Dark Matter

ASTR 333/433
Spring 2026
TR 11:30am–12:45pm
Sears 552

http://astroweb.case.edu/ssm/ASTR333/

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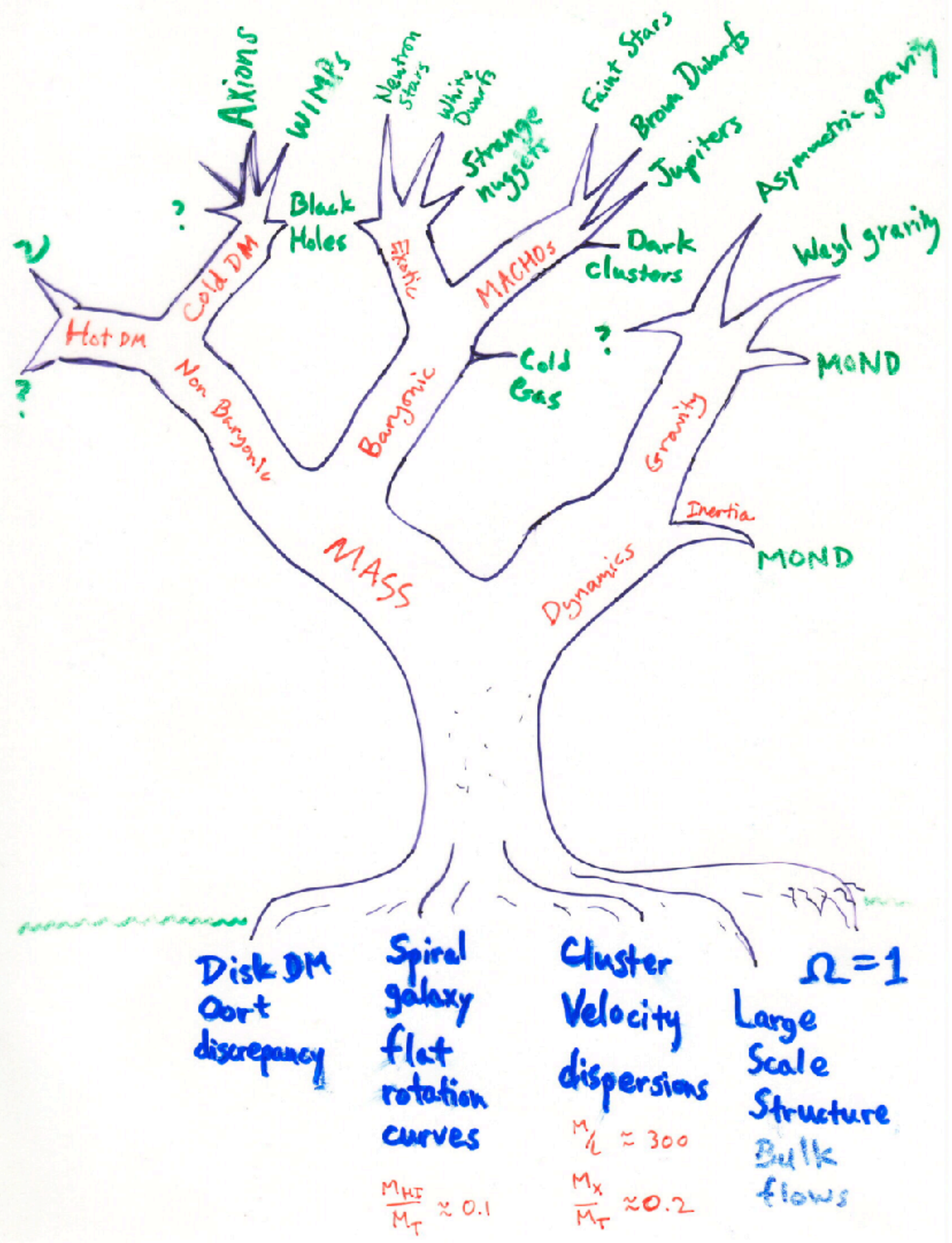


Case Western Reserve University EST. 1826

Empirical Laws of Galactic Rotation

- Flat rotation curves (Rubin-Bosma Law)

  Rotation curves tend asymptotically towards a constant rotation velocity that persists to indefinitely large radii: $V(R \to \infty) \to V_f$

- Tully-Fisher relation (Luminous, Stellar Mass, and Baryonic TF relations)

  The baryonic mass of galaxies scales as the fourth power of the flat rotation velocity: $M_b = AV_f^4$

- Central density relation (lower surface brightness galaxies exhibit larger mass discrepancies)

  The central dynamical surface densities of galaxies is related to their central surface brightnesses: $\Sigma_{dyn}(R \to 0) = f[\Sigma_*(R \to 0)]$

- Renzo's rule (Sancisi's Law)

  "For any feature in the luminosity profile there is a corresponding feature in the rotation curve and vice versa." (Sancisi 2004).

