

CONFRONTATION OF MODIFIED NEWTONIAN DYNAMICS PREDICTIONS WITH *WILKINSON MICROWAVE ANISOTROPY PROBE* FIRST YEAR DATA

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Received 2003 December 21; accepted 2004 April 14

ABSTRACT

I present a model devoid of nonbaryonic cold dark matter (CDM) that provides an acceptable fit to the *Wilkinson Microwave Anisotropy Probe* (*WMAP*) data for the power spectrum of temperature fluctuations in the cosmic background radiation. An a priori prediction of such no-CDM models was a first-to-second peak amplitude ratio $A_{1,2} \approx 2.4$. *WMAP* measures $A_{1,2} = 2.34 \pm 0.09$. The baryon content is the dominant factor in fixing this ratio; no-CDM models that are consistent with the *WMAP* data are also consistent with constraints on the baryon density from the primordial abundances of ^2H , ^4He , and ^7Li . However, in order to match the modest width of the acoustic peaks observed by *WMAP*, a substantial neutrino mass is implied: $m_\nu \approx 1$ eV. Even with such a heavy neutrino, structure is expected to form rapidly under the influence of modified Newtonian dynamics. Consequently, the epoch of reionization should occur earlier than is nominally expected in ΛCDM . This prediction is realized in the polarization signal measured by *WMAP*. An outstanding test is in the amplitude of the third acoustic peak. Experiments that probe high l appear to favor a third peak that is larger than predicted by the no-CDM model.

Subject headings: cosmic microwave background — cosmology: observations — dark matter — gravitation

Online material: color figures

1. INTRODUCTION

The past decade has seen a remarkable convergence toward a standard cosmological model, ΛCDM . A great variety of different and independent data seem to point to very nearly the same set of cosmological parameters. The high-precision measurement of the temperature fluctuations in the microwave background by the *Wilkinson Microwave Anisotropy Probe* (*WMAP*) satellite (Bennett et al. 2003) has given further impetus to this picture.

While ΛCDM does provide a fairly consistent cosmology, there do remain nagging problems, some of which are potentially quite serious. Perhaps the most embarrassing¹ aspect of our modern cosmology is the dominance of invisible components. Dark matter and dark energy comprise $\geq 95\%$ of the mass-energy content of the universe, yet we have only ideas about what they are.

In the context of cosmology, it is essential for the dark matter to be nonbaryonic (in order to satisfy $\Omega_m \gg \Omega_b$) and dynamically cold (in order to grow structure). This led to the development of the cold dark matter (CDM) paradigm, in conjunction with the suggestion from particle physics that a class of supersymmetric, weakly interacting massive particles might have the right properties to be the CDM. Our modern cosmology absolutely requires CDM, but as yet we have no compelling laboratory evidence that such particles actually exist.

The CDM hypothesis faces serious problems on the scale of individual galaxies (Sellwood & Kosowsky 2001). One widely debated problem is that dark matter halos should have steep central density cusps (e.g., Navarro et al. 1997) but appear not

to (e.g., de Blok et al. 2003; Swaters et al. 2003; for further references see McGaugh et al. 2003). While this is an important point, the cusp problem itself is only one aspect of the severe fine-tuning problems one encounters in trying to understand the systematic properties of the rotation curves of spiral galaxies (McGaugh & de Blok 1998a; McGaugh 2004).

The difficulty for CDM encountered in the dynamics of individual galaxies stems from the close coupling observed between baryonic and dark matter components. The distribution of the observed baryons is completely predictive of that of the dynamically dominant dark matter (Sancisi 2004; McGaugh 1999b, 2000a, 2004). One does not naturally expect, and finds difficult to impart (McGaugh & de Blok 1998a), such a close coupling between a small, dynamically cold, thin disk of baryons and a large, dynamically hot, quasi-spherical halo of nonbaryonic dark matter.

Into this troubling mix comes the surprising success of the modified Newtonian dynamics (MOND) proposed by Milgrom (1983). Long known to provide good fits to the rotation curves of high surface brightness spiral galaxies (e.g., Begeman et al. 1991; Sanders 1996), MOND also works in systems ranging from tiny dwarf spheroidal galaxies² (Sanders & McGaugh 2002) to giant elliptical galaxies (Milgrom & Sanders 2003). While there is a genuine puzzle in rich clusters of galaxies (Aguirre et al. 2001), it is not obvious that this problem is more serious than those faced by CDM. One should also bear in mind the surprising success of the very specific a priori predictions made for low surface brightness (LSB) galaxies by

¹ For those of us who found the inflationary solution to the coincidence problem so satisfying, the presently inferred acceleration of the expansion of the universe—which makes the coincidence problem worse than it had been in an open universe—is also rather embarrassing.

² Ursa Minor and Draco are often cited as problematic cases for MOND. This is true, but it also happens that for both of these cases, it is unclear (at $< 1 \sigma$) which MOND mass estimator is appropriate: both are very close to the dividing line between internal and external field domination. For the other eight cases with adequate data (Carina, Fornax, LGS3, Leo I, Leo II, Sagittarius, Sculptor, and Sextans) where the choice of mass estimator is not ambiguous, the results are consistent with MOND.

Milgrom (1983), which were realized in great detail (McGaugh & de Blok 1998b). All of the things that are so puzzling about the dynamics of these systems from a conventional point of view stem fundamentally from their adherence to the MOND force law. This should not happen if CDM is correct.

As stressed by McGaugh & de Blok (1998b), the dynamical data admit only two interpretations. Either MOND is correct (presumably as the appropriate limit of some more general theory), or the processes of galaxy formation somehow lead to MOND-like behavior. The latter case is discussed elsewhere (McGaugh 1999b, 2000a, 2004) and provides quite stringent empirical constraints on conventional galaxy formation theory. However, should MOND be correct, how else might we tell? The cosmic background radiation (CBR) provides a further opportunity to distinguish between the very different cases of a “preposterous” universe filled with dark matter and dark energy and an even more mysterious one in which the mass discrepancies observed in extragalactic systems are caused by a modification of dynamical laws.

MOND itself is not a generally covariant theory. This makes it impossible (not merely daunting!) to derive a proper cosmology and compute the power spectrum of the CBR. However, as is often the case in such situations, a simple *Ansatz* can be very illuminating.

McGaugh (1999c) made the *Ansatz* that the acceleration scale a_0 of MOND does not vary with time. This is a common assumption (Felten 1984; Sanders 1998) with the consequence that the early universe is not in the MOND regime. In this case, things are normal until sufficiently late times that the usual early-universe results (e.g., big bang nucleosynthesis [BBN]) are retained.

The point of MOND is to obviate the need for dark matter. If it is essentially correct, then CDM does not exist. This suggests a minimum difference approach: if MOND is active in the early universe, then presumably the effects on the CBR would be larger than what we would infer under the *Ansatz*. If MOND is not active at the time of recombination and there is no CDM, then the CBR power spectrum should have a distinctive shape. Without CDM, baryonic drag dominates, and one should see only the resultant damping, with each peak being lower in amplitude than the preceding one. If CDM does exist in the required mass density, it must provide some net forcing term for the acoustic oscillations that counteracts the baryonic drag: one should never see a pure damping spectrum in a Λ CDM universe.

Since we use only conventional physics under our *Ansatz*, MOND itself is not tested. Thus, we cannot exclude MOND on the basis of any CBR data simply for lack of a definite prediction. However, we can potentially exclude the existence of CDM. It is very important to have such a test, for once we have convinced ourselves of the need for an all-pervasive, invisible, and perhaps undetectable mass component, it is virtually impossible to falsify the notion should it happen to be wrong (Davis et al. 1992).

To this end, McGaugh (1999c) proposed the no-CDM model as a stand-in for MOND. This provides a minimal first approximation of what we might expect in MOND. Of course, it should fail at some level. We have known for a long time that a purely Newtonian, baryonic model fails—we must have either CDM or MOND. Even under the *Ansatz* that MOND is not important at the time of recombination, there can be late-time effects that modify the observed CBR. Given these considerations, McGaugh (1999c) also discussed some of the deviations from a pure no-CDM model that are likely to be caused

by MOND. For example, rapid structure formation is natural to MOND (Sanders 1998; McGaugh 1999a). This should lead to early reionization and consequently a large polarization signal. It may also enhance the integrated Sachs-Wolfe (ISW) effect. These are indicators of MOND-like physics beyond the simple no-CDM model.

The amplitudes of the peaks expected in the CBR power spectrum in the absence of CDM are discussed in § 2. These are tied to the baryon density, for which independent constraints from BBN are critical. A specific no-CDM model that fits the *WMAP* data is given, and its predictions for the third and subsequent peaks are checked against higher l data from other experiments. Features of the CBR power spectrum that might suggest MOND-like physics beyond the simple no-CDM model are discussed in § 3. Further predictions that could help to distinguish between CDM and MOND are discussed in § 4.

2. THE NO-CDM MODEL

2.1. The Amplitude of the Second Peak

The difference between a standard Λ CDM model and the MOND-inspired no-CDM model turns out to be rather subtle. The first difference is a modest one in the amplitude of the second peak relative to the first: the second peak should be somewhat smaller without CDM for the same baryon density. Quite high quality data are required to distinguish between CDM and MOND when only the first two peaks are observed.

McGaugh (1999c) suggested two robust measures. The first is the ratio of the absolute amplitude of the first to second peak:

$$A_{1:2} = \frac{A_1}{A_2} = \frac{C_{l,1}}{C_{l,2}}, \quad (1)$$

where $C_{l,N}$ is the peak amplitude of the N th peak. (A similar but inverted notation was adopted by Hu et al. 2001: their $H_2 = A_{1:2}^{-1}$.) The second measure is the peak amplitude ratio relative to the intervening trough ($C_{l,t}$):

$$R_{1:2} = \frac{R_1}{R_2} = \frac{C_{l,1} - C_{l,t}}{C_{l,2} - C_{l,t}}. \quad (2)$$

Though both ratios $A_{1:2}$ and $R_{1:2}$ contain much the same information, the peak amplitude ratio relative to the intervening trough ($R_{1:2}$) gives a stronger segregation between the model predictions. However, it is sensitive to uncertainties in the trough amplitude, which is harder to measure than the peaks.

