lithosphere undergoes a small degree of partial melting and dehydration. The resulting hydrous siliceous melts percolate through the overlying depleted mantle wedge. They thus metasomatize and 'refertilize' the depleted peridotites, and impart on them the trace-element source characteristics of typical island-arc magmas. This fertilized peridotitic region then melts to produce arc basalts and andesites. ☐

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The observational case for a low-density Universe with a non-zero cosmological constant

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OBSERVATIONS are providing progressively tighter constraints on cosmological models advanced to explain the formation of large-scale structure in the Universe. These include recent determinations of the Hubble constant[1-3] (which quantifies the present expansion rate of the Universe) and measurements of the anisotropy of the cosmic microwave background[4,5]. Although the limits imposed by these diverse observations have occasionally led to suggestions[6] that cosmology is facing a crisis, we show here that there remains a wide range of cosmological models in good concordance with these constraints. The combined observations point to models in which the matter density of the Universe falls well below the critical energy density required to halt its expansion. But they also permit a substantial contribution to the energy density from the vacuum itself (a positive 'cosmological constant'), sufficient to recover the critical density favoured by the simplest inflationary models. The observations do not yet rule out the possibility that we live in an ever-expanding 'open' Universe, but a Universe having the critical energy density and a large cosmological constant appears to be favoured.

Cosmological models can be categorized according to their mechanism for generating seeds for the formation of large-scale structure. The standard Big Bang model successfully explains the Hubble expansion, the primordial formation of the elements, and the origin of cosmic background radiation. However, it offers no explanation for how structure formed. Recognizing that the Big Bang picture is incomplete, cosmologists have put forth various theoretical proposals to address that issue.

Our focus will be on a leading candidate, the inflationary model of the Universe[7-9] although our analysis also extends to current alternatives. The inflationary model proposes that the seeds for large-scale structure formation were produced by microscopic quantum fluctuations in the energy density during the first instants after the Big Bang[10-13]. There was subsequently a burst of spectacular, superluminal expansion (inflation) that stretched the Universe and the fluctuations to cosmic dimensions. The resulting spectrum of primordial fluctuations is nearly scale-invariant: if the fluctuation in density over space is expressed as a Fourier sum of waves with amplitude $\delta(\lambda)$, the waves have nearly equal amplitude independent of wavelength, λ. Cosmologists parametrize the spectrum in terms of a spectral index n, defined by the relation $\delta \propto \lambda^{(1-n)/2}$. In the early 1970s, before the inflationary model was proposed, Harrison[14], Zel'dovich[15] and Peebles and Yu[16] had argued that a scale-invariant spectrum ($n=1$) is the most plausible because the amplitude did not diverge at small wavelengths, which would produce too many black holes, or at large wavelengths, which would produce too much distortion in the cosmic background radiation. Hence, it was regarded as a major triumph when it was discovered that inflation naturally generates a nearly scale-invariant spectrum.

It should be emphasized, however, that inflation does not predict a precisely $n=1$ spectrum. Rather, depending on the rate of inflation and the details of how inflation ends, the spectral index can take values between roughly $n=0.7$ and 1.2 (refs 5, 17). It is an important aspect of our tests that we do not fix the spectral index ab initio, but rather treat it as a free parameter to be constrained by observational data. In particular, we have found cases in which models have been judged inconsistent with large-scale observations under the strict assumption that $n=1$ (ref. 18). Yet a relatively modest deviation of n from unity, well within the bounds permitted by inflation, brings the model back into concordance.