- Radial acceleration relation

  The observed centripetal acceleration is related to that predicted by the observed distribution of baryons: $\mathrm{g_{obs}} = \mathcal{F}(\mathrm{g_{bar}})$

These systematic properties involve a critical acceleration scale.

- Baryonic Tully-Fisher Relation

$$\mathrm{g}_{\dagger}^{\mathrm{BTFR}} = 1.24 \pm 0.14 \times 10^{-10}\ \mathrm{m\,s^{-2}}$$

(McGaugh 2011)

- Central Density Relation

$$\mathrm{g}_{\dagger}^{\mathrm{CDR}} = G\Sigma_{\dagger} = 1.27 \pm 0.05 \times 10^{-10}\ \mathrm{m\,s^{-2}}$$

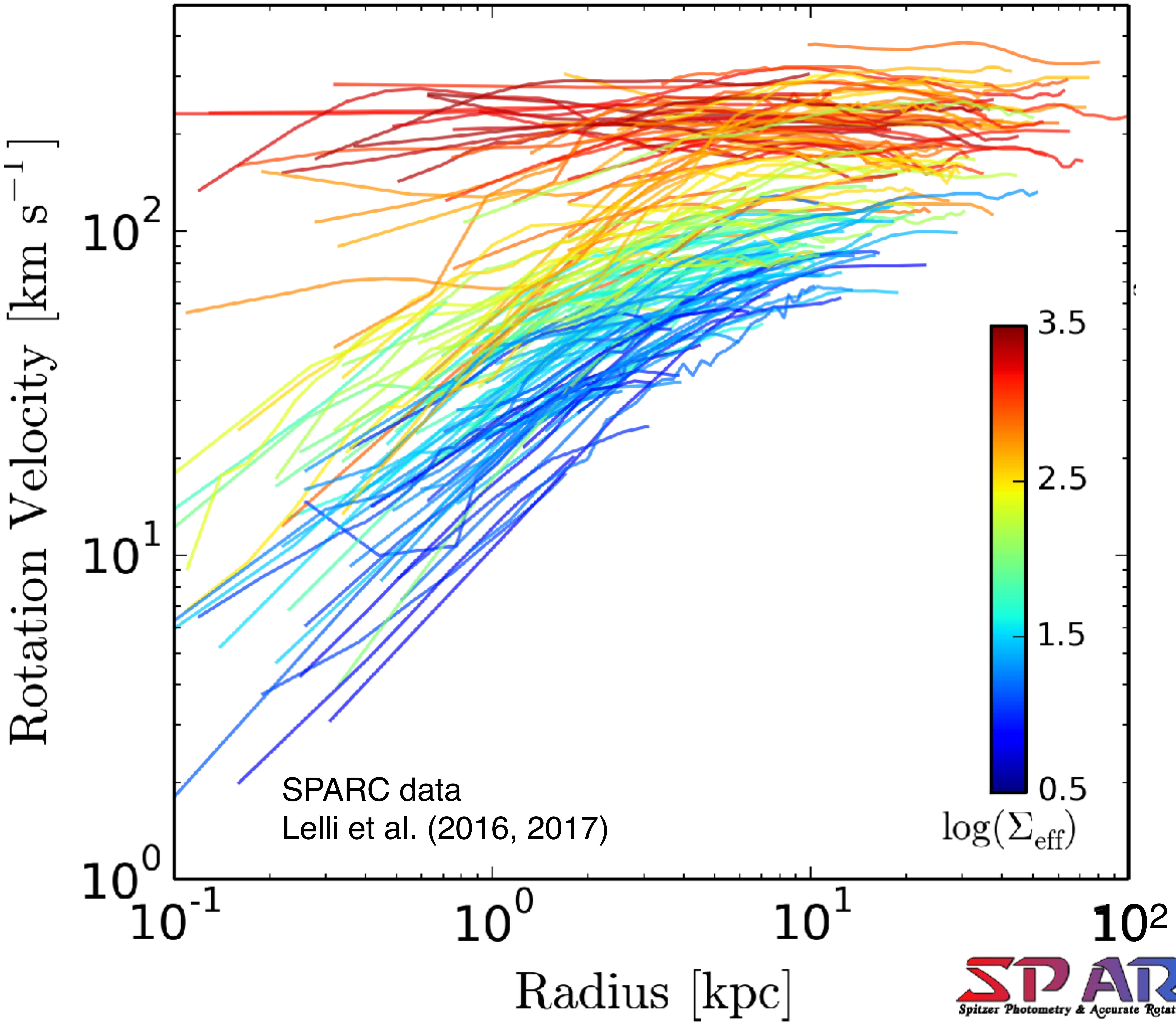
(Lelli et al. 2016)

