

TESTING THE HYPOTHESIS OF MODIFIED DYNAMICS WITH LOW SURFACE BRIGHTNESS GALAXIES AND OTHER EVIDENCE

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ABSTRACT

The rotation curves of low surface brightness galaxies provide a unique data set with which to test alternative theories of gravitation over a large dynamic range in size, mass, surface density, and acceleration. Many clearly fail, including any in which the mass discrepancy appears at a particular length scale. One hypothesis, Modified Newtonian Dynamics (MOND), is consistent with the data. Indeed, it accurately predicts the observed behavior. We find no evidence on any scale that clearly contradicts MOND and much that supports it.

Subject headings: dark matter — galaxies: formation — galaxies: halos — galaxies: kinematics and dynamics — galaxies: structure — gravitation

When you have eliminated the impossible, whatever remains, however improbable, must be the truth. Sherlock Holmes

1. INTRODUCTION

There exists clear evidence for mass discrepancies in extragalactic systems. Application of the usual Newtonian dynamical equations to the observed luminous mass does not predict the observed motions. This leads to the inference of dynamically dominant amounts of dark matter.

In a preceding paper, we described the many difficulties that arise in trying to understand the data in terms of dark matter (McGaugh & de Blok 1998; hereafter Paper I). In this paper, we consider the alternative of a change in the fundamental equations of motion as the cause of the observed mass discrepancies. Many alternative theories of gravity have been posited to this end, but most have been ruled out (see, e.g., Sanders 1986). A few that may still appear to be viable are not in the light of data for low surface brightness (LSB) galaxies. Length-scale dependent alteration of the inverse square law (e.g., the generic cases discussed by Liboff 1992 or the nonsymmetric gravity of Moffat & Sokolov 1996) cannot explain the large variation in the scale on which the mass discrepancy appears (Fig. 3 of Paper I) unless the length scale of the theory is allowed to vary from galaxy to galaxy. Similarly, the linear potential theory explored by Mannheim & Kazanas (1989) predicts rotation curves that should ultimately rise rather than remain flat. This is obviously not consistent with extant rotation curve data (Carlson & Lowenstein 1996), and the rotation curves of at least some LSB galaxies (e.g., UGC 128) remain flat well beyond the point where the upturn should be observed. Only the Modified Newtonian Dynamics (MOND) proposed by Milgrom (1983a, 1983b, 1983c) appears empirically viable, so we focus on testing it.

The rotation curves of disk galaxies derived from gaseous tracers (H α and H I) provide the strongest tests of alterna-

tive force laws (see, e.g., Kent 1987; Begeman, Broeils, & Sanders 1991). With only the assumption of circular motion, it is possible to directly equate the centripetal acceleration

$$a_c = \frac{V^2}{R} \quad (1)$$

with the gravitational acceleration

$$g = -\frac{\partial\varphi}{\partial R} \quad (2)$$

determined from the Poisson equation

$$\nabla^2\varphi = 4\pi G\rho \quad (3)$$

or any hypothesized alternative, for example

$$\nabla \cdot \left[\mu \left(\frac{|\nabla\varphi|}{a_0} \right) \nabla\varphi \right] = 4\pi G\rho \quad (4)$$

(Bekenstein & Milgrom 1984). In no other type of system are tests so direct and free of assumptions.

To test MOND, we employ the data compiled in Paper I (mostly from Broeils 1992; van der Hulst et al. 1993; and de Blok, McGaugh, & van der Hulst 1996). These are rotation curves of disk galaxies spanning a large range in size, luminosity, and surface brightness. Luminosity and surface brightness must trace mass and mass surface density modulo only the mass-to-light ratio of the stars in this alternative to dark matter.

Previous tests of MOND have given mixed results. MOND does generally do a good job of fitting rotation curves (Begeman et al. 1991), but it is less clear that it works in systems other than disks (e.g., clusters of galaxies, The & White 1988). As emphasized by Milgrom (1983b, 1988), LSB galaxies provide particularly strong tests. Some attempts to test MOND with LSB dwarf galaxies found that it failed (see, e.g., Lake 1989). Milgrom (1991) pointed out some uncertainties and limitations in Lake's analysis, so it

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remains unclear whether these cases constitute contradictions of MOND. Other attempts to fit a limited number of LSB galaxies reported success (Milgrom & Braun 1988; Begeman et al. 1991; Sanders 1996).

We test MOND with the substantial amount of new data we have accumulated on LSB galaxies (van der Hulst et al. 1993; de Blok et al. 1996). Section 2 describes the predictions of MOND relevant to LSB galaxies and tests each. Section 3 examines the viability of MOND in other systems. A summary of the results and some discussion of their implications is given in § 4. Detailed MOND fits to the rotation curves of LSB galaxies are given by de Blok & McGaugh (1998; hereafter Paper III). Symbols and definitions follow the conventions of Paper I. We adopt $H_0 = 75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ throughout.

2. THE MODIFIED DYNAMICS

Modified Newtonian Dynamics is an empirically motivated force law that can be interpreted either as a modification of gravity or of the law of inertia (see review by Milgrom 1994). We are interested here not in which interpretation is preferable, but rather in testing whether MOND is indeed the correct force law. In this context, it should be realized that MOND has not yet been developed into a full theory in the sense of General Relativity. However, as a force law it makes very precise and testable predictions just as the inverse square law did well before the adornment of subsequent theoretical elaboration.

The empirical motivation for MOND is the observation that the rotation curves of high surface brightness (HSB) spiral galaxies are asymptotically flat. One must take care not to test only the fact the force law was constructed to realize. The properties of LSB galaxies were largely unknown at the time and were not part of the input motivating MOND. Milgrom (1983b) stated that “disk galaxies with low surface brightness provide particularly strong tests” and made a series of specific predictions about LSB galaxies that constitute genuine tests of the MOND force law.

2.1. The Modified Force Law

The MOND force law is

$$a = \sqrt{g_N a_0}, \quad (5)$$

where g_N is the usual Newtonian acceleration and a_0 is a universal constant. This applies only for $a \ll a_0$; for $a \gg a_0$ the behavior is purely Newtonian. There is an interpolation function $\mu(x)$ with $x = a/a_0$ for connecting the two regimes (Milgrom 1983a). The function is required to have the asymptotic behavior $\mu(x \gg 1) \rightarrow 1$ and $\mu(x \ll 1) \rightarrow x$. We will not specify $\mu(x)$ here, and we restrict our tests to the deep MOND regime where equation (5) is the effective force law. In practice this means $x < \frac{1}{2}$, where the precise form of $\mu(x)$ ceases to matter {i.e., $[\mu(x) - x]/x < 10\%$ }. The MOND limit occurs only for extremely small accelerations: $a_0 = 1.2 \times 10^{-10} \text{ ms}^{-2}$ (Milgrom & Braun 1988; Begeman et al. 1991). This is roughly 1 \AA s^{-2} , or $10^{-11} g_\oplus$.

For circular orbits about a point mass, \mathcal{M} , in the MOND limit,

$$\frac{V_c^2}{R} = \sqrt{\frac{G\mathcal{M}}{R^2}} a_0. \quad (6)$$

This gives an asymptotically constant rotation velocity V_c independent of R :

$$V_c^4 = a_0 G\mathcal{M}. \quad (7)$$

It is this behavior that gives rise to asymptotically flat rotation curves and the Tully-Fisher relation (Tully & Fisher 1977).

Since MOND is a force law based on a modification at a particular acceleration scale, the strongest tests of MOND are provided by systems with the lowest accelerations. The stars in HSB galaxies experience centripetal accelerations of order a_0 . Low surface brightness galaxies have very low mean accelerations, typically $\langle a \rangle \approx 10^{-11} \text{ ms}^{-2} \sim a_0/10$. As a consequence, LSB galaxies provide strong tests of MOND.

2.2. Testing MOND

Solar system tests can be very precise but do not probe the MOND regime. The transition between Newtonian and MOND regimes allows us to define a transition radius where $a = a_0$,

$$R_t = \sqrt{G\mathcal{M}/a_0}. \quad (8)$$

For the sun, $R_t \approx 0.1 \text{ lt-yr}$. One would not expect to notice MOND effects until this radius, at which point the field of the Milky Way becomes significant. Classic solar system tests near to the sun are restricted to the regime $a \gg a_0$ and test only the deviation of the interpolation function from its asymptotic limit (Milgrom 1983a). The same holds true for binary pulsars, which experience large accelerations well removed from the MOND regime.

Extragalactic systems are the only laboratories where the MOND regime is clearly probed. In fitting MOND to galaxy data, there are several adjustable parameters. One is the acceleration constant itself. This must be a universal constant: once measured accurately in one galaxy, the same value must apply in all galaxies. We adopt $a_0 = 1.2 \text{ \AA s}^{-2}$ and keep it fixed. This was determined by Begeman et al. (1991) to be the best fit to a sample of galaxies with high-quality H I rotation curves and that has no overlap with our new LSB galaxy data. Slight adjustments to the distance to a galaxy can sometimes improve a MOND fit (Begeman et al. 1991). This occurs because of the interplay between the MOND mass, which depends only on the distance-independent circular velocity, and the gas mass, which varies as D^2 . We keep D fixed at the value determined from the assumed H_0 . For other choices of H_0 , there is some limited freedom to adjust a_0 to compensate. As with a_0 and D , adjusting the inclination of a disk galaxy can improve some fits. Inclination is crucial since MOND masses depend on $V^4/\sin^4(i)$. In this paper we keep i fixed; see Paper III for further examination of this point.

So far we have named three parameters that could in principle be adjusted. In practice, there is very little freedom to do this, and we will keep all of them fixed. The only truly free parameter is the conversion factor from light to mass, Υ_* . We will treat this as a value to be determined and compared to the expectations for stellar populations.

In the following subsections, we discuss particular tests of MOND with LSB galaxies.

2.3. The Tully-Fisher Relation

A very strong prediction of MOND is a single universal Tully-Fisher relation. “The relation between asymptotic

velocity and the mass of the galaxy is an absolute one” (Milgrom 1983b). This follows from equation (7), which gives

$$V_c^4 = a_0 G \mathcal{M} = a_0 G \left(\frac{\mathcal{M}_b}{L} \right) L \propto \Upsilon_b L, \quad (9)$$

where the baryonic mass-to-light ratio $\Upsilon_b = \mathcal{M}_b/L$. Note that R does not appear in this equation, nor do the dark matter galaxy formation parameters of equation (13) in Paper I.

That R does not appear in equation (9) is of fundamental significance. In the case of purely Newtonian dynamics, $V^2 = G\mathcal{M}/R$. Since LSB galaxies have, by definition, larger R than HSB galaxies of the same luminosity, they should not fall on the same Tully-Fisher relation, yet they do (Sprayberry et al. 1995; Zwaan et al. 1995). Invoking a length scale for the mass that is different from that of the light helps not at all: serious fine-tuning problems occur in the dark matter picture (Paper I). In MOND the Tully-Fisher relation follows simply and directly from the form of the force law.

