The collision velocity of the bullet cluster in conventional and modified dynamics

G. W. Angus¹, S. S. McGaugh²
¹SUPA, School of Physics and Astronomy, University of St. Andrews, Scotland KY16 9SS
²Department of Astronomy, University of Maryland, College Park, MD 20742-242 USA

ABSTRACT
We consider the orbit of the bullet cluster 1E 0657-56 in both CDM and MOND using accurate mass models appropriate to each case in order to ascertain the maximum plausible collision velocity. Impact velocities consistent with the shock velocity (∼ 4700 km s⁻¹) occur naturally in MOND. CDM can generate collision velocities of at most ∼ 3800 km s⁻¹, and is only consistent with the data provided that the shock velocity has been substantially enhanced by hydrodynamical effects.

Key words: gravitation - dark matter - galaxies: clusters: individual (1E 0657-56)

1 INTRODUCTION
Many lines of observational evidence now oblige us to believe that the universe is filled with a novel, invisible form of mass that dominates gravitationally over normal baryonic matter. In addition, a dark energy component which exerts negative pressure to accelerate the expansion of the universe is also necessary. Though this ΛCDM paradigm is well established, we still have only ideas about what these dark components might be, and no laboratory detections thereof.

One possible alternative to ΛCDM is the Modified Newtonian Dynamics (MOND; Milgrom 1983a,b,c). This hypothesis has been more successful than seems to be widely appreciated (McGaugh & de Blok 1998; Sanders & McGaugh 2002), and has received a theoretical boost from the introduction of generally covariant formulations (Bekenstein 2004; Sanders 2005; Zlosnik, Ferreira, & Starkman 2007). The dark matter and alternative gravity paradigms are radically different, so every observation that might distinguish between them is valuable.

ACDM is known to work well on large scales (e.g., Spergel et al. 2006) while MOND is known to work well in individual galaxies (Sanders & McGaugh 2002). This success extends over five decades in mass (Fig.1) ranging from tiny dwarfs (e.g., Milgrom & Sanders 2007) through spirals of low surface brightness (de Blok & McGaugh 1998) and high surface brightness (Sanders 1996; Sanders & Noordermeer 2007) to massive ellipticals (Milgrom & Sanders 2003). However, MOND persistently fails to completely explain the mass discrepancy in rich clusters of galaxies. Consequently, clusters require substantial amounts of non-luminous matter in MOND.

That rich clusters contain more mass than meets the eye in MOND goes back to Milgrom's original papers (Milgrom 1983c). At the time, the discrepancy was very much larger than it is today, as it was not then widely appreciated how much baryonic mass resides in the intra-cluster medium. Further work on the X-ray gas (e.g., Sanders 1994, 1999) and with velocity dispersions (McGaugh & de Blok 1998) showed that MOND was at least within a factor of a few, but close inspection revealed a persistent discrepancy of a factor of two or three in mass (e.g., Gerbal et al. 1992; The & White 1998; Pointecouteau & Silk 2005). Weak gravitational lensing (Angus et al. 2007a; Takahashi & Chiba 2007) provides a similar result.

To make matters worse, the distribution of the unseen mass does not trace that of either the galaxies or the X-ray gas (Aguirre et al. 2001; Sanders 2003; Angus et al. 2007b; Sanders 2007). The unseen mass is clustered within the inner 100 or 300kpc depending on temperature. In Fig. 1 we plot the baryonic mass of many spiral galaxies and clusters against their circular velocity together with the predictions of MOND and CDM. MOND is missing mass at the cluster scale. CDM suffers an analogous missing baryon problem on the scale of individual galaxies.

The colliding bullet cluster 1E-0657-56 (Clowe et al. 2004,2006, Bradac et al. 2006, Markevitch et al. 2003, 2007) illustrates in a spectacular way the residual mass discrepancy in MOND. While certainly problematic for MOND as a theory, it does not constitute a falsification thereof. Indeed, given that the need for extra mass in clusters was already well established, it would have been surprising had this effect not also manifested itself in the bullet cluster. The new
information the bullet cluster provides is that if there is indeed more mass in clusters as required by MOND, it must be in some collision-less form.