Models can be further distinguished by the values of other cosmological parameters such as the Hubble expansion rate, H_0, the density of baryons (ordinary matter), the total matter density including any dark matter, and the vacuum energy density or, equivalently, the cosmological constant (Λ). The symbols Ω_B,
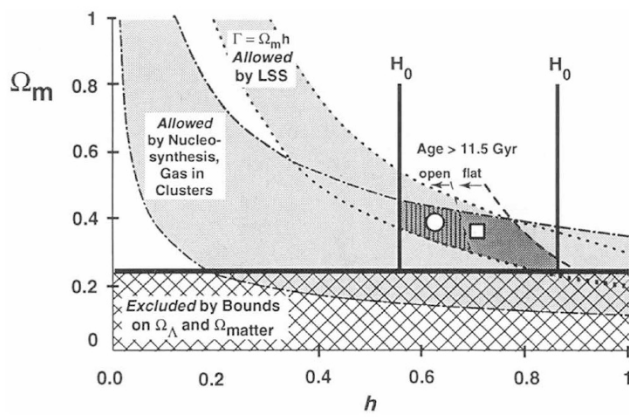
FIG. 1 The range of models in concordance with the best known astronomical observations. The entire darkly shaded region (with and without vertical stripes) is the concordance domain for flat models with $\Omega_m + \Omega_\Lambda = 1$; the vertically striped subregion alone applies to open models with $\Omega_\Lambda = 0$. (The age constraint—which permits the region only to the left of the dashed line—differs in the two cases.) The square indicates a central, representative flat model with $h = 0.7$, $\Omega_m = 0.35$ and $\Omega_\Lambda = 0.65$. The circle represents a representative open model, $h = 0.625$, $\Omega_m = 0.375$. See text for discussion of the constraints.

Ω_m, and Ω_Λ, are used to represent the ratio of baryons, total matter, or vacuum energy density to the critical energy density. The baryon density is constrained to $\Omega_B < 10\%$ by fitting Big Bang nucleosynthesis to the observed nuclear abundances[19]. An important property of inflationary expansion is that it produces sufficient energy to drive the Universe towards the critical energy density, $\Omega_{total} = \Omega_m + \Omega_\Lambda = 1$. Models with critical energy density are termed 'flat' because Einstein's theory of gravity predicts that these universes possess no overall curvature. Within this framework, conventional models have typically assumed that $\Omega_\Lambda = 0$ and $\Omega_{total} = \Omega_m = 1$, where the matter density consists of normal, baryonic matter plus cold (non-relativistic) dark matter, hot dark matter (relativistic at the onset of galaxy formation), or mixtures of the two[20]. A second possibility is that $\Omega_{total} = 1$, with a non-zero cosmological constant, $\Omega_\Lambda > 0$, and Ω_m significantly less than unity. Yet a third possibility is that Ω_{total} is less than unity and the Universe is open. Although inflation is an extraordinarily efficient mechanism for driving Ω_{total} towards unity, some have considered 'open inflation' models[21,22] in which the expansion is delicately adjusted to bring the Universe just short of critical density.

Our approach has been to apply objectively the best known observational constraints to the leading models and map out the permitted range of parameters. We first consider key astronomical measurements that delimit the plane of Ω_m versus H_0 (Fig. 1).

Measurements of H_0. Most recent observations[23] are in the range $H_0 = 70 \pm 15$ kilometres per second per megaparsec (km s^{-1} Mpc^{-1}), the weighted average of the recent Hubble Space Telescope study using classical Cepheid variables[1] ($H_0 = 82 \pm 17$ km s^{-1} Mpc^{-1}) and studies using type I supernovae[2] ($H_0 = 67 \pm 7$ km s^{-1} Mpc^{-1}) as standard candles to infer distance.

Age of the Universe. The lower bound on the age of the Universe is based on re-evaluations of the ages of the oldest globular clusters: $t_0 = 15.8 \pm 2.1$ billion years based on the main-sequence turnoff[6] and $t_0 = 13.5 \pm 2.0$ billion years using giant-branch fitting[24]. The first limit, which suggests $t_0 > 13.7$ billion years at the 1σ level, is more stringent and, as it virtually excludes open models, is stronger support for our conclusions. However, to be conservative, we adopt the latter lower bound, 11.5 billion years, in Fig. 1.

Ω_Λ **and** Ω_m. Ω_Λ is constrained by many tests[25], but most directly by the measures of the number of gravitationally lensed quasars

versus redshift[26,27]. For increasing Ω_Λ, the physical volume associated with an interval of redshift is larger, and hence the probability of encountering a lensing galaxy is greater. The bound $\Omega_\Lambda < 0.75$ is also consistent with the lower bound $0.2-0.3 < \Omega_m$ based on observed light density and cluster mass-to-light ratios[28] or by utilization of large-scale structure measurements[29]. The two limits are represented as a cross-hatched region in Fig. 1.