The error budget for scatter in the Tully-Fisher relation in magnitudes is

$$|\delta M|^2 = 18.86 \left| \frac{\delta V_c}{V_c} \right|^2 + 1.18 \left| \frac{\delta \Upsilon_b}{\Upsilon_b} \right|^2. \quad (10)$$

Comparison of this with equation 12 of Paper I shows that the term involving the central surface brightness has disappeared. The real difference is larger, though. In the dark matter picture, there should be some intrinsic variation in V_c from halo to halo. In MOND, mass and velocity are strictly related; the term for the error in V_c in equation (10) now refers only to observational uncertainties and not also to intrinsic variations. All but one of the terms that should contribute intrinsic variance in the dark matter picture disappear. To obtain a single universal Tully-Fisher relation with little scatter, all that is really required in MOND is that \mathcal{M}_b/L be roughly constant: $\delta M = 1.086 |\delta \Upsilon_b / \Upsilon_b|$. Small intrinsic scatter in the Tully-Fisher relation is much easier to understand with MOND than with dark matter.

The slope of the Tully-Fisher relation, $L \propto V^y$ with $y = 4$ (Aaronson, Huchra, & Mould 1979; Tully & Verheijen 1997; Paper I), is dictated by the form of the force law in MOND. No particular slope is required in the case of dark matter. A slope of $y = 4$ is sometimes attributed to the virial theorem (see, e.g., Silk 1997), but a slope closer to $y = 3$ has also been suggested as the value arising in plausible disk galaxy formation scenarios (see, e.g., Mo, Mao, & White 1998). In any case, a constant disk plus halo mass surface density must be arranged. Multicomponent disk-halo models generally predict that some signature of the disk component should be visible in Tully-Fisher residuals, but none are present (Courteau & Rix 1997). More generally, there is no signature of the transition from disk to halo domination (the “disk-halo conspiracy”). These all occur naturally if there is no halo and the disk is the only component, as in MOND.

The Tully-Fisher relation of LSB galaxies provides a new test of MOND. That HSB galaxies obey the Tully-Fisher relation was known when MOND was developed, so this does not itself constitute a test. That LSB galaxies fall on the same relation with the same normalization was not known. That they should constitutes a genuine prediction

(Milgrom 1983b): “We predict, for example, that the proportionality factor in the $\mathcal{M} \propto V_c^4$ relation for these galaxies is the same as for high surface density galaxies.”

2.4. The Υ - Σ Conspiracy

If the MOND force law is the basis of the Tully-Fisher relation, it should be possible to derive from MOND the Υ - Σ conspiracy inferred with conventional dynamics (Zwaan et al. 1995; Paper I). This requires $\Upsilon_o^2 \Sigma \propto \text{constant}$, where Υ_o is the conventional dynamical mass-to-light ratio. In MOND, the typical acceleration of a disk is proportional to the square root of its characteristic surface density, so that

$$\langle a \rangle^2 \propto \sigma = \Upsilon_b \Sigma \quad (11)$$

(eq. [6] of Milgrom 1983b and eq. [19]).

The severity of the inferred Newtonian mass discrepancy $\mathcal{M}_N/\mathcal{M}_b$ depends on the extent to which the acceleration is in the MOND regime,

$$\frac{\mathcal{M}_N}{\mathcal{M}_b} = \frac{1}{\mu(x)} = \frac{\Upsilon_o}{\Upsilon_b} \approx \frac{a_0}{\langle a \rangle}. \quad (12)$$

The approximation effectively becomes an equality for $\langle a \rangle < a_0/2$. As a result, $\Upsilon_o = \Upsilon_b a_0 / \langle a \rangle$. Using equation (11), this becomes $\Upsilon_o^2 \propto (a_0^2/\sigma) \Upsilon_b^2 = a_0^2 \Upsilon_b / \Sigma$, i.e.,

$$\Upsilon_o^2 \Sigma \propto \Upsilon_b a_0^2 \sim \text{constant}. \quad (13)$$

The Υ - Σ conspiracy does follow from MOND.

2.5. Stellar Mass-to-Light Ratios

Another test of MOND is provided by the values required for the mass-to-light ratio of stellar populations, Υ_* . These should be consistent with what we know about stars. Since there is no dark matter, the kinematic data provide a direct measure of the luminous mass.

The only free parameter in equation (9) is Υ_* . The constants a_0 and G are known, V_c , L , and the gas mass \mathcal{M}_g are measured. Hence, $\mathcal{M}_* = \mathcal{M} - \mathcal{M}_g$ and $\Upsilon_* = \mathcal{M}_*/L$, where \mathcal{M} is the total dynamical mass indicated by MOND. For spiral disks, we expect Υ_* in the range 1–2 in the B -band, with the range $0.5 < \Upsilon_* < 4$ being credible (see, e.g., Larson & Tinsley 1978; Bruzual & Charlot 1993).

The gas mass is inferred directly from the 21 cm flux multiplied by a conversion factor to account for helium and metals: $\mathcal{M}_g = 1.4 \mathcal{M}_{\text{HI}}$. The conversion factor is probably closer to the primordial value 1.32 for LSB galaxies since these systems are very metal poor (McGaugh 1994; Rönnback & Bergvall 1995). This difference is very small compared to the accuracy with which Υ_* can be estimated. We assume molecular gas is not a major mass component. High surface brightness galaxies are gas poor with molecular gas mass being a small fraction of the stellar mass (Young & Knezek 1989; McGaugh & de Blok 1997). Low surface brightness galaxies are H I rich but apparently have very little molecular gas (Schombert et al. 1990; de Blok 1997). Any molecular mass that is present will be attributed to stars, resulting in Υ_* that are slightly too high. Again, this is a small effect compared to the uncertainty in Υ_* .

Strictly speaking, equation (7) holds precisely only for point masses. Disk galaxies are reasonably well approximated by an exponential mass distribution for which

$$\mathcal{M}(R) = 2\pi\sigma_0 h^2 \left[1 - e^{-R/h} \left(1 + \frac{R}{h} \right) \right] \quad (14)$$

and

$$g_N(R) = \pi G \sigma_0 \frac{R}{h} \left[I_0\left(\frac{R}{2h}\right) K_0\left(\frac{R}{2h}\right) - I_1\left(\frac{R}{2h}\right) K_1\left(\frac{R}{2h}\right) \right], \quad (15)$$

where I_n and K_n are modified Bessel functions of the first and second kind (Freeman 1970). Combining these and following the MOND prescription $a = (g_N a_0)^{1/2}$ gives a dimensionless factor $\chi(R)$ correcting the simple point mass formula for the disk geometry:

$$\chi(R) = \frac{2h^3 [1 - e^{-R/h}(1 + R/h)]}{R^3 [I_0(R/2h)K_0(R/2h) - I_1(R/2h)K_1(R/2h)]}, \quad (16)$$

so that

$$\mathcal{M}(R) = \chi(R) \frac{V^4(R)}{a_0 G}. \quad (17)$$

This is a fairly mild correction, as $\chi(R)$ does not deviate far from unity at most radii. For consistency with Paper I, we evaluate it at $R = 4h$, where $\chi(4h) = 0.76$. The effect is to reduce the mass somewhat as a disk geometry rotates faster than the equivalent spherical mass distribution.

The stellar mass is computed as

$$\mathcal{M}_* = 0.76 \frac{V_c^4}{a_0 G} - 1.4 \mathcal{M}_{H1}. \quad (18)$$

We have determined Υ_* in this fashion for all galaxies meeting the requirements stipulated in Paper I (essentially everything for which we have a measurement of V_c , L , μ_0 , and h). The results are given in Table 1 and plotted in Figure 1, which is analogous to Figure 4 of Paper I.

The stellar mass-to-light ratios inferred from MOND are consistent with those expected for disk population stars. In the mean, $\langle \Upsilon_* \rangle = 1.9$, and $\Upsilon_* = 1.6$ in the median. There is a fair amount of scatter about the mean value. No trend of Υ_* with μ_0 is obvious in Figure 1*b*. Slight trends of Υ_* with M_B and h are perceptible in Figures 1*a* and 1*c* but are not highly significant as they are strongly influenced by the low Υ_* values of a few small galaxies. The directions of these trends are nevertheless consistent with those of color-magnitude and color-size relations.

MOND can be quite sensitive to the parameters we have held fixed, a_0 , D , and i . It has not been necessary to adjust any of these parameters to get reasonable results from the global approach taken here. By global, we mean that we use only the asymptotic flat circular velocity V_c to determine \mathcal{M} . In Paper III we present fits to the full shape of $V(R)$. This is a different exercise that gives slightly different results for the mass-to-light ratios of individual galaxies.

The precise distance to a galaxy is sometimes important because of the different D -dependence of gas mass and total MOND mass. Total mass $\mathcal{M} \propto V_c^4$ is measured independent of distance (for small redshifts), but $\mathcal{M}_g \propto D^2$. An overestimate of the distance can lead to a situation in which it appears that the gas mass exceeds the total dynamical mass. For a sample with many gas rich galaxies, it would be surprising if this did not occasionally manifest itself. Indeed, it is well known in the case of DDO 154 (Milgrom & Braun 1988; Begeman et al. 1991; Sanders 1996). Examination of Table 1 shows that F571-V1 and F574-1 might be similar

TABLE 1
MOND Υ_*

Galaxy	Υ_*	ξ
F563-1	2.6	0.8
F563-V2	1.4	1.6
F568-1	1.2	0.7
F568-3	1.8	1.0
F568-V1	3.7	1.3
F571-V1	-0.2 ^a	0.4
F574-1	0.01 ^b	0.6
F583-1	0.9	1.3
F583-4	0.2	0.5
UGC 128	1.0	0.5
UGC 6614	2.7	0.7
DDO 154	~0 ^c	1.2
DDO 168	0.8	0.9
NGC 55	0.3	1.3
NGC 247	2.2	1.2
NGC 300	1.6	1.2
NGC 801	1.2	1.1
NGC 1560	2.0	1.3
NGC 2403	1.5	2.4
NGC 2841	~7 ^c	6.1
NGC 2903	4.8	5.5
NGC 2998	0.8	2.3
NGC 3109	0.4	0.8
NGC 3198	2.5	2.6
NGC 5033	5.4	2.3
NGC 5533	4.1	1.8
NGC 5585	0.9	1.6
NGC 6503	1.7	2.3
NGC 6674	2.9	2.3
NGC 7331	2.6	3.5
UGC 2259	2.7	1.7
UGC 2885	1.6	1.8

^a ± 2.5 .

^b ± 1.4 .

^c Distance sensitive.

cases, as both are inferred to have unreasonably small or negative Υ_* . A 20% reduction in the assumed distance to these galaxies would have the effect $\Upsilon_*(F571-V1) \rightarrow 0.5$ and $\Upsilon_*(F574-1) \rightarrow 0.8$. Given the uncertainty in the distance scale, neither case provides a clear counterexample to MOND.

MOND is also very sensitive to the inclination since the observed line of sight velocity must be corrected by $\sin^{-4}(i)$. Inclination determinations can be difficult and are sometimes quite uncertain for LSB galaxies (McGaugh & Bothun 1994; de Blok, van der Hulst, & Bothun 1995; de Blok et al. 1996). In the case of F571-V1, Υ_* becomes positive if $i = 35 \rightarrow 34$. We certainly would not claim to be able to determine inclinations this accurately by information independent of MOND (Paper III).