The definition of these ratios is illustrated in Figure 1 with the no-CDM models of McGaugh (1999c) and the *WMAP* peak location estimates of Page et al. (2003). Note that the no-CDM models are consistent with the *WMAP* peak amplitude estimates, having very similar peak ratios for plausible baryon densities. Coeval Λ CDM models generically had larger second peaks.

In order to clarify the Λ CDM expectation value for $A_{1:2}$ and $R_{1:2}$ as it existed prior to CBR constraints, I have produced new models using “ Λ CDM 1999” parameters (e.g., Turner 1999). These are completely “vanilla” models with reasonable parameters for the time, i.e., $\Omega_m = 0.3$, $\Omega_\Lambda = 0.7$, and $n = 1$. The only significant difference from the Λ CDM models discussed by McGaugh (1999c) or, for that matter, from current Λ CDM models is the baryon density. This I fix to the value given by Tytler et al. (2000): $\omega_b \equiv \Omega_b h^2 = 0.019$. This is a little higher than the baryon densities I had considered in 1999, which were chosen to sample the range suggested by prior BBN reviews (Walker et al. 1991; Copi et al. 1995). A higher baryon density

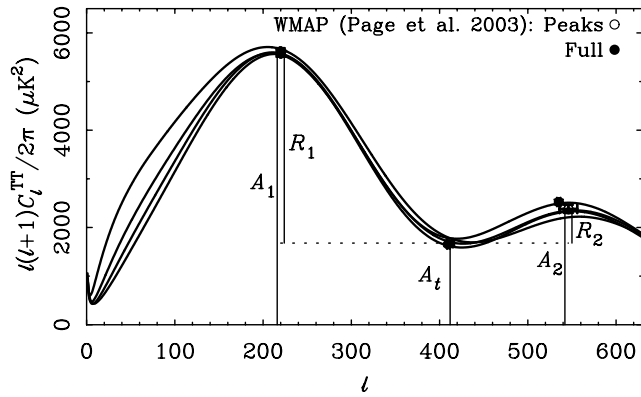


FIG. 1.—No-CDM models of McGaugh (1999c) together with the peak positions and amplitudes measured by *WMAP* as reported by Page et al. (2003). Also shown are the peak amplitudes used in defining the ratios $A_{1:2} = A_1/A_2$ and $R_{1:2} = R_1/R_2$. No-CDM models predict a very narrow range of these ratios for plausible (BBN) baryon densities, consistent with the *WMAP* observations. The specific models shown (from top to bottom at low l) are the $\Omega_b = 0.01, 0.02, 0.03$, and $\Omega_\nu = \Omega_b = 0.02$ from McGaugh (1999c). [See the electronic edition of the *Journal* for a color version of this figure.]

produces a lower second peak, which is more favorable for Λ CDM in the context of subsequent CBR observations. This being the case, I also give a model with $\omega_b = 0.0214$, which is the 95% c.l. upper limit of Tytler et al. (2000). The result is an upper limit on the Λ CDM prior expectation for the amplitude ratio of $A_{1:2} < 2.06$ (Table 1). This is as generous as it is reasonable to be to the Λ CDM paradigm as it existed before stringent CBR constraints, when there was no cause to consider higher baryon densities or exotic effects such as a running tilt to the power spectrum.

The first data constraining the first and second peaks were reported by BOOMERANG (de Bernardis et al. 2000). These data could immediately be seen to be more consistent with the no-CDM prediction than with the expectations of Λ CDM as they existed at the time (McGaugh 2000b). The second BOOMERANG data release (de Bernardis et al. 2002) measured $A_{1:2} = 2.45 \pm 0.79$. This is to be compared with the prior expectation of $A_{1:2} < 2.06$ for Λ CDM and $A_{1:2} \approx 2.40$ for no-CDM (Table 1).

With the release of the *WMAP* first year data both $A_{1:2}$ and $R_{1:2}$ are measured to high accuracy (Page et al. 2003). The *WMAP* data are in good agreement with the predictions of McGaugh (1999c) for the case of no-CDM (Table 1). They are, in fact, bang on: $A_{1:2} = 2.40$ (predicted) versus $A_{1:2} = 2.34 \pm 0.09$ (measured) and $R_{1:2} = 5.41$ (predicted) versus $R_{1:2} = 5.56 \pm 0.75$ (measured). It is striking that the peak amplitude ratio is exactly that predicted in advance by the no-CDM model.

2.2. Constraints on the Baryon Density

The parameter with the most leverage on the amplitude of the second peak is the baryon content. Increasing the baryon fraction depresses the amplitude of the second peak relative to the first, as the importance of baryonic drag is increased relative to the forcing of the oscillations by CDM. The observed peak amplitude ratio can thus be obtained by decreasing the CDM density (to zero) at fixed baryon density or by increasing the baryon density while retaining CDM. In order to be able to distinguish between Λ CDM and no-CDM, we must have a robust independent constraint on the baryon density from BBN.

After the initial BOOMERANG measurements (de Bernardis et al. 2000) failed to detect the large second peak anticipated in Λ CDM, it was common to attribute this to a large baryon density ($\omega_b = 0.031$; Tegmark & Zaldarriaga 2000). This was considerably larger than implied by deuterium measurements at the time ($\omega_b = 0.019$; Tytler et al. 2000), which were themselves surprisingly high compared with previous BBN estimates ($\omega_b = 0.0125$; Walker et al. 1991). Later, the baryon density implied by Λ CDM analyses of the CBR data dropped somewhat (de Bernardis et al. 2002). This was not because the second peak grew larger; it merely resolved out at the amplitude predicted by no-CDM. However, the lower limit on the amplitude of the peak was considerably improved, excluding the very high baryon densities that would be required to completely suppress the second peak. That these had been consistent with the earliest CBR data somewhat skewed our perspective. After the second peak was clearly detected, the preferred baryon density was still larger than the BBN value, but only by a small amount, and not one that was significant once systematic errors (e.g., tilt) were considered (Netterfield et al. 2002).

The *WMAP* best-fit model (Spergel et al. 2003) fits the small second peak with a baryon density of $\omega_b = 0.0224 \pm 0.0009$ and a tilt of $n = 0.93$ with a large running of n with scale. Tilting the power spectrum in this manner helps to suppress the second peak relative to the first. The test I had proposed should maintain $n = 1$, which is perfectly consistent with the *WMAP* data themselves.³ In this case, the baryon density must be higher: $\omega_b = 0.024 \pm 0.001$. It is interesting to compare these with independent BBN constraints.

Estimates of the baryon density ω_b inferred from the light elements ^2H , ^4He , and ^7Li over the past decade are tabulated in Table 2 and plotted in Figure 2. BBN was already a very well developed field prior to 1995; earlier work is represented by the compilations of Walker et al. (1991) and Copi et al. (1995). The baryon densities implied by analysis of the CBR with CDM are also shown. Without CDM the CBR is not a precision baryometer, as the peak amplitude ratios are very similar for the plausible range of baryon densities (Fig. 1). The *WMAP* peak data are consistent with no-CDM models in the range $0.010 \leq \omega_b \leq 0.022$.

The BBN literature contains a wealth of information. When authors have quoted a value for ω_b , it is given in Table 2.

³ Running is inferred when other types of data are included in the analysis, especially the power spectrum from Ly α clouds.

TABLE 1
PEAK RATIOS

Model	ω_b	$A_{1:2}$	$R_{1:2}$
Λ CDM ^a (1999)	0.019	1.95	3.79
	<0.0214	<2.06	<4.05
No CDM ^a	0.0056	2.57	7.61
	0.011	2.37	5.72
	0.017	2.40	5.41
<i>WMAP</i> ^b	2.34	5.56

^a Values expected prior to any observational constraints (1999). Λ CDM models are optimistic in that they represent the largest values of the ratios considered plausible at the time. The three no-CDM models are the $\Omega_b = 0.01, 0.02$, and 0.03 predictions from McGaugh (1999c).

^b Page et al. (2003). The uncertainty in $A_{1:2}$ is ± 0.09 and that in $R_{1:2}$ is ± 0.75 .

TABLE 2
BARYON DENSITY

Measurement	Ω_b^a	$\pm\sigma$	Reference
Compilation.....	0.0125	0.0025	1
	0.0145	0.0055	2 ^b
	0.0066	0.0010	3
	0.0190	0.0012	4
² H.....	0.006	0.001	5
	0.0193	0.0014	6
	0.019	0.001	7
	0.0188	0.0010	8
	0.025	0.001	9
	0.0205	0.0018	10
	0.0214	0.0020	11
⁴ He.....	0.0058	0.0012	12
	0.0125	0.0025	13
	0.0066	0.0010	14
	0.0145	0.0020	15
	0.017	0.0035	16
	0.0068	0.0013	17
	0.0052	0.0009	18
	0.0170	0.0025	19
	0.009	0.002	20
	0.0095	0.0015	21
⁷ Li ^c	0.0125	0.0025	22
	0.0146	0.0030	23
	0.010	0.004	24
	0.015	0.003	25
	0.0102	0.0038	26
CBR: Λ CDM.....	0.016	0.004	27
	0.031	0.0045	28
	0.022	0.003	29
	0.024	0.001	30 ^d
	0.0224	0.0009	30 ^e
CBR: no CDM.....	0.016	0.006	31 ^b

^a $\omega_b \equiv \Omega_b h^2$. The calibration of Burles et al. (1999a, 1999b) has been used.

