Observational Indications of the External Field Effect

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• Galaxies: some relevant properties
  • kinematic laws
• MOND
• Equivalence Principles
  • Weak, Strong, Einstein
• External Field Effect
  • detections in Local Group dwarfs
  • detections in spiral galaxies
Galaxies exist over a huge dynamic range in

Rotation velocity
$20 < V_f < 300 \text{ km/s}$

Luminosity
$1 \times 10^7 < L_{[3.6]} < 5 \times 10^{11} \text{ L}_\odot$

Gas mass
$1 \times 10^7 < M^* < 5 \times 10^{10} \text{ M}_\odot$

Surface brightness
$5 < \mu_e < 3 \times 10^3 \text{ L}_\odot \text{ pc}^{-2}$

Gas fraction
$0.03 < f_g < 0.97$
Galaxies are made of gas as well as stars

NGC 6946 stars & gas

near infrared

atomic gas
NGC 6946

Rotation curve data from
Boomsma et al (2008) [H\textsc{i}]
Daigle et al (2006) [H\alpha]
Blais-Ouellette et al (2004) [H\alpha]

The sinusoidal variation of velocity in each ring measures the position angle, inclination, and rotation curve $V_c(R)$.

$V \sin i = V_{sys} + V_c \cos \theta + V_r \sin \theta$
Surface photometry quantifies the variation of surface brightness with radius. These data are used to numerically solve the Poisson equation to obtain the gravitational potential of the stars.

\[ \nabla^2 \Phi_* = 4\pi G \rho_* \]

\[ \mu \rightarrow \Sigma_* \]

\[ \Sigma_* \rightarrow \rho_* \]
Progressive approximations in mass modeling

- Point Mass
- Spherical Cow
- thin exponential disk
- thick exponential disk
- surface density $\Sigma(R)$
- 2D $\Sigma(R, \phi)$ [e.g., bars]
- 3D $\rho(R, \phi, z)$
- 3D + non-equilibrium

Examples for the size and mass of NGC 6946

$M_* = 3.3 \times 10^{10} \ M_\odot$
$R_d = 2.44 \ kpc$

We numerically solve the Poisson equation to obtain the gravitational potential $\Phi_*$ from the observed surface density $\Sigma_*(R)$

$$V_{\text{exp disk}}^2 = \frac{2GM_*}{R_d} y^2(\text{ikik})$$

$$y = \frac{R}{2R_d}$$

$$\text{ikik} = [I_0(y)K_0(y) - I_1(y)K_1(y)]$$
Now have

Surface density for
stars
gas
and corresponding rotation
curves for each component

Observed rotation curve

NGC 6946

Surface density for
stars
gas
and corresponding rotation
curves for each component

Observed rotation curve

\[ V_{\text{bar}}^2 = V_{\text{disk}}^2 + V_{\text{bulge}}^2 + V_{\text{gas}}^2 \]
Photometric Scaling Relations are weak, have lots of intrinsic scatter.

Kinematic Scaling Relations

1. Flat Rotation Curves
2. Renzo’s Rule
3. Baryonic Tully-Fisher Relation
4. Central Density Relation
5. Radial Acceleration Relation

Kinematic Scaling Relations are strong, have little intrinsic scatter.
1. Rotation curves become flat at large radii


Fig. 3.—Mean velocities in the plane of the galaxy, as a function of linear radius for 23 Sb galaxies, arranged approximately according to increasing luminosity. Adopted curve is rotation curve formed from the mean of velocities on both sides of the major axis. Vertical bar marks the location of $R_{25}$, the isophote of 25 mag arcsec$^{-2}$, corrected for effects of internal extinction and inclination. Regions with no measured velocities are indicated by dashed lines.
2. Renzo’s Rule

For every feature in the luminosity profile, there is a corresponding feature in the rotation curve, and vice-versa.

Sancisi (2004)

High surface brightness galaxies near maximal (the stars suffice to explain the velocity at small radii).