It is a logical fallacy to conclude that because extra mass is required by MOND in clusters, that dark matter is required throughout the entire universe. While undeniably problematic, the residual mass discrepancy in MOND is limited to rich clusters of galaxies: these are the only systems in which it systematically fails to remedy the dynamical mass discrepancy (see discussion in Sanders 2003). Could we be absolutely certain that we had accounted for all the baryons in clusters, then MOND would indeed be falsified. But CDM suffers an analogous missing baryon problem in galaxies (Fig.1) in addition to the usual dynamical mass discrepancy, yet this is not widely perceived to be problematic. In either case we are obliged to invoke the existence of some hidden mass (HM) which is presumably baryonic (or perhaps neutrinos) in the case of MOND. In neither case is there any danger of violating big bang nucleosynthesis constraints. The integrated baryonic mass density of rich clusters is much less than that of all baryons; having the required mass of baryons in clusters would be the proverbial drop in the bucket with regards to the global missing baryon problem.

A pressing question is the apparently high relative velocity between the two clusters that comprise the bullet cluster 1E 0657-56 (Clowe et al. 2006, Bradac et al. 2006, Markevitch et al. 2003, 2007). The relative velocity derived from the gas shockwave is \( v_{\text{rel}} = 4740 \pm 550 \) km s\(^{-1}\) (Clowe et al. 2006). Taken at face value, this is very high, and seems difficult to reconcile with CDM (Hyashi & White 2006). The problem is sufficiently large that it has been used to argue that the gas shockwave is the only long range force in the dark sector (Farrar & Rosen 2007). Here we examine the possibility of such a high relative velocity in the case of both CDM and MOND.

One critical point that has only very recently been addressed is how the shock velocity relates to the collision velocity of the clusters. Naively, one might expect the dissipationless collision of the gas clouds to slow things down so that the shock speed would provide a lower limit on the collision speed. Recent hydrodynamical simulations (Springel & Farrar 2007; Milosavljevic et al. 2007) suggest the opposite. A combination of effects in the two hydrodynamical simulations show that the shock velocity may be higher than the impact velocity. The results of the two independent hydrodynamical simulations do not seem to be in perfect concordance, and the precise result seems to be rather model dependent. Nevertheless, it seems that the actual relative velocity lies somewhere in the range 3500-4500 km s\(^{-1}\).

The difficulties posed by a high collision velocity for CDM have been discussed previously by Hayashi & White (2006) and Farrar & Rosen (2007). Whereas Springel & Farrar (2007) and Milosavljevic et al. (2007) consider the complex hydrodynamic response of the two gas clouds during the ongoing collision, here we investigate the ability of two clusters like those comprising the bullet cluster to accelerate to such a high relative velocity in the case of both CDM and MOND prior to the merger. We compute a simple free fall model for the two clusters in an expanding universe, and ask whether the observed collision velocity can be generated within the time available. We take care to match the mass models to the specific observed properties of the system appropriate to each flavor of gravity in order to realistically evaluate the orbit of the clusters prior to their collision.

2 MODELING THE FREEFALL

We wish to address a simple question. Given the observed masses of the two clusters, is it possible to account for the measured relative velocity from their gravitational freefall? The expansion of the universe mitigates against large velocities, since the clusters must decouple from the Hubble flow before falling together. Presumably it takes some time to form such massive objects, though this is expected to occur earlier in MOND than in ACDM (Sanders 1998, 2001; McGaugh 1999, 2004; Nusser 2002; Stachniewicz & Kutschera 2002; Knebe & Gibson 2004; Dodelson & Liguori 2006). The clusters are observed at \( z = 0.3 \), giving at most 10Gyrs for them to accelerate towards each other. This imposes an upper limit to the velocity that can be generated gravitationally. Without doing the calculation, it is not obvious whether the larger masses of the clusters in CDM or the stronger long range force in MOND will induce larger relative velocities.

Since we know the state of the system directly prior to collision, it makes sense to begin our simulations from the final state and work backwards in time towards when the relative velocity was zero. This point, where the clusters have zero relative velocity, is when they turned around from the Hubble flow and began their long journey gravitating towards each other. Working backwards in time leads to potentially counter-intuitive discussions (such as Hubble contraction), which we try to limit.