^b In these cases, $\pm\sigma$ gives the plausible range.

^c The higher Ω_b has been taken when double valued.

^d Power-law Λ CDM fit.

^e Λ CDM fit with a running spectral index.

REFERENCES.—(1) Walker et al. 1991; (2) Copi et al. 1995; (3) Fields et al. 1996; (4) Tytler et al. 2000; (5) Rugers & Hogan 1996; (6) Burles & Tytler 1998a; (7) Burles & Tytler 1998b; (8) Burles et al. 1999a; (9) Pettini & Bowen 2001; (10) O’Meara et al. 2001; (11) Kirkman et al. 2003; (12) Olive & Steigman 1995; (13) Izotov et al. 1997; (14) Olive et al. 1997; (15) Thuan & Izotov 1998; (16) Izotov et al. 1999; (17) Peimbert et al. 2000; (18) Olive & Skillman 2001; (19) Thuan & Izotov 2002; (20) Peimbert et al. 2002; (21) Luridiana et al. 2003; (22) Izotov & Thuan 2004; (23) Bonifacio & Molaro 1997; (24) Ryan et al. 2000; (25) Vangioni-Flam et al. 2000; (26) Suzuki et al. 2000; (27) Bonifacio et al. 2002; (28) Tegmark & Zaldarriaga 2000; (29) Netterfield et al. 2002; (30) Spergel et al. 2003; (31) This work.

Often, only a measurement of the abundance of a particular element is reported. In these cases, I have translated the result into a baryon density using the calibration of Burles et al. (1999b). For lithium, this determination can be double valued; in such cases I have adopted the higher ω_b .

Figure 2 shows that the bulk of the data from all three elements fall in the range $0.005 < \omega_b < 0.02$. Taking the data at face value, both Gaussian and biweight statistics give $\langle \omega_b \rangle = 0.014 \pm 0.05$. This value comes from the individual elemental abundances only and does not include previous compilations or CBR data. This mean is fairly robust to how the data are treated. It does not change if extremal points are rejected, nor if we accept only a single measurement from each independent group.

No-CDM models are consistent with all three of the light elements, to the extent to which they are consistent with each

other. In contrast, the baryon densities implied by Λ CDM analyses of the CBR all have $\omega_b > 0.020$. No measurement of any element ever implied a baryon density this high until after the first CBR data appeared (see also Steigman et al. 2002). So while it is true that the *WMAP* Λ CDM analysis (Spergel et al. 2003) is consistent with the latest deuterium estimate (Kirkman et al. 2003), the bulk of BBN data are more consistent with no-CDM.

2.3. CBR Data and Models: 2000–2003

Figure 3 shows the impressive improvement of CBR data over the past few years, together with the models advanced to explain them. The top panels of Figure 3 show the initial data from the BOOMERANG (de Bernardis et al. 2000) and MAXIMA-1 (Hanany et al. 2000) experiments. The BOOMERANG data are shown both as published (*open triangles*) and adjusted to agree with the MAXIMA-1 calibration (*filled triangles*; McGaugh 2001). On the left the $\omega_b = 0.019$ “ Λ CDM 1999” model from Table 1 is shown. This case is very similar to the high baryon fraction Λ CDM model considered by McGaugh (1999c) and is chosen for illustration as the a priori model that is least inconsistent with the data. A quite generic prediction of Λ CDM models had been a substantially larger second peak than was subsequently observed.

In contrast, the low amplitude of the second peak is quite natural for the no-CDM model. The $\Omega_b = 0.03$ model from McGaugh 1999c is shown at right. Parameters of the no-CDM models shown in Figure 3 are given in Table 3. Note that all of the no-CDM models shown in Figure 3 existed before the data they are shown with.

The middle panel of Figure 3 shows the second data release from BOOMERANG (de Bernardis et al. 2002) and the first data from the Degree Angular Scale Interferometer (DASI; Halverson et al. 2002). On the left the best-fit Λ CDM model (with strong priors) of Netterfield et al. (2002) is shown. This

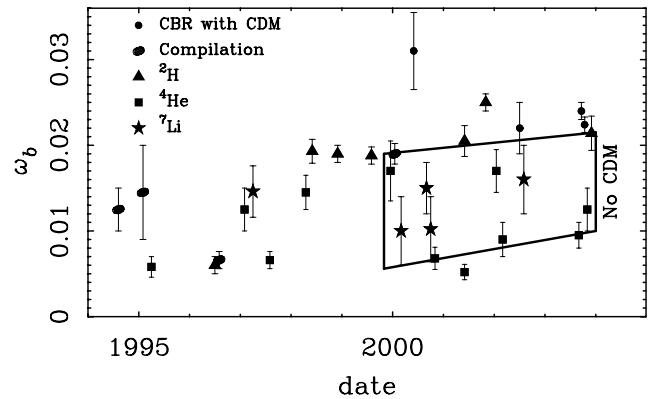


FIG. 2.—Recent measurements of the baryon density $\omega_b = \Omega_b h^2$, as given in Table 2. The type of measurement is given by the various symbols, including those works that are compilations of multiple isotopes. BBN was a well-developed field well before the start of this graph; earlier work is represented by the compilation of Walker et al. (1991; *leftmost point*) and Copi et al. (1995). All error bars are 1σ except as noted in Table 2. There is a dichotomy at $\omega_b = 0.020$. The power-law Λ CDM fit from *WMAP* gives $\omega_b = 0.024 \pm 0.001$. Yet there were no BBN measurements that suggested $\omega_b > 0.020$ prior to the first constraints from the CBR. No-CDM models are consistent with a range of lower Ω_b (*box*). The left edge of the no-CDM box represents the range of Ω_b considered by McGaugh (1999c), which was based on the plausible range of Walker et al. (1991). The right edge represents the range of no-CDM models consistent with the *WMAP* first-to-second peak amplitude ratio. This range is consistent with independent estimates from all elements. [See the electronic edition of the *Journal* for a color version of this figure.]

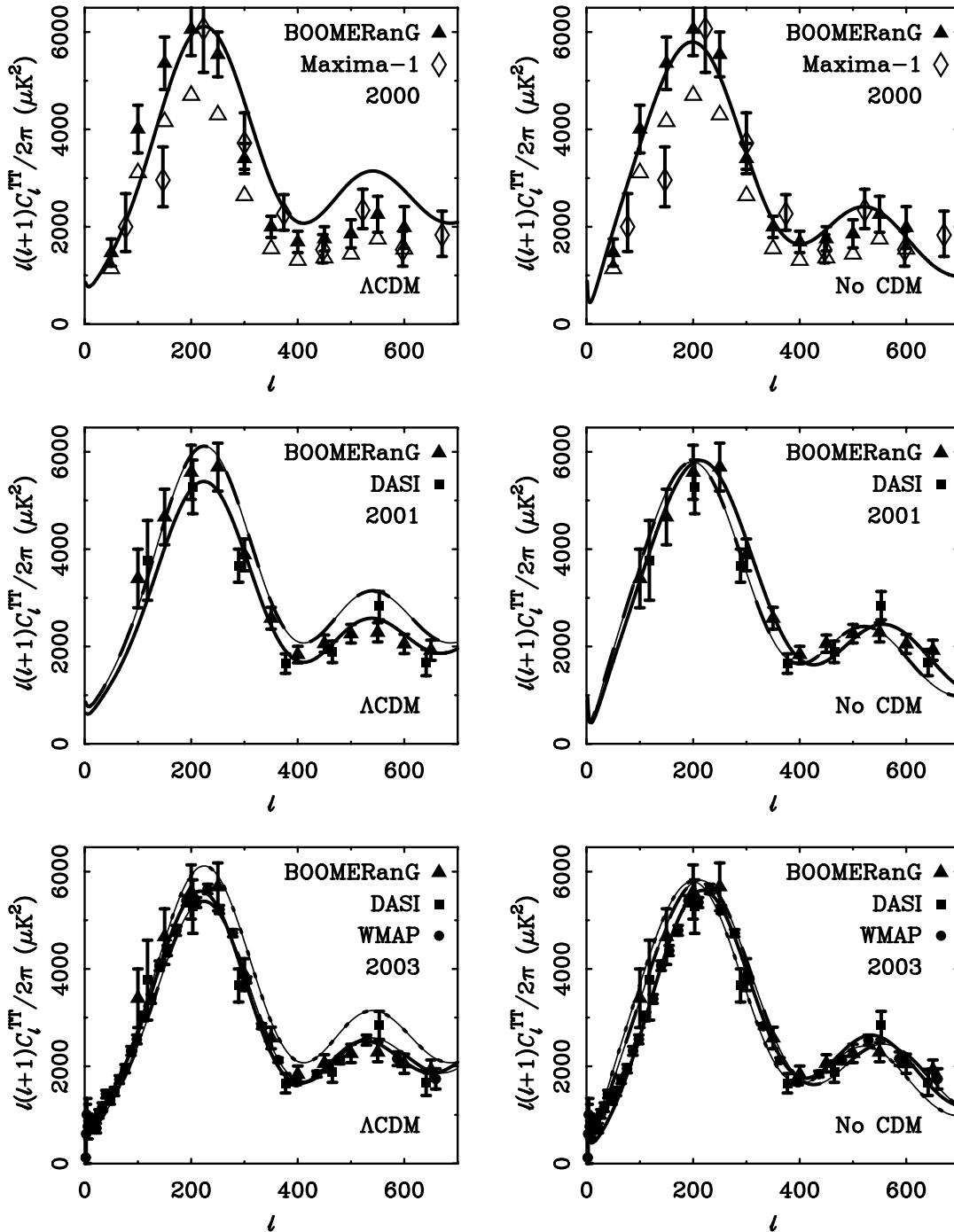


FIG. 3.—Power spectrum of temperature fluctuations in the microwave background, as observed by recent experiments. Lines show model predictions and fits at the corresponding times, with Λ CDM models on the left and no-CDM models on the right. The previous edition of each model is retained as dashed then dotted lines in subsequent panels. The no-CDM model provided a more accurate prediction of the first-to-second peak amplitude ratio than did Λ CDM, and it has evolved considerably less in response to improving data. All of the no-CDM models shown in this figure existed before the corresponding data (see text). [See the electronic edition of the Journal for a color version of this figure.]

has changed substantially from the prior expectation of Λ CDM yet still provides a worse description of the data than does the no-CDM model (Fig. 3, right). The model is only acceptable because of the systematic uncertainty in the beam size, which allows for some play in the tilt of the power spectrum.