Low surface brightness galaxies are far from maximal (nevertheless, Renzo’s rule holds).

amplitude of rotation speed correlates with mass

flat rotation speed minimizes scatter
3. Tully-Fisher relations
amplitude of flat rotation correlates with mass

Lelli et al (2019)
Spitzer [3.6]
$M_g > M_*$
The fundamental relation is between **baryonic mass** and the amplitude of the **flat rotation speed**

\[
M_b = M_* + M_g = AV_f^4
\]

\[A = 48.5 \pm 3.3 \, \text{M}_\odot (\text{km s}^{-1})^{-4}\]

there is remarkably little **intrinsic** scatter

\[
\sigma_M < 0.11 \, \text{dex}
\]

which is about what is expected for scatter in stellar population $M^*/L$
Example application:
Calibrate BTFR with 50 galaxies having distances that are known via either Cepheids of Tip of the Red Giant Branch measurements.

Applied to ~100 galaxies with high quality rotation curves, this provides a local measurement of the Hubble constant:

\[ H_0 = 75.1 \pm 2.3 \text{ (stat)} \pm 1.5 \text{ (sys) km s}^{-1} \text{ Mpc}^{-1} \]

This is consistent with the application of the traditional luminosity-linewidth Tully-Fisher relation to a much larger sample of ~10,000 galaxies.

\[ H_0 = 75.1 \pm 0.2 \text{ (stat)} \pm 3 \text{ (sys) km s}^{-1} \text{ Mpc}^{-1} \]
The shape of the rotation curve correlates with the stellar surface density. 

Rotation curve shape correlates with baryonic surface density

SPARC data
Lelli et al. (2016, 2017)

BTFR only probes outer flat velocity
Rotation curve shape correlates with baryonic surface density

![Graph showing rotation velocity vs. radius with SPARC data from Lelli et al. (2016, 2017)]

What about inner parts?

SPARC data
Lelli et al. (2016, 2017)
The dynamical central mass surface density correlates with the central surface brightness of stars in galaxies.

\begin{equation}
\Sigma_{\text{dyn}}(0) = \frac{1}{2\pi G} \int_0^\infty \frac{V^2(R)}{r^2} dR
\end{equation}

\[\Sigma_\dagger \approx 900 \, \text{M}_\odot \, \text{pc}^{-2}\]
Rotation curve shape correlates with baryonic surface density.

SPARC data
Lelli et al. (2016, 2017)

What about everything in between?
What about everything in between?

The observed centripetal acceleration is linked to that predicted by the observed distribution of baryons.
\[ g_{\text{obs}} = \frac{V^2}{R} \]
determined from rotation curve

\[ g_{\text{bar}} = \left| \frac{\partial \Phi}{\partial R} \right| \]
determined from baryon distribution

independent quantities
1999 DATA:
26 galaxies
356 points
2004 DATA:
74 galaxies
1145 points
2016 DATA:
153 galaxies
2693 points
5. Radial Acceleration Relation

The observed acceleration correlates with that predicted by the baryons

The data are well fit by

$$g_{\text{obs}} = \frac{g_{\text{bar}}}{1 - e^{-\sqrt{g_{\text{bar}} / g^\dagger}}}$$

$$g^\dagger = 1.20 \times 10^{-10} \text{ m s}^{-2}$$

±0.02 (random) ± 0.24 (systematic)

The data are consistent with zero intrinsic scatter

observed rms scatter

scatter expected from observational errors

The data are well fit by

$$\pm 0.02 \pm 0.24$$

The data are consistent with zero intrinsic scatter
Kinematic Scaling Relations

1. Flat Rotation Curves
2. Renzo’s Rule
3. Baryonic Tully-Fisher Relation
4. Central Density Relation
5. Radial Acceleration Relation

Kinematic Scaling Relations are strong, have little intrinsic scatter.
The quantitative relations involve a critical acceleration scale.

This acceleration scale is ubiquitous in galaxy data.