We must account for the Hubble expansion in a manner representing the universe before \( z = 0.3 \). The detailed form of the expansion history of the universe \( a(t) \) is not known in the case of MOND, nor is it particularly important for this calculation. The important aspect is the basic fact that...
the universe is expanding and the mutual attraction of the clusters must overcome this before they can plunge together at high velocity. For our purposes, it suffices to approximate the scale factor as $a(t) = (t/10\text{Gyr})^{2/3}$ since the clusters are observed at a redshift before which the universe was matter dominated. We use this in both the CDM and MOND simulations.

We implement the scale factor in the simulations through the equation of motion
\[
\frac{1}{a(t)} \frac{da(t)v}{dt} = g. \tag{1}
\]
Solving this numerically, from time step to time step we calculate the ratio of the scale factor in the previous time step to the current time step (i.e. $a(t_{i-1})/a(t_i)$; we use negative time steps to move backwards in time from the presently known configuration, so $a(t_{i-1})/a(t_i) > 1$ (higher $i$ means earlier universe). Eq 1 differs in MOND and CDM not only because the law of gravity is altered, but also because the gravitating masses are higher in CDM.

The initial conditions are the crux of the problem, with at least 4 unknowns. These include the masses of the two clusters, the relative velocity of the clusters, and the distance of separation between the two when they had this relative velocity. The separation is the same in MOND and Newtonian gravity, but the Newtonian mass is higher.

The relative velocity of the two clusters can be measured because in the last few 100Myrs cluster 2 (the less massive sub cluster) has passed through the centre of cluster 1 (the more massive main cluster). The ram pressure has imposed a smooth bow shock (Markevitch et al. 2004; Markevitch et al. 2007) on the gas of cluster 2. Since the relative velocity is the foundation of the problem we leave it free and try many variations of it. In our simulation, we think it sensible to consider the separation of the two clusters (i.e. of the two centres of mass) when they had the calculated relative velocity to be when the leading edge of cluster 2's gas cloud began to pass through the dense region of gas belonging to cluster 1 and separate from the dark matter. It appears that the centre of cluster 2's gas cloud (the location of the bullet) is preceeded by the bow shock by around 180kpc further in the direction of travel. We take 180kpc to also be the radius within which the gas of cluster 1 was dense enough to imprint the bow shock. Indeed, the gas mass of cluster 1 (cluster 2) is only measured out to 180kpc (100kpc) and could not be found further detailed in the literature. Therefore, we take when the centre of cluster 1 and cluster 2 are separated by 360kpc as when the two pre-collision clusters had the relative velocity of $v_{rel}$ related to the shock velocity $4740^{+3590}_{-550}$ km s$^{-1}$. Now of course, they are on the opposite sides on the sky after having passed through each other and the gas has been offset from the HM.

3 THE COLLISION IN CDM

In the CDM universe we can run accurate N-body simulations once the density profiles of the two clusters are specified. The best fit to the convergence map by Clowe et al. (2006) with an NFW model (Navarro, Frenk & White 1996,1997) containing all the mass out to $r_{200} \sim 2100$ kpc (the radius where the density is 200 $\times$ the cosmic mean) is

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{figure2.png}
\caption{The total enclosed masses for the two clusters in CDM (black) and MOND (blue). Cluster 1 sits above cluster 2 in both gravities, but their mass ratio is different. In CDM, cluster 1 outweighs cluster 2 by 10:1 whereas in MOND it is 3:1. The latter is more consistent with the observed gas mass ratio (Brownstein & Moffat 2007).}
\end{figure}

some 6$\times$ larger than the mass we infer by integrating our best fit up to 250kpc (Angus et al. 2007a). We clearly cannot neglect this mass because it will make a huge difference to the deceleration and subsequent acceleration of the other cluster.

We run N-body ($N=10^4$ for each cluster) simulations using the parameters from the best fit NFW models of Clowe et al. (2006). For cluster 1 they give $M_{200} = 1.5 \times 10^{15} M_\odot$, $r_{200} = 2100$ kpc and concentration $c=194$. At present, cluster 1 appears to be the most massive cluster known in the entire universe. For cluster 2, $M_{200} = 1.5 \times 10^{14} M_\odot$, $r_{200} = 1000$ kpc and $c=7.12$. We augment the CDM with a baryon fraction of 17% (Spergel et al. 2006). We generated both clusters with $10^4$ particles and offset the initial centre of cluster 1 by 360kpc (before the gas clouds `touch'). We ran (negative) time steps of 0.15Myr until 10Gyr had elapsed using the tree code of Vine & Sigurdsson (1998).