In contrast, the improved data have simply moved closer to the no-CDM model as fitted to the initial BOOMERANG data release (McGaugh 2000b) and normalized to MAXIMA-1 (McGaugh 2001). There is no need to iterate the fit at all.

Indeed, the revised calibration of BOOMERANG was well predicted by the preexisting no-CDM model.

The bottom panel of Figure 3 shows these data and the first year *WMAP* data. The fit to the *WMAP* data (Spergel et al. 2003; left) has a larger $A_{1;2}$ than the fit of Netterfield et al. (2002), matching that of the no-CDM prediction (right). The *WMAP* data are good enough to require the first tweak to the no-CDM fit, as the first peak in the *WMAP* data is slightly narrower than indicated by previous data. This small difference

TABLE 3
NO-CDM MODELS

Model in	ω_b	Ω_Λ	α^a	f_ν	n_t	τ	A^b	Reference
Figure 3 (<i>top</i>).....	0.017	0.97	0.66	0.0	0.0	0.0	0.59	M1999b
Figure 3 (<i>middle</i>).....	0.019	1.006	1.0	0.0	0.0	0.0	0.61	M2000b
Figure 3 (<i>bottom</i>).....	0.019	0.90	0.89	0.61	0.0	0.0	0.55	
Figures 4–8.....	0.019	0.918	1.0	0.65	0.04	0.17	1.01	This work

^a Flat model scaled by $l \rightarrow \alpha l$ (McGaugh 2000b).

^b Amplitude normalization $C_l \rightarrow AC_l$.

has the interesting consequence of implying a significant neutrino mass.

2.4. A No-CDM Model for the *WMAP* Data

Providing a detailed fit to CBR data is beyond the ambition of the no-CDM model. We expect the new physics entailed by MOND to render standard calculation of the temperature fluctuations incorrect at some level. The peak amplitude ratios are the most robust aspect of the no-CDM prediction, since there is no obvious mechanism to deviate from a pure damping spectrum in the absence of CDM.

Nevertheless, preexisting models do give a tolerable match to the detailed shape of the power spectrum (Fig. 3). It is interesting to see whether it is possible to find a no-CDM model that matches the details of the *WMAP* data, not just the peak amplitude ratio. Such a model is shown in Figure 4, with details in Table 3.

As a matter of principle, I restrict the free parameters of no-CDM as much as possible with very strong priors. For example, I fix the baryon density to the value advocated by Tytler et al. (2000): $\omega_b = 0.019$. Other baryon densities are certainly viable, but the question here is not what baryon density the CBR prefers. Rather, we wish to know whether there is a no-CDM model motivated by BBN that is consistent with *WMAP*. Similarly, I fix $H_0 = 72 \text{ km s}^{-1} \text{ Mpc}^{-1}$ (Freedman et al. 2001) and $\tau = 0.17$ (Kogut et al. 2003). Of course, the most important prior for a no-CDM model is $\Omega_{\text{CDM}} = 0$.

Three free parameters are required to fit the *WMAP* data. The aspects of the data that need to be matched are the position (in l) of the first peak, the amplitude of the temperature fluctuations (C_l), and the width of the peaks. The positions of the peaks are controlled by geometry. The amplitude of the fluctuations is

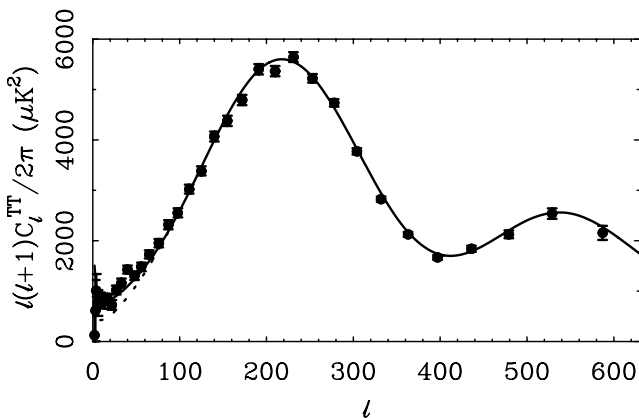


FIG. 4.—No-CDM model that matches the *WMAP* first year data. The solid line is the model discussed in the text. The dashed line is the closest preexisting no-CDM model (Table 3). The two are indistinguishable for $l > 100$. [See the electronic edition of the *Journal* for a color version of this figure.]

somewhat arbitrary in CMBFAST models and can be addressed in several ways. In order to match the width of the peaks, it appears necessary to invoke massive neutrinos.

It is perhaps too strong a term to describe what I have done as a “fit” to the *WMAP* data. In order to match the data, I have adjusted three parameters: Ω_Λ , n_t , and f_ν . The value of Ω_Λ is adjusted to match the location of the first peak. A modest tensor component $n_t = 0.04$ is used to match the amplitude at small l . The neutrino fraction f_ν is used to slim the peaks, which are a bit too fat in purely baryonic models. I have only adjusted these far enough to find a tolerable match to the data. I have not attempted an exhaustive exploration of parameter space; modest improvements are no doubt possible. However, I do not believe that this would be a meaningful exercise, since I expect MOND to cause real deviations from a pure no-CDM model, especially at $l < 100$. Each of these three parameters is discussed in turn below.

2.4.1. Geometry

The geometry of an FRW universe is well specified by the matter content Ω_m and Ω_Λ . A great success of the inflationary paradigm is the apparent flatness of the universe ($\Omega_m + \Omega_\Lambda \approx 1$) as indicated by the position of the first acoustic peak $l_1 = 220$ (Page et al. 2003). As noted earlier, an oddity of a universe with a significant cosmological constant is that while it is consistent with the inflationary prediction of geometric flatness, it does not solve the coincidence problem, which was one of the important motivations for inflation. This is not important here; one can have either or both MOND and inflation. Neither requires the other, nor are they mutually exclusive.

While the cosmic geometry of an FRW model is understood, that in MOND is not. Thus, while a robust prediction for $A_{1:2}$ can be made, there is no prior expectation for the location of the first peak. This is treated as a parameter to be fitted.

There are two approaches that can be used to fit the peak location. A model that is flat in the usual Robertson-Walker sense can be scaled by a multiplicative factor α to match the peak position, so that $l_1 = \alpha l_{\text{model}}$ (McGaugh 2000b). Alternatively, one may treat Ω_Λ as a free parameter and find the value that places the peak in the right location. Either way, there is a single fit parameter. I have adopted the latter approach here.

In effect, I am using the cosmological constant as a fudge factor to express our ignorance about the geometry in MOND. This is not particularly different from the role it currently plays in standard cosmology. As such, we should not invest too much importance in the particular numerical value obtained.

One curious note is that while the geometry is *close* to flat, no-CDM models do require a slight but significant positive spatial curvature. Such a model is closed in standard cosmology, but the significance of this fact in MOND is unclear. Perhaps it contains a clue to the nature of the underlying theory, or perhaps it is just a coincidence.

The coincidence problem may be eased by the proximity of the no-CDM geometry to the de Sitter solution, which is an attractor in Ω_m - Ω_Λ space. It is worth noting that in the absence of a repulsive term (Λ), a MOND universe may eventually recollapse for any Ω_m (Felten 1984). Since Λ is essentially just a fudge factor encapsulating our ignorance of the geometry, it remains conceivable that we live in such a Felten universe where there is no critical value of Ω_m and hence no coincidence problem.

2.4.2. Amplitude

As with geometry, the amplitude of a model can be scaled by a multiplicative factor to match the data: $C_l = AC_{l,\text{model}}$. There are many uncertainties in the normalization of CBR models, so this is fair within plausible bounds. This can be seen in the various models presented by Spergel et al. (2003).

Various factors affect the normalization, including the optical depth and neutrino mass. If we get these things right, we should be able to get the normalization right ($A = 1$). Traditionally, CBR models have been normalized to *COBE* at $l = 10$. This is not the best choice for a no-CDM model, as one of the predicted post-recombination effects of MOND is an enhancement of the ISW effect (McGaugh 1999c). This should be most pronounced on large scales.

The ISW effect arises from the variation of metric fluctuations during structure formation. Unfortunately, it is not presently possible to make an explicit calculation of this effect in MOND. However, we do expect MOND to cause rapid structure formation. The rapid variation of the effective potential should enhance the ISW. This will distort the shape of the CBR power spectrum so that there is an excess of power at $l < 100$ over that in a pure no-CDM model, which is normalized to the amplitude of the peaks (at $l \approx 220$).

The preexisting no-CDM model shown in Figure 3 (*bottom*) fits the *WMAP* data well at $l > 100$. It underpredicts the observed power at $l < 100$ (Fig. 5). This is, qualitatively at least, the expected signature of the ISW in MOND. However, it is interesting to see whether we can obtain a fit to all of the data. In order to make up the power on large scales, we invoke a modest tensor contribution. This is the only significant difference between the fit in Figure 4 and the old no-CDM model with neutrinos.