- Radial Acceleration Relation

$$\mathrm{g}_{\dagger}^{\mathrm{RAR}} = 1.20 \pm 0.02 \times 10^{-10}\ \mathrm{m\,s^{-2}}$$

(McGaugh et al. 2016)

Applications: both qualitative and quantitative



Applications

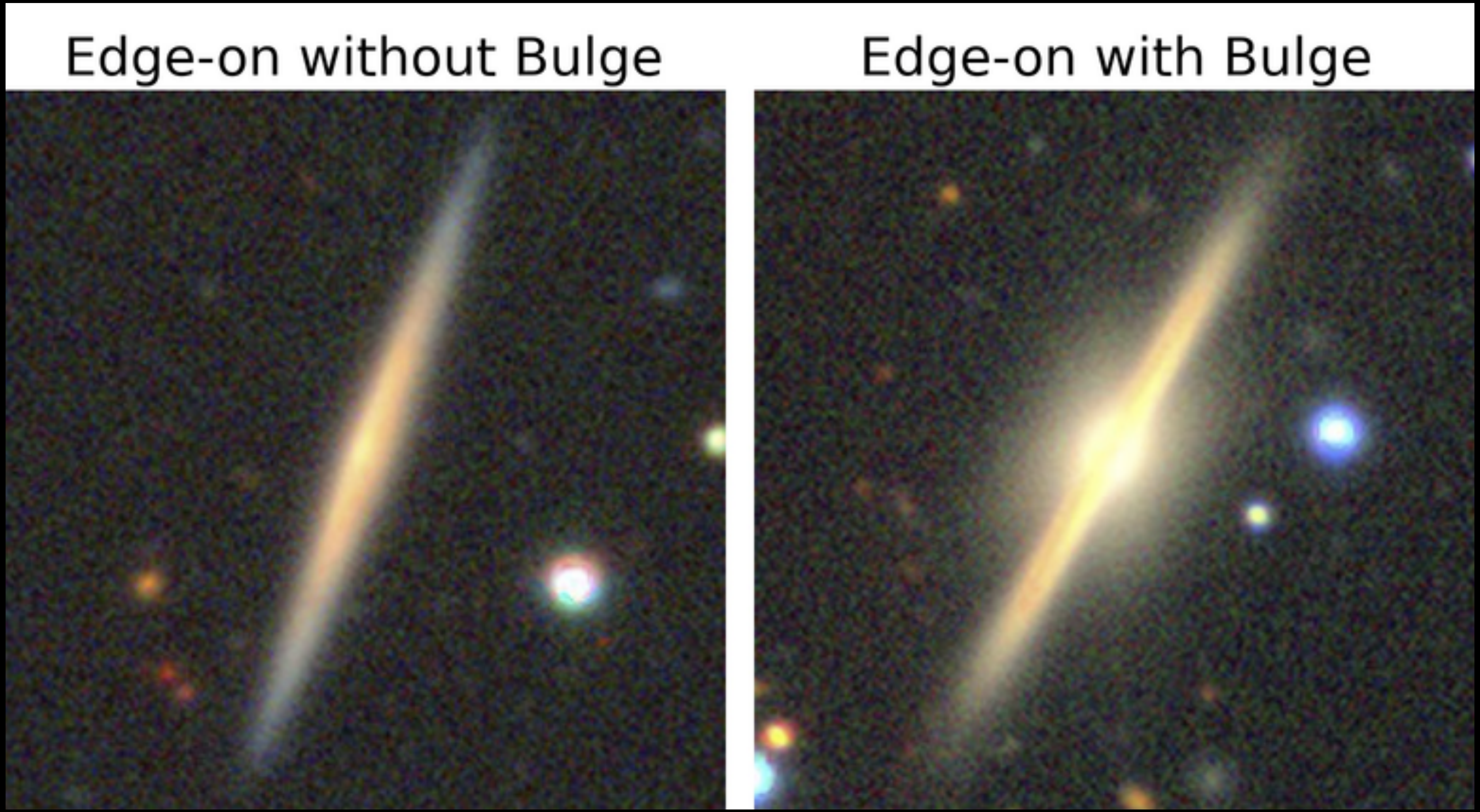
Central Density Relation: lower surface brightness means slower rotation curve rise

