Monthly Notices of the ROYAL ASTRONOMICAL SOCIETY

MNRAS **480,** 473–476 (2018)
Advance Access publication 2018 July 11

doi:10.1093/mnras/sty1884

MOND and the dynamics of NGC 1052−DF2

B. Famaey,[1★] S. McGaugh[2] and M. Milgrom[3]

[1] *Université de Strasbourg, CNRS UMR 7550, Observatoire astronomique de Strasbourg, F-67000 Strasbourg, France*
[2] *Department of Astronomy, Case Western Reserve University, Cleveland, OH 44106, USA*
[3] *Department of Particle Physics and Astrophysics, Weizmann Institute of Science, Rehovot 76100, Israel*

Accepted 2018 July 7. Received 2018 June 08; in original form 2018 April 11

ABSTRACT

The dwarf galaxy NGC 1052−DF2 has recently been identified as potentially lacking dark matter. If correct, this could be a challenge for modified Newtonian dynamics (MOND), which predicts that low surface brightness galaxies should evince large mass discrepancies. However, the correct prediction of MOND depends on both the internal field of the dwarf and the external field caused by its proximity to the giant elliptical NGC 1052. Taking both into consideration, we find $\sigma_{\rm MOND} = 13.4^{+4.8}_{-3.7}$ km s^{-1} – where the quoted error only reflects the uncertainty on the stellar mass-to-light ratio – if the dwarf is at the projected distance from its host and the MOND interpolating function is the so-called 'simple' one. More generally, we find that acceptable values of $\sigma_{\rm MOND}$ range from 8.9 to 19 km s^{-1} depending on the interpolating function, stellar mass-to-light ratio, and three-dimensional distance to the host. We also discuss a few caveats on both the observational and theoretical side. On the theory side, the internal virialization time in this dwarf may be longer than the time-scale of variation of the external field. On the observational side, the paucity of data and their large uncertainties call for further analysis of the velocity dispersion of NGC 1052−DF2, to check whether it poses a challenge to MOND or is a success thereof.

Key words: galaxies: dwarf – dark matter.

1 INTRODUCTION

In a recent paper, van Dokkum et al. (2018) report the line-of-sight velocities of 10 globular clusters around the dwarf galaxy NGC 1052−DF2. They infer from these a surprisingly low velocity dispersion for such a galaxy: $\sigma < 10.5$ km s^{-1} at 90 per cent confidence. They then deduce that the total mass within the outermost radius of 7.6 kpc would be $< 3.4 \times 10^8$ M$_\odot$. The stellar mass that van Dokkum et al. (2018) assume for this galaxy, $M_* \approx 2 \times 10^8$ M$_\odot$, is based on a stellar mass-to-light ratio of $\Upsilon_* \approx 2$ M$_\odot$ L$_\odot^{-1}$. This stellar mass is close to the dynamical mass, implying no need for dark matter. If all these deductions are correct, this could in principle contradict modified Newtonian dynamics (MOND). This finding has generated some discussions in the literature (Laporte, Agnello & Navarro 2018; Martin et al. 2018) regarding its deductions and its interpretation in the standard context. In the following, we concentrate on the MOND prediction for the velocity dispersion.

MOND (Milgrom 1983; Famaey & McGaugh 2012; Milgrom 2014) is a paradigm predicting the dynamics of galaxies directly from their baryonic mass distribution. The postulate is that, for gravitational accelerations below $a_0 \approx 10^{-10}$ m s^{-2} the actual gravitational attraction approaches $(g_{\rm N} a_0)^{1/2}$ where $g_{\rm N}$ is the usual Newtonian gravitational attraction.

The successes of MOND are best known for galaxies that are isolated, axisymmetric, and rotationally supported (e.g. Sanders & McGaugh 2002; Gentile, Famaey & de Blok 2011). In such systems, there is a clear and direct connection between the baryonic mass distribution and the rotation curve (McGaugh, Lelli & Schombert 2016; Lelli et al. 2017). This empirical relation is indistinguishable from MOND (Li et al. 2018).