The tensor contribution is the standard inflationary complement to a mildly tilted spectrum: $n_t = 1 - n = 0.04$. Such a tensor contribution would be consistent with a broad range of inflationary models (e.g., Tegmark et al. 2004). As inflation occurs very early and the effects of MOND are only manifest after recombination, it is quite possible to have both.

While it is conceivable that there literally is a tensor component, really this is just a fudge to show that it is possible to fit the low- l data with a no-CDM model. For the reasons described above, we do not expect to be able to fit a no-CDM MOND proxy model to $l \lesssim 100$. Instead, the deviation at small l illustrated in Figure 5 should be viewed as an empirical constraint on the ISW in MOND. How well low-redshift structure should correlate with the temperature fluctuations in the CBR is unclear because of the highly nonlinear nature of MOND. But one would naively expect a strong signal in this scenario.

An interesting consequence of this approach is that further scaling (by A) is hardly necessary. That is, n_t not only fits the low- l data but also fixes the overall normalization to well within the uncertainty of the optical depth (fixed to $\tau = 0.17$; Kogut et al. 2003). It would perhaps be better to normalize no-CDM models at $l = 220$ than at $l = 10$.

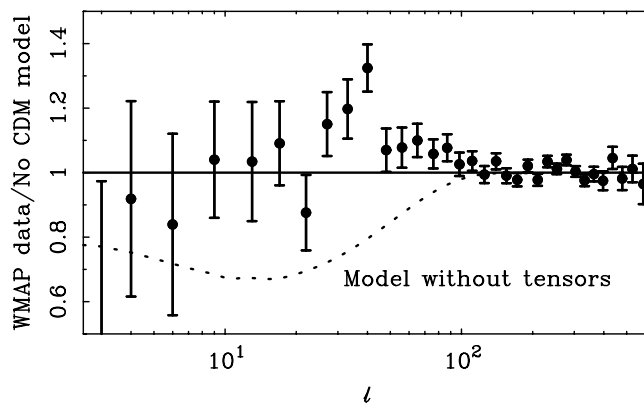


FIG. 5.—Ratio of *WMAP* data to the best-fitting no-CDM model, with a logarithmic abscissa to emphasize small l . The scalar-only no-CDM model (*dotted line*) fits well for $l > 100$ but systematically underpredicts the power at small l . The excess power is suggestive of an amplified ISW effect. This is one predicted signature of MOND-like physics beyond the simple no-CDM model. [See the electronic edition of the *Journal* for a color version of this figure.]

2.4.3. Neutrino Mass

One peculiar aspect of the fit shown in Figure 3 is a high neutrino fraction, $f_\nu \approx 0.65$. In order to obtain a detailed no-CDM fit to the *WMAP* first year data, it is helpful to have heavy neutrinos. The CBR parameter space is very large, so it is difficult to say whether these are required. However, models with neutrino densities comparable to or slightly in excess of the baryon density do seem to be preferred.

The need for heavy neutrinos is brought on entirely by the narrowness of the first peak. This is rather subtle, being the difference between the models in the bottom right panel of Figure 3. While McGaugh (1999c) noted that such an effect was possible, it is only perceptible in data of *WMAP* quality.

The model in Figure 3 has three degenerate neutrinos with $m_\nu = 1.1$ eV. The neutrino mass⁴ is not well determined. One issue is a degeneracy between neutrino mass and optical depth. Both affect the absolute amplitude of the observed temperature fluctuations in similar ways. If the optical depth were found to be larger than 0.17, the neutrino mass would go down. Another effect is due to the baryon density. The strongest constraint is on the neutrino fraction; the neutrino mass scales roughly with the baryon density. Lower baryon densities are tolerable, so lower neutrino masses would be as well.

There are other indications of a finite neutrino mass in MOND. Perhaps the most significant is the residual mass discrepancy in clusters of galaxies (Aguirre et al. 2001; Sanders 2003). A neutrino mass of 1–2 eV would be about right to explain this. Neutrinos of this mass would not be trapped by galaxy-scale potentials but would be by clusters and would provide about the right amount of mass.

While the neutrino mass is not well constrained at present, it is subject to independent laboratory tests. The prospects for measuring a mass as high as $m_\nu \sim 1$ eV in upcoming experiments are good; there is already a claim that $m_\nu \approx 0.39$, albeit with a large uncertainty (Klapdor-Kleingrothaus et al. 2001). Such a measurement would give us another means of distinguishing between scenarios. Structure can only form in Λ CDM if $m_\nu < 0.23$ eV (Spergel et al. 2003). A firm laboratory

⁴ The inferred neutrino mass is also very sensitive to the physics of recombination, changing upward by 20% between CMBFAST versions 4.0 and 4.3.

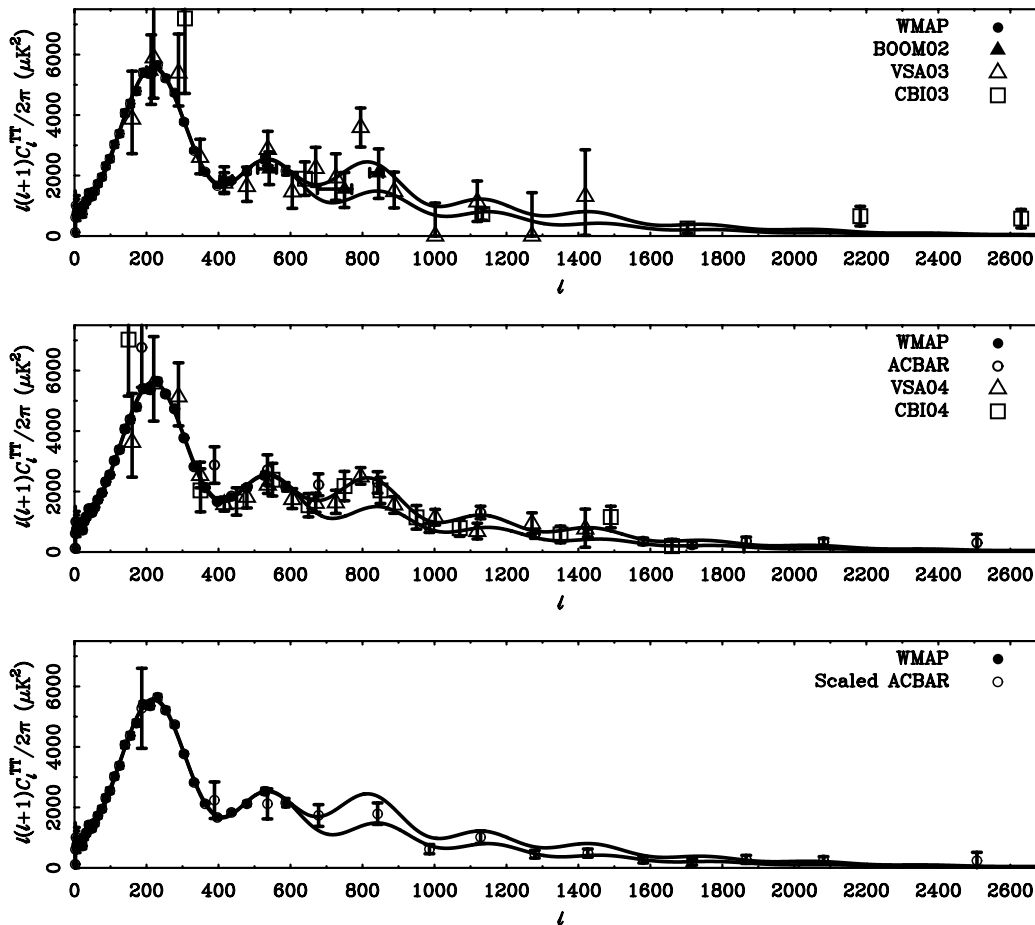


FIG. 6.—CBR power spectrum extending to large l . *Top*: Data from CBI and VSA, as reported in 2003, together with the *WMAP* data and the BOOMERANG peak locations. *Middle*: ACBAR data and the updated (2004) versions of the VSA and CBI data. *Bottom*: ACBAR data scaled to match the *WMAP* data in the range of l where the two overlap. Note that the ACBAR data closely follow the predictions of the model lines, though which one depends on the scaling. The lines are the power-law *WMAP* Λ CDM model and the no-CDM model. These are indistinguishable for $l < 600$. The models diverge for $l > 600$, with Λ CDM predicting the larger third peak and no-CDM the smaller. [See the electronic edition of the *Journal* for a color version of this figure.]

measurement of a neutrino mass heavier than this would constitute a clear falsification of Λ CDM. The rapid structure formation in MOND is not so adversely affected to this limit, and some power suppression from heavy neutrinos might even be desirable to prevent MOND from overproducing structure (Nusser 2002; Knebe & Gibson 2004).

Presumably, if MOND is correct, it is only the approximate expression of a more general theory from which the proper cosmic geometry should be derived. By that token, the Robertson-Walker geometry would also be an approximation of the deeper theory. We should not expect that one fudge factor (Λ) can necessarily connect both. It is conceivable that the approximation to the geometry we are making is inadequate for simultaneously matching the locations and widths of the peaks. These are extremely sensitive to the geometry. It is therefore difficult to exclude the possibility that a deeper, relativistic theory could fit the observations without heavy neutrinos. However, they do seem to be the most obvious way to explain the modest width of the acoustic peaks without CDM.

2.5. The Third Peak

The *WMAP* first year data drop to a signal-to-noise ratio $S/N < 1$ for $l > 660$ (Bennett et al. 2003), so do not, as yet, provide any useful constraint on the amplitude of the third peak (Page et al. 2003). Figure 6 shows the data from *WMAP* to-

gether with other experiments that probe smaller angular scales: the Cosmic Background Imager (CBI; Mason et al. 2003), the Arcminute Cosmology Bolometer Array Receiver⁵ (ACBAR; Kuo et al. 2004), and the Very Small Array (VSA; Grainge et al. 2003). The other experiments provide some hints but do not yet define the third peak to the quality required for a clear test.