High surface brightness

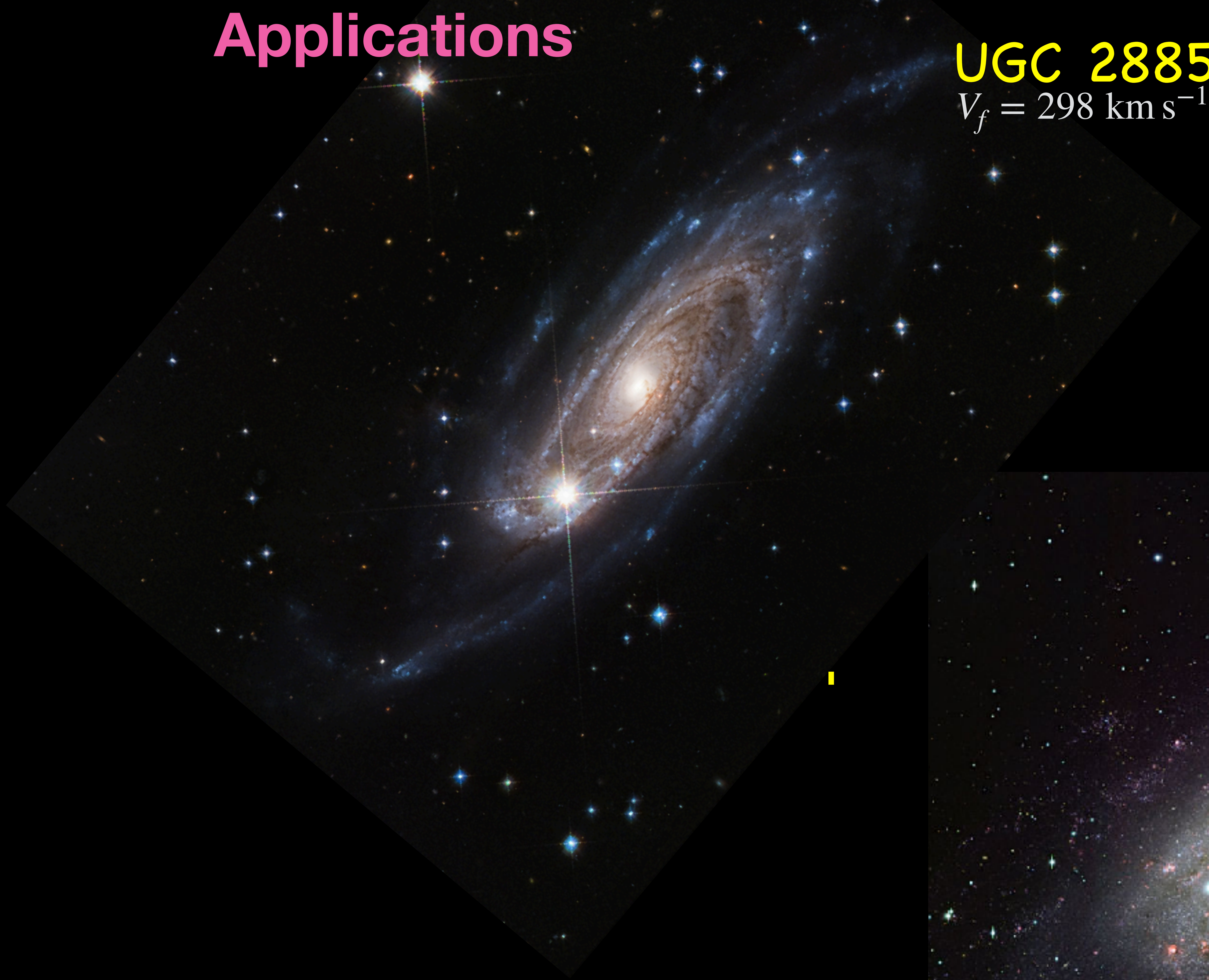
Low surface brightness

Applications

Renzo's rule: features in the light map to features in the rotation curve



Applications



UGC 2885

$V_f = 298 \text{ km s}^{-1}$

What is the baryonic mass of each galaxy?

Which relation to use?