MOND also has a good track record of predictive success for pressure-supported systems like NGC 1052−DF2 (McGaugh & Milgrom 2013a,b; Pawlowski & McGaugh 2014; McGaugh 2016). The analysis in such cases is complicated by the same uncertainties as in Newtonian analyses, such as that in the stellar mass-to-light ratio and the unknown anisotropy in the velocity tracers. Unique to MOND is the external field effect (EFE; Milgrom 1983; Bekenstein & Milgrom 1984; Famaey & McGaugh 2012; Haghi et al. 2016; Hees et al. 2016). Because of the non-linearity of MOND, the internal dynamics of a system can be affected by the external gravitational field in which it is immersed. When the external field dominates over the internal one, the amplitude of the MOND effect, and the corresponding amount of dark matter inferred, is reduced. Some interesting effects unique to MOND are related to the EFE, such as the prediction of asymmetric tidal streams of globular clusters (Thomas et al. 2018).

★ E-mail: benoit.famaey@astro.unistra.fr

An essential consequence of the EFE in MOND is that the predicted velocity dispersion of a dwarf galaxy depends on its environment. An object in isolation is expected to have a higher velocity dispersion than the same object in orbit around a massive host. This difference is perceptible in pairs of photometrically indistinguishable dwarf satellites of Andromeda (McGaugh & Milgrom 2013b). Indeed, the EFE was essential to the correct a priori prediction (McGaugh & Milgrom 2013a) of the velocity dispersions of the dwarfs And XIX, And XXI, and And XXV. These cases are notable for their large scale lengths and low velocity dispersions (Collins et al. 2013) – properties that were surprising in the context of dark matter but are natural in MOND. A further example is provided by the recently discovered Milky Way satellite Crater 2 (Torrealba et al. 2016). McGaugh (2016) predicted that this object would have a velocity dispersion of $2.1^{+0.9}_{-0.6}$ km s^{-1}, much lower than the nominal expectation in the context of dark matter. Caldwell et al. (2017) subsequently observed 2.7 ± 0.3 km s^{-1}. In the standard context, this must be explained through tidal stripping (Fattahi et al. 2018; Sanders, Evans & Dehnen 2018).

Here, we take the external field of the host galaxy NGC 1052 into account to predict the expected velocity dispersion of NGC 1052−DF2 in MOND. In Section 2, we first adopt the simple MOND interpolating function, the projected distance to the host, and a nominal mass-to-light ratio of $\Upsilon_* = 2\,\mathrm{M}_\odot\,\mathrm{L}_\odot^{-1}$ to obtain $\sigma_{\mathrm{MOND}} \approx 13.4$ km s^{-1}. For a factor of two uncertainty in the V-band stellar mass-to-light ratio, the allowed range becomes $9.7 \leq \sigma_{\mathrm{MOND}} \leq 18.2$ km s^{-1}. This range of variation follows simply from plausible uncertainty in the conversion of luminosity to stellar mass. Varying also the interpolating function and the three-dimensional (3D) distance to the host, we find that the typically allowed range extends to $8.9 \leq \sigma_{\mathrm{MOND}} \leq 19$ km s^{-1}. For instance, the velocity dispersion for a distance to the host larger by a factor $\sim\sqrt{2}$ than the projected distance, the standard interpolating function, and $\Upsilon_* = 2\,\mathrm{M}_\odot/\mathrm{L}_\odot$, is $\sigma_{\mathrm{MOND}} \approx 14$ km s^{-1}. Other potentially relevant uncertainties are discussed further in Section 3. Brief conclusions are given in Section 4.

