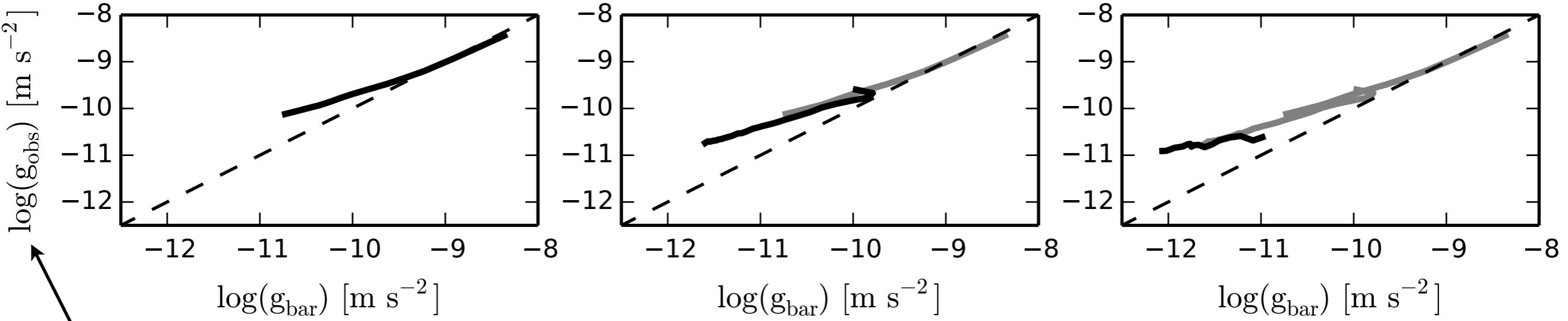
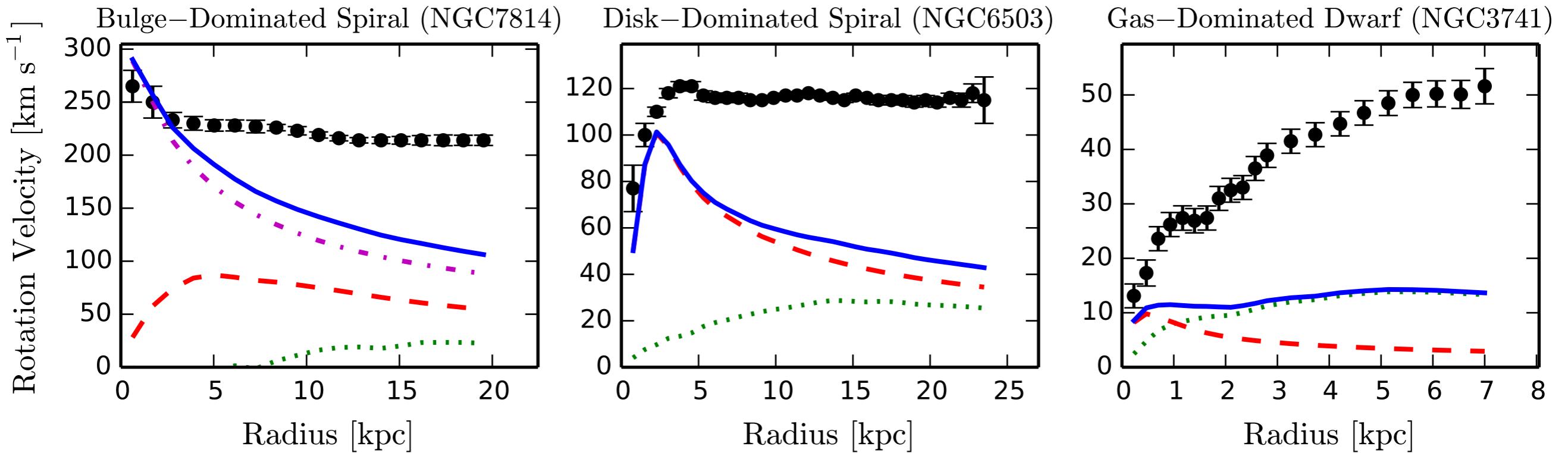
# Rotation curve shape correlates with baryonic surface density



What about everything in between?