The mass distributions as functions of radius for the two clusters in CDM and the ones used in the MOND simulations are shown in Fig 2. A subtle point about the total masses of the two clusters is that we do not expect the mass to remain constant as we go back in time. It would be more realistic to have the two clusters losing mass as we go to earlier times to account for their hierarchical merging history. This tends to impede their freefall, reducing the maximum collision velocity to $\sim 2900$ km s$^{-1}$ by the estimate of Farrar & Rosen (2007).

For the CDM scenario, we ran a series of simulations with different relative velocities ranging from 2800-4200 km s$^{-1}$ which can be seen plotted against time in Fig 3. It was found that relative velocities up to 3800 km s$^{-1}$ could be achieved under favourable conditions in our frame-
work. However, 3600 km s$^{-1}$ is a more realistic upper limit to the collision velocity attainable in CDM as it allows for a finite formation and turn-around time. This is in good accord with other results (Hayashi & White 2006; Farrar & In MOND, the basic modification of purely Newtonian dynamics is

\[ \mu(x)g = g_N, \]

where $g_N$ is the Newtonian acceleration computed in the usual way from the baryonic mass distribution, $g$ is the actual acceleration (including the effective amplification due to MOND conventionally ascribed to DM), $a_o$ is the characteristic acceleration at which the modification becomes effective ($\sim 10^{-10}$ m s$^{-2}$), $x = g/a_o$, and $\mu(x)$ is an interpolation function smoothly connecting the Newtonian and MOND regimes. In the limit of large accelerations, $g \gg a_o$, $\mu \to 1$ and the Newtonian limit is obtained; everything behaves normally. The MOND limit occurs only for exceedingly low accelerations, with $\mu \to x$ for $g \ll a_o$. We implement two possible versions of the interpolation function: the ‘standard’ function traditionally used in fitting rotation curves:

\[ \mu = \frac{x}{\sqrt{1 + x^2}} \]

(e.g., Sanders & McGaugh 2002), and the ‘simple’ function found by Famaey & Binney (2005) to provide a good fit the terminal velocity curve of the Galaxy:

\[ \mu = \frac{x}{1 + x} \]

A well known problem with implementing the MOND force law in numerical computations is that the original formulation (Eq 2) does not conserve momentum (Felten 1984; Bekenstein 2007). This was corrected with the introduction of a Lagrangian formulation of MOND (Bekenstein & Milgrom 1984; Milgrom 1986) which has the modified Poisson equation

\[ \nabla \cdot \left[ \mu \left(\frac{v}{a_o}\right) \nabla \Phi \right] = 4\pi G\rho. \]

This formulation has been shown to obey the necessary conservation laws (Bekenstein & Milgrom 1984; Bekenstein 2007). With some rearrangement, it leads to

\[ \mu(x)g = g_N + \nabla \times h, \]

which we recognize as Eq 2 with the addition of a curl field.

Unfortunately, implementing a numerical formulation of the modified Poisson equation is not a simple one-line change to typical N-body codes: this fails to obey the conservation laws. Instead, one needs an entirely different numerical approach than is commonly employed. Progress has been made along these lines (e.g., Brada & Milgrom 1995, 1999; Ciotti, Londrillo, & Nipoti 2006; Nipoti, Londrillo & Ciotti 2007; Tiret & Combes 2007; see also Nusser 2002; Knebe & Gibson 2004), but we do not seek here a full N-body treatment of complex systems. Rather, we wish to develop and apply a simple tool (Angus & McGaugh, in preparation) that can provide some physical insight into basic problems. For the specific case of the large collision velocity of the bullet cluster, it suffices to treat the curl field as a small correction to the center of mass motion (Milgrom 1986). The external field effect (see Bekenstein 2007) is crudely approximated as a constant of appropriate magnitude (McGaugh 2004). We check the effect of varying the external field, which is modest. It is not possible to do better without complete knowledge of the mass distribution in the environment of the clusters.

When modeling the bullet cluster in MOND, Angus et al. (2007a) fitted the convergence map of Clowe et al. (2006) using spherical potential models for the four mass components. Their best fit gives masses for all four components in MOND and standard gravity. The HM is well constrained and the gas mass is known regardless of gravity. Our best fits showed the HM to be around $9 \times 10^{13} M_\odot$ for cluster 1 and $6.8 \times 10^{13} M_\odot$ for cluster 2 in MOND, integrated out to a radius of 250kpc in both clusters. Here we needed to know it as a function of radius for still greater radii in cluster 1, which we now integrate up to a radius of 340kpc.