2 PREDICTING THE VELOCITY DISPERSION OF NGC 1052−DF2 IN MOND

2.1 Isolated prediction

McGaugh & Milgrom (2013a) outlined the procedure by which the velocity dispersions of satellite galaxies like NGC 1052−DF2 can be predicted. For a spherical, isotropic, isolated system,

$$\sigma_{\mathrm{iso}} = \left(\frac{4}{81} G M a_0\right)^{1/4} \simeq \left(\frac{\Upsilon_*}{2}\right)^{1/4} 20\ \mathrm{km\,s^{-1}} \quad \text{for NGC 1052−DF2,} \tag{1}$$

or $16.8 \leq \sigma_{\mathrm{iso}} \leq 23.8$ km s^{-1}. This is basically the same result obtained by van Dokkum et al. (2018) for the same assumptions. However, this is not the correct MOND prediction, as this dwarf is not isolated: the external field due to the giant host NGC 1052 is not negligible.

2.2 Instantaneous external field effect prediction

The isolated MOND boost to Newtonian gravity is predicted to be observed only in systems where the *absolute* value of the gravity both internal, g, and external, g_{e} (from a host galaxy, or large-scale structure), is less than a_0. If $g_{\mathrm{e}} < g < a_0$, then we have isolated MOND effects. If instead $g < g_{\mathrm{e}} < a_0$, then the system is quasi-Newtonian. The usual Newtonian formula applies, but with a renormalized gravitational constant. When $g_{\mathrm{e}} \ll a_0$, the renormalizing factor is simply a_0/g_{e}.

The velocity dispersion estimator is simple (McGaugh & Milgrom 2013a) when $g \gg g_{\mathrm{e}}$ (isolated) or $g \ll g_{\mathrm{e}}$ (EFE) in that it depends only on the dominant acceleration (g or g_{e}). When $g \approx g_{\mathrm{e}}$, both must be taken into consideration. This turns out to be the case for NGC 1052−DF2.

To estimate these quantities, we make the same assumptions as van Dokkum et al. (2018). The galaxy and its host are located at a distance of 20 Mpc, and the projected separation from the host is 80 kpc. For a V-band stellar mass-to-light ratio of $\Upsilon_* \simeq 2$, the baryonic mass is $2 \times 10^8\,\mathrm{M}_\odot$. At the deprojected 3D half-light radius $r_{1/2} = (4/3)R_{\mathrm{e}} = 2.9$ kpc, the internal Newtonian acceleration is $g_{\mathrm{N}} = 1.34 \times 10^{-2} a_0$. This corresponds to $g_{\mathrm{i}} \sim 0.12 a_0$ for an isolated object in MOND.

For the external field, we assume $g_{\mathrm{e}} = V^2/D$ with a flat rotation curve[1] for the host NGC 1052 of $V = 210$ km s^{-1}, in accordance (within the MOND context) with the stellar mass of the host measured by Forbes et al. (2017), $M_* = 10^{11.02}\,\mathrm{M}_\odot$ – see also table 2 of Bellstedt et al. (2018) – and consistent with the kinematic observations of van Gorkom et al. (1986). At the projected distance of NGC 1052−DF2 from its host, this yields an external field $g_{\mathrm{e}} = 0.15 a_0$. This estimate of the external acceleration is rather uncertain for observational, not theoretical, reasons. For specificity, we first adopt $g_{\mathrm{e}} = 0.15 a_0$ here, and discuss the uncertainties further below.

The internal isolated MOND acceleration $g_{\mathrm{i}} \sim 0.12 a_0$ and the external one from the host $g_{\mathrm{e}} \sim 0.15 a_0$ are thus of the same order of magnitude. The exact calculation should then be made with a numerical Poisson solver. Nevertheless, a good ansatz in such a case is to consider the net MOND effect in one dimension (equation 59 of Famaey & McGaugh 2012):

$$(g + g_{\mathrm{e}})\,\mu\left(\frac{g + g_{\mathrm{e}}}{a_0}\right) = g_{\mathrm{N}} + g_{\mathrm{e}}\mu\left(\frac{g_{\mathrm{e}}}{a_0}\right), \tag{2}$$

Here, μ is the MOND interpolating function, g is the norm of the internal gravitational field one is looking for, g_{e} is the external field from the host, and g_{N} is the Newtonian internal gravitational field. Adopting here the 'simple' interpolating function of MOND, which is known to provide good fits to galaxy rotation curves (Famaey & Binney 2005; Gentile et al. 2011), this equation can easily be solved for g. Wolf et al. (2010) provide us with a mass estimator at the deprojected 3D half-light radius $r_{1/2}$, where we can consider that the system feels the effect of the renormalized gravitational constant in MOND, $G_{\mathrm{eff}} = G \times [g(r_{1/2})/g_{\mathrm{N}}(r_{1/2})]$.