In the case of F574-1, it is not obvious that the asymptotically flat part of the rotation curve has been reached (de Blok et al. 1996). If V_c is underestimated, the MOND mass will be too small. The stellar mass-to-light ratio becomes more reasonable ($\Upsilon_* > 0.5$) if $V_c = 91 \rightarrow 99 \text{ km s}^{-1}$, which is possible within the errors. It is also possible that the factor $\chi(R)$ we have assumed could be an underestimate in this case.

In sum, MOND yields reasonable values for the mass-to-light ratios of stellar populations. This is not a trivial result. Since $\Upsilon_* \propto V_c^4$, even small errors in V_c can make Υ_* seem incorrect. There are a few individual galaxies for which Υ_* may seem a bit unreasonable, but is this surprising given the nature of astronomical data?

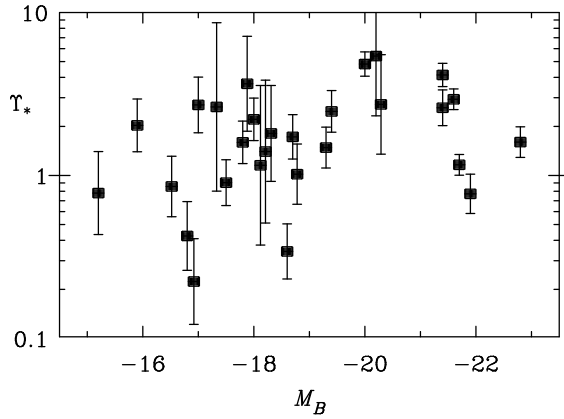


FIG. 1a

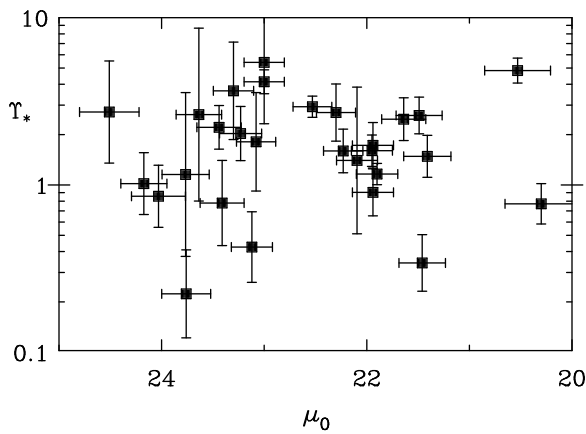


FIG. 1b

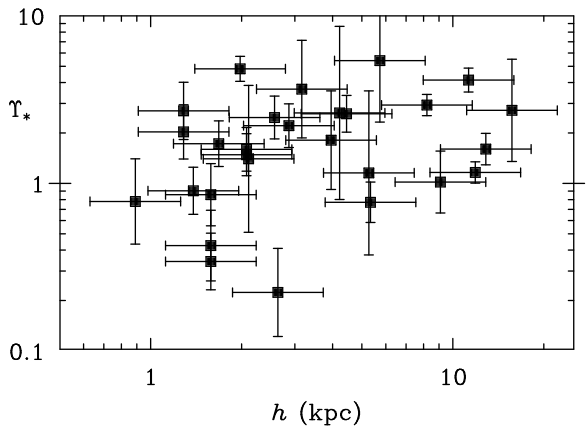


FIG. 1c

FIG. 1.—Stellar mass-to-light ratios of spiral galaxies determined from MOND, plotted against (a) absolute magnitude, (b) central surface brightness, and (c) scale length. The median value is $\Upsilon_* = 1.6$. This figure is analogous to Fig. 4 of Paper I. As was done there, error bars in Υ_* have been computed assuming a nominal inclination uncertainty of 3° . The errors are larger than those in Fig. 4 of Paper I because MOND masses are proportional to the fourth power of the velocity, and so have a factor \sin^{-4} (i) contributing to the error instead of just \sin^{-2} (i), as in the conventional case.

2.6. Mass Surface Densities

Since there is no dark matter in the MOND hypothesis, mass surface density must be correlated with luminous surface density. A way to see this without invoking stellar

populations is through the characteristic parameter ξ defined by Milgrom (1983b) for exponential disks,

$$\xi \equiv \frac{V_c^2}{a_0 h} = \sqrt{\frac{G\mathcal{M}}{a_0 h^2}}. \quad (19)$$

This is proportional to the characteristic acceleration V_c^2/h of a disk and to the square root of its mass surface density ($\sigma \propto \mathcal{M}/h^2$).

By using the observed velocity and scale length, we can construct the quantity ξ , which provides a direct dynamical estimate of the mass surface density. Values computed for ξ are given in Table 1. The mass surface density indicated dynamically by ξ must be correlated with the mass surface density of ordinary matter, indicated by the central surface brightness μ_0 . These quantities are plotted against each other in Figure 2. There is a strong correlation, with regression coefficient $\mathcal{R} = 0.82$. The slope and normalization of the relation is consistent with that expected in MOND. From equation (19), one can derive

$$\log(\xi) = -\frac{1}{5}(\mu_0 - 27) + \frac{1}{2} \log\left(\frac{2\pi G}{a_0} \Upsilon_b\right), \quad (20)$$

assuming an exponential mass distribution with $\mathcal{M} = 2\pi\sigma_0 h^2$. A formal fit to the data gives a slope of -0.21 , consistent with the expected -0.2 . The normalization is

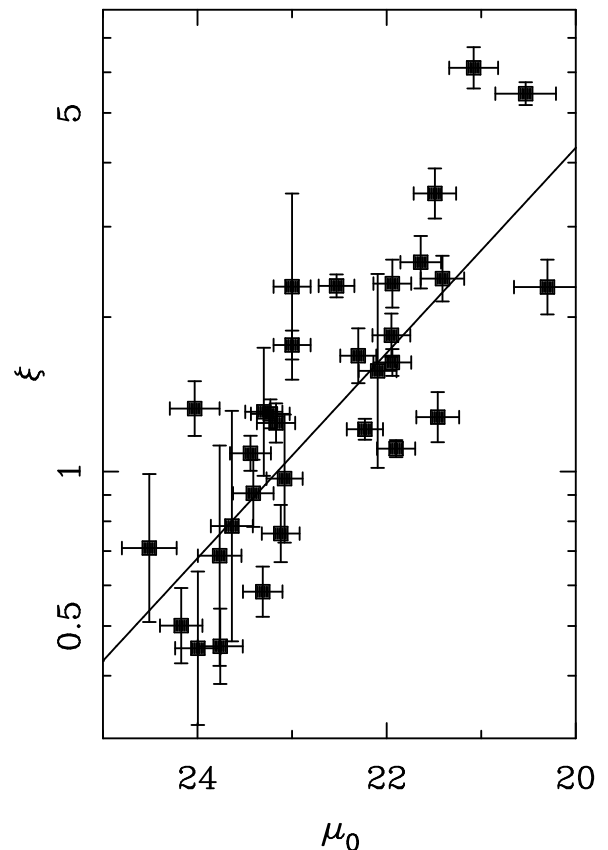


FIG. 2.—Mass surface density ξ indicated by MOND plotted against surface brightness. The two are strongly correlated, as expected in the absence of dark matter. The line is not a fit to the data. It illustrates the slope expected from the form of the MOND force law. The normalization is also predicted modulo the baryonic mass-to-light ratio Υ_b (eq. [20]). The line is drawn for $\Upsilon_b = 4$.

also correct for $\Upsilon_b \approx 4$, a reasonable value given the assumption of an exponential mass distribution. Deviations from this idealized case will alter the factor 2π in equation (20) and hence the precise value of Υ_b that is inferred.

2.7. Conventional Quantities

The effects of MOND, when interpreted in terms of conventional dynamics, lead to specific predictions for the behavior of conventional quantities. For example, the Newtonian mass-to-light ratio in any given galaxy should show no mass discrepancy until beyond the transition radius defined by equation (8). After this point, $\Upsilon_o(R)$ should increase as R increases. As one examines galaxies of decreasing central surface brightness, two effects should be apparent: the severity of the discrepancy should grow, and the radius (measured in scale lengths) at which the discrepancy sets in should shrink. If at every point in a galaxy $a(R) \ll a_o$, the mass discrepancy should be apparent at essentially $R = 0$ (Milgrom 1983b).

In Figure 3 we plot Υ_o against R/h for all available galaxies (Tables 1 and 2 of Paper I). The mass-to-light ratio is computed as the accumulated mass within radius R divided by the accumulated luminosity at the same point. Mass is assumed to be distributed spherically [$\mathcal{M} \propto RV^2(R)$], which is an adequate approximation for present purposes if not strictly true at small radii. We have subdivided the data in Figure 3 into bins of 1 mag in surface brightness. The rise of Υ_o does appear to become more pronounced as μ_0 declines. There is no clearly defined transition radius, but all disk galaxies have $a \lesssim a_o$. This may be a requirement for disk stability (see § 3.3).

What really matters is the acceleration. Another way of stating the prediction of MOND is that $\langle a \rangle$ should decrease with surface density (eq. [19]). In Figure 4 we plot the rotation curves as the centripetal acceleration required

to produce them, $a = V^2(R)/R$. There is some scatter induced by inclination uncertainties and intrinsic scatter in Υ_* . Nevertheless, there is a clear trend for $\langle a \rangle$ to decline with surface brightness, as expected in MOND.

2.8. Rotation Curve Shapes

The shapes of the rotation curves of exponential disks is controlled in MOND by the parameter ξ , which is closely related to the surface brightness. Milgrom (1983b) predicts “a correlation between the value of the average surface density (or brightness) of a galaxy and the steepness with which the rotational velocity rises to its asymptotic value. Small surface densities imply slow rise of V .” We noted in Paper I the slow rate of rise of the rotation curves of LSB galaxies (fact 2). Our empirical statement is essentially identical to the prediction of Milgrom (1983b). As a measure of this effect, Milgrom (1983b) suggested the radius (in scale lengths) at which $V(R/h) = V_c/2$. This contains the same information as $R_{3,4}$ in Paper I, which should thus be correlated with surface brightness. That is, Figure 12b of Paper I is in fact a prediction of MOND.

We illustrate this further by plotting the rotation curves logarithmically to show their shape (Fig. 5), binned by surface brightness as before. The rotation curves of the highest surface brightness galaxies rise rapidly, often reaching the asymptotic velocity within 1 scale length. The rate of rise becomes slower as surface brightness decreases. The asymptotic flat velocity is reached only at 2 or more scale lengths in the lowest surface brightness galaxies, as expected in MOND.

Another systematic of rotation curves is that the rate of rise of $V(R/h)$ is well correlated with absolute magnitude (Fig. 12a of Paper I). That this occurs is not predicted or required by MOND. Hence it neither supports nor contradicts MOND.