Gas in clusters and nucleosynthesis. Assuming that galaxy clusters contain a representative baryonic and matter density, X-ray measurements[30] of gas combined with estimates of the total virial masses[30] place a limit on the ratio of baryonic matter to total mass, $\Omega_B h^{3/2}/\Omega_m = 0.07 \pm 0.03$, where $H_0 \equiv 100h$ km s^{-1} Mpc^{-1}. Light-element nucleosynthesis[19] constrains $\Omega_B h^2 = 0.015 \pm 0.005$. Together, these imply $1 - \Omega_\Lambda = \Omega_m = (0.21 \pm 0.12)h^{-1/2}$.

$\Gamma \equiv \Omega_m h$. The formation of large-scale structure depends on the primordial spectrum of inhomogeneities and also on cosmological parameters that determine how those seeds evolve in time. Fits assuming Ω_m consists of cold dark matter[31] to the observed spectrum of mass fluctuations constrain the combination, $\Gamma \equiv \Omega_m h = 0.25 \pm 0.05$.

The range of concordance consistent with these constraints is indicated in Fig. 1. The heavily shaded region, both with and without vertical stripes, is for flat models with $\Omega_\Lambda > 0$. The vertical stripes indicate the somewhat more restricted subregion that applies for open models with $\Omega_\Lambda = 0$, with the reduction being due to a different age constraint. Two conclusions are worth noting here: (1) a substantial permitted area does exist, and (2) removal of any one of the observational sets of limits does not significantly enlarge the permitted area. Or, put differently, recovering $\Omega_m = 1$, as assumed in the standard cold, hot, and mixed dark-matter models, requires a combination of several observations to change in a coherent fashion. Independent analyses using some of the same astronomical tests have reached similar conclusions[32-37].

A great leap forward in testing cosmological models is now possible due to the recently acquired ability to measure the cosmic background radiation (CBR) anistropy. Because the CBR emanates from earlier epochs and greater distances than are probed by astronomical tests, the anisotropy provides truly independent information capable of discriminating among the surviving possibilities.

The amplitude of the CBR power spectrum, as determined by the Cosmic Background Explorer (COBE) satellite[4], can be used to determine the spectrum of primordial fluctuations that seeded large-scale structure. The spectrum is specified by the spectral index n, which fixes the spectral shape, and an overall normalization, conventionally chosen to be σ_8, the root-mean-square mass fluctuations averaged over eight h^{-1} Mpc spheres. The value of σ_8 required to explain the distribution of large-scale structure and velocities[38] over $1-100$ Mpc scales is $\sigma_8 = (0.56 \pm 0.06) (\Omega_m)^{-0.56}$. The COBE-determined amplitude is another measure of normalization, but based on the greater distances probed by the CBR. The extrapolation from COBE to a normalization at shorter distance scales depends on n, the fractional contribution of energy density fluctuations to CBR fluctuations, and the values of Ω_m, Ω_Λ, H_0, and Ω_B (ref. 39). The latter parameters determine how fluctuations on $8h^{-1}$ Mpc scales evolve compared to COBE scales. Any given point in the concordance region of Fig. 1 corresponds to fixed Ω_m, Ω_Λ and H_0, and Big Bang nucleosynthesis then determines Ω_B, by the constraints given above. Because inflation sets a relation between n and the energy density fluctuation contribution[5,17], matching COBE normalization to the astronomical normalization fixes the remaining undetermined parameter, the spectral index n.