Solving equation (2) yields $G_{\mathrm{eff}} = 3.64G$ at the half-light radius. Using this and solving the mass estimator of Wolf et al. (2010) for the velocity dispersion leads to

$$\sigma_{\mathrm{MOND}} \approx 13.4\ \mathrm{km\,s^{-1}}. \tag{3}$$

This is lower than the isolated case (20 km s^{-1}) considered by van Dokkum et al. (2018), as is always the case when the EFE is important. This analytic estimate of the predicted velocity dispersion in MOND is in good agreement with an independent estimate by Kroupa and collaborators (private communication) based on fits to numerical simulations by Haghi et al. (2009) of stellar systems embedded in an external field.

[1]This assumption of a flat rotation curve is the only self-consistent one to make in MOND as it is one of the central tenets of the paradigm.

To estimate an uncertainty on this prediction, we consider a factor of two variation in the *V*-band mass-to-light ratio. That is, we obtain $\sigma_{\rm MOND} = 9.7\,{\rm km\,s^{-1}}$ for $\Upsilon_* = 1\,{\rm M_\odot\,L_\odot^{-1}}$, and $\sigma_{\rm MOND} = 18.2\,{\rm km\,s^{-1}}$ for $\Upsilon_* = 4\,{\rm M_\odot\,L_\odot^{-1}}$. Whether this range is appropriate for this particular dwarf is pure supposition that we must make in any theory. Note also that if we apply the same range to the mass of the host galaxy, this largely compensates for possible variations in distance of NGC 1052−DF2 with respect to the host.

We nevertheless check hereafter that the choice of MOND interpolating function and the 3D distance to the host do not radically affect the above prediction. For the 'standard' MOND interpolating function (for a definition, see e.g. Famaey & Binney 2005; Famaey & McGaugh 2012), we find that the prediction becomes $\sigma_{\rm MOND} = 12.2^{+4.3}_{-3.3}\,{\rm km\,s^{-1}}$ for the same assumption on the value of the external field. Finally, letting the external field decrease to $g_{\rm e} = 0.1a_0$, namely, decreasing it by a factor $\sim\sqrt{2}$ to account for the uncertainty in the 3D distance from the host, leads to $\sigma_{\rm MOND} = 15^{+4}_{-3.9}\,{\rm km\,s^{-1}}$ and $\sigma_{\rm MOND} = 14^{+4.5}_{-3.6}\,{\rm km\,s^{-1}}$ in the simple and standard case, respectively. In summary, given the large observational uncertainty in the external field, the acceptable values of $\sigma_{\rm MOND}$ range from 8.9 to 19 $\,{\rm km\,s^{-1}}$.

3 CAVEATS

The prediction of velocity dispersions in MOND is most clean and simple in the isolated, low-acceleration case, depending only on the stellar mass given the usual assumptions of spherical symmetry, isotropy, and dynamical equilibrium. Examples of isolated dwarfs are provided by And XXVIII (McGaugh & Milgrom 2013a,b) and Cetus (Pawlowski & McGaugh 2014). The problem becomes more involved when the EFE is relevant. In addition to the stellar mass of the dwarf, the extent of the stellar distribution matters to the determination of the internal acceleration. Diffuse objects[2] like NGC 1052−DF2 or Crater 2 are more subject to the EFE than compact objects like globular clusters. In addition to the internal field, we also need to measure the external field. This depends on the mass of the host and the 3D distance separating the host from its dwarf satellite. In dense environments where there may be multiple massive systems, matters obviously become more complicated still.