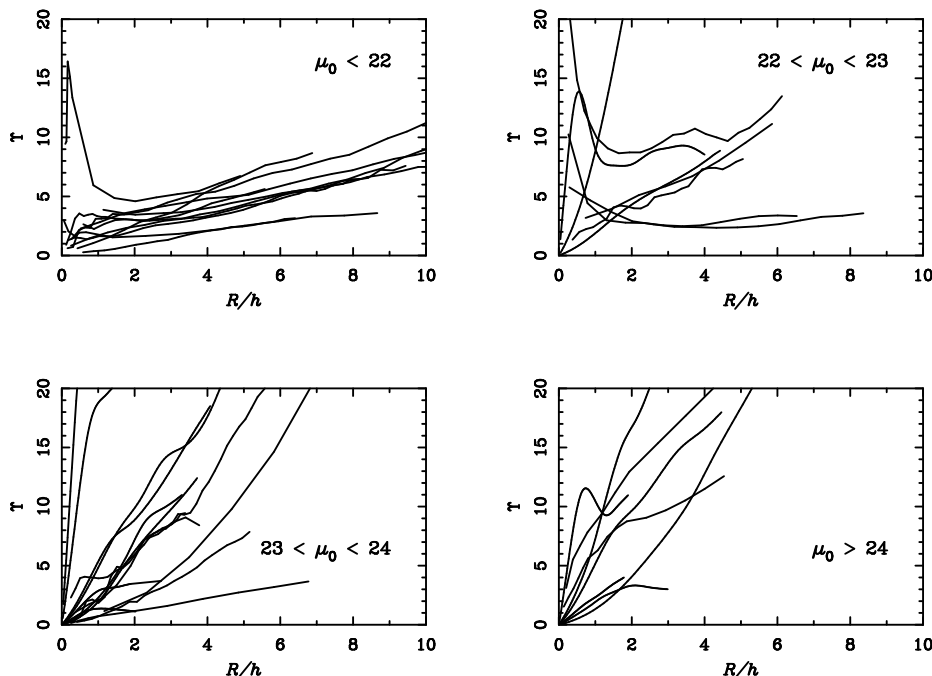


FIG. 3.—Accumulated conventional dynamical mass-to-light ratio ($\Upsilon_o \propto V^2 R/L$) as a function of radius. The plot is divided into bins of different central surface brightness as indicated in each panel. All available data (Tables 1 and 2 of Paper I) have been used. In MOND, one expects the mass discrepancy indicated by Υ_o to be larger and to set in at smaller radii in galaxies of lower surface brightness.

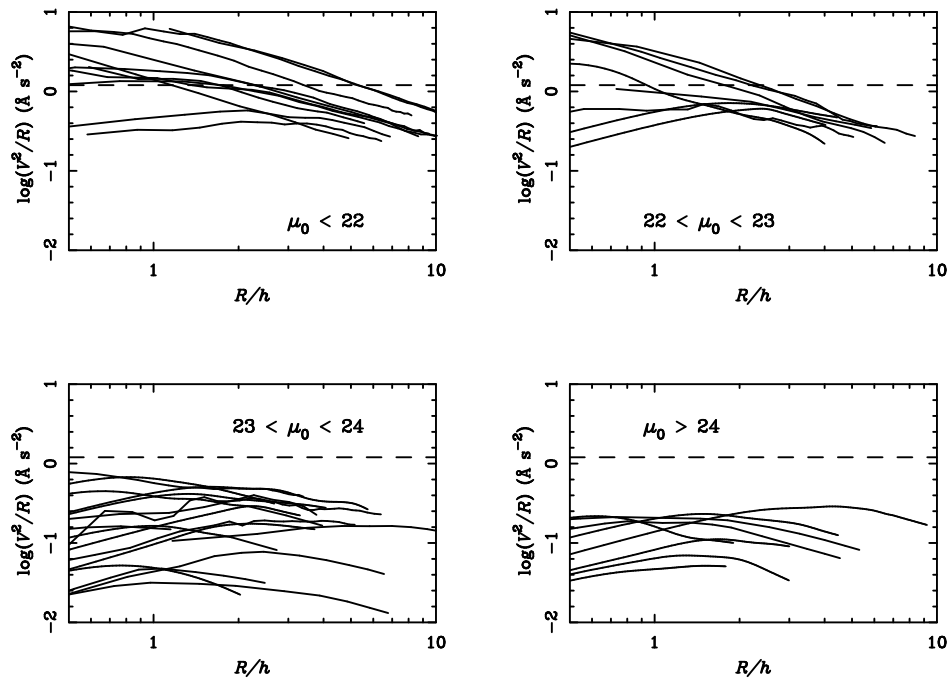


FIG. 4.—Rotation curves plotted in terms of the requisite centripetal acceleration $a = V^2(R)/R$. The plot is divided into different bins in central surface brightness, and the value of a_0 is marked by the dashed line. In MOND, one expects the acceleration to decline with surface brightness.

Though the rotation curve shapes are consistent with MOND, the predictions discussed in this section are less strong than previous ones. The systematic trend of $R_{3.4}/h$ with ξ is predicted to be fairly weak, and the presence of a bulge component can have an additional effect on the shape of $V(R)$ not accounted for by the quantity ξ defined for exponential disks. Though bulges are generally anemic in LSB galaxies, they are present in the higher surface bright-

ness galaxies in Figure 5. The question then becomes whether MOND can fit the rotation curves in detail given the actual observed luminous mass distribution and not just the exponential disk approximation.

2.9. Residuals of Rotation Curve Fits

MOND is known to work well in fitting rotation curves, particularly in HSB galaxies (Begeman et al. 1991; Sanders

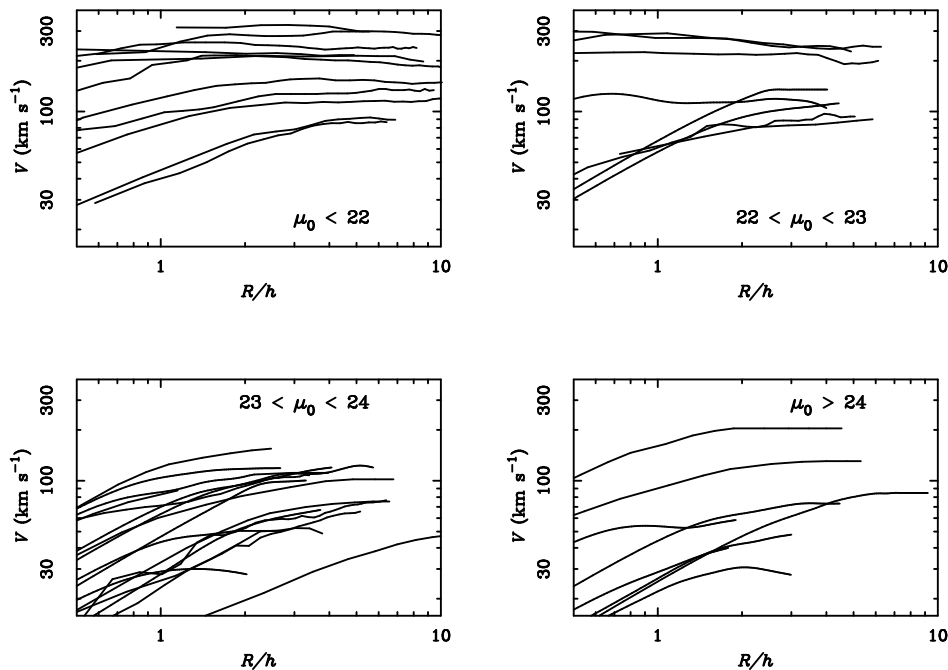


FIG. 5.—Rotation curves plotted logarithmically to illustrate their shapes. This test is not inclination dependent. In MOND, one expects high surface brightness galaxies to have rapidly rising rotation curves that fall gradually to the asymptotically flat value. Low surface brightness galaxies should have rotation curves that rise slowly to the asymptotically flat value.

1996). Except for a few dwarfs in those samples, $V(R)$ reaches the asymptotic flat velocity very rapidly. This is not true in LSB galaxies; MOND must give not only an asymptotically flat rotation curve but also the gradual observed rise. It was not contrived to do this, so detailed fits to LSB galaxy rotation curves provide strong tests.

An example of a detailed MOND fit to an LSB galaxy is given in Figure 6. Fits for the entire sample are the subject of Paper III. We make use of those results here with only a few comments. The prediction of $V(R)$ follows with one fittable parameter, Υ_* . The freedom in adjusting Υ_* is very limited. It does not have a strong effect on the fit ($V \propto \Upsilon_*^{1/4}$), and it must return a reasonable value for a stellar population. The shape of the stellar mass distribution is constrained to be the same as that of the light; Υ_* acts only as a normalization factor and is not allowed to vary with radius. The gas mass is comparable to the stellar mass in these galaxies. There is very little uncertainty in the conversion factor from 21 cm flux to H I mass, and we have no freedom to adjust it. The importance of the gas component often constrains Υ_* so tightly that in many cases the “fits” are effectively parameter free. For nine LSB galaxies, good fits were found immediately with one-parameter (Υ_* only) fits. For six others, achieving tolerable fits required adjustment of the inclination (Paper III). MOND is very sensitive to this since it enters through $\sin^{-4}(i)$ and a number of the LSB galaxies in our sample are fairly face-on. Two-parameter (Υ_* , i) MOND fits remain tightly constrained, and both parameters are subject to independent checks. In contrast, dark matter fits require at least three parameters (Υ_* and two halo parameters). The two halo parameters are not constrained by any data independent of the rotation curves, leading to notorious degeneracies (Athanasoula, Bosma, & Papaioannou 1987; Kent 1987; de Blok & McGaugh 1997).

The residuals to the MOND fits to the rotation curves of LSB galaxies from Paper III are shown in Figure 7 as a function of the critical parameter $x = a/a_0$. The data extend over an order of magnitude range, all with $x < 1$ and most with $x < \frac{1}{2}$. (See Sanders 1996 for fits to galaxies with $x > 1$ as well as $x < 1$.) Each point is a measured point from a rotation curve; the residuals of all LSB galaxy fits are

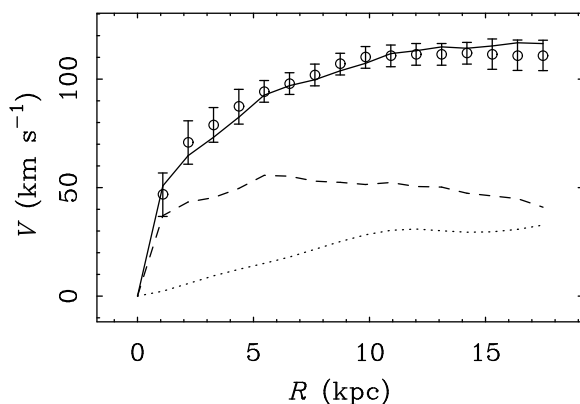


FIG. 6.—Example of the MOND fits to the rotation curves of low surface brightness galaxies from Paper III. The data points are for the galaxy F563-1. The dashed line shows the Newtonian rotation curve of the stellar disk, and the dotted line that of the gas. The solid line is the resulting MOND fit to the entire rotation curve. This follows directly when the MOND force law is applied to the observed luminous mass distribution.