A critical issue with mixing data from different experiments is the mutual consistency of their calibrations. This is an ancient problem in astronomy from which CBR experiments are not immune. Indeed, a prominent example of the problems faced in comparing such data has already been provided in the first data release by BOOMERANG and MAXIMA-1. Even though the *shape* of the power spectrum measured by the two experiments has always been consistent, their absolute calibration was clearly offset in the first data release. There is little reason to expect that the calibrations of all the independent experiments are in perfect accord.

It is possible to place the various experiments on a mutually consistent scale by normalizing them to a common reference. Once a reference calibration is adopted, the other experiments can be brought onto a common scale by a multiplicative factor

⁵ Data obtained from <http://cosmology.berkeley.edu/group/swlh/acbar/data/index.html>.

TABLE 4
SCALING FACTORS AND χ^2_ν

Experiment	\mathcal{F}^a	$\mathcal{F}_{\text{obs}}^b$	χ^2_ν	$\chi^2_{\nu,\text{modif}}^c$	Modification
BOOMERANG.....	1.29	1.32	1.65	1.19	Excludes $l = 700$
MAXIMA.....	1.29
DASI.....	1.95
CBI.....	0.76	0.94	1.25
VSA.....	0.95	0.92	1.76	0.93	Excludes $l = 795$
ACBAR.....	0.78	...	2.40	0.73	Uses $\mathcal{F} = 0.78$
ARCHEOPS.....	1.18
WMAP.....	1.60	1.17	Excludes $l = 40$
All.....	1.68	1.18	All of the above

^a Scaling factor that reconciles the calibrations of each experiment: $C_l \rightarrow \mathcal{F}C_l$.

^b Observed correction to calibration postdating the determination of \mathcal{F} .

^c $\chi^2_{\nu,\text{modif}}$ shows the effect of the noted modification.

\mathcal{F} , which maps $C_l \rightarrow \mathcal{F}C_l$. The value of \mathcal{F} is determined by minimizing χ^2 for the difference in C_l between each experiment and the reference over the range of l where they overlap.

This method was employed by McGaugh (2001) to bring the first BOOMERANG data into accord with the MAXIMA-1 calibration. The result was successful in anticipating the subsequent recalibration of BOOMERANG (Fig. 3). The obvious choice of reference calibration now is *WMAP*. The scaling \mathcal{F} determined relative to *WMAP* for each experiment is given in Table 4. For the most part, these imply that the normalizations of the experiments that probe high l need to be reduced somewhat to be consistent with *WMAP*. This can be seen by eye in Figure 6 (top), where the CBI and ACBAR data fall above the *WMAP* data in the range of l where they overlap. This was also noted by Ödman (2003), who pointed out that there is some tension between low- l and high- l experiments.

After the initial submission of this paper, new data were reported by CBI (Readhead et al. 2004) and VSA (Dickinson et al. 2004). These are shown in Figure 6 (middle). The new results included an explicit evaluation of the calibration difference of these experiments with respect to *WMAP* based on their observations of Jupiter. This allows us to compute the effective “observed” correction factor \mathcal{F}_{obs} as a check on the previously computed \mathcal{F} . In the case of the VSA, as with BOOMERANG previously, the two are consistent. In the case of CBI, \mathcal{F} is somewhat less than \mathcal{F}_{obs} . This appears simply to be due to the large uncertainty in \mathcal{F} in this one case: the initial CBI data only have two independent points overlapping the range of l constrained by *WMAP*, and one of those has a very large error bar. The procedure of scaling by \mathcal{F} does therefore appear to be valid.

The reduced χ^2_ν of the no-CDM model with respect to *WMAP* and the other CBR experiments are given in Table 4. The match to the *WMAP* TT data is treated as predictive of what the other experiments should observe: no further adjustment is made to the model. A plot of the χ^2 values of the individual data points is given in Figure 7.

A few badly fitting points stand out. The single most deviant point is that of *WMAP* at $l = 40$. Like the point at $l = 210$, which gives a notched appearance to the first peak, this point at $l = 40$ is deviant from any smooth model. Since the use of a tensor contribution to mimic an excess ISW effect from MOND is far from perfect, this is not of great concern. To show the impact of this and the other stand-out points on the χ^2 budget, Table 4 also gives $\chi^2_{\nu,\text{mod}}$ without them.

A deviant point of greater concern is that of the VSA at $l = 795$. This indicates a third peak significantly larger than

predicted by the no-CDM model. The absolute value of C_l of this point dropped in 2004 from its previous value in 2003, but the error bar also shrank considerably, making the statistical significance of the deviation greater. Still, the overall fit with respect to this experiment is not terrible, and it is rather odd that the lion’s share of the χ^2 budget is due to this single point.

The situation is similar with respect to BOOMERANG. The χ^2 budget is dominated by the point at $l = 700$. The overall fit to this experiment is nevertheless acceptable. Indeed, my χ^2 do not account for the tilt in the power spectrum allowed by the systematic uncertainty in the beam size (Netterfield et al. 2002), so would come down somewhat if this were taken into consideration. The no-CDM model is clearly acceptable to the empirical peak location estimates of de Bernardis et al. (2002) (Fig. 6, top).

The fit with respect to MAXIMA (Lee et al. 2001) and Archeops (Benoit et al. 2003) is acceptable without caveats. That with respect to DASI is rather poor. This is in part because of a hint of a large third peak in those data, but roughly half the χ^2 budget is contributed by a point at small l where the *WMAP* data have been given precedence. This would seem to tell us more about the difficulty of error estimation in real experiments than about the model.

The worst χ^2_ν is that of ACBAR, with the calibration as published. However, if we scale the ACBAR data by $\mathcal{F} = 0.78$ to match the *WMAP* data in the range where they overlap, $\chi^2_{\nu,\text{mod}}$ is quite acceptable. Indeed, once this scaling factor is accounted for, the no-CDM model does an excellent job of predicting the undulations of the ACBAR data, which appear to hit the fourth

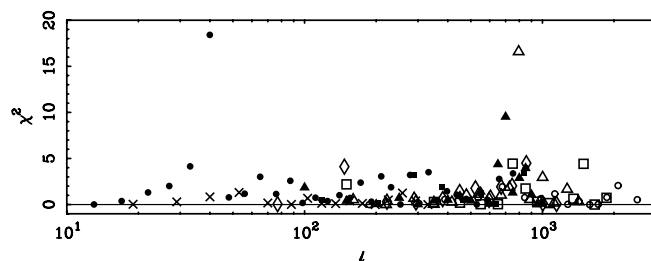


FIG. 7.— χ^2 of individual data points for each of the experiments listed in Table 4. Symbols are the same as in previous figures. ACBAR is plotted scaled with $\mathcal{F} = 0.78$. The χ^2 budget is dominated by the *WMAP* point at $l = 40$ and the VSA point at $l = 795$. There is a clear indication of power in excess of the no-CDM prediction in the vicinity of the third peak, but only the single point from VSA is highly significant. [See the electronic edition of the *Journal* for a color version of this figure.]

and fifth peaks and their preceding troughs. Whether these data indicate a large or small third peak hinges entirely on the true value of \mathcal{F} . There are four points with tolerable error bars overlapping the *WMAP* data, so \mathcal{F} is better determined than for CBI but not as well as for the other experiments.

Multiple experiments measure power in the vicinity of the third peak in excess of that predicted by the no-CDM model. The agreement between independent experiments is encouraging, but the data are not yet adequate to clearly reject this aspect of the no-CDM model. One must weigh the merits of all the predictions together; as yet, the amplitude of the third peak remains the least well constrained of the tests discussed here. To be really sure, we would like to see the first three peaks all measured by the same experiment with high quality so that the third peak is at least as well defined as the second one is in the *WMAP* first year data.

3. SIGNATURES OF MOND-LIKE PHYSICS

While the no-CDM *Ansatz* is a good first approximation to what we might expect for the CBR in MOND, it must fail at some level. We have known for a long time that the growth factor between the epoch of recombination and the present epoch is too large to be explained by gravitational collapse with normal gravity and the observed baryons. This does not in itself require CDM: one could also imagine a change to the force law that results in a faster growth rate.

Recently, the formation of structure under MOND has been considered by a number of authors (Sanders 1998, 2001; McGaugh 1999a; Nusser 2002; Stachniewicz & Kutschera 2002; Knebe & Gibson 2004). A variety of assumptions and approximations have been made in these works, but one generic result is that MOND causes structure to form fast. The onset of structure formation is necessarily delayed until after recombination (there is no CDM component immune to radiation pressure, that can clump up earlier) and matter domination (which occurs after recombination for $\Omega_m = \Omega_b$). Once these conditions are met and the perturbations enter the MOND regime, the nonlinear MOND force law causes structure formation to proceed rapidly. Bright ($\sim L_*$) galaxies can be in place by $z \sim 10$; clusters, by $z \sim 3$ (Sanders 1998). The growth of structure slows or even saturates in accelerating universes (Sanders 2000), so the resulting picture is one of a very quiescent early universe undergoing a dramatic period of rapid structure formation that then eases off (see Fig. 1 of Sanders 2000). This contrasts sharply with the steady, gradual buildup of structure in CDM models and should leave subtle but recognizable signatures in the CBR.