Unlike CDM, the gas mass is very important in MOND as it comprises a sizeable fraction of the total mass. According to C06, the gas mass for cluster 1 is $2 \times 10^{13} M_\odot$ within a radius of 180kpc and $6 \times 10^{13} M_\odot$ for cluster 2 within 100kpc. It is difficult to judge from the X-ray maps in C06 where a
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**Figure 4.** Shows the relative velocity of the two clusters plotted against time in MOND. Time=0Myr is the current (z=0.3) relative velocity of the two clusters with larger times corresponding to higher redshifts. On the left we plot the results from simulations using MOND with the standard \( \mu \)-function, the simple \( \mu \)-function is on the right. The different lines correspond to a variety of initial relative velocities which are slightly different for each gravity. For the standard \( \mu \) we have we have \( v_{\text{rel}} = 3600-4500 \) km s\(^{-1}\) in intervals of 100 km s\(^{-1}\). For the simple \( \mu \) we have \( v_{\text{rel}} = 3800-4700 \) km s\(^{-1}\) in intervals of 100 km s\(^{-1}\). The black lines correspond to relative velocities which can be achieved and the red lines are for those which cannot. The 4 dashed lines are the predicted relative velocities according to the mean and 1\( \sigma \) error of the original relative velocity from Markevitch (2007) in blue, the simulations of Milosavljevic (2007) in green and Springel & Farrar (2007) in turquoise. The high observed collision velocity is more readily obtained in MOND than CDM.

**Figure 5.** Shows the dependancy of the MOND simulations on the two least constrained parameters: the magnitude of the external field and the initial separation of the two cluster centres of mass. For both comparisons we use simulations with the simple \( \mu \) function with initial relative velocity of 4200 km s\(^{-1}\). For (a) we keep the initial separation of 360kpc and try external gravities of magnitude \( g_{\text{ex}} = a_0/100 \) (black), \( a_0/50 \) (blue), \( a_0/30 \) (red) and \( a_0/10 \) (green). Only when we reach high external gravities such as \( a_0/10 \) does the effect become pronounced. In (b) we keep \( g_{\text{ex}} = a_0/100 \) constant and try initial separations of 300 (black), 360 (blue), 420 (red), 480 (green) and 540kpc (black).
sensible truncation (probably \(\sim 500-750\text{kpc}\)) to the gas mass would be since the plotted quantity is proportional to the square of the gas density. Nevertheless, integrating to only 180kpc likely neglects a factor of 3-4 of the gas mass. In MOND, the X-ray gas generally accounts for between 25-50% of the total mass, the rest being the cluster hidden matter (See Sanders 2003, Angus, Famaey & Buote 2007b; Sanders 2007). Therefore, we assign a gas mass to each cluster which is half the mass of the hidden matter contained within 750kpc for cluster 1 and 500kpc for cluster 2. We add a small extra component of hidden mass to each cluster between the last integrated radius (340kpc and 250kpc for clusters 1 and 2 respectively) and the gas truncation radius. This is to avoid divergence from integrating our model well beyond the range over which the fit to the converg ence map is valid. The total enclosed mass for each cluster is shown in Fig.2.

In order to compute the Newtonian gravity (to input into the MOND equation) of cluster 1 on cluster 2 (and vice-versa) we used the following scheme: we calculated the distance between the two centres of mass, which is initially 360kpc and is increasing backwards in time. Then we computed the enclosed mass within half this distance for both clusters. This radius gives mass shells for both clusters which do not overlap: while the mass distributions of the two clusters overlap, we approximate the mutual gravity by assuming all the non overlapping mass is concentrated at a point. We have a series of known masses for given radii and linearly interpolate to find the correct mass for a given radius which is used in Eq.7. The clusters move away from each other quickly so that there is no mass left out due to overlap.