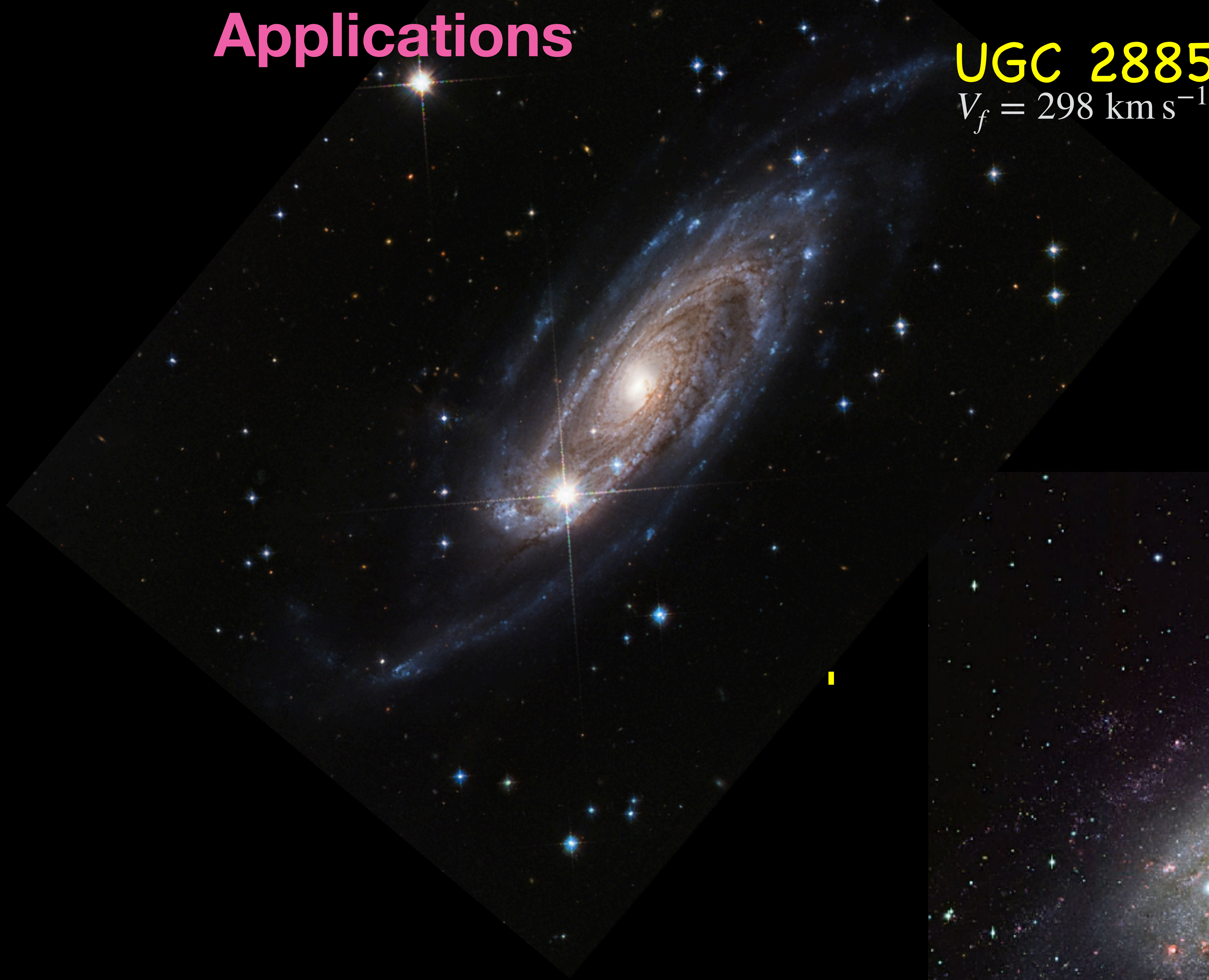
NGC 2403

$V_f = 131 \text{ km s}^{-1}$

DDO 154

$V_f = 53 \text{ km s}^{-1}$

Applications



UGC 2885
$V_f = 298\ \mathrm{km\,s^{-1}}$

Which relation to use?
The baryonic Tully-Fisher relation!