3.1. Early Reionization

Perhaps the most obvious signature of MOND-induced structure formation is an early onset of reionization. In CDM models, no significant mass should form in stars before $z \sim 7$ (e.g., White & Frenk 1991; Stachniewicz & Kutschera 2003). Consequently, reionization of the universe was expected to occur fairly late (Loeb & Barkana 2001), giving little opportunity for scattering of the CBR by free electrons. This translates to the expectation that there should be little polarization of the CBR and low optical depth to the surface of last scattering. The situation shortly before the *WMAP* data release was summed up by Peacock (2003): “For reionization at redshift 8, we would have $\tau \approx 0.05$; it is unlikely that τ can be hugely larger.”

Kogut et al. (2003) report a surprisingly large polarization signal of $\tau = 0.17 \pm 0.04$. This pushes the epoch of reioni-

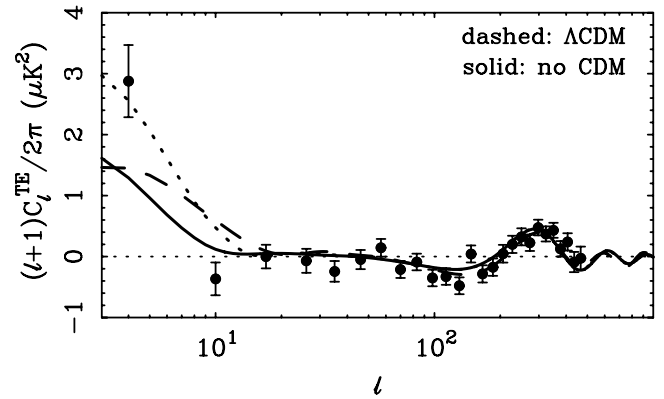


FIG. 8.—TE polarization signal in the *WMAP* data (points). Also shown is the Λ CDM model from the *WMAP* team (dashed line) and two no-CDM models. The solid line is the same model as in previous plots, with optical depth fixed to $\tau = 0.17$. The dotted line is a similar model with $\tau = 0.30$. This is at the upper edge of the confidence interval quoted by Kogut et al. (2003) but is not obviously inconsistent with the data in this plot. An early epoch of structure formation, leading to relatively high optical depth, is natural in MOND and was one of the a priori predictions of McGaugh (1999c). [See the electronic edition of the *Journal* for a color version of this figure.]

zation toward $z \approx 17$, much earlier than nominally expected in Λ CDM. While the difference between $\tau = 0.05$ and 0.17 may not sound large, bear in mind the inverse relation between time and redshift. The universe is ~ 500 Myr old at $z = 8$; one of the advantages of Λ CDM is that it forms structure this “early” (Mo & Fukugita 1996). At $z = 17$ the universe is only 180 Myr old, a very early time by the standard of any prior expectation in the context of CDM. The observed optical depth is hugely larger than expected.

In contrast, early structure formation is a natural consequence of MOND. There is no need to invoke supermassive Population III stars as is currently popular in Λ CDM models. Stars with a normal initial mass function (IMF) simply form sooner. This distinction was noted by McGaugh (1999c), where it was predicted that the polarization signal would be higher than nominally expected in Λ CDM.

While the qualitative prediction of early reionization is clearly realized in the *WMAP* data, it is rather more difficult to estimate a specific number for the optical depth. This depends not only on the timing of structure formation but also on details of the star formation process and the liberation of ionizing photons, which are uncertain in either case. Still, we can make a crude estimate based on the work of Sanders (1998, 2000) and Stachniewicz & Kutschera (2002). Roughly speaking, L_* galaxies may be in place by $z \approx 10$, with smaller (globular cluster) scale lumps forming even earlier, perhaps as early as $z \approx 100$ (Stachniewicz & Kutschera 2002). Stars take a finite amount of time to form, and it remains uncertain how long it takes to form a significant number. However, it seems quite plausible for reionization to have occurred at $z \gtrsim 15$. MOND is therefore quite consistent with $\tau \gtrsim 0.15$ and might conceivably produce even larger optical depths (e.g., $\tau \approx 0.3$; Fig. 8). This fits nicely with the reionization epoch determined from the cumulative mass function of Ly α systems, $z = 24 \pm 4$ (Popa et al. 2004). It might also help to explain the excess power at $l > 2000$ reported by CBI (Bond et al. 2002), as more clusters will be in place earlier than in Λ CDM, enhancing the SZ effect.

3.1.1. Are Supermassive Population III Stars Viable?

If the optical depth is really as large as 17%, reionization must occur early and structure must form fast. One way to

achieve this in Λ CDM is to increase the amplitude of the power spectrum on small scales. However, this would contradict the limits on power in the fluctuation spectrum on small scales at $z = 0$ (Zentner & Bullock 2002; McGaugh et al. 2003; Somerville et al. 2003; van den Bosch et al. 2003).

Ordinary stars and quasars will not suffice to produce the observed optical depth within the limits on the power spectrum. It is necessary to invoke supermassive Population III stars or some other object that is very efficient at producing ionizing radiation. The earliest objects to collapse must be ~ 50 times as efficient at converting mass to ionizing photons as are collapsed objects at the present time (Sokasian et al. 2003).

Obtaining the required ionizing flux early enough is quite a stretch. Fukugita & Kawasaki (2003) argue that the earliest reionization can occur is $z \approx 13.5$, with an upper limit on the optical depth of $\tau < 0.17$ —right at the observed value. Ricotti & Ostriker (2004) find that $\tau > 0.12$ can only be achieved with great contrivance even with Population III. One requisite detail is a remarkably high UV escape fraction (Sokasian et al. 2004). These considerations make one wonder whether it is reasonable to invoke supermassive Population III stars.

Population III stars have long been sought as the first generation of stars with primordial abundances. While very metal-poor stars have been discovered (e.g., Norris et al. 2000), no examples of metal-free stars have ever been found. At $[\text{Fe}/\text{H}] = -3.7$, the star discussed by Norris et al. (2000) is consistent with enrichment by the supernova of a single metal-free $\sim 30 M_{\odot}$ star (Woosley & Weaver 1995). Since the formation time for low-mass stars exceeds the lifetime of massive ones, it seems plausible that long-lived stars of the first generation were enriched by the massive stars of the first generation to a sufficient degree that no metal-free stars remain today.

A $\sim 30 M_{\odot}$ star is perfectly normal and will not suffice to achieve the early reionization of the universe. Metal-free stars have less opacity than Population I stars and should therefore be more efficient producers of UV radiation, but the net gain due to this effect is only a factor of ~ 3 (Schaerer 2003). To obtain the desired factor of ~ 50 increase, Population III must have had a peculiar IMF that produced many supermassive (200–500 M_{\odot}) stars.

It is commonly thought reasonable that Population III have an IMF skewed toward massive stars. The line of argument is that since Population III stars form from metal-free material, gas cooling is less efficient, leading to a larger Jeans mass and hence a larger characteristic stellar mass. At this hand-waving level, supermassive Population III stars seem fairly natural. If we proceed with the logic of this argument, the IMF should be a strong function of metallicity.

A good deal is known about stars. As noted in the review by Hillenbrand (2004), “there is little evidence for substantial variations in the IMF from region to region, or over time.” The IMF appears to be universal, with no evidence for the expected dependence on metallicity (Kroupa 2002). This includes the IMF of massive stars (Oey et al. 2004). There is no indication that the early-time IMF was different from what it is now, either from direct counts of stars in ancient, low-metallicity systems (Wyse et al. 2002) or from the integrated cosmic star formation history (Baldry & Glazebrook 2003). Moreover, dynamical constraints, especially the small scatter in the Tully-Fisher relation, allow very little room to consider a variable IMF (McGaugh et al. 2000; Bell & de Jong 2001; Verheijen 2001; McGaugh 2004), at least when averaged over the scales of galaxies. The hypothesis that metallicity affects the IMF can be tested directly in low-metallicity star-forming regions like the

Magellanic clouds, where again there is no evidence for a systematic variation of the IMF (Massey 2002). There *is* good evidence that the UV spectral hardness increases with declining metallicity, but this is entirely explained by the lower opacity of lower metallicity stars with the same IMF (McGaugh 1991).

A considerable and diverse body of evidence disfavors the notion that the IMF varies. From what we know empirically about stars over a broad range of metallicity, it seems that the Jeans scale does not play a strong role in determining the IMF. From this perspective, there is no reason to suppose that Population III stars would have formed with a radically top-heavy IMF. One can of course imagine some discontinuity at zero metallicity, and there are certainly reasons why such a discontinuity might occur (e.g., Bromm & Loeb 2003). However, this is no guarantee that such stars would form with the required properties. Supermassive Population III stars are only inferred to exist in order to cause what for Λ CDM is abnormally early reionization. This is not sufficient evidence to prove their existence.

In contrast, early structure formation is natural to MOND. Normal stars suffice; they simply form earlier. There is no need to invoke a population of supermassive Population III stars. It remains to be seen whether substantial improvements to the polarization measurement of Kogut et al. (2003) can be made or whether any such refinements would lead to grossly different interpretations. A drop in the optical depth to $\tau < 0.12$ would favor CDM, while $\tau > 0.12$ favors MOND.

4. FURTHER TESTS

Recent measurements of temperature fluctuations in the CBR are a vast advance over the situation of only a few years ago. However, they are not yet sufficient to distinguish between the very different world models of Λ CDM and MOND. On the one hand, the Λ CDM models discussed by Spergel et al. (2003) provide an excellent fit to the *WMAP* data. On the other hand, the second peak is much smaller than it was predicted to be by Λ CDM prior to its measurement, while being exactly the amplitude that was predicted by McGaugh (1999c) for the case of no-CDM. In addition, the surprisingly large polarization signal measured by *WMAP* (Kogut et al. 2003) is also expected from early structure formation with MOND. The confirmation of these two a priori predictions of McGaugh (1999c) would be a remarkable coincidence if Λ CDM were correct.