The mutual gravity imposed upon cluster 1 by cluster 2 is

\[
g_1\mu\left(\frac{|g_1 + g_{ex}|}{a_o}\right) = g_{n1} = -\frac{GM_2}{(r_1 - r_2)^2} \tag{7}
\]

and we simply swap the numbers around to find the mutual gravity of cluster 1 upon cluster 2. Following on from above, \(r_1 - r_2\) is the distance between the two centres of mass and \(M_2\) is the enclosed mass within a radius \(|r_1 - r_2|/2\). The \(g_{ex}\) is the external field limiting the MOND correction which comes from large scale structure and is always assumed orthogonal to the direction of \(g_1\), making the argument of the \(\mu\) function more easily expressed as \(\sqrt{\frac{x^2 + y^2}{a_o^2}}\). The direction and amplitude of \(g_{ex}\) is unknown at all times. The MONDian additional acceleration becomes minor when the acceleration drops below \(g_{ex}\). We use \(g_{ex} = a_o/100\) (Aguirre et al. 2001; McGaugh 2004) for most models, but consider the effect of higher external fields in §5.1.

Just like in the CDM simulations we have knowledge of all the final parameters of the system and we have the equations of motion, therefore, it made sense to use negative time steps (of 0.4Myr) to work backwards towards the beginning of the freefall of the two clusters. For the standard (Eq 3) and simple (Eq 4) \(\mu\)-functions, we ran a series of simulations with different initial relative velocities just as per the CDM case. They can be seen plotted as velocity versus time in Fig 4.

The maximum relative velocity attainable by the standard function is 4100 km s\(^{-1}\) which is consistent with the uncertainty of the original (pre-hydrodynamical simulations) relative velocity. With the simple function it is possible to reach more than 4400 km s\(^{-1}\). This speed is bang on the obvious interpretation of the shock speed; there is no need to invoke hydrodynamical effects. The simple function approaches the deep-MOND regime more swiftly than does the standard function which means intermediate gravities are amplified marginally more so than with the standard function. This feature is what allows the simple function to generate higher relative velocities. Recent work by Famaey et al. (2007), Zhao & Famaey (2006) and Sanders & Noordermeer (2007) hints at the simple function being as good and potentially better at fitting of galactic rotation curves. Irrespective of the true form of the interpolation function, comparatively high collision velocities follow naturally from MOND.

**5 RESULTS**

The ability of the two clusters that comprise the bullet to bring each other to a halt at a finite time in the past is sensitive to both the flavor of gravity at work and the true relative velocity. In Figs. 3 and 4 we plot the relative velocity of the two clusters as a function of time for a series of initial relative velocities in Newtonian gravity and MOND. The maximum relative velocity CDM, standard and simple \(\mu\) can generate are 3800, 4100 and 4400 km s\(^{-1}\). Velocities a few 100 km s\(^{-1}\) below the maximum can be seen to be quite easily attained. For velocities larger than the maximum, the relative velocity never reaches zero and increase sharply at early times (many Gyr ago). The two clusters do not gravitate strongly
enough to generate such high velocities and would have to have had a huge relative velocity towards each other in the early universe in order to overcome the Hubble expansion and fall together with such a high relative velocity at z=0.3.

5.1 Sensitivity to the external field

As a check on our assumed initial conditions in the MOND computations, we use the simulations with the simple $\mu$ function to investigate the importance of the strength of the external field. The external field is used here to effectively cut off the MOND effect between the two clusters when the two body acceleration drops below the value of the background gravity imposed by large scale structure. Calculations by McGaugh (2004) and Famaey et al. (2007) have shown this to be around $a_o/30 - a_o/100$. We plot in Fig.5(a) the results from a set of simulations using a series of external field gravities, while keeping all other parameters constant. Clearly, the effect is minimal between $a_o/30 - a_o/100$, but using $g_{e x} = a_o/10$ makes it considerably more difficult to attain large relative velocities. There are many galaxies with rotation curves that are observed to extend to $\sim a_o/10$ at their last measured point (Sanders 1996), so this represents a firm upper limit to the strength of the external field.

5.2 Initial separation

Another important variable was the initial separation of the two clusters when they had their defined relative velocity. We used simulations for the simple $\mu$ function keeping all other parameters constant and varying the initial separation from 300-540kpc (the standard separation is 360kpc). Within sensible variations of the separation, the effect is modest, making the infall time $\approx 4 \pm 1\,\text{Gyr}$ for separations of $360 \pm 60\,\text{kpc}$; which can be seen in Fig.5(b).