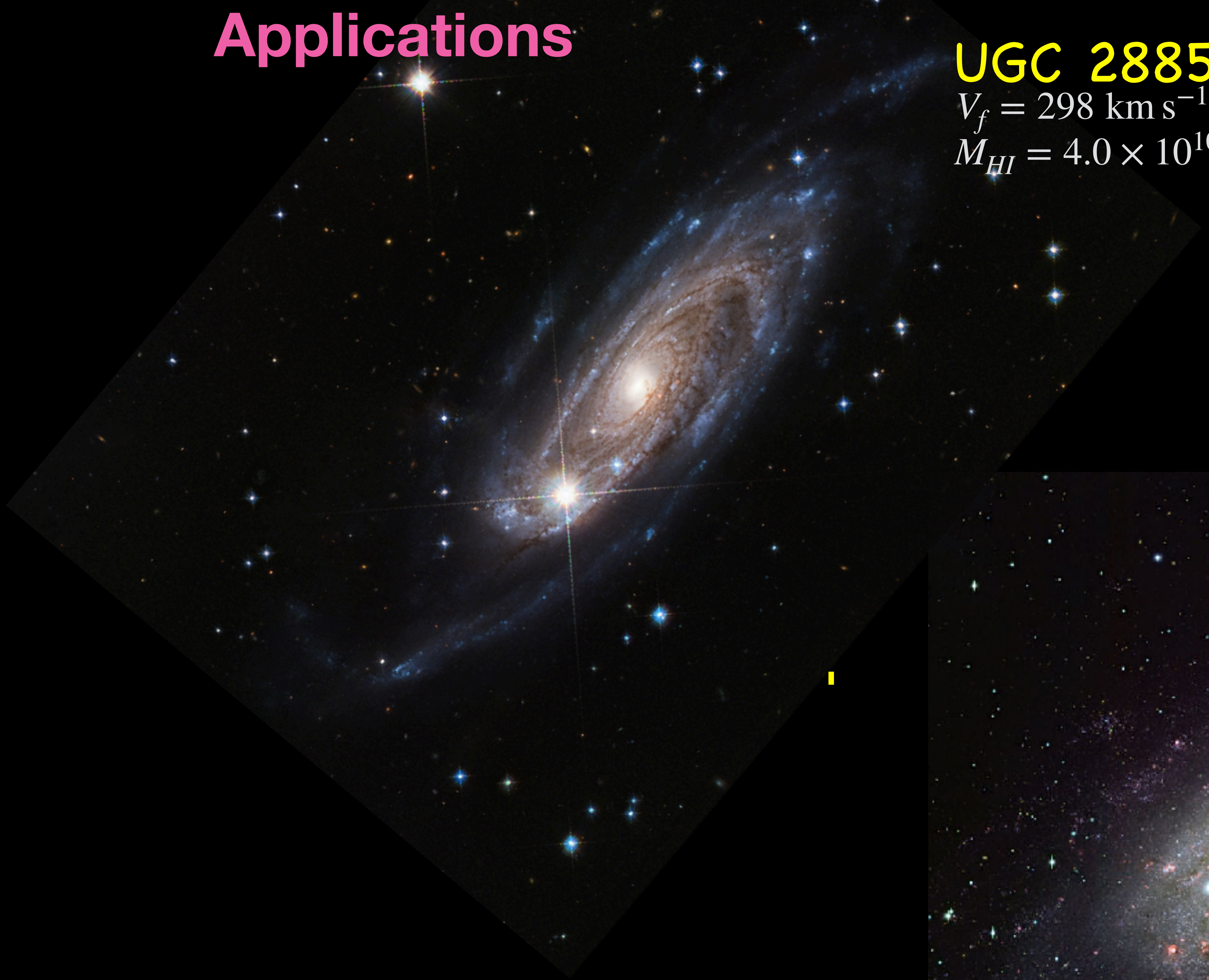
$$M_b = A V_f^4$$

$$A = 47\ \mathrm{M_\odot\,km^{-4}\,s^4}$$

NGC 2403
$V_f = 131\ \mathrm{km\,s^{-1}}$

DDO 154
$V_f = 54\ \mathrm{km\,s^{-1}}$

Applications



UGC 2885

$V_f = 298\ \mathrm{km\,s^{-1}}$

$M_{HI} = 4.0 \times 10^{10}\ \mathrm{M_\odot}$

What is the stellar mass of each galaxy?