The observed centripetal acceleration is linked to that predicted by the observed distribution of baryons.



$$g_{\text{obs}} = \frac{V^2}{R}$$

determined from rotation curve

independent quantities

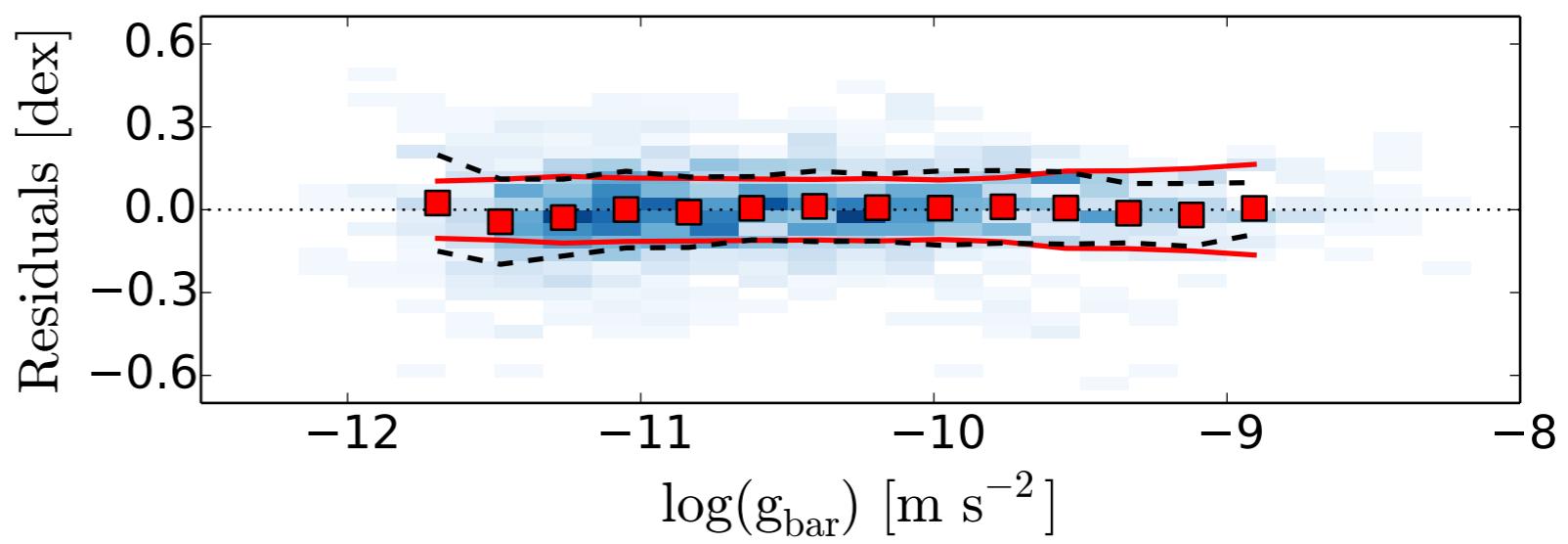
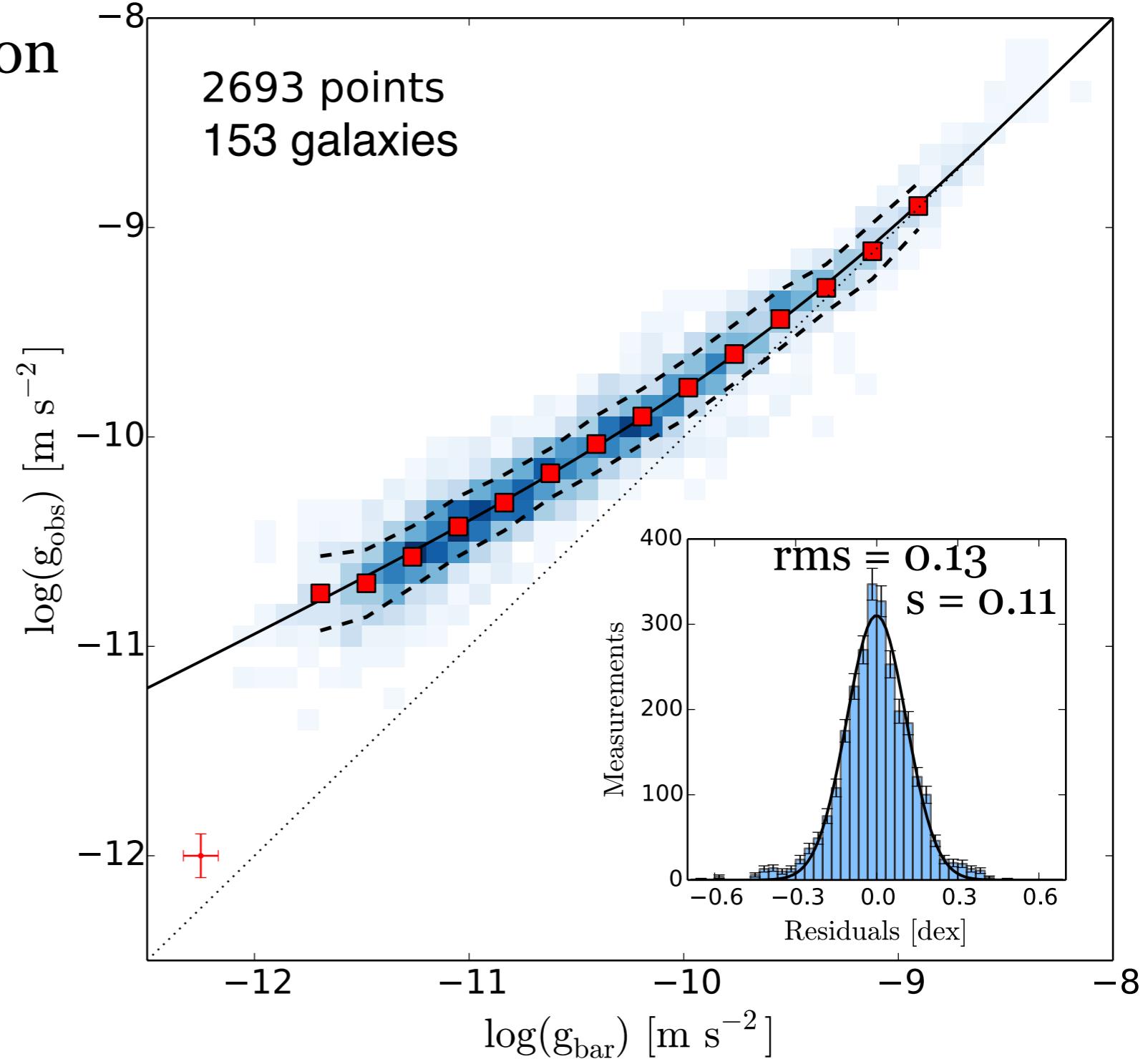
$$g_{\text{bar}} = -\frac{\partial \Phi_{\text{bar}}}{\partial R}$$

