Chapter 13
32) Which method could detect a planet in an orbit that is face-on to the Earth?
c) Astrometric method. The astrometric method measures the tangential motion of a star, and a face-on planetary orbit would force the star to move in the tangential direction. In contrast, both the Doppler and transit method require that the planet orbit partially edge on to measure a radial velocity or eclipse, respectively.

51) The star 51 Pegasi has about the same mass as our Sun. A planet discovered orbiting it has an orbital period of 4.23 days. The mass of the planet is estimated to be 0.6 times the mass of Jupiter. Use Kepler’s third law to find the planet’s average distance from the star. (Hint: Because the mass of 51 Pegasi is about the same as the mass of our Sun, you can use Kepler’s third law in its original form, \( p^2 = a^3 \). Be sure to convert the period into years before using this equation).

This problem can be solved using Kepler’s third law \( p^2 = a^3 \) where \( a \) is in AU and \( p \) is in years, since the mass of the planet is much less than the mass of its orbiting star and the mass of the star is approximately 1 solar mass. Converting 4.23 days to years, the orbital period of the planet is 0.0116 years. Solving for the semimajor axis \( a \), \( a = p^2 = 0.051 \) AU, gives the average distance from the planet to the star. Therefore, the planet around 51 Pegasi is approximately eight times closer than Mercury is to the Sun.

53) The Doppler method allows us to find a planet’s semimajor axis using just the orbital period and the star’s mass (Mathematical Insight 13.1). a) Imagine that a new planet is discovered orbiting a 2\( \text{M}_\odot \) star with a period of 5 days. What is its semimajor axis?

b) Another planet is discovered orbiting a 0.5\( \text{M}_\odot \) star with a period of 100 days. What is its semimajor axis?

a) From Mathematical Insight 13.1 the semi-major axis \( a \) is equal to \( a = \sqrt[3]{\frac{GM_{\text{star}}}{4\pi^2} p_{\text{planet}}^2} \) when the mass of the star is much larger than the mass of the planet. The problem gives \( M_{\text{star}} \) to be 2\( \text{M}_\odot \) and \( p \) equals 5 days. Instead of worrying about constants, we can normalize by the Earth’s orbit around the Sun to solve this problem \( a_{\text{Earth}} = \sqrt[3]{\frac{GM_{\odot}}{4\pi^2} p_{\text{Earth}}^2} \).

Therefore, \( a_{\text{planet}} = \sqrt[3]{\frac{M_{\text{star}}}{M_{\odot}} \left( \frac{p_{\text{planet}}}{p_{\text{Earth}}} \right)^2} \). Plugging in the parameters given in the problem, the period of the planet is five days, so 0.0137 years (and don’t forget the period of Earth is 1 year by definition) and the mass of the star is 2\( \text{M}_\odot \). Therefore, \( a_{\text{planet}} = \sqrt[3]{2 \times 0.0137^2} = 0.072 \). Recalling that \( a_{\text{Earth}} \) is 1 AU by definition, the semimajor axis is \( a_{\text{planet}} = 0.072 a_{\text{Earth}} \).

b) Using the same equation as part a \( a_{\text{planet}} = \sqrt[3]{\frac{M_{\text{star}}}{M_{\odot}} \left( \frac{p_{\text{planet}}}{p_{\text{Earth}}} \right)^2} \) we can find the semimajor axis for this planet. Using the values of 0.5\( \text{M}_\odot \) for \( M_{\text{star}} \) and 0.2738 years for \( p_{\text{planet}} \), the semi-major axis is 0.335 AU.

Chapter 14
29) Which of these group of particles has the greatest mass? a) 4 protons. A helium nucleus (4.003 amu) is composed of two protons and two neutrons, but is created from the fusion of 4 protons (4.029 amu). When a helium nucleus is created energy is release which comes from the conversion of mass, so a helium nucleus must be less massive than four protons. Finally, electrons have a mass less than about 1000 times that of a proton or neutron, so
the mass of 4 electrons is insignificant.