Estimating the external field can be challenging. We discuss here a few of the issues that arise, both observational (how well do the data constrain $g_{\rm e}$) and theoretical (is it fair to assume the instantaneous value of the EFE at this moment in the dwarf's orbit). The estimates made above adopt reasonable assumptions consistent with those of van Dokkum et al. (2018), but these need not be correct.

3.1 Observational caveats

First, let us note that the use of a biweight dispersion to account for potential contamination actually leads to the omission of a single velocity tracer by van Dokkum et al. (2018, the object labelled 98 in their fig. 2). Once excluded, van Dokkum et al. (2018) argued that the best-fitting intrinsic velocity dispersion would be as low as $3.2\,{\rm km\,s^{-1}}$, at which point we would find ourselves in a perverse situation where there would not be enough dynamical mass to explain the observed stars. It was of course beyond the scope of the present work, focused on the MOND prediction, to reanalyse these data, but after our original submission, Martin et al. (2018) revised the estimation of the velocity dispersion of NGC 1052−DF2 by building a generative model and sampling the posterior probability distribution function of the intrinsic velocity dispersion. Assuming a uniform contamination model, this method yielded $\sigma = 9.2^{+4.8}_{-3.6}\,{\rm km\,s^{-1}}$ ($< 17.3\,{\rm km\,s^{-1}}$ at the 90 per cent confidence level).

Second, we note that, when comparing this value to our MOND prediction, one might also be concerned whether this tracer population is representative of the mass-weighted velocity dispersion (see discussion in Milgrom 2010, and references therein), or if nearly face-on rotation of the globular cluster system could play a role.

Finally, a key uncertainty in our analysis is the distance, which is required to determine the absolute acceleration scale. Distances are notoriously difficult to constrain for individual galaxies. We largely agree with the assessment of the distance laid out by van Dokkum et al. (2018), but for completeness point out where this matters most.

The absolute distance to NGC 1052−DF2 matters in so far as this determines its association with the putative giant host NGC 1052. If the two are not associated, then the object is likely isolated. Even that is not guaranteed given the proximity of NGC 1042 along the line of sight. Setting that aside, a closer distance would be more consistent with the isolated MOND case as the stellar mass shrinks as D^2 and the predicted velocity dispersion along with it. Conversely, a significantly larger distance (beyond NGC 1052) would be problematic for MOND as the object would then be more massive and should have a higher velocity dispersion.

The absolute 3D distance relative to the host NGC 1052 also matters, as it affects the strength of the external field. While the projected separation of 80 kpc is well determined for the assumed distance of 20 Mpc, the additional distance along the line of sight is not. As this increases, the EFE weakens. As explained above in Section 2, this is largely degenerate with uncertainties in the mass of the host, and varying this distance by a typical $\sqrt{2}$ factor leads to an increase of $\sim 1.5\,{\rm km\,s^{-1}}$ in the predicted velocity dispersion. This is also slightly degenerate with the choice of interpolating function, as changing from the simple to the standard interpolating function leads to a decrease of $\sim 1\,{\rm km\,s^{-1}}$ in the predicted velocity dispersion.

3.2 Theoretical caveats

MOND is a non-linear theory. The approximation we have adopted to predict the velocity dispersion implicitly assumes that the external field is effectively constant. In reality, the amplitude and vector direction of the external field varies as the dwarf orbits its host. The issue then becomes whether the internal dynamics of the dwarf have time to come into equilibrium with the continually changing external field (Brada & Milgrom 2000). If so, the prediction we obtain is valid. If not, it is no more valid than if the condition of dynamical equilibrium is not obtained in Newtonian dynamics, as would be the case if the dwarf were being tidally stripped of its dark matter.

