Laws of Galactic Rotation

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Galaxies

“Island Universes”

Gravitationally self-bound entities composed of stars, gas, dust, [& dark matter(?)]

Huge dynamic range in size
0.1 - 300 kpc

stellar mass
$10^6 - 10^{12} \, M_\odot$

gas fraction
0 - 99%

surface brightness
$10 - 10^4 \, L_\odot^2$

dynamical mass
$10^9 - 10^{13} \, M_\odot$

$V_c = 72$

$V_c = 300$

$V_c = 134$

$V_c = 38$
3 Laws of Galactic Rotation

1. Rotation curves tend towards asymptotic flatness \( V_f \rightarrow \text{constant} \)

2. Baryonic mass scales as the fourth power of rotation velocity (Baryonic Tully-Fisher)

\[ M_b \propto V_f^4 \]

3. Gravitational force correlates with baryonic surface density

\[ -\frac{\partial \Phi}{\partial R} \propto \Sigma_b^{1/2} \]
1. Rotation curves tend to become flat at large radii

\[ V \propto \text{const} \]

\[ M \propto R \]

\[ \rho \propto R^{-2} \]


...and stay flat to the largest radii probed
Mass modeling

NGC 6946
Mass modeling

NGC 6946
Must estimate mass-to-light ratio $M^*/L$ for stars.

Not necessary for gas - conversion from 21 cm to gas mass known from physics of spin flip transition.
NGC 6946 velocity field

\[ V_{\text{sini}} = V_{\text{sys}} + V_c \cos \theta + V_r \sin \theta \]

THINGS (Walter et al. 2008; de Blok et al. 2008)
**Mass model**

Solve Possion’s Eqn for the observed surface density distribution

$$\nabla^2 \Phi = 4\pi G \rho$$

assuming a nominal disk thickness

$$\rho(R, z) = \Sigma(R) \nu(z)$$

Equate centripetal acceleration with gravitational force

$$\frac{V^2}{R} = -\frac{\partial \Phi}{\partial R}$$

To find expected $V(R)$ for each baryonic component
- stars in bulge
- stars in disk
- gas
**Mass model**

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To find expected \( V(R) \) for each baryonic component
- stars in bulge
- stars in disk
- gas
2. Rotation curves amplitude correlates with mass:

Flat rotation curves continue to occur in quite small systems ($V_{\text{flat}} \sim 20 \text{ km/s}$).
Luminosity and line-width are presumably proxies for stellar mass and rotation velocity.
Stellar Mass Tully-Fisher relation

nominal $M^*/L$ (Kroupa IMF)

$$M_* = \left( \frac{M_*}{L} \right) L$$

Stellar Mass

$M_*(M_*)$

$10^{12}$

$10^{11}$

$10^{10}$

$10^{11}$

$10^{9}$

$10^{8}$

$10^{7}$

$10^{6}$

$10^{5}$

$10^{4}$

$10^{3}$

$10^{2}$

$\frac{1}{2} W_{20} \ (\text{km} \ s^{-1})$

line-width

Bothun et al. (1985)

Sakai et al. (2001)
...but stellar mass is completely dependent on choice of mass-to-light ratio (and degenerate with distance)

double $M^*/L$
Stellar Mass Tully-Fisher relation

nominal $M^*/L$

...but stellar mass is completely dependent on choice of mass-to-light ratio (and degenerate with distance)
Stellar Mass Tully-Fisher relation

...but stellar mass is completely dependent on choice of mass-to-light ratio (and degenerate with distance)

half $M^*/L$
Scatter in TF relation reduced with resolved rotation curves (Verheijen 2001)
Low mass galaxies tend to fall below extrapolation of linear fit to fast rotators (Matthews, van Driel, & Gallagher 1998; Freeman 1999)

\[ M_* = \left( \frac{M_*}{L} \right) L \]
Adding gas to stellar mass restores a single continuous relation for all rotators.

\[ M_b = M_* + M_g \]

Baryonic mass is the important physical quantity. It doesn’t matter whether the mass is in stars or in gas.
Twice Nominal $M^*/L$

Now instead of a translation, the slope pivots as we vary $M^*/L$.

Scatter increases as we diverge from the nominal $M^*/L$. 
Nominal $M^*/L$

Now instead of a translation, the slope pivots as we vary $M^*/L$.

Scatter increases as we diverge from the nominal $M^*/L$. 
Half Nominal $M^*/L$

Now instead of a translation, the slope pivots as we vary $M^*/L$.

Scatter increases as we diverge from the nominal $M^*/L$. 
Quarter Nominal $M^*/L$

Now instead of a translation, the slope pivots as we vary $M^*/L$.

Scatter increases as we diverge from the nominal $M^*/L$. 

$V_f$ (km s$^{-1}$) 
outer (flat) velocity
Now instead of a translation, the slope pivots as we vary $M^*/L$. Scatter increases as we diverge from the nominal $M^*/L$. 

Zero $M^*/L$
Gas dominated galaxies provide an absolute calibration of the mass scale.