- **Baryonic Tully-Fisher Relation**
  \[ g^\text{BTFR}_{\dagger} = \frac{\chi V_f^4}{GM_b} = 1.24 \times 10^{-10} \pm 0.14 \text{ m s}^{-2} \]
  (McGaugh 2011)

- **Central Density Relation**
  \[ g^\text{CDR}_{\dagger} = G\Sigma_{\dagger} = 1.27 \pm 0.05 \times 10^{-10} \text{ m s}^{-2} \]
  (Lelli et al. 2016)

- **Radial Acceleration Relation**
  \[ g^\text{RAR}_{\dagger} = 1.20 \pm 0.02 \times 10^{-10} \text{ m s}^{-2} \]
  (McGaugh et al. 2016)
Baryonic Tully-Fisher Relation

Can construct a characteristic acceleration for each galaxy

\[ g^\dagger = \frac{\chi V_f^4}{GM_b} \]

Galaxies closely follow a single, universal acceleration.

\( \chi \) is a factor of order unity that accounts for the geometry of disk galaxies, which are not spherical cows. We adopt \( \chi = 0.8 \) (McGaugh & de Blok 1998; McGaugh 2005).

Baryonic Mass

Flat Rotation Velocity
Over 25 decades in acceleration, galaxies only exist around $1 \text{Å/s/s}$.

$g^\dagger$ is a special value.
Laws of Galactic Rotation

1. Flat Rotation Curves
2. Renzo’s Rule
3. Baryonic Tully-Fisher Relation
4. Central Density Relation
5. Radial Acceleration Relation

There is a ubiquitous acceleration scale in the data: \( g_t = 1.2 \times 10^{-10} \, \text{m} \, \text{s}^{-2} \)

Modified gravity rather than dark matter?
MOND - predicted exactly what we see
**MOND**

Modified Newtonian Dynamics
introduced by Moti Milgrom in 1983

http://www.scholarpedia.org/article/The_MOND_paradigm_of_modified_dynamics

Modify the force law at an acceleration scale *(not a length scale)*

Above a critical acceleration $a_0$ everything is normal. Below that scale, gravity in, effect becomes stronger.

$$g_N = \mu(a/a_0)a$$

Newtonian and MOND regimes joined by smooth interpolation function $\mu(a/a_0)$ with asymptotic limits

$$a \to g_N \quad \text{for} \quad a \gg a_0$$

$$a \to \sqrt{g_N a_0} \quad \text{for} \quad a \ll a_0$$

$a_0 = 1.2 \times 10^{-10}$ m s$^{-2}$
MOND
Modified Newtonian Dynamics
introduced by Moti Milgrom in 1983
http://www.scholarpedia.org/article/The_MOND_paradigm_of_modified_dynamics

MOND can be interpreted as either a modification of gravity or inertia

modify inertia
\[ F = m_i a \]
\[ m_i \neq m_g \]

modify gravity
\[ F = \frac{G m_1 m_2}{r^2} \]

\[ g_N = \mu \left( \frac{a}{a_0} \right) a \]

Newtonian and MOND regimes joined by smooth interpolation function \( \mu(a/a_0) \) with asymptotic limits

\[ a \rightarrow g_N \quad \text{for} \quad a \gg a_0 \]

\[ a \rightarrow \sqrt{g_N a_0} \quad \text{for} \quad a \ll a_0 \]

\[ a_0 = 1.2 \times 10^{-10} \text{ m s}^{-2} \]
MOND

Modified Newtonian Dynamics

introduced by Moti Milgrom in 1983

http://www.scholarpedia.org/article/The_MOND_paradigm_of_modified_dynamics

First theory derived from a Lagrangian by Bekenstein & Milgrom (1984)

\[ \nabla^2 \Phi = 4\pi G \rho \quad \text{modified Poisson equation to} \quad \nabla \cdot \left[ \mu \left( \frac{|\nabla \Phi|}{a_0} \right) \nabla \Phi \right] = 4\pi G \rho \]

Generalizes Newton but not Einstein

\[ g_N = \mu(a/a_0)a \]

Newtonian and MOND regimes joined by smooth interpolation function \( \mu(a/a_0) \) with asymptotic limits

\[ a \rightarrow g_N \quad \text{for} \quad a \gg a_0 \]

\[ a \rightarrow \sqrt{g_N a_0} \quad \text{for} \quad a \ll a_0 \]

\[ a_0 = 1.2 \times 10^{-10} \, \text{m} \, \text{s}^{-2} \]
Table 1. MOND Predictions and Tests.

