1. The Local Group

3 largest members: M31 (Andromeda) \[ \text{all spirals} \]
                 Milky Way
                 M33

Other types of galaxies: dwarf ellipticals
                        dwarf irregulars
                        dwarf spheroidals

The bulk of the mass is associated with the two largest galaxies (M31 & the Milky Way). The rest of the galaxies add up to rather little by comparison.
2. Large Scale Structure

Galaxies are not randomly distributed in space; they cluster together. Small groups of galaxies like the Local Group are common, each containing only a few bright members. Larger groups, clusters, and rich clusters contain more bright galaxies—hundreds, or even thousands—but are rare. These structures are organized into long filaments and walls surrounding large, empty voids.
3. Regularities in the dynamics of spiral galaxies

- Rotation curves tend to become approximately flat at large radii.

- Maximum disk suffices to explain the inner shapes of rotation curves.

- The stellar disk and dark matter halo are coupled. (The “disk-halo conspiracy”)

- Spirals (and all rotating disk galaxies) obey a relation between their baryonic mass and characteristic rotation velocity (Tully-Fisher).

- The shape of the rotation curve depends on the distribution of light (e.g., Reitz's rule; the Universal Rotation Curve).

- The need for dark matter appears at a particular acceleration scale. (The mass discrepancy - acceleration relation; MONDian phenomenology)
4. Dark Matter

a) Astronomical evidence for dark matter:
   e.g.  
   - rotation curve being flat
   - velocity dispersions of rich clusters of galaxies
   - the Local Group timing argument

   etc. - many answers are valid
   (e.g., gravitationally lensing, X-ray gas in clusters,
   the velocity dispersions of dwarf spheroidals, etc...)

b) The two primary reasons for invoking non-baryonic cold dark matter are:

1) $\Omega_m > \Omega_b$

2) Structure formation

c) Three methods for WIMP detection

1) Direct detection in the lab (e.g., CDMS; XENON100)

2) Indirect detection of decay products ($\gamma$-rays; cosmic rays)
   with satellites like Fermi or balloon borne experiments.

3) Production in particle colliders (e.g., the LHC)
5. \[ \Omega_m = \frac{\rho}{\rho_{\text{crit}}} \]

\[ \left( \frac{M}{L} \right)_{\text{cluster}} = 300 \frac{M_\odot}{L_\odot} \]

\[ j = 10^8 \frac{L_\odot}{Mpc^{-3}} \]

\[ \rho = \left( \frac{M}{L} \right) j = \left( 300 \frac{M_\odot}{L_\odot} \right) 10^8 \frac{L_\odot}{Mpc^{-3}} = 3 \times 10^{10} \frac{M_\odot}{Mpc^{-3}} \]

\[ \rho_{\text{crit}} = \frac{3H_0^2}{8\pi G} \]

\[ H_0 = 72 \frac{\text{km}}{\text{s} \ Mpc^{-1}} \]

\[ G = 1.327 \times 10^{11} \frac{\text{km}^3}{\text{s}^2 \ M_\odot^{-1}} \]

\[ 1 \ Mpc = 3.086 \times 10^{19} \text{ km} \]

\[ H_0 = \frac{72 \frac{\text{km}}{\text{s} \ Mpc^{-1}}}{3.086 \times 10^{19} \frac{\text{km}}{\text{Mpc}}} = 2.33 \times 10^{-18} \text{ s}^{-1} \]

\[ G = \frac{1.327 \times 10^{11} \frac{\text{km}^3}{\text{s}^2 M_\odot^{-1}}}{\left(3.086 \times 10^{19} \frac{\text{km}}{\text{Mpc}}\right)^3} = 4.52 \times 10^{-48} \frac{Mpc^3}{s^2 M_\odot^{-1}} \]

\[ \rho_{\text{crit}} = \frac{3 \left(2.33 \times 10^{-18} \text{ s}^{-1}\right)^2}{8\pi \left(4.52 \times 10^{-48} \frac{Mpc^3}{s^2 M_\odot^{-1}}\right)} \]

\[ \rho_{\text{crit}} = 1.44 \times 10^6 \frac{M_\odot}{Mpc^{-3}} \]

\[ \Omega_m = \frac{\rho}{\rho_{\text{crit}}} = \frac{3 \times 10^{10} \frac{M_\odot}{Mpc^{-3}}}{1.44 \times 10^6 \frac{M_\odot}{Mpc^{-3}}} \]

\[ \Omega_m \approx 0.2 \]
6. Evolution of a solar mass star

Main sequence (longest lived phase)