In MOND, the internal dynamics of an object depends on the external field in which it is immersed as soon as the external field dominates over the internal one. If the external field does not vary quickly in time with respect to the time it takes for the object to settle back to virial equilibrium, the 'instantaneous' external field approximation that we have applied is appropriate. This works well for quasi-circular orbits, as the amplitude of the external field is steady even though the orientation vector varies. In general, the external field is expected to change over time-scales D/V, where D is the distance to NGC 1052 and V is its circular velocity. This

[2]The quantity of merit in MOND is the surface density, not the mass. Lower surface density means lower acceleration. That NGC 1052−DF2 is massive for a dwarf does not shield it from the EFE as it is a diffuse object.

time-scale is of the order of 3×10^8 yr. The relevant internal time is the time it takes for the system (in the present case, the system of globular clusters being utilized as velocity tracers) to come back to virial equilibrium. If this would be just half a revolution around NGC 1052−DF2 at a typical radius of 5 kpc for the globular clusters, and for a typical velocity of 10 km s^{-1}, this time would be 1.6×10^9 yr. So the dynamical state of the system of globular clusters has not obviously had time to equilibrate. Consequently, it might retain a memory of a previous value of the external field, and the system would then basically be out of equilibrium. Much depends on the details of its orbit: whether it is on a radial or circular orbit, and when its last pericentre passage was.

The stability of dwarfs orbiting massive hosts has been investigated numerically by Brada & Milgrom (2000). Applying their criteria as in McGaugh & Wolf (2010), it indeed appears that NGC 1052−DF2 may be on the fringe where dynamical equilibrium ceases to apply for the ultrafaint dwarfs of the Milky Way. Basically, the parameter γ of equation (6) of McGaugh & Wolf (2010) is the number of orbits a star should make within a dwarf at the half-light radius for every orbit the dwarf makes about its host, and roughly quantifies how adiabatic the effects of the external field are. A typical minimum value for the adiabatic condition to hold should be of the order of $\gamma \sim 7$ or 8. For NGC 1052−DF2 and the standard parameters assumed in the rest of the paper, we find $\gamma = 6.5$, hence on the fringe.

More generally, Brada & Milgrom (2000) describe how an orbit dwarf may experience oscillations in velocity dispersion and scale length as it orbits from pericentre to apocentre. It is easy to imagine that NGC 1052−DF2 is in a phase where its velocity dispersion is minimized while the scale length is maximized. While there is no way to ascertain if this is the case, this is an anticipated effect in MOND, and illustrates some of the subtle non-linear effects that can arise.

We may also estimate the tidal radius in MOND (equation 5 of McGaugh & Wolf 2010). At the projected distance from the host, 80 kpc, this is ∼8 kpc. Intriguingly, this is just beyond the outermost globular cluster. Consequently, the limit on the distance from the host obtained from the Jacobi radius by van Dokkum et al. (2018) does not apply in MOND.

The caveats discussed in this subsection are theoretical: non-linear theories can be sensitive, and great care must be taken to extract the correct prediction. That said, we see no reason to expect the bulk velocity dispersion to vary outside the bounds stipulated by the plausible range in the stellar mass-to-light ratio, at least for our calculations referenced to the half-light radius. It would be interesting to compute the detailed radial variation of the velocity dispersion (Alexander et al. 2017), but such a computation is well beyond the scope of this letter, and indeed, beyond the scope of foreseeable observations to test for this remote dwarf galaxy.

4 CONCLUSIONS

We have predicted the velocity dispersion of the dwarf galaxy NGC 1052−DF2 (van Dokkum et al. 2018) in MOND. For this, one must take the EFE into consideration. The acceleration of stars within the dwarf is comparable to the acceleration of the dwarf in its orbit around its host galaxy, NGC 1052. Consequently, neither field may be ignored, and the net MOND effect is reduced from the isolated case. This, in turn, reduces the predicted velocity dispersion compared to the isolated case considered in the original MOND estimate of van Dokkum et al. (2018). Taking the external field into account, as well as uncertainties in the MOND interpolating function, stellar mass-to-light ratio, and 3D distance to the host, we find that acceptable values of $\sigma_{\rm MOND}$ range from 8.9 to 19 km s^{-1}. This is currently not in conflict with the velocity dispersion measurement of van Dokkum et al. (2018), nor with the value re-evaluated by Martin et al. (2018) for their baseline model including possible contamination. We note that a key uncertainty in our analysis is the distance, which is required to determine the absolute acceleration scale. Here, we chose to largely trust the distance estimated by van Dokkum et al. (2018). In any case, further observational data will thus be needed to check whether NGC 1052−DF2 poses a challenge to MOND or is a success thereof.