<table>
<thead>
<tr>
<th>Prediction</th>
<th>Test Positive?</th>
<th>A Priori?</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>MASR (Tully–Fisher)</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Property 1. Normalization</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Property 2. Slope</td>
<td>Yes</td>
<td>★ No</td>
</tr>
<tr>
<td>Property 3. Mass &amp; Asymptotic Speed</td>
<td>Yes</td>
<td>★ Yes</td>
</tr>
<tr>
<td>Property 4. Surface Brightness Independence</td>
<td>Yes</td>
<td>★ Yes</td>
</tr>
<tr>
<td><strong>Rotation Curves</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Property 5. Flat Rotation Curves (First Law)</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>Property 6. Acceleration Discrepancy (RAR)</td>
<td>Yes</td>
<td>★ Yes</td>
</tr>
<tr>
<td>Property 7. Rotation Curve Shapes</td>
<td>Yes</td>
<td>★ Yes</td>
</tr>
<tr>
<td>Property 8. Surface Brightness &amp; Density (Central Density rel’n)</td>
<td>Yes</td>
<td>★ Yes</td>
</tr>
<tr>
<td>Property 9. Detailed Fits</td>
<td>Yes</td>
<td>★ No</td>
</tr>
<tr>
<td>Property 10. Stellar Population Y_*</td>
<td>Yes</td>
<td>—</td>
</tr>
<tr>
<td>Property 11. Feature Correspondence (Renzo’s rule)</td>
<td>Yes</td>
<td>—</td>
</tr>
<tr>
<td><strong>Disk Stability</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Property 12. Freeman Limit</td>
<td>Yes</td>
<td>★ No</td>
</tr>
<tr>
<td>Property 13. Vertical Velocity Dispersions</td>
<td>?</td>
<td>No</td>
</tr>
<tr>
<td>Property 14. LSB Galaxy Morphology</td>
<td>Yes</td>
<td>★ Yes</td>
</tr>
</tbody>
</table>

Not expected with DM
Contradicts DM?
Newtonian regime

\[ g_{in} > a_0 \]

\[ M = \frac{RV^2}{G} \]

e.g., surface of the Earth

External Field dominant Newtonian regime

\[ g_{in} < a_0 < g_{ex} \]

\[ M = \frac{RV^2}{G} \]

e.g., Eotvos-type experiment on the surface of the Earth

ISO

\[ g_{in} < a_0 \]

\[ M = \frac{V^4}{a_0 G} \]

e.g., remote dwarf Leo I

EFE

External Field dominant quasi-Newtonian regime

\[ g_{in} < g_{ex} < a_0 \]

\[ M = \frac{g_{ex} RV^2}{a_0 G} \]

e.g., nearby dwarf Segue 1

MOND regime

\[ g_{in} < a_0 \]

\[ M = \frac{V^4}{a_0 G} \]
Equivalence Principles


• Weak Equivalence Principle
  • universality of free fall \textit{the motion of a particle is independent of its internal structure or composition}

• Strong Equivalence Principle \( m_i \equiv m_g \)
  • WEP + Lorenz Invariance + Local Position Invariance

• Einstein Equivalence Principle
  • SEP but \textit{excluding} gravity from Local Position Invariance
  • Doesn’t matter where you do an E&M experiment, but it might matter where you do a gravitational experiment.
**Deep MOND Regime**

practically isolated; internal field dominates

\[ g_{ex} < g_{in} < a_0 \]

\[
\sigma_{iso} = \left( \frac{4}{81} a_0 G M_* \right)^{1/4}
\]

**External Field Effect**

External field stronger than internal field

\[ g_{in} < g_{ex} < a_0 \]

\[
\sigma_{efe} = \left( \frac{a_0 G M_*}{3 g_{ex} r^{1/2}} \right)^{1/2}
\]
Some dwarf galaxies in the Local Group obey the Tully-Fisher relation; others don’t. Is this a sign of the EFE?