We find that $-0.15 < n - 1 < 0.2$ for the flat models with $\Omega_\Lambda > 0$ which lie in the concordance region. This lies totally within the range of n determined by COBE[40] from comparing anisotropy averaged over a range of angular scales. It is also totally within the range that is achievable in inflationary models, n between
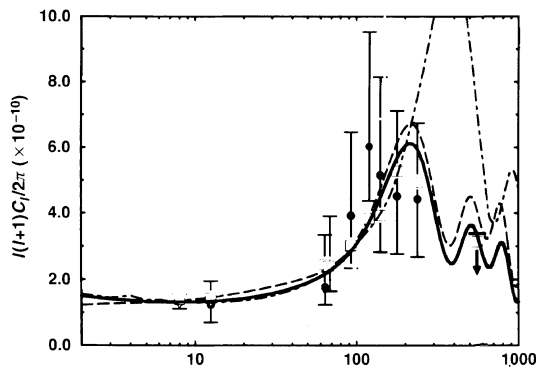
FIG. 2 Predicted CBR power spectrum for representative models: standard cold dark matter ($n=1$, $h=0.5$, $\Omega_\Lambda=0$; dashed line); a flat model in the concordance region of Fig. 1 ($n=0.96$, $h=0.65$, $\Omega_m=0.35$, $\Omega_\Lambda=0.65$, $\sigma_8=0.87$; solid line); and approximate results for an open model[43] in the concordance region, ($n=1.15$, $h=0.625$, $\Omega_m=0.375$; dot-dashed curve). The boxes represent the theoretical predictions for present CBR experiments for the flat model. The error bars represent 1σ detections, and the arrow indicates 95% upper confidence (no detection) limits. (See ref. 5 for details.) Note that the power spectrum for the flat model is remarkably similar to the prediction for standard cold dark matter, except at smaller angular scales ($l>250$), where the flat model is marginally more consistent with present observational limits. Open models predict that the leftmost peak occurs at higher l and, consequently, has more power at small angular scales[43].

0.7 and 1.2 (refs 5, 17). For open models, the COBE fit[41] requires shifting to n greater than unity, $+0.05 < n-1 < 1.4$. The larger value of n is required to compensate for the fact that there is less amplitude growth in open models compared to flat models. Requiring $n>1$ is a new constraint on open models stemming from this analysis.

With Ω_m, H_0 and n determined for each type of model, CBR anisotropy measurements at intermediate scales (0.5–$2°$) can distinguish between the remaining models. The measurements determine the temperature autocorrelation function, $C(\theta)$, the sky-averaged product of the CBR temperature along two directions separated by angle θ. If $C(\theta)$ is re-expressed in Legendre polynomial series, $C(\theta)=(4\pi)^{-1}\Sigma(2\ell+1)C_\ell P_\ell(\cos\theta)$, the coefficients C_ℓ are called multipoles and a plot of $l(l+1)C_l$ is the CBR power spectrum. Roughly speaking, a given C_l is determined by temperature variations across the sky with mean wavelength π/l. The shape of the CBR power spectrum is very sensitive to models and their parameters. Figure 2 shows the CBR power spectrum for the standard cold dark matter model and for two representative models from the middle of the concordance regions, a flat model with $\Omega_\Lambda > 0$ (open square in Fig. 1) and an open model (open circle in Fig. 1)[42]. The important discriminating feature is the large Doppler peak, whose position along the abscissa is a sensitive test of $\Omega_m+\Omega_\Lambda$ and whose magnitude for the open concordance model is substantially greater than the Doppler peak for flat models[22]. The current results favour the flat models over the open models, although definitive conclusions

should be deferred until after the significant experimental improvements anticipated over the next few years.

Anisotropy measurements at yet smaller angular scales, down to 5-arcmin resolution or better, provide important corroborating evidence[5,42]. An interesting feature, illustrated in Fig. 2, is that the predicted CBR power spectrum for $l>250$ in standard ($\Omega_\Lambda=0$) cold dark matter models and in our flat concordance models are virtually indistinguishable and in equal agreement with observations, despite their substantially different cosmological parameter values. However, for $l>250$ (spanning 10–30 arcmin), representative flat concordance models predict somewhat less power than standard cold dark matter models, marginally more consistent with current measurements by the OVRO, White Dish and MSAM experiments[5]. The open concordance models predict substantially more power than does the standard cold dark matter model[5]. This example is strong motivation for improved CBR experiments with angular resolution of 5–40 arcmin.

We would be interested to hear if a serious observational problem can be identified with the low-density concordance models illustrated in Figs 1 and 2 and, in particular, with the flat model which has a substantial cosmological constant. If not, perhaps we have already identified models which, in broad outline, capture the essential properties of the large-scale Universe. Should this be the case, a challenge arises: how can we explain the non-zero value of the cosmological constant from a theoretical point of view? □

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