REFERENCES

Alexander S. G., Walentosky M. J., Messinger J., Staron A., Blankartz B., Clark T., 2017, ApJ, 835, 233
Bekenstein J., Milgrom M., 1984, ApJ, 286, 7
Bellstedt S. et al., 2018, MNRAS, 476, 4543
Brada R., Milgrom M., 2000, ApJ, 541, 556
Caldwell N. et al., 2017, ApJ, 839, 20
Collins M. L. M. et al., 2013, ApJ, 768, 172
Famaey B., Binney J., 2005, MNRAS, 363, 603
Famaey B., McGaugh S. S., 2012, Living Rev. Relativ., 15, 10
Fattahi A., Navarro J. F., Frenk C. S., Oman K. A., Sawala T., Schaller M., 2018, MNRAS, 476, 3816
Forbes D. A., Sinpetru L., Savorgnan G., Romanowsky A. J., Usher C., Brodie J., 2017, MNRAS, 464, 4611
Gentile G., Famaey B., de Blok W. J. G., 2011, A&A, 527, A76
Haghi H., Baumgardt H., Kroupa P., Grebel E. K., Hilker M., Jordi K., 2009, MNRAS, 395, 1549
Haghi H., Bazkiaei A. E., Zonoozi A. H., Kroupa P., 2016, MNRAS, 458, 4172
Hees A., Famaey B., Angus G. W., Gentile G., 2016, MNRAS, 455, 449
Laporte C. F. P., Agnello A., Navarro J. F., 2018, MNRAS, preprint (arXiv: 1804.04139)
Lelli F., McGaugh S. S., Schombert J. M., Pawlowski M. S., 2017, ApJ, 836, 152
Li P., Lelli F., McGaugh S., Schormbert J., 2018, A&A, in press (arXiv: 1803.00022)
Martin N. F., Collins M. L. M., Longeard N., Tollerud E., 2018, ApJ, 859, L5
McGaugh S., Milgrom M., 2013a, ApJ, 766, 22
McGaugh S., Milgrom M., 2013b, ApJ, 775, 139
McGaugh S. S., 2016, ApJ, 832, L8
McGaugh S. S., Lelli F., Schombert J. M., 2016, Phys. Rev. Lett., 117, 201101
McGaugh S. S., Wolf J., 2010, ApJ, 722, 248
Milgrom M., 1983, ApJ, 270, 365
Milgrom M., 2010, MNRAS, 403, 886
Milgrom M., 2014, Scholarpedia, 9, 31410
Pawlowski M. S., McGaugh S. S., 2014, MNRAS, 440, 908
Sanders J. L., Evans N. W., Dehnen W., 2018, MNRAS, 478, 3879
Sanders R. H., McGaugh S. S., 2002, ARA&A, 40, 263
Thomas G. F., Famaey B., Ibata R., Renaud F., Martin N. F., Kroupa P., 2018, A&A, 609, A44
Torrealba G., Koposov S. E., Belokurov V., Irwin M., 2016, MNRAS, 459, 2370
van Dokkum P. et al., 2018, Nature, 555, 629
van Gorkom J. H., Knapp G. R., Raimond E., Faber S. M., Gallagher J. S., 1986, AJ, 91, 791
Wolf J., Martinez G. D., Bullock J. S., Kaplinghat M., Geha M., Muñoz R. R., Simon J. D., Avedo F. F., 2010, MNRAS, 406, 1220

This paper has been typeset from a TEX/LATEX file prepared by the author.