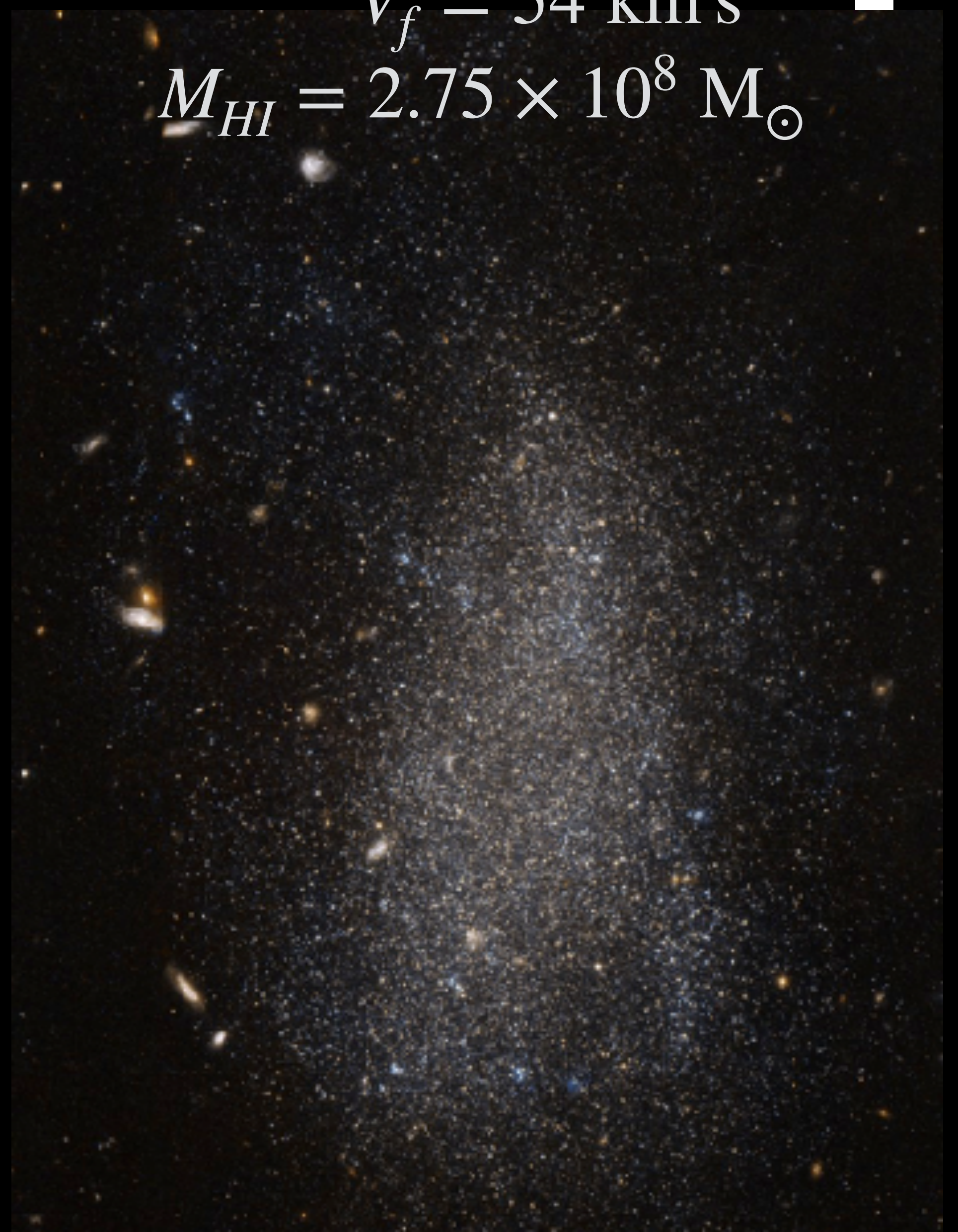
$$M_b = M_* + M_g$$

$$M_g = 1.4\, M_{HI}$$

NGC 2403

$V_f = 131\ \mathrm{km\,s^{-1}}$

$M_{HI} = 3.2 \times 10^{9}\ \mathrm{M_\odot}$



DDO 154

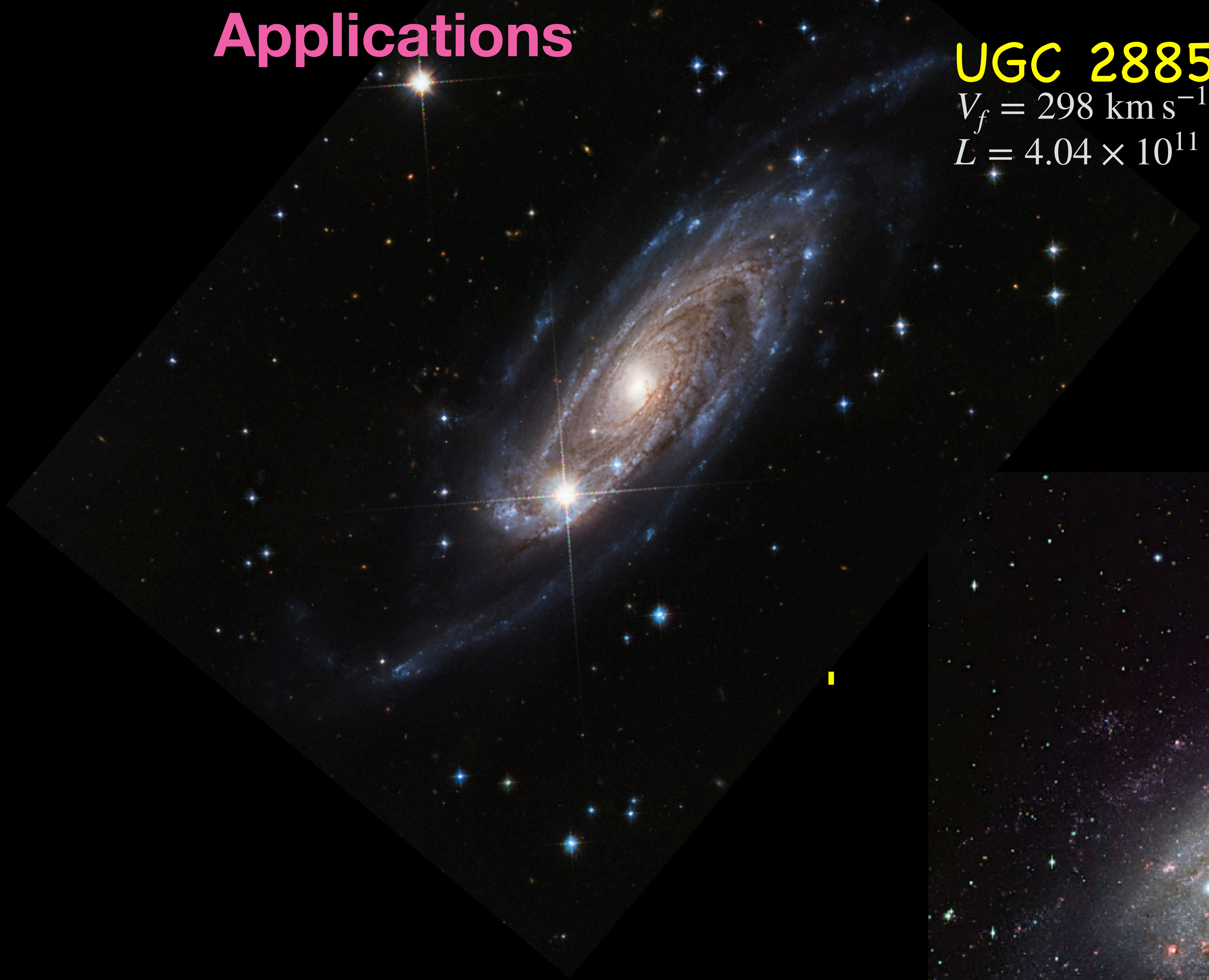
$V_f = 54\ \mathrm{km\,s^{-1}}$