Red giant

$\Rightarrow$ Helium Flash $\Leftarrow$

ignition of the burning

Horizontal branch

Asymptotic giant

$\Rightarrow$ mass loss during brief planetary nebula phase

White dwarf

Remnant of core

supported by electron degeneracy pressure —
no further fusion reactions.
7. Active Galactic Nuclei

a) AGN were long thought to be supermassive black holes because

i) they are very small: size \( \lesssim 1\) ct
   where the variability time scale at is days never hours.

ii) they are very bright: \( \gtrsim L^*\)
   Need enormous energy density.

b) Kepler's generalized 3rd Law applies

\[ MP^2 = a^3 \quad \text{in solar units} \]
\[ \quad \text{(} M_\odot, \text{years, AU)} \]
\[ \frac{p^2}{M} = \frac{a^3}{M_\odot} \]
\[ p = \sqrt{\frac{(0.01 \text{ pc}) (206265 \text{ Au pc}^{-1})^3}{3 \times 10^6 M_\odot}} \]
\[ p = 54 \text{ yr} \]

\[ R_0 = 8 \text{ kpc} = 8000 \text{ au} \]
\[ 1 \text{ radian} = 206265 \text{ arcsec} \]

\[ V = \frac{2\pi r}{p} = \frac{2\pi (0.01 \text{ pc})}{54 \text{ yr}} = 0.0012 \text{ pc yr}^{-1} \]

\[ \mu = \frac{V}{R_0} = \frac{0.0012 \text{ pc yr}^{-1}}{8000 \text{ pc}} = (1.45 \times 10^7 \text{ radians yr}^{-1}) (206265 \text{ arcsec radians}) \]
\[ \mu = 0.03 \text{ arcsec yr}^{-1} \]

d) This is small but measurable with patience – two independent groups have done it!
8. Tidal Limit

\[ R_t = D \left( \frac{m}{2M} \right)^{1/3} \]

typical satellite has \( m = 2 \times 10^7 \, M_\odot \)
\( r = 300 \, \text{pc} \)

Milky Way \( \frac{V_f}{c} \approx 200 \, \text{km s}^{-1} \)

enclosed mass of the Milky Way:

\[ M = \frac{\frac{V_f^2 D}{G}}{\text{within distance } D \text{ to the satellite}} \]

The satellite will be subject to disruption if it approaches close enough to the center of the Milky Way that its size becomes equal to the tidal radius:

\[ R_t = D \left( \frac{m}{2M} \right)^{1/3} = D \left( \frac{Gm}{2D V_f^2} \right)^{1/3} = D^{2/3} \left( \frac{Gm}{2V_f^2} \right)^{1/3} \]

\[ D^{2/3} = \left( \frac{2V_f^2}{Gm} \right)^{1/3} R_t \]

\[ D = \left( \frac{2V_f^2}{Gm} \right)^{1/2} R_t^{3/2} = \left( \frac{2}{Gm} \right)^{1/2} \frac{r}{R_t} \]

\[ D = 2^{1/2} (200 \, \text{km s}^{-1}) \left[ \frac{(300 \, \text{pc}) \left( 3.086 \times 10^{19} \, \text{km} \cdot \text{pc}^{-1} \right)}{\left( 1.327 \times 10^{11} \, \text{km}^2 \text{s}^{-2} \text{M}_\odot^{-1} \right) \left( 2 \times 10^7 \, \text{M}_\odot \right)} \right]^{3/2} \]

\[ D = 1.55 \times 10^{17} \, \text{km} = 5 \, \text{kpc} \]

That's pretty close - inside the orbit of the sun.

If the numbers were slightly different, perhaps no disruption would occur.