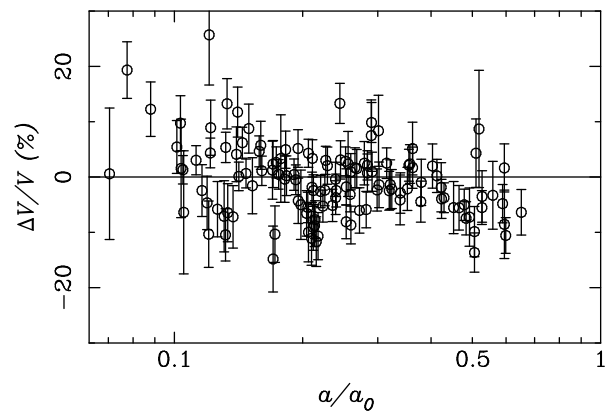


FIG. 7.—Residuals (in percent) of all the MOND fits to the rotation curves of LSB galaxies. The difference between the observed velocity and that predicted by MOND from the observed luminous mass distribution ($\Delta V = V_{\text{obs}} - V_{\text{MOND}}$) is shown as a function of the critical parameter $x = a/a_0$, where in practice $a = V^2/R$. Each point represents one measured point from the rotation-curve fits described in Paper III. The residuals for all points from all LSB galaxies are shown together. This represents 15 new LSB galaxy fits with nearly 100 total independent measured points.

plotted together. The data closely follow the line of zero residual with $\sim 10\%$ scatter, about what is expected for our errors. In absolute terms, the mean deviation $\Delta V = V_{\text{obs}} - V_{\text{MOND}} = 0.6 \pm 5.6 \text{ km s}^{-1}$.

The strongest test is provided by the points with $x < \frac{1}{2}$, where the form of the interpolation function $\mu(x)$ is insignificant. In this part of the diagram, not only is the mean residual small, but also there is no systematic trend of the residuals with x . For $x > \frac{1}{2}$, there is a hint of a systematic deviation. The difference is not highly significant, but it might indicate that the actual form of the interpolation function is slightly different than the one assumed. The explanation may be more prosaic, as some of the data are modestly affected by beam smearing (see de Blok & McGaugh 1997 for an extensive discussion).

There is one further consequence of Figure 7. Even though the mass discrepancy does not appear at a particular length scale, it does appear at a particular acceleration scale. MOND is the *effective* force law in disk galaxies.

3. OTHER EVIDENCE

The data for LSB galaxies are consistent with MOND. Indeed, each of the specific predictions Milgrom (1983b) made about LSB galaxies is confirmed. This means something. Is it possible that MOND, not dark matter, is the solution to the mass discrepancy problem?

There are many other systems besides disks in which the mass discrepancy is apparent. If MOND is correct, it must also be consistent with tests from these other systems. Tests are provided by any system with $\langle a \rangle \ll a_0$; in systems where $\langle a \rangle \gg a_0$, no mass discrepancy should be inferred unless there is genuine hidden mass. Here we review the evidence provided by other types of systems on different scales.

3.1. Dwarf Spheroidals

A class of galaxies that are low in surface brightness but quite distinct from the LSB galaxies we have so far dis-

cussed are the dwarf spheroidals (dSph) found in the Local Group. They are distinct for a number of reasons, most importantly in that they are three-dimensional systems supported by the velocity dispersion of their constituent stars, not rotating disks. As LSB systems, they should have low accelerations and provide a strong test of MOND: “Effects of the modified dynamics are predicted to be particularly strong in dwarf elliptical galaxies” (Milgrom 1983b).

That dSph galaxies have large mass discrepancies is clear (Mateo et al. 1993; Vogt et al. 1995; Mateo 1997). However, it is less clear that the discrepancies are of the correct magnitude for consistency with MOND. Gerhard & Spergel (1992) and Gerhard (1994) have applied this test. In some cases, MOND seems to do well, giving stellar mass-to-light ratios that are reasonable for the stellar populations of these galaxies. In other cases, the results are less satisfactory.

Dwarf spheroidals provide a strong test in the sense that the acceleration is low, comparable to LSB galaxies. However, they do not provide as clean a test. What one really needs to know is the three-dimensional velocity ellipsoid. All that is possible, of course, is to obtain a line-of-sight velocity dispersion ζ . Additional assumptions are therefore required to interpret the observations. We must assume that the MOND equivalent of the virial theorem applies (Milgrom 1984, 1995; Gerhard & Spergel 1992) and that each system is either spherical or isotropic.

In general, the mass of an isolated system in the MOND regime is

$$\mathcal{M} = \frac{9}{4} \frac{\langle V_{\text{rms}}^2 \rangle^2}{a_0 G} \quad (21)$$

(Milgrom 1995). The assumption of sphericity or isotropy is necessary to relate the rms velocity to the observable line-of-sight velocity $\langle V_{\text{rms}}^2 \rangle = 3\zeta^2$. Thus the effective equation is

$$\mathcal{M} = \frac{81}{4} \frac{\zeta^4}{a_0 G}. \quad (22)$$

Note that the assumed relation between rms and line-of-sight velocity has a significant impact in the geometric constant relating mass to velocity. Since $\mathcal{M} \propto \zeta^4$, a 20% error in ζ will lead to a factor of 2 error in mass. This need not arise as an error. It might occur even with perfect observations simply because the true velocity ellipsoid may not have the form we are obliged to assume.

Gerhard & Spergel (1992) and Gerhard (1994) concluded that MOND fails because it yields unreasonable Υ_* for some dwarf spheroidals, especially Fornax and Ursa Minor. By reasonable, we mean $1 < \Upsilon_* < 6$ in the V -band. There is considerable uncertainty in the stellar content of these galaxies. Originally thought to be old, implying high Υ_* , at least some appear surprisingly young (see, e.g., Mateo et al. 1991; Smecker-Hane et al. 1994). The Υ_* of Gerhard (1994) are plotted in Figure 8. One can immediately see two things: there are no error estimates, and many of the values are in fact quite reasonable. So, do the unreasonable values falsify MOND, or are the errors simply very large in those cases?

Data of the sort required for this analysis have accumulated rapidly in the period during which these analyses were performed. As a result, rather better data exist now than at the time of these analyses. Milgrom (1995) has revisited this

issue. The results he obtained using the same method and equations as Gerhard & Spergel (1992) are also plotted in Figure 8. Within the errors, the results give reasonable values for Υ_* . The only thing that has really changed is that the data have improved—data that come from the same sources.

The data continue to improve, so we repeat this analysis using the compilation of data provided by Mateo (1997). Equation (22) above applies only for isolated spheres. When a system is subject to a dominant external field (e.g., that of the Milky Way), the MOND behavior is quasi-Newtonian,

$$\mathcal{M} = 2G_{\text{eff}}^{-1} \zeta^2 R_V, \quad (23)$$

with a modified effective constant of gravitation, $G_{\text{eff}} \rightarrow a_0 G / \langle a \rangle$. Here, R_V is the virial radius (eq. [10] of Gerhard & Spergel 1992). Which equation applies depends on the relative accelerations imposed by the internal and external fields:

$$\eta = \frac{3\zeta^2/2R_C}{V_{\text{MW}}^2/D}, \quad (24)$$

where R_C is the core radius of the King profile that fits the light distribution of the dSph in question, D is its Galactocentric distance, and V_{MW} is the asymptotic velocity of the Milky Way. For the latter we adopt $V_{\text{MW}} = 220 \text{ km s}^{-1}$. If $\eta > 1$ the internal field dominates and the object can be treated as isolated (eq. [22]). If $\eta < 1$, the external field due to the Milky Way is dominant, and the behavior is quasi-Newtonian (eq. [23]). As can be seen in Table 2, most dSph galaxies for which good kinematic data exist are probably influenced by the external field of the Milky Way. When $\eta \approx 1$, neither of these limits are really appropriate. This is the case for many objects when the uncertainties in the many parameters determining η are considered. A significant external field can elongate the internal field and hence affect the assumed geometry, so there are significant uncertainties besetting this analysis regardless of the precision of the data. Only Leo I provides a test in a reasonably isolated system.

Where Mateo (1997) gives different values for ζ , we adopt the one based on the greater number of stars. In most cases, ζ has been measured only in the central regions. A global mean is more appropriate, so in the case of Fornax, for which a velocity dispersion profile is available, we use it to estimate $\zeta \approx 12 \text{ km s}^{-1}$. We do not compute Υ_* for two dwarfs for which velocity dispersions are listed, LGS 3 and Sagittarius. The velocity dispersion listed for LGS 3 gives $\Upsilon_* = 2.5$ but is based on only four stars, so the uncertainties render this meaningless. In the case of Sagittarius, the very close interaction with the Milky Way undermines all of the assumptions on which the analysis is based.

Results of our analysis of the data compiled by Mateo (1997) are listed in Table 2 and shown in Figure 8 together with the previous determinations of Gerhard (1994) and of Milgrom (1996). Our error bars are based only on the error in ζ given by Mateo (1997); they do not include the large uncertainties in the luminosity, surface brightness, and distance to these galaxies. Neither do we attempt to estimate any of the systematic uncertainties discussed above. Hence the error bars plotted in Figure 8 are a lower limit to the true range of uncertainty. Nevertheless, they do show that our determinations are consistent with previous ones, when the errors are considered.

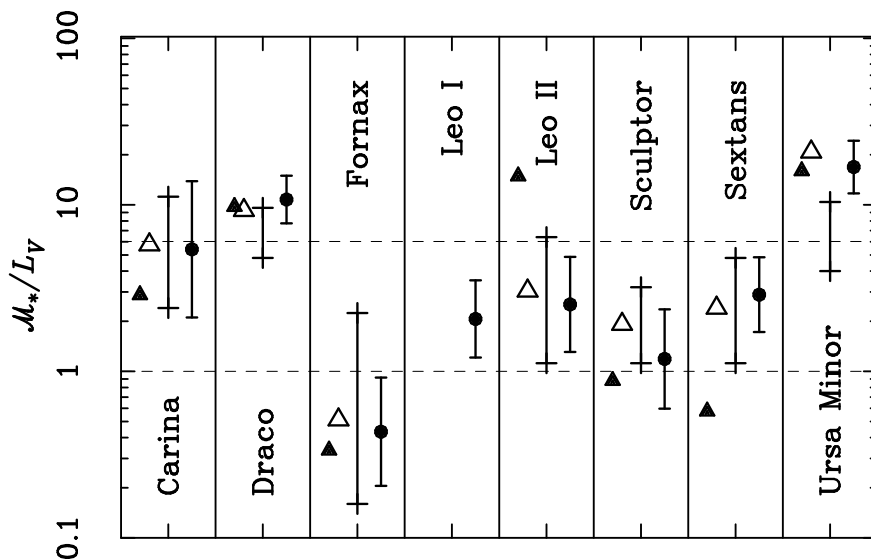


FIG. 8.—MOND mass-to-light ratios inferred for Local Group dwarf spheroidal galaxies. Triangles are the determinations of Gerhard (1994). Solid triangles are the values he finds for the isolated case appropriate if $\eta > 1$ (eq. [24]), and open triangles are his quasi-Newtonian values that are more appropriate when the external field is dominant ($\eta < 1$). Lines capped by crosses illustrate the range of allowable values determined by Milgrom (1995). The results of Gerhard and of Milgrom have been scaled to the value of a_0 adopted here, which increases their Υ_* by a factor of 1.6. Solid circles are our own determinations based on the data compiled by Mateo (1997). Dashed lines delimit the most plausible range, $1 < \Upsilon_* < 6$. Most galaxies fall in this range according to all three independent determinations. In only one case (Ursa Minor) is there a marginally significant deviation from the most plausible range.