We can use MOND to predict the velocity dispersions of dwarf satellite galaxies in advance of their observation.
MOND

Modify gravity at an acceleration scale
Hypothesized by Milgrom (1983)

\[ a \gg a_0 \quad a \to g_N \]
\[ a \ll a_0 \quad a \to \sqrt{g_N a_0} \]

\[ a_0 = 1.2 \times 10^{-10} \text{ m s}^{-2} \]

Unique & unsettling feature of MOND: the external field matters (violates Strong Equivalence)

<table>
<thead>
<tr>
<th>Isolated MOND regime</th>
<th>ISO</th>
<th>( g_{ex} &lt; g_{in} &lt; a_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Internal gravity of dwarf dominates</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \sigma_{iso} = \left( \frac{4}{81} a_0 GM_* \right)^{1/4} \]

Prediction depends only on stellar mass

<table>
<thead>
<tr>
<th>External Field Effect</th>
<th>EFE</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g_{in} &lt; g_{ex} &lt; a_0 )</td>
<td></td>
</tr>
<tr>
<td>External gravity of host dominates</td>
<td></td>
</tr>
</tbody>
</table>

\[ \sigma_{efe} = \left( \frac{a_0 GM_*}{3g_{ex} r_{1/2}} \right)^{1/2} \]

Prediction also depends on the size of dwarf and mass of and distance from host galaxy
The dwarf satellites of Andromeda (McGaugh & Milgrom 2013a,b)
### Quantities used to predict the velocity dispersion

<table>
<thead>
<tr>
<th>ISO</th>
<th>( \sigma_{iso} = \left( \frac{4}{81} a_0 G M_* \right)^{1/4} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Luminosity</td>
<td>( M_* = \gamma_* L_V )</td>
</tr>
<tr>
<td></td>
<td>( \gamma_*^V = 2^{+2}<em>{-1} , M</em>\odot / L_\odot )</td>
</tr>
</tbody>
</table>

**EFE**

<table>
<thead>
<tr>
<th>Luminosity</th>
<th>( M_* = \gamma_* L_V )</th>
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<tbody>
<tr>
<td>( \gamma_*^V )</td>
<td>( 2^{+2}<em>{-1} , M</em>\odot / L_\odot )</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Half-light radius</th>
<th>( r_{1/2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \sigma_{efe} )</td>
<td>( \left( \frac{a_0 G M_*}{3 g_{ex} r_{1/2}} \right)^{1/2} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>External field</th>
<th>( g_{ex} = \frac{V_f^2}{D} )</th>
</tr>
</thead>
</table>

- \( V_f = 230 \, \text{km s}^{-1} \)

Range of plausible stellar mass-to-light ratio

---

MOND correctly predicts the velocity dispersion for most of the M31 dwarfs (silver stars).

The prediction is completely a priori in many cases (gold stars).

**MOND** Predictions of isolated case are GREEN circles; open circles are the predictions for when the EFE dominates.
### Matched pairs of dwarfs - indistinguishable except for whether they’re affected by the EFE or not.

<table>
<thead>
<tr>
<th>Name</th>
<th>Luminosity</th>
<th>$R_e$</th>
<th>$\sigma_{\text{obs}}$</th>
<th>$\sigma_{\text{pred}}$</th>
<th>Affected by EFE</th>
</tr>
</thead>
<tbody>
<tr>
<td>And XVII</td>
<td>2.60E+05</td>
<td>381</td>
<td>2.9</td>
<td>2.5</td>
<td>EFE</td>
</tr>
<tr>
<td>And XXVIII</td>
<td>2.10E+05</td>
<td>284</td>
<td>4.9</td>
<td>4.3</td>
<td>isolated</td>
</tr>
</tbody>
</table>

#### MOND

Matched pairs of dwarfs - indistinguishable except for whether they’re affected by the EFE or not.
Pairs of photometrically identical dwarfs should have different velocity dispersion depending on whether they are isolated or dominated by the external field effect.