50) The total mass of the Sun is about $2 \times 10^{30}$ kilograms, of which about 70% was hydrogen when the Sun formed. However, only about 13% of this hydrogen ever becomes available for fusion in the core. The rest remains in layers of the Sun where the temperature is too low for fusion. **a** Use the given data to calculate the total mass of hydrogen available for fusion over the lifetime of the Sun. **b** The Sun fuses about 600 billion kilograms of hydrogen each second. Based on your result from part a, calculate how long the Sun’s initial supply of hydrogen can last. Give your answer in both seconds and year. **c** Given that our solar system is now about 4.6 billion years old, when will we need to worry about the Sun running out of hydrogen for fusion?

**a** The mass of Hydrogen available for fusion is $2 \times 10^{30}$kg multiplied by the mass fraction of hydrogen (0.70) multiplied by the mass fraction of hydrogen available for fusion (0.13). Plugging in the numbers gives a mass of $1.82 \times 10^{29}$kg of Hydrogen available for fusion. **b** The simplest way to answer and think of this problem is by using unit analysis. The mass available for fusion divided by the mass fusion rate gives a unit of time ($[\text{kg}] / [\text{kg/s}] = [\text{s}]$). Plugging in the numbers gives $1.82 \times 10^{29} / 600 \times 10^9 = 3.0333 \times 10^{17}$s or 9.6 billions years. **c** The remaining time for the is the total fusion time minus the age of the Sun. Since the age of the Sun is 4.6 billion years, the remaining time fusion time is approximately 5 billion years. Or the Sun has exhausted approximately half of its fuel and we have a while before we need to worry about the Sun running out of hydrogen. Although we still do need to worry about the Sun before then. In the next hundred million to billion years the Sun’s core will contract to fuse hydrogen more efficiently as its core hydrogen deletes. The smaller core will cause an increase in the temperature of the core, which will increase the luminosity output from the Sun and bake away the Earth’s atmosphere and any life on the surface.

53) The Sun’s average surface temperature is about 5800K. Use Wien’s law (see Mathematical Insight 5.2) to calculate the wavelength of peak thermal emission from the Sun. What color does this wavelength corresponds to in the visible-light spectrum? Why do you think the Sun appears white or yellow to our eyes?

Using Wien’s law $\lambda_{\text{max}} = \frac{2,900,000 \text{nm}}{T(\text{K})}$ where $T = 5800$ K. This gives a peak wavelength of 500nm, which is green light. However, we observe white because your eye smudges all of the light from the Sun together.

Chapter 24

51) Suppose we have a spaceship about the size of a typical ocean cruise ship today, which means it has a mass of about 100 million kilograms, and we want to accelerate the ship to a speed of 10% of the speed of light. **a** How much energy would be required? (Hint: You can find the answer simply by calculating the kinetic energy when it reaches cruising speed; because 10% of the speed of light is still small compared to the speed of light, you can use the formula that tells us that kinetic energy = $\frac{1}{2}mv^2$.) **b** How does your answer compare to the total energy use at present, which is about $5 \times 10^{22}$ joules per year? **c** The typical cost of energy is roughly 5 cents per 1 million joules. At this price, how much would it cost to generate the energy needed by this spaceship?
Using the kinetic energy equation the energy required to accelerate to 10% the speed of light is $\frac{1}{2} m v^2$ or $4.50 \times 10^{22}$ joules. This is approximately the same as the energy produced by the Earth in a year. Therefore, we could not easily produce the energy to run the proposed spaceship. The price per joule is the cost for a million joules (0.05 dollars) multiplied by the number of million of joules needed to accelerate the ship ($4.5 \times 10^{22}$ joules/10^6 joules) and the total cost would be $2.25 \times 10^{15}$ dollars. Therefore, the ship would cost 2.25 petadollars to accelerate to 10% the speed of light in an ideal situation.