Systematic errors in $M^*/L$ no longer dominate the error budget for galaxies with $M_g > M^*$. 
\[ M_b = AV_f^4 \]

\[ A = 47 \pm 3 \text{ (stat)} \pm 5 \text{ (sys)} \, \text{M}_\odot \, \text{km}^{-4} \, \text{s}^4 \]
The data specify a particular acceleration scale: \( a = \frac{V_f^4}{GM_b} \)

The data are consistent with zero intrinsic scatter. 

\[ a_\dagger \approx \Lambda^{1/2} \]

The data are consistent with zero intrinsic scatter.
The BTFR is just the zeroth moment, as it were - total baryonic mass vs. characteristic circular velocity. There is more information in the distribution of mass.
No residuals from TF with size or surface density

Same (M,V) but very different size and surface density

which is strange, since \( V^2 = \frac{GM}{R} \)
No residuals from TF with size or surface density for disks

\[ V^2 = \frac{GM}{R} \rightarrow \frac{\delta \log(V)}{\delta \log(R)} = -\frac{1}{2} \text{ expected slope (dotted line)} \]

Note: large range in size at a given mass or velocity

A contradiction to conventional dynamics?
2nd Law: the Baryonic Tully-Fisher Relation

- Fundamentally a relation between the baryonic mass of a galaxy and its rotation velocity
  \[ M_b = M^* + M_g = 47 V_f^4 \] (McGaugh 2012)
- doesn’t matter if it is stars or gas
- Intrinsic scatter negligibly small
- Can mostly be accounted for by the expected variation in stellar \( M^*/L \)
- Physical basis of the relation remains unclear

Relation has real physical units if slope has integer value -
Slope appears to be 4 if Vflat is used.
Rotation curve amplitude and shape correlate with luminosity
Universal Rotation curve (Persic & Salucci 1996)

$V(R/R_{\text{opt}})$ correlates with Luminosity.

NOT just $V(R)$ - must be normalized by optical size $R_{\text{opt}}$. 

*Figure 4.* The universal rotation curve of spiral galaxies. Radii are in units of $R_{\text{opt}}$. 
Remember our TF pair?

Radius in physical units (kpc)
The dynamics knows about the distribution of baryons, not just their total mass.
Rotation curve shapes depend on luminosity and surface density
The mass discrepancy sets in sooner and is more severe in LSB galaxies.
At all radii, the baryonic surface density correlates with the acceleration (gravitational force per unit mass)

with the acceleration (gravitational force per unit mass)
Just looking at the peak radius

\[ a \sim \Sigma_b^{1/2} \]

Gravitational force is related to the baryonic surface density
Devil in the details...

Renzo’s Rule: (2004 IAU; 1995 private communication) “When you see a feature in the light, you see a corresponding feature in the rotation curve.”
In NGC 6946, a tiny bulge (just 4% of the total light) leaves a distinctive mark.
Renzo’s Rule:
“When you see a feature in the light, you see a corresponding feature in the rotation curve.”

In NGC 1560, a marked feature in the gas is reflected in the kinematics, even though it accounts for little of the dynamical mass.

Gentile et al. (2010)
Renzo’s Rule:

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The Mass Discrepancy correlates with acceleration and baryonic surface density
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Quantify by defining the Mass Discrepancy:

\[ \mathcal{D} = \frac{V^2}{V_b^2} = \frac{V^2}{\Upsilon_* \nu_*^2 + V_g^2} \]

The Mass Discrepancy correlates with acceleration and baryonic surface density.
74 galaxies
> 1000 points
(all data)

60 galaxies
> 600 points
(errors < 5%)

radius

orbital frequency

acceleration

Gravitational force is related to the baryonic surface density

\[ \tau_*^K = 0.6 \, M_\odot / L_\odot \]
Empirical calibration of the mass discrepancy-acceleration relation

Can fit a fcn to the data $D(g_b/a_\dagger)$

\[
D = \left(1 - e^{-\sqrt{g_b/a_\dagger}}\right)^{-1}
\]

with $a_\dagger = 1.23 \pm 0.03$ Å s\(^{-2}\)

or equivalently, $\Sigma_\dagger = a_\dagger / G = 880$ M\(_\odot\) pc\(^{-2}\) $\approx 1.8$ kg m\(^{-2}\)

- a stiff piece of construction paper
Can do it lots of times. Works for bright galaxies and faint,
and those of low surface brightness where baryons are everywhere sub-dominant.
Residuals from mapping baryonic rotation curve to the total rotation curve with mass-discrepancy acceleration relation.
Light and Mass

• Many indications of a strong connection between the distribution of baryons and the dynamics:
  
• Rotation curve shape correlates with luminosity (Rubin et al. 1980)
  
• Universal Rotation Curve (Persic & Salucci 1996)
  
• Renzo’s Rule (Sancisi 2004)
  
• Mass Discrepancy-Acceleration Relation (McGaugh 2004)
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What does it mean?

- **IF** these are true Natural Laws, it makes more sense to explain them with gravity than with dark matter.
- MOND was motivated by the First Law
- MOND predicted the other two laws a priori
- Need a relativistic theory encompassing GR and MOND
- There is something special about the scale \( a_+ = 1.2 \, \text{Å} \, \text{s}^{-2} \)

\[
\Sigma_+ = a_+ / G = 880 \, \text{M}_\odot \, \text{pc}^{-2} \approx 1.8 \, \text{kg} \, \text{m}^{-2}
\]