There is no EFE in dark matter - this is a unique signature of MOND.
It has become conventional to attribute anomalous cases to tidal disruption in CDM. MOND is very good at predicting which objects we’ll need to invoke this for.

Cases noted by Collins et al. (2014) as having anomalously low velocity dispersions for their large sizes were naturally predicted by the EFE.
The recently discovered, ultra-diffuse Crater 2 provides another test. LCDM anticipates 10 - 17 km/s (abundance matching; size-v. disp. rel'n)

**MOND predicts** 2.1 +0.9/-0.6 km/s (in EFE regime  arXiv:1610.06189)

**Subsequently observed:** 2.7 ± 0.3 km/s (Caldwell et al. arXiv:1612.06398)

Consistent with a priori MOND prediction

Very hard to understand in the context of ΛCDM - incredibly low velocity at a very large radius.

If the universe is made of cold dark matter, why does MOND get any prediction right?
Galaxies are never completely isolated

Every dot pictured here is a galaxy, color coded by redshift

Can we compute the environmental EFE at any point from everything else in the universe?
An early estimate: “Taking these numbers at face value leads to $<a> \sim 0.026 \, \text{Å} \, \text{s}^{-2}$“ (McGaugh & de Blok 1998)

That’s a mean environmental acceleration of about 2% of $a_0$.

The External Field estimated from the observed galaxy distribution (Desmond, private communication)
$0.021 \pm 0.008 \quad 0.090 \pm 0.013 \quad 0.024 \pm 0.009 \quad 0.103 \pm 0.018$

$p_{\text{binom}} = 0.171 \quad 0.119 \quad 0.151 \quad 0.025$

$(N = 77) \quad (N = 19) \quad (N = 16) \quad (N = 17)$

$\frac{e_N}{\sqrt{|e_N|}}$

$\log_{10}(D/\text{Mpc})$

- environmental fields (max clustering)
- environmental fields (no clustering)
- individual EFE fits of 129 RCs
- CfA2 Great Wall: $0.111 \pm 0.021$
- Pisces-Perseus supercluster: $0.100 \pm 0.009$
- median of EFE fits in each bin
- median of all points: $0.040 \pm 0.006$
The EFE in the Local Group is relatively strong for satellites of Andromeda and the Milky Way.

There should be a weak EFE everywhere that affects the outskirts of all galaxies. Can we detect this?

\[ g_{ex} < g_{in} < a_0 \]

but

\[ |g_{in} + g_{ex}| > |g_{in}| \]

so

\[ \mu( |g_{in}| / a_0 ) \rightarrow \mu( |g_{in} + g_{ex}| / a_0 ) \]

\[ |g_{in} + g_{ex}| \approx |g_{in}| + |g_{ex}| \]
0.052 ± 0.011
The missing baryon problem

Cosmic baryon budget
(Shull et al arXiv:1112.2706)

@ z = 0

All the baryons in the universe contribute to the EFE, but galaxies contain < 10% of them!

- WHIM (OVI) 15±4%
- WHIM (Lyα) 14±7%
- photoionized (Lyα forest) 28±11%
- missing 29±13%
- galaxies 7±2%
- CGM 5±3%
- ICM 4±1.5%
- cold gas 1.7±0.4%
Conclusions

• Galaxies obey strict kinematic scaling laws

• The observed laws were predicted by MOND

• A further prediction of MOND is the External Field Effect (EFE)

• violates the SEP, but not the WEP or EEP

• The EFE appears in two distinct sets of observations

• Local Group dwarfs

• Outskirts of spirals

• Outstanding EFE uncertainty: where are all the baryons?