Most dwarf spheroidals do in fact lie in the range $1 < \Upsilon_* < 6$, according to all three independent analyses. Marginal cases are Draco and Ursa Minor (Υ_* too high) and Fornax (Υ_* too low). The deviation of Draco and Fornax from the reasonable range is marginal ($\sim 1 \sigma$). Draco has η very near to unity, which will tend to cause a modest overestimate of Υ_* (Milgrom 1995). Ursa Minor is the most problematic case, but even there the deviation is only $\sim 2 \sigma$. There is a complication in this case in that some of the velocity dispersion may result from rotation (Mateo 1997). This can affect the analysis since rotational velocity enters the calculation with less weight than does velocity dispersion. Also, Mateo (1997) lists two somewhat different velocity dispersions for this object, both based on a sizable number of stars. Armandroff, Olszewski, & Prior (1995) give $\zeta = 8.8 \pm 0.8 \text{ km s}^{-1}$ while Hargreaves et al. (1994) give $\zeta = 6.7 \pm 1 \text{ km s}^{-1}$. Using the latter value gives $\Upsilon_*(\text{Ursa Minor}) = 10^{+6}_{-4}$, not significantly unreasonable.

Another complication is the systematic uncertainty in the parameters of the Milky Way. Since η is close to unity in most cases, a difference in the assumed strength of the exter-

nal field of the Milky Way might make a difference. It has been suggested that V_{MW} could be as low as 180 km s^{-1} (Olling & Merrifield 1998). If we redo the analysis with this value, little changes. In most cases there is a slight shift toward lower Υ_* . The exception is Ursa Minor, where it makes a substantial difference. For the higher velocity dispersion of Armandroff et al. (1995), $\Upsilon_* = 17 \rightarrow 10$. In principle, uncertainties in the distance to individual dwarfs can have similar effects.

Gerhard (1994) stresses that Υ_* varies by a factor of 20 between Fornax and Ursa Minor. It is true that this is a lot, but it is also true that these are the two most extreme points on either side of a sensible mean value. This is bound to happen at some level, and the degree to which it occurs here does not seem outrageous given the uncertainty in the data and the analysis.

In sum, there is no evidence that clearly contradicts MOND in the data for dSph galaxies. Agreement between these data and MOND has generally improved as the data have improved. The qualitative prediction of MOND, that dSph galaxies should have high mass discrepancies, is not in dispute. MOND also appears to work quantitatively.

TABLE 2

DWARF SPHEROIDALS

Galaxy	Υ_*	η
Carina	5.4	0.5
Draco	10.8	0.9
Fornax	0.4	0.6
Leo I	2.1	3.8
Leo II	2.5	1.4
Sculptor	1.2	0.6
Sextans	2.9	0.3
Ursa Minor	16.9 ^a	0.5

^a $\Upsilon_* = 10^{+6}_{-4}$ for an alternative measurement of ζ .

3.2. Giant Elliptical Galaxies

The kinematic data that exist for giant elliptical galaxies is rather limited. This is because observations of these systems are generally restricted to absorption lines that can typically be measured only within 1 half-light radius (R_e), where the surface brightness is high (see, e.g., Bertin et al. 1994). Application of these data as a test of any hypothesized force law is limited by the limited range of radii probed. It is further hindered by the fact that ellipticals are quite complicated systems. The brightest ellipticals are predominantly pressure supported but often have significant rotation as well (Davies et al. 1983). Even in the absence of rotation, anisotropies in the velocity ellipsoid must exist to

give rise to the observed range of shapes of ellipticals. This is quite problematic for applying equation (22); we simply do not have adequate knowledge of the geometry of the orbits.

For testing MOND, an even more severe constraint applies. The typical acceleration at radii for which measurements are available (Bertin et al. 1994) is

$$a \approx \frac{3}{2} \frac{\zeta^2}{R_e} \gtrsim 3a_0. \quad (25)$$

Extant data do not probe the MOND regime. Giant ellipticals therefore provide no direct test of MOND. They do provide several indirect tests, however. One is that there should be no apparent mass discrepancy where $a > a_0$. This is consistent with the persistent lack of a clear need for dark matter in elliptical galaxies (van der Marel 1991; Bertin et al. 1994).

Though the data may not probe the MOND regime, MOND effects must matter at some level in elliptical galaxies. One might expect some critical phenomenon associated with the MOND scale a_0 . In the simple case of spherical galaxies in the MOND limit, Milgrom (1983b) expects a Faber-Jackson (Faber & Jackson 1976) relation of the form $\mathcal{M} \propto \zeta^4$. If ellipticals are approximately isothermal, then Milgrom (1984) also expects a Fish (1964) law. MOND may therefore manifest itself in the regularity of the fundamental plane (Djorgovski & Davis 1987). The analog of the fundamental plane for disks is the Tully-Fisher relation. With μ_0 or R_e as a third parameter, the “fundamental plane” of disks is perpendicular to the luminosity-velocity plane, resulting in a narrow Tully-Fisher relation for galaxies of all surface brightnesses. Similarly, the fundamental plane of ellipticals is viewed nearly edge-on in the luminosity-velocity dispersion plane. This suggests that the same effect is at work. That there is a modest tilt to the fundamental plane merely indicates some modest systematic trend of Υ_* or the degree of pressure support with luminosity.

A generalization of the fundamental plane by Burstein et al. (1997) finds that essentially all extragalactic systems form narrow structures in “ κ -space.” This is indicative of a universal scale like a_0 . The κ -structures exhibit little surface density dependence, the signature of the MOND force law.

Elliptical galaxies with shell systems provide an interesting probe to large radii. Hernquist & Quinn (1987) use the remarkably regular shell system NGC 3923 to argue that MOND predicts the wrong number of shells for the periods implied by the observed relative radii, and also that the slope of the radius-shell number relation is incorrect (their Fig. 2). The latter argument is based on normalization to the outermost shell ($N_{\text{shell}} = 1$). This is a little unfair, as only the shape is predicted. The predicted shape is approximately correct for $1 < N_{\text{shell}} < 17$ —it is the outer shell that is deviant. Unlike the majority of the shells in this system, this outermost shell is substantially offset from the major axis. Alignment with the major axis is central to the argument, which assumes the shell system was created by a simple radial phase-wrapping merger event (Quinn 1984). Any deviations from this simple idealized case complicate the interpretation.

The periodicity argument that Hernquist & Quinn (1987) make is more persuasive. For a phase-wrapped system, MOND should have formed more shells than they count.

This requires that phase wrapping is an adequate model and that nearly all shells have been detected. Hernquist & Quinn (1987) argue that it would be difficult to miss the additional shells that seem to be required by their MOND argument. However, Prieur (1988) did discover additional shells that had previously been missed.

Though a good argument, the period-number relation of Hernquist & Quinn (1987) does not constitute the direct test of the MOND force law. The argument is based on a very idealized realization of simple phase wrapping as the result of a minor merger. Prieur (1988) shows that NGC 3923 is not as clean a system as required by this picture. Indeed, Hernquist & Spergel (1992) suggest that such rippled systems can result from *major* mergers (as already suggested from observational evidence by Schweizer 1982 and McGaugh & Bothun 1990). They specifically cite NGC 3923 as a likely example. Even in the conventional context, they abandon the minor-merger phase-wrapping hypothesis, which is the basis of the argument against MOND.

Another interesting argument involving an elliptical galaxy is the apparent difference between the shape and orientation of the optical and X-ray isophotes in NGC 720 (Buote & Canizares 1994). These authors point out that in any modified theory of gravity, the isopotential surface presumably traced by the gas should not differ from that determined by the dominant stars. Any significant difference would impose a geometrical requirement for a dark mass component.

In other elliptical galaxies that Buote & Canizares (1994) examine, there is no apparent difference in this sense. However, in the case of NGC 720, the stars have an ellipticity $\epsilon_* = 0.4$ and the gas $\epsilon_g = 0.2$ – 0.3 with a difference in position angle of $30^\circ \pm 15^\circ$ (Buote & Canizares 1994, 1996). The X-ray isophotes twist and appear to become more pointy than the optical isophotes outside $\sim 1R_e$. Buote & Canizares (1994) argue that the potential due to the stars can only become rounder with increasing radius, so the pointy X-ray isophotes provide geometrical evidence in favor of dark matter.

This situation is rather puzzling even with dark matter, since the stars should still contribute substantially to the mass over the observed region. Even though the X-ray isophotes appear to become more elongated than those of the stars, Buote & Canizares (1997) derive a dark matter mass potential that is more round than either stars or gas. This is less easy to accomplish in MOND since there is no freedom to vary the shape and position angle of the dominant mass as there is with dark matter. Nevertheless, both dark matter and MOND imply potentials consistent with or slightly rounder than the isophotes of the stars and rounder than the X-ray isophotes. The interpretation of NGC 720 seems difficult with either dark matter or MOND if we accept the data at face value.

The basic argument of Buote & Canizares (1994) is based on the difference between the X-ray and optical isophotes. The X-ray isophotes are very ragged (see Fig. 2 of Buote & Canizares 1994 or Fig. 1 of Buote & Canizares 1996) and do not obviously provide a strong constraint. Paper III provides several examples of how misleading the shapes of ragged isophotes can be.

The geometrical argument of Buote & Canizares (1994) is valid in principle. Observing many more galaxies with considerably higher signal-to-noise ratios would prove interesting. At the present time, these and other data for giant

elliptical galaxies provide only weak and indirect tests of MOND. None of these data clearly contradict it.

3.3. Disk Stability

Another indication of a mass discrepancy is in the long-term existence of dynamically cold spiral disks. Purely Newtonian disks are subject to global instabilities that rapidly lead to their demise unless stabilized by dominant dark halos (Ostriker & Peebles 1973). What really matters here is the ratio of binding to kinetic energy; this can be achieved either with dark matter or by altering the force law. Milgrom (1989) showed analytically that MOND disks are somewhat more stable than purely Newtonian ones.

It turns out to be very difficult to adopt standard N -body codes to address this problem properly (J. C. Mihos, private communication). Brada (1996) developed an alternative approach based on the multigrid algorithm. This supports and extends the analytic conclusions reached by Milgrom (1989): MOND disks are more stable than purely Newtonian disks both locally and globally. The additional stability is fairly modest, roughly equivalent to that provided by a halo with $\mathcal{M}(R = 5h) \approx 3\mathcal{M}_{\text{disk}}$ (Brada 1996).

This leads to a further test. The stability properties predicted by dark matter and MOND diverge as surface brightness decreases. In the case of dark matter, the halo mass enclosed by the disk increases systematically with decreasing surface brightness (Fig. 4b of Paper I). Self-gravity in the dynamically cold disk is the driving force for bars and spiral arms but becomes progressively less important. At some point such features should be completely suppressed by the dominant hot halo. In contrast, the amount of additional stability provided by MOND depends only weakly on surface density and the self-gravity of the disk always matters.

Spiral features appear feeble in LSB galaxies but are clearly present (Schombert et al. 1992; McGaugh et al. 1995; de Blok et al. 1995; Impey et al. 1997). Brada (1996) predicts the growth rate of the $m = 2$ mode for both dark matter and MOND (his Fig. 11). Using the observed accelerations as a scale, LSB galaxies correspond to his mass models with $\mathcal{M} < 0.1$. Below this level, instabilities are almost completely suppressed in the dark matter case (see also Mihos, McGaugh, & de Blok 1997).

In order to generate spiral structure internally, the disk needs to be rather heavy (Athanasoula et al. 1987). In LSB disks, it is conceivable that the minimum disk mass required to generate spiral arms might exceed the maximum disk mass allowed by the rotation curve (see Quillen & Pickering 1997). This may provide a further test.