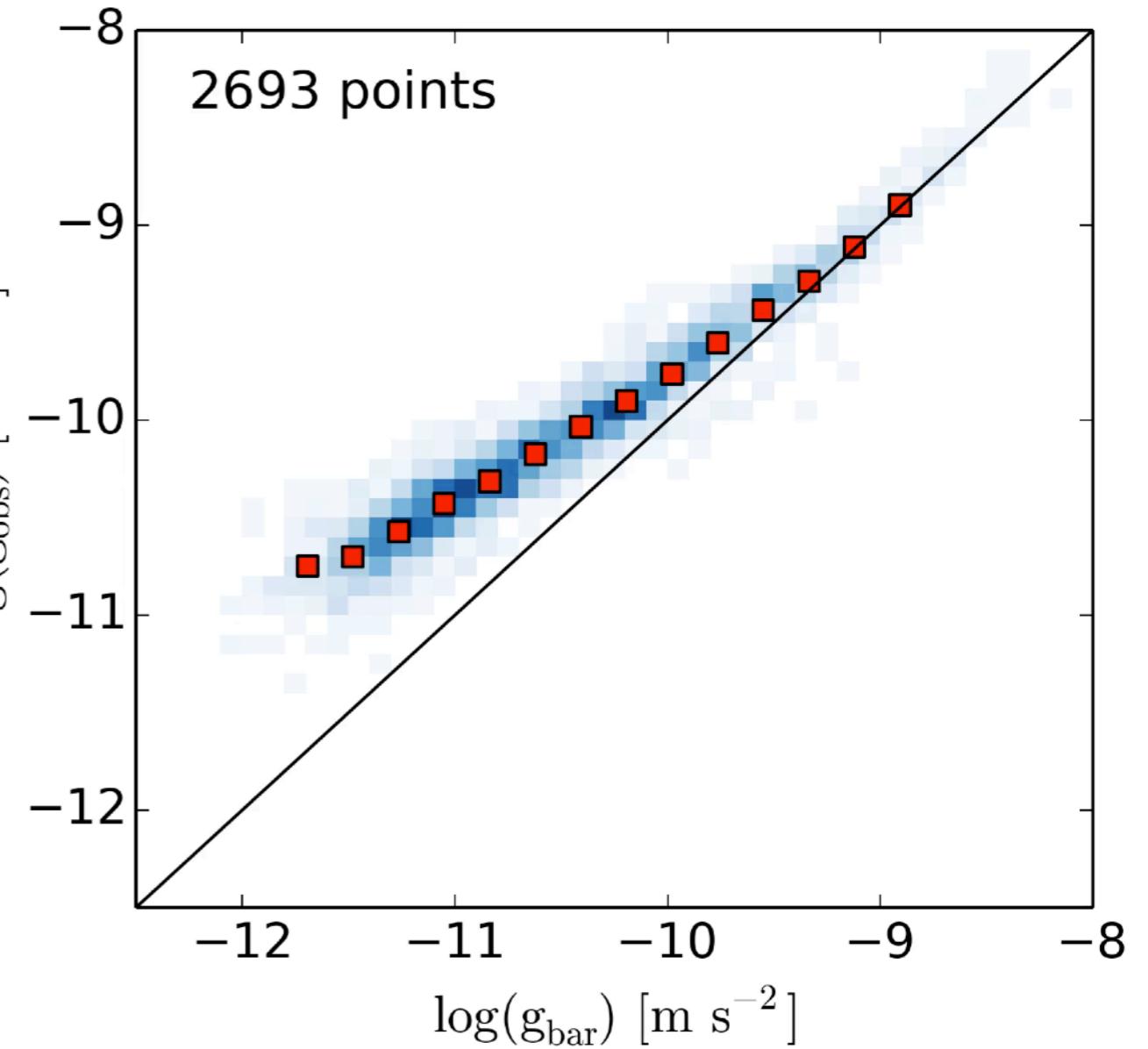