5.3 Cluster mass

The cluster masses in MOND are minimal. Virtually no extra mass has been inferred beyond the scope of the convergence map. Conversely, in the NFW fits to the central region is justified since the mass sheet degeneracy is broken by constraining the mass at the edges of the fit based on the slope of the profile in the inner regions - but if the mass profile is wrong then it could lead to the completely wrong measurement for the value of the mass sheet (Clowe, de Lucia and King 2004). Another concern is that the clusters are unlikely to be spherically symmetric and are presumably elongated in the direction of motion.

There have been some doubts over the concentration parameter of cluster 1 ($c=2$) which seems low, but this is probably due to the fact that cluster 2 is double cored, forcing azimuthal averaging over the two cores after centering on one (D. Clowe private communication).

To contend with all these points and that the actual convergence map may change when strong lensing observations are added (Bradac et al. 2006), we show in Fig 6 the variation of the infall time with total mass of cluster 1. Basically, we ran four extra simulations in which we kept all parameters constant except for the mass of cluster 1. This we varied from the standard value of $1.5 \times 10^{15} M_\odot$ (multiplied by 1.17 to include the gas) by trying 30% and 15% less mass and 15% and 30% more mass. Clearly, adding extra mass makes it considerably easier to freefall, but removing just 15% of the total mass makes even 3600 km s$^{-1}$ an unachievable relative velocity. It seems highly improbable that the total mass of cluster 1 could increase beyond its already enormous mass; indeed, much of the total mass is merely inferred from the notion extrapolation of the NFW profile that is fit to the well constrained inner parts of the clusters. Conversely, in MOND we use the bare minimum of matter.

6 SUMMARY

We have constructed specific mass models for the bullet cluster in both CDM and MOND. We integrate backwards from the observed conditions to check whether the large ($\sim 4700\,\text{km s}^{-1}$) apparent transverse velocity can be attained in either context. We find that it is difficult to achieve $v_{rel} > 4400\,\text{km s}^{-1}$ under any conditions. Nevertheless, within the range of the uncertainties, the appropriate velocity occurs fairly naturally in MOND. In contrast, ACDM models can at most attain $\sim 3800\,\text{km s}^{-1}$ and are more comfortable with considerably smaller velocities.

Taken at face value, a collision velocity of 4700 km s$^{-1}$ constitutes a direct contradiction to ACDM. Ironically, this cluster, widely advertised as a fatal observation to MOND because of the residual mass discrepancy it shows, seems to pose a comparably serious problem for ACDM. It has often been the case that observations which are claimed to falsify MOND turn out to make no more sense in terms of dark matter.

The critical outstanding issue is how the observed shock velocity relates to the actual collision velocity of the two gravitating masses. The recent simulations of Springel & Farrar (2007) and Milosavljevic et al. (2007) seem to suggest that, contrary to naive expectations, hydrodynamic effects reduce the relative velocity of the mass with respect to the shock. A combination of effects is responsible, being just barely sufficient to reconcile the data with ACDM. Hydrodynamical simulations are notoriously difficult, and indeed these two recent ones do not agree in detail. It would be good to see a fully self-consistent simulation including both hydrodynamical effects and a proper mass model and orbital computation like that presented here.

There are a number of puzzling aspects to the hydrodynamical simulations. For instance, Springel & Farrar (2007) find that the morphology of the bullet is reproduced only for a remarkably dead head-on collision. If the impact parameter is even 12kpc — a target smaller than the diameter of the Milky Way — quite noticeable morphological differences ensue. This can be avoided if the separation of mass centres happens to be along our line of sight — quite a coincidence in a system already remarkable for having the vector of its collision velocity almost entirely in the plane of the sky. Fur-
thermore, the mass models require significant tweaking from that inferred from the convergence map and are unable to reproduce the currently observed, post merger positions of the gas and HM. It appears to us that only the first rather than the last chapter has been written on this subject. Getting this right is of the utmost importance, as the validity of both paradigms rests on the edge of a knife, separated by just a few hundred km s\(^{-1}\).

More generally, the frequency of bullet-like clusters may provide an additional test. The probability of high collision velocities drops with dramatic rapidity in CDM (Hayashi & White 2006). In contrast, somewhat higher velocities seem natural to MOND. Naively it would seem that high impact velocity systems like the bullet would be part and parcel of what might be expected of a MOND universe.

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