It may be that spiral arms do not have an origin internal to the dynamics of the disk. However, it is difficult to invoke an external trigger in LSB galaxies since they are quite isolated systems (Bothun et al. 1993; Mo, McGaugh, & Bothun 1994). If spiral structure has nothing to do with disk kinematics, then of course there is no test one way or the other.

The form of the surface brightness distribution in the field (McGaugh 1996; Fig. 8 of Paper I) may also hold a clue. Milgrom (1989) argued that the stability of disks would lead to a transition at the critical surface density $\sigma_0^* \approx a_0 G^{-1} = 880 \mathcal{M}_{\odot} \text{pc}^{-2}$. Above this, the typical acceleration of a disk will exceed a_0 and one expects purely Newtonian behavior. Since bare Newtonian disks are subject to self-destructive instabilities, they should not survive. Below the critical

surface density, MOND lends the extra stability discussed above and a disk may assume any $\sigma_0 < \sigma_0^*$.

There should therefore exist a sharp cutoff in the surface brightness distribution at $\Sigma_0^* = \sigma_0^*/\Upsilon_*$. Such a sharp cutoff is observed, though its precise position is rather uncertain (McGaugh 1996). For $\Upsilon_* = 2$, one expects $\mu_0^* = 20.4$, consistent with the bright edge of the surface brightness distribution determined by de Jong (1996).

The distinction between giant ellipticals and spirals may be dictated by the critical σ_0^* . Gas dissipation tends naturally to form disk systems. However, these are only stable if $\sigma_0 < \sigma_0^*$. An elliptical galaxy may result when, whether through initial conditions or subsequent mergers, a system exceeds the critical surface density. It will not be stable as a disk, but since there is no objection to three-dimensional pressure-supported Newtonian systems, an elliptical galaxy seems a natural result. The collapse of rotational support presumably has drastic consequences for the gaseous content of such a system, so one might expect a starburst to consume the gas roughly coeval with the instability event. This would leave the signature of a population dating to said event, so that at present elliptical galaxies would appear gas poor and old.

This same process could also occur with conventional dynamics and dark matter halos. Should self-gravity become sufficiently great in high surface density disks, the halos will no longer suffice to stabilize them. However, the dark matter picture offers no reason why this should happen at the particular scale $a_0 G^{-1}$ natural to MOND.

The thickness and velocity dispersion of disks may provide a further test. In the Newtonian case, the dark mass is arranged in a halo, so the thickness of the disk is determined solely by the mass of the stars and the usual conventional dynamical equations. For a vertical density distribution

$$\rho(z) = \rho(0) \text{sech}^2\left(\frac{z}{z_0}\right) \quad (26)$$

(Spitzer 1942),

$$z_0 = \frac{\zeta_z^2}{\pi G \sigma}. \quad (27)$$

This would lead one to expect the disks of LSB galaxies either to be relatively thick or to have very low vertical velocity dispersions since the low surface mass density disks have little self-gravity. In contrast, MOND increases the binding force over the Newtonian prediction in a way that increases with decreasing surface density. The modified version of the expression for disk thickness (Milgrom 1983b) becomes

$$z_0 = \mu(x) \frac{\zeta_z^2}{\pi G \sigma}, \quad (28)$$

where $\mu(x)$ is the interpolation function of MOND. Disks in the far MOND regime should be thinner than the Newtonian equivalent by the factor $\mu(x) \approx x = a/a_0$ for a given velocity dispersion. For a given disk thickness, MOND disks can support a velocity dispersion a factor of $[\mu(x)]^{1/2}$ higher.

This could provide a strong test. Low surface brightness disks appear to be quite thin (Dalcanton & Schectman

1996; Kudrya et al. 1994). If they are Newtonian they must have quite small velocity dispersions. MOND disks of the same thickness would have distinctly higher ζ_z .

Moreover, there comes a surface density where Newtonian disks cease to be disks at all. To illustrate this, consider the disk thickness resulting from equations (27) and (28) with velocity dispersions plausible for the central regions of low surface brightness disks. The only difference between these two equations is the MOND interpolation function $\mu(x)$. For plotting convenience we adopt $\mu(x) = x/(1+x^2)^{1/2}$ (Milgrom 1983a). The interesting effects occur in the asymptotic regime $\mu(x) \approx x$, where the assumed form of $\mu(x)$ is irrelevant. Assuming the central regions of the disk can be approximated as a plane-parallel slab where $V^2/R \ll \zeta_z^2/z_0$ so that the vertical restoring force dominates the acceleration allows us to approximate x as $x \approx (\sigma_0/\sigma_0^*)^{1/2}$. This is useful for illustrating the dependence of the disk thickness on the central surface brightnesses and velocity dispersion without stipulating a specific form of $V(R)$. In general, the precise MOND prediction depends on the total acceleration, and radial, tangential, and vertical components can all contribute. The approximation made here using only the vertical component is adequate for illustrating the relevant effects near the centers of disks. The most interesting effect is the behavior of purely Newtonian disks of very low surface mass density.

The disk thickness is plotted as a function of disk central mass surface density in Figure 9 for plausible values of the velocity dispersion and disk scale length. Note that as the central mass surface density decreases, there comes a point (which also depends on ζ_z) at which Newtonian disks rapidly become intolerably thick. Unless ζ_z is quite low for very LSB disks, these objects should not be disks at all. Disks remain reasonably thin in MOND because the restoring force of the disk is larger.

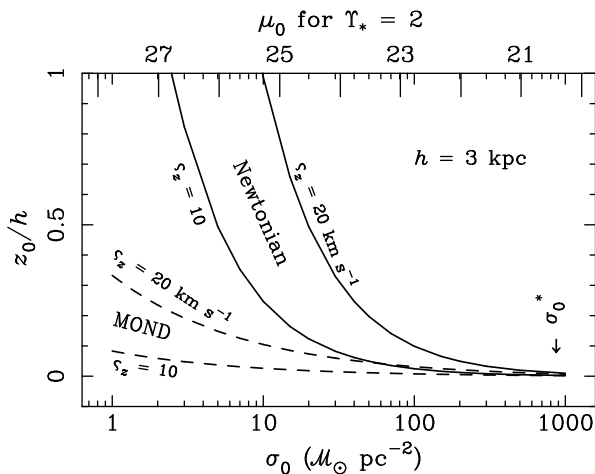


FIG. 9.—Thickness z_0/h expected for disks of various central surface densities σ_0 . Shown along the top axis is the equivalent B -band central surface brightness μ_0 for $Y_* = 2$. Parameters chosen for illustration are noted in the figure (a typical scale length h and two choices of central vertical velocity dispersion ζ_z). Other plausible values give similar results (eqs. [27] and [28]). The solid lines are the Newtonian expectation and the dashed lines that of MOND. The only difference between the two cases is the MOND interpolation function $\mu(x) = x/(1+x^2)^{1/2}$ with $x = (\sigma_0/\sigma_0^*)^{1/2}$. The Newtonian and MOND cases are similar at high surface densities but differ enormously at low surface densities. Newtonian disks become unacceptably thick unless LSB disks are very cold ($\zeta_z < 10 \text{ km s}^{-1}$). In contrast, MOND disks remain reasonably thin to quite low surface density.

Consider the actual numbers for the case illustrated in Figure 9. A normal sized ($h = 3 \text{ kpc}$) disk with a central velocity dispersion $\zeta_z = 20 \text{ km s}^{-1}$ is respectably thin in both Newtonian and MOND cases for $\sigma_0 > 80 \mathcal{M}_\odot \text{ pc}^{-2}$ ($\mu_0 \approx 23$ for $Y_* = 2$). Below this surface density, the two predictions diverge. MOND disks remain credibly thin ($z_0/h < \frac{1}{4}$) down to $\sigma_0 \approx 2 \mathcal{M}_\odot \text{ pc}^{-2}$ ($\mu_0 \approx 27$). In the Newtonian case, $z_0/h = \frac{1}{4}$ occurs at $\sigma_0 \approx 30 \mathcal{M}_\odot \text{ pc}^{-2}$ ($\mu_0 \approx 24$) and by $\sigma_0 = 10 \mathcal{M}_\odot \text{ pc}^{-2}$ ($\mu_0 \approx 25$) the object ceases to be a disk at all, with $z_0 \approx h$. In order to have such low surface brightness disks (which do exist), the central velocity dispersion must be very low; $\zeta_z < 10 \text{ km s}^{-1}$ is required to postpone the Newtonian divergence to the regime $\mu_0 > 25$ not yet observed. Global stability is not the only problem for purely Newtonian disks; it is also difficult to explain the existence of cold, thin disks of low surface brightness with a purely Newtonian force law.

Unfortunately, no stellar velocity dispersion data exist for LSB galaxies. This would be an extremely difficult observation but would provide a powerful test. Velocity dispersion data do reach to large radii in some high surface brightness disks (Olling 1996a). If interpreted in terms of dark matter, the MOND signature in such data will be a requirement either for a large amount of disk dark matter or a flattened halo. Data that reach far enough to imply a large mass discrepancy should make it necessary to put a lot of dark mass in a distribution close to that of the disk (see, e.g., Olling 1996b). In very LSB disks, there may come a point where the dark matter required to bring this about would exceed that allowed by the maximum disk solution.

A related observation is the Oort discrepancy in the Milky Way. Kuijken & Gilmore (1989) test MOND in this context and find that it tends to overcorrect somewhat. However, they used a value of a_0 nearly 4 times larger than currently measured. This will cause a mass discrepancy to be implied by the MOND equations before it actually should be, leading to an apparent overcorrection.

3.4. Clusters of Galaxies

Perhaps the strongest observational argument against MOND at the time of its introduction was that while it worked in galaxies, it failed in clusters of galaxies. The apparent mass discrepancies $\mathcal{M}_N/\mathcal{M}_* \gtrsim 100$ were too large to be explained by the typical accelerations, $a_0/\langle a \rangle \sim 10$. Even with MOND, substantial amounts of dark matter seemed to be required, an unacceptable situation.

The basic picture that held for many years is that the amount of dark matter increased with increasing scale. Mass discrepancies in galaxies were a factor of 10, those in clusters were factors of hundreds, and factors of even more were required to close the universe. This picture of ever more dark matter on ever larger scales has changed dramatically in recent years. X-ray observations of rich clusters of galaxies have shown that much or even most of the baryonic matter is in the form of hot gas (see, e.g., David et al. 1990; White & Fabian 1995). Rather than having more dark matter than individual galaxies, clusters are in fact more baryon dominated (Paper I). There were in effect two missing mass problems in clusters: the usual dynamical mass discrepancy and the fact that many of the baryons are in the form of hot gas rather than stars.

The need for an additional mass component that seemed so unacceptable for MOND would now appear to be a successful prediction. It is thus important to reanalyze

MOND in light of this new knowledge. The & White (1988) did this for the case of the Coma cluster, and found that MOND still seemed to be off by a factor of ~ 2 . Recall, however, the many uncertainties associated with tests in quasi-spherical systems discussed in the case of dwarf spheroidals. Given the uncertainty in the underlying assumptions of sphericity and virial equilibrium upon which the analysis is based, is a factor of 2 a problem or a success?