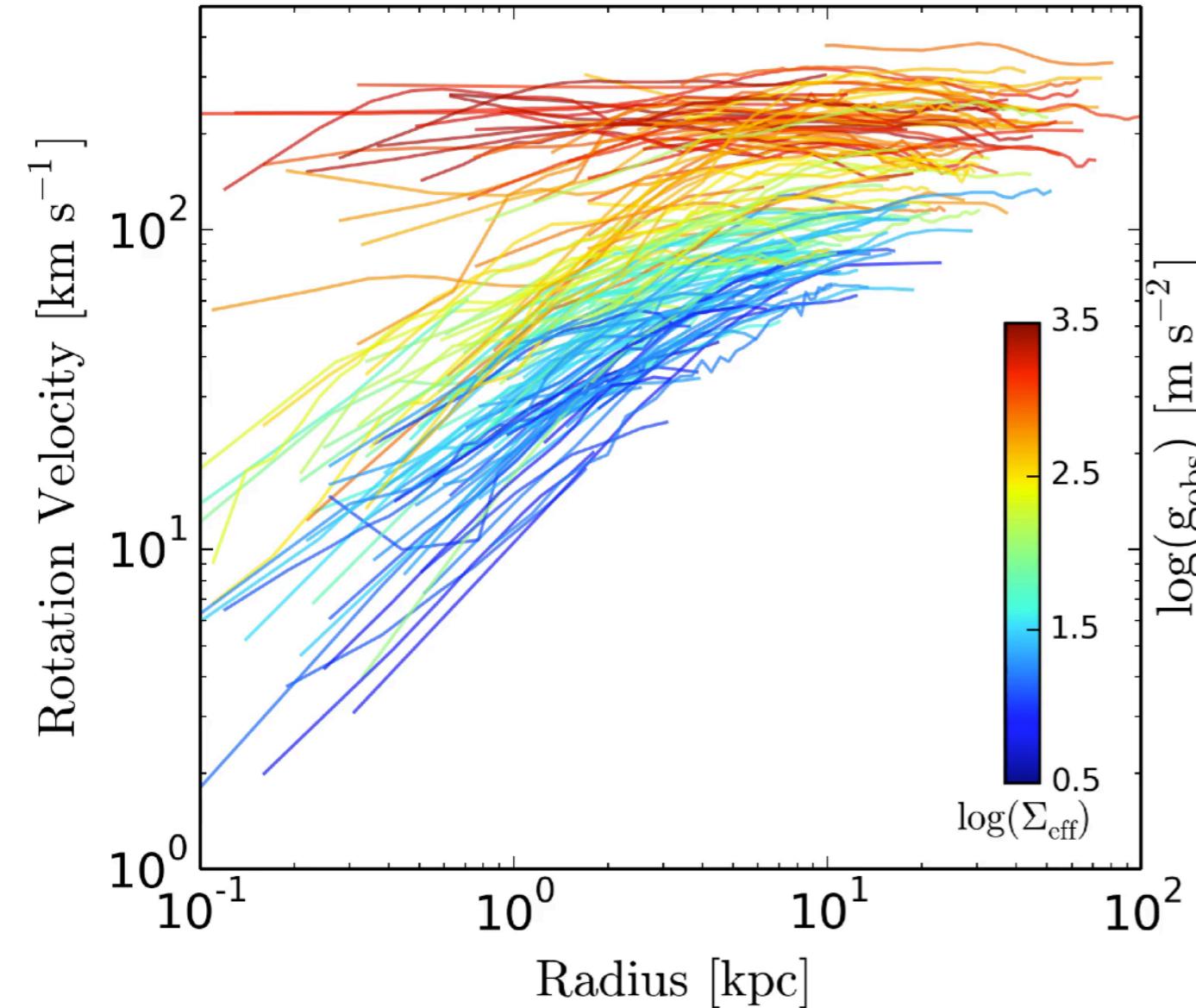
determined from baryon distribution