One might think it also a remarkable coincidence that Λ CDM fits the *WMAP* data so well if it is incorrect. This is not really the case. Until much better constraints on the third peak become available, one expects to be able to fit a Λ CDM model to the data even if MOND is the correct underlying physics. Indeed, because of the deviation from a pure no-CDM model expected from early structure formation in MOND, conventional fits *should* imply the need for some CDM.

There are other observations that might help to distinguish between Λ CDM and MOND. A few of these are explored in this section. This is by no means a comprehensive list. It is merely intended to give expectation values that can be tested against new data.

4.1. Baryonic Features in the Galaxy Power Spectrum

The acoustic oscillations observed in the CBR will be frozen into the power spectrum if not washed out by dark matter. This predicts (McGaugh 1999a; Sanders 2001) that there will be sharp spikes in the galaxy power spectrum $P(k)$ at low redshift. On small scales, these may well be wiped out by the

highly nonlinear growth of structure in MOND, which may lead to the exchange of power between wavenumbers (it cannot be assumed that individual k will remain independent). Indeed, Nusser (2002) finds convergence to a unique slope of the power spectrum irrespective of initial conditions. An overall shape for the power spectrum similar to that of Λ CDM is found by Sanders (2001), with a bit more power on large scales relative to small ones.

There is some hope that the baryonic features on the largest scales will survive the nonlinear evolution. If so, they will appear as sharp, narrow down-pointing spikes in a power spectrum with an otherwise smooth envelope (McGaugh 1999a; Sanders 2001). Tegmark et al. (2004) note that similar bumps and wiggles are present in three independent data sets: the IRAS Point Source Catalog Redshift Survey, the Two-Degree Field Galaxy Redshift Survey, and the Sloan Digital Sky Survey (see their Fig. 36). These are suggestive of the predicted baryonic features but are hardly convincing detections.

Detecting the narrow spikes predicted as baryonic features is extremely challenging experimentally. It requires not only large-scale surveys but also very fine resolution in k -space. The window functions typically used in power spectrum analyses are broader than the predicted features and will act to wash out their appearance if present. The experiment may be easier to perform at high redshift (Blake & Glazebrook 2003), though the rapid growth of structure in MOND may mitigate against this.

Nevertheless, the presence of sharp, strong baryonic features provides another means to discriminate between CDM and MOND. $P(k)$ should be smooth and featureless in CDM. In MOND it should contain sharply pronounced features on large scales.

4.2. Structures at High Redshift

Another observation that may help to distinguish between CDM and MOND is the amount of structure at high redshift. As noted by Sanders (1998, 2001) and McGaugh (1999a), structure should form considerably more rapidly in MOND than in CDM. There are many indications of early structure formation, and indeed, this was one of the many indicators that pushed us toward Λ CDM. The presence of a finite cosmological constant provides more time to form structure early relative to previously conventional CDM models (e.g., Mo & Fukugita 1996).

Early structure formation is quite natural in MOND (Sanders 1998, 2001; McGaugh 1999a; Nusser 2002; Stachiewicz & Kutschera 2002), so having structure in place at high redshift is quite reasonable. Indeed, the problem MOND suffers may be overproducing structure (Sanders 1998; Nusser 2002; Knebe & Gibson 2004), perhaps by a factor of ~ 2 . This is sensitive to a variety of assumptions, and the subject is not yet well developed (Sanders & McGaugh 2002), so it is unclear how serious a problem this is. Moreover, it may be mitigated if neutrinos turn out to be heavy. Late times are harder to predict in both CDM and MOND, so the strongest test is at high redshift.

The evidence for structure at high z has grown considerably in recent years. Lyman break galaxies at $z \sim 3$ are already highly clustered (Steidel et al. 2000, 2003), leading to the inference that they are very highly biased ($b \sim 6$ as opposed to ~ 1 for galaxies of comparable mass at $z = 0$), or not massive at all, with luminosities vastly enhanced by collisional star bursts (Somerville et al. 2001). In MOND, these would simply be the early stages of normal galaxies. Chen et al. (2004) find

that massive galaxies are surprisingly abundant and metal-rich at $z > 1$. Rocca-Volmerange et al. (2004) find that large ($M_* \approx 10^{12} M_\odot$) galaxies are already in place at $z \approx 4$, much earlier than should be the case with CDM. L_* galaxies can form as early as $z \approx 10$ in MOND (Sanders 1998), so these observations present no puzzle. Not only should massive objects form early, but they should also form into large-scale structures early. Palunas et al. (2004) present evidence for large voids being present in the galaxy distribution already at $z \approx 3$, for which they find only a $\sim 1\%$ probability in Λ CDM.

The high-redshift universe appears to be rich in structure, perhaps surprisingly so for our expectations from CDM. Such early structure formation is quite natural in MOND. If the latter is correct, we would expect to continue to find large numbers of massive galaxies already in place at $z > 3$, and rich clusters of galaxies at $z > 1$. Of course, the stellar content of these systems must evolve in either case, and mass evolution from merging may also occur in both. But the expectation is for there to be more structure sooner in MOND than one would nominally expect with CDM.

5. CONCLUSIONS

McGaugh (1999c) predicted various features of the power spectrum of temperature fluctuations in the CBR for no-CDM universes inspired by MOND. These predict

1. a specific first-to-second peak amplitude ratio, $A_{1,2} = 2.4$, and
2. a third peak smaller than the second.

These stem from the stipulation that there should be no CDM in a MOND universe so that baryonic damping dominates the power spectrum. The specific value of $A_{1,2}$ follows from the baryon density as given by BBN, with very little variation for plausible values of ω_b . Beyond the no-CDM model, one does expect some signature of MOND in the CBR. MOND-specific predictions are

3. an early epoch of reionization leading to a large polarization signal and
4. an excess of power on large ($l < 100$) scales.

I have tested these predictions with the *WMAP* first year data and other recent CBR experiments.

WMAP provides tests of all of these predictions except the third peak, to which its data do not yet extend ($S/N < 1$ for $l > 660$; Bennett et al. 2003). For the three items for which *WMAP* alone provides a strong test, all of the MOND predictions are confirmed. Other data must be invoked to test the third peak. These do not confirm the no-CDM prediction but do not clearly reject it (§ 2.5).

The first-to-second peak amplitude ratio measured by *WMAP* is $A_{1,2} = 2.34 \pm 0.09$ (Page et al. 2003). The amplitude of the second peak is considerably smaller than had been anticipated by Λ CDM models prior to observational constraints on its value. Yet it is accurately predicted with no free parameters by no-CDM (McGaugh 1999c). It follows simply from setting $\Omega_{\text{CDM}} = 0$ and Ω_b to its BBN value.

This peak ratio $A_{1,2}$ is very sensitive to the baryon content of the universe. The range of baryon densities acceptable to no-CDM models is in good agreement with BBN constraints from ^2H , ^4He , and ^7Li . Λ CDM requires a significantly higher Ω_b than was indicated by BBN prior to measurement of the second peak (§ 2.2). Our plain “vanilla” Λ CDM has been spiked with extra baryons.

The large polarization signal reported by *WMAP* (Kogut et al. 2003) is surprising in the context of Λ CDM but not in MOND. MOND is expected to grow structure much more rapidly than the conventional $\delta \sim t^{2/3}$. Consequently, the first stars can readily form by $z \gtrsim 15$. The high optical depth reported by *WMAP* confirms this prediction. There is no need to invoke supermassive Population III stars. A review of constraints on the sensitivity of the IMF to metallicity suggests that these are rather contrived (§ 3.1.1).

Early reionization is one clear indication of MONDian physics beyond the simple no-CDM model used to predict the peak amplitude ratios. Another possible signature of MOND is the excess of power at $l < 100$ over the nominal no-CDM prediction. This may be due to a strong ISW effect as anticipated by McGaugh (1999c). Though the data are qualitatively consistent with this prediction, a quantitative computation of this effect in MOND remains lacking.

Obtaining a detailed fit to the *WMAP* data (not just the peak ratio) requires three fit parameters: one for the position of the first peak, one for its width, and one for the amplitude. One combination of parameters that works is $\Omega_\Lambda = 0.92$, $f_\nu = 0.65$, and a tensor component $n_t = 0.04$. These should not be taken too literally, as they may simply encapsulate our ignorance of the underlying theory. The apparent tensor contribution, in particular, is simply a fudge to fit the data at $l < 100$, where a deviation from the standard calculation is expected from an enhanced ISW effect.

One interesting aspect of the detailed no-CDM fit is the apparent need for a massive neutrino ($m_\nu \sim 1$ eV). There are a

variety of effects that might cause this inference to be incorrect (see discussion in § 2.4.3). While a neutrino mass this high may seem unappealing, it provides an additional means of distinguishing between CDM and MOND. Λ CDM models cannot form sufficient structure if $m_\nu > 0.23$ eV (Spergel et al. 2003), so a robust laboratory measurement in excess of this value would in principle falsify CDM. While a smaller neutrino mass would not exclude MOND, it would make it difficult to understand rich clusters of galaxies (Aguirre et al. 2001; Sanders 2003).

Three of the four predictions listed above have been realized in the *WMAP* data. The prediction of a low third peak does not appear to be realized in other data sets. If improved measurements confirm a second-to-third peak amplitude ratio $A_{2,3} \lesssim 1$, it would favor CDM. In this case, the success of the other MOND predictions may just be a remarkable fluke. On the other hand, a definitive measurement of $A_{2,3} > 1.5$ would provide the clearest possible falsification of the existence of nonbaryonic cold dark matter.

I would also like to thank Bob Sanders for his encouragement and persistent questioning on these issues. I am also grateful to the referee for suggestions that improved the organization of this text. The author acknowledges the support of the University of Maryland through a semester General Research Board award. The work of S. S. M. is supported in part by NSF grant AST 02-06078 and NASA grant NAG5-13108.

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