Andernach & Tago (1998) have compiled a good deal of cluster redshift data and quote a median cluster velocity dispersion of 695 km s^{-1} . For a spherical virialized system, this corresponds to a MOND mass of $3 \times 10^{14} M_{\odot}$ (eq. [22]). The typical hot gas mass of clusters in the compilation of White & Fabian (1995) is $\sim 10^{14} M_{\odot}$. So even a crude calculation comes pretty close, especially if there is a comparable amount of mass in stars.

Sanders (1994) readdressed the problem in greater detail. For 16 clusters, he shows that the MOND mass is strongly correlated with the X-ray gas mass. The MOND masses are a bit greater than the gas mass (his Fig. 3) indicating some additional mass in stars.

A more thorough investigation of this particular test would require a great deal more data than are available to us, such as combined X-ray and optical observations giving good estimates of the mass in each component and accurate velocity dispersions. Obtaining the full optical luminosity of a cluster is far from trivial (Impey, Bothun, & Malin 1988), and X-ray observations still lack the spatial resolution to guarantee the validity of the assumption of isothermality. Obtaining a reliable velocity dispersion from a sample of galaxies that are truly virialized is also challenging. Interlopers and substructure could play a strong role in distorting the mass-indicative velocity dispersion. A 40% overestimate of ζ would imply a MOND mass a factor of 4 too great. This would appear disastrous if this uncertainty is not considered and properly propagated. It is important to test MOND in clusters as adequate data become available, keeping in mind the many uncertainties.

3.5. Gravitational Lensing

Another indication of a mass discrepancy independent of kinematic data is gravitational lensing, both in individual galaxies (see, e.g., Kochanek 1995) and clusters (see, e.g., Tyson, Wenk, & Valdes 1990). Since there is not yet a relativistic extension of MOND, there is no clear prediction for gravitational lensing. That lensing occurs must ultimately be explained, but at present it provides no test of the validity of the MOND force law.

Some progress has nevertheless been made. If MOND is interpreted as an alteration of the law of inertia, it is fairly successful at explaining weak lensing in clusters. However, it does then predict that additional, as yet undetected baryonic mass resides in the cores of a few of the most X-ray rich clusters (Milgrom 1996). This is apparently the only place where MOND does not remedy the mass discrepancy problem in the sense that significant additional mass remains hidden. This is a very limited missing mass problem, restricted to the central Newtonian cores of some rich clusters. These are generally cooling flow clusters, so at least some additional baryonic mass is already inferred to reside there.

Gravitational lensing observations have allowed entire classes of theories to be ruled out as the possible basis of combining relativity and MOND (Bekenstein & Sanders

1994). This does not mean none can exist. One attempt at such a theory (Sanders 1997) has an interesting consequence. In this case, the mass distribution that would be inferred when interpreted in terms of dark matter is the same for both lensing and kinematical observations. Hence, observational agreement between such observations does not uniquely require a dark matter interpretation.

3.6. Large-Scale Structure

Observations of the motions of galaxies now extend over very large scales. These do not provide strong tests of MOND, since we must make crude assumptions about the mass distribution. We can, however, address the consistency of MOND with current observations on a qualitative level.

That there are large ($\gtrsim 30 \text{ Mpc}$) voids and filaments in the large-scale galaxy distribution is now regularly reproduced by simulations of structure formation. It is worth recalling that these features initially came as a great surprise (de Lapparent, Geller, & Huchra 1986): the universe was supposed to be homogeneous on these large scales. That these large, sharp features exist makes a reasonable amount of sense in the presence of a force law that is effectively enhanced on large scales, as does the occasional inference of an excessive mass concentration like the Great Attractor (Lynden-Bell et al. 1988).

Where dynamical measurements exist, these large scale structures can be used as crude tests of MOND. Milgrom (1997) presents an analysis of the Perseus-Pisces filament and derives $\Upsilon \sim 10$. This is fairly reasonable considering the crude assumptions that must be made (i.e., that the structure is a virialized linear feature) and that it may well contain substantial amounts of gas.

The expansion of voids could provide another test. As underdense regions, voids *must* expand in the conventional picture. This need not be the case in MOND, for which the effective potential is logarithmic on large scales. A spherical shell mass distribution will eventually turn around and collapse regardless of its interior density (Felten 1984), so one might expect to see some voids expanding and others contracting. In the one case where careful distance as well as redshift measurements have been made (Bothun et al. 1992), no expansion is detected ($\lesssim 5\%$ of the void diameter in km s^{-1}).

On the largest scales, MOND does require a universe composed entirely of baryons. It may therefore seem troubling that dynamical estimates of the mass density are persistently around $\Omega \approx 0.3$, much higher than the baryon density allowed by primordial nucleosynthesis, $0.01 \leq \Omega_b \leq 0.03$. These estimates of Ω are of course based on the usual Newtonian equations, and MOND will require much less mass. As with all mass discrepancies, the amount by which Ω is overestimated depends on the typical acceleration scale probed:

$$\Omega_{\text{MOND}} = \frac{\langle a \rangle}{a_0} \Omega . \quad (29)$$

It is difficult to estimate $\langle a \rangle$ here, but as an example we make the usual assumption of homogeneity so that the density field may be approximated as a constant. The Poisson equation with the usual $\Omega = 8\pi G\rho/3H_0^2$ then gives

$$\langle a \rangle \approx |\nabla\phi| = \frac{1}{2} H_0^2 \Omega R , \quad (30)$$

where R is the scale that the observations cover. Davis, Nusser, & Willick (1996) give a value $\Omega^{0.6b} = 0.6$ from data that extend reliably out to $R \approx 5000 \text{ km s}^{-1}$. There should be no mass biasing in MOND (though different populations of galaxies may be biased relative to one another), so we assume $b = 1$. This gives a conventional $\Omega = 0.43$. Taking these numbers at face value leads to $\langle a \rangle \approx 0.026 \text{ \AA s}^{-2}$ and

$$\Omega_{\text{MOND}} \approx 0.01 . \quad (31)$$

Given the nature of the data and the necessary assumptions, this is probably uncertain by at least a factor of a few. Nevertheless, it is striking that

$$\Omega_{\text{MOND}} \approx \Omega_b . \quad (32)$$

MOND appears to adequately address the dynamical mass discrepancy problem on even the largest scale.

4. DISCUSSION

We have taken care to review previous analyses that have found fault with MOND. There is no evidence on any scale that clearly contradicts MOND. Some data that are cited as contradicting MOND actually appear to support it (e.g., that for dwarf spheroidals). It should be noted that not all previous independent analyses of MOND have been negative. Begeman et al. (1991), Morishima & Saio (1995), and Sanders (1996) all report positive tests of the theory. The new data for LSB galaxies we have collected obey all the predictions made by Milgrom (1983b).

The observational tests of MOND we have discussed are summarized in Table 3. That LSB galaxies fall on the Tully-Fisher relation is a strong prediction of MOND (§ 2.3). The Υ - Σ conspiracy that occurs when interpreting the LSB galaxy Tully-Fisher relation with dark matter (Zwaan et al. 1995; Paper I) can be derived from MOND (§ 2.4). Stellar mass-to-light ratios that agree well with what is expected for stars can be computed directly from the observations with MOND (§ 2.5). The mass surface density implied by the kinematic measure V_c^2/h is strongly correlated with the disk surface brightness μ_0 with the slope expected in MOND (§ 2.6). The radial variation of the dynamical mass-to-light ratio computed conventionally behaves in a manner consistent with the predictions of Milgrom (1983b). Simi-

TABLE 3
TESTS OF PREDICTIONS

Observational Test	MOND
LSBG Tully-Fisher relation	Y
Υ - Σ relation	Y
Stellar mass-to-light ratios	Y
Mass surface densities	Y
Conventional $\Upsilon_*(R)$	Y
Transition radii	Y
Characteristic accelerations	Y
Rotation curve shapes	Y
Rotation curve rate of rise	Y
Rotation curve fits	Y
Disk stability	?
Dwarf spheroidal galaxies	Y
Giant elliptical galaxies	NT
Galaxy clusters	?
Gravitational lensing	NP
Large-scale structure	?
$\Omega = \Omega_b$?	?

NOTE.—“Y” = prediction confirmed; “N” = prediction falsified; “?” = remains uncertain; “NP” = no prediction; and “NT” = no test.

larly, the radii of transition to apparent dark matter domination and the typical accelerations observed in disks vary with surface brightness as expected in MOND (§ 2.7). The systematic dependence of the shape of rotation curves on surface brightness is also predicted by MOND (§ 2.8). Taken in sum, the data are well fit by MOND (§ 2.9; Paper III). Indeed, all of the empirical facts we identified in Paper I describing the systematic properties of the rotation curves of disks as a function of surface brightness were anticipated by Milgrom (1983b).

MOND also survives tests in systems other than disks. Dwarf spheroidal galaxies are an important example. Once thought to fail there (Gerhard & Spergel 1992), MOND now appears to do well with improved data (§ 3.1). On the other hand, giant elliptical galaxies provide no useful test of MOND since the accelerations they experience do not probe the MOND regime, at least not with current observations (§ 3.2). The stability properties of disks appear consistent with MOND, and the velocity dispersions of thin LSB disks could provide a very strong test (§ 3.3). Galaxy clusters provide another important test. Originally seeming to require additional dark matter in clusters (Milgrom 1983c), the detection of large amounts of hot X-ray emitting gas in clusters generally brings these into consistency with MOND (§ 3.4; Sanders 1994). A definitive prediction for gravitational lensing requires a relativistic generalization of MOND that does not yet exist (§ 3.5). Some types of theories can be ruled out on this basis (Bekenstein & Sanders 1994) while others remain possible (Sanders 1997). On the largest scales, MOND does as well as can be expected given the applicability of the available data (§ 3.6). A very low density ($\Omega \approx 0.01$), purely baryonic universe is roughly consistent with the dynamical data that constrain Ω .

Empirically, MOND is the effective force law in disk galaxies. It appears that this may also be the case in other systems. The reason for this phenomenology needs to be understood.

There data allow two possible interpretations:

1. MOND is correct.
2. Dark matter mimics the behavior of MOND, at least in disks.

The second possibility implies a unique and powerful coupling between dark and luminous matter. It is possible to write down an equation that directly links the dark matter-dominated dynamics to the detailed distribution of the luminous matter. This provides a new observational test of theories of disk galaxy formation within the standard dark matter paradigm. Since MOND always fits disk galaxy rotation curves, it must be possible to take the luminous mass distribution predicted for any given disk by a dark matter galaxy formation theory, apply the MOND procedure to the luminous mass only, and thereby obtain the correct rotation curve. If this cannot be done, the theory has failed to produce a realistic disk.

There are no clear empirical objections to the first possibility. Milgrom (1983b) did accurately predict numerous aspects of the kinematical properties of LSB galaxies. This seems unlikely to have occurred by accident, so the possibility that MOND is correct should be considered seriously.

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Note added in proof.—It has been pointed out to us that the globular cluster system of the elliptical galaxy NGC 720 shows no discrepancy in ellipticity or position angle with that of the stars (M. Kissler-Patig, T. Richtler, & M. Hilker, *A&A*, 308, 704 [1996]). Reanalysis of the X-ray data (B. Dirsch, diploma thesis, Sternwarte Bonn [1996]) indicates a position angle that is consistent with that of the stars as with that reported by Buote & Canizares (1994). It appears that considerably better X-ray data are required to place useful constraints on MOND.