# Radial Acceleration Relation (RAR)

Constructed from 153 galaxies with 21cm rotation curves and near-IR surface photometry from the *Spitzer* space telescope.

Apparently the mass-to-light ratio in the near-IR is close to constant: individual galaxies do not stand out in this relation.





<http://astroweb.case.edu/SPARC/RARmovie.mp4>

## Empirical Laws of Galactic Rotation

- Flat rotation curves (Rubin-Bosma Law)

Rotation curves tend asymptotically towards a constant rotation velocity that persists to indefinitely large radii:  $V(R \rightarrow \infty) \rightarrow V_f$

- Tully-Fisher relation (Luminous, Stellar Mass, and Baryonic TF relations)

The baryonic mass of galaxies scales as the fourth power of the flat rotation velocity:  $M_b = A V_f^4$

- Central density relation (lower surface brightness galaxies exhibit larger mass discrepancies)

The central dynamical surface densities of galaxies is related to their central surface brightnesses:  $\Sigma_{dyn}(R \rightarrow 0) = f[\Sigma_*(R \rightarrow 0)]$

- Renzo's rule (Sancisi's Law)

“For any feature in the luminosity profile there is a corresponding feature in the rotation curve and vice versa.” (Sancisi 2004).

- Radial acceleration relation

The observed centripetal acceleration is related to that predicted by the observed distribution of baryons:  $g_{obs} = \mathcal{F}(g_{bar})$