$M_{HI} = 2.75 \times 10^{8}\ \mathrm{M_\odot}$

Applications

What is the stellar mass-to-light ratio of each galaxy?

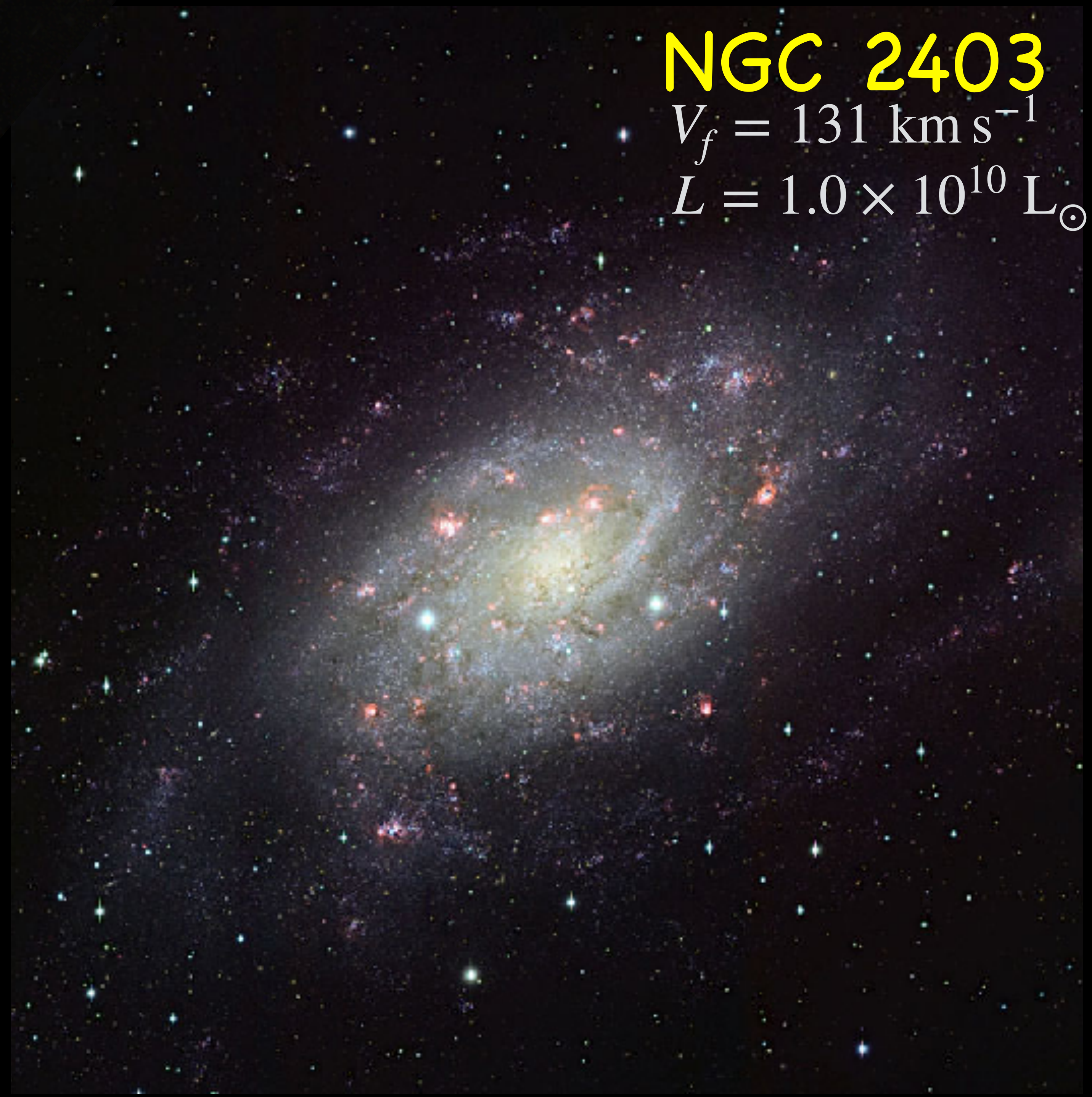
$$\Upsilon_* = M_*/L$$



UGC 2885

$V_f = 298\ \mathrm{km\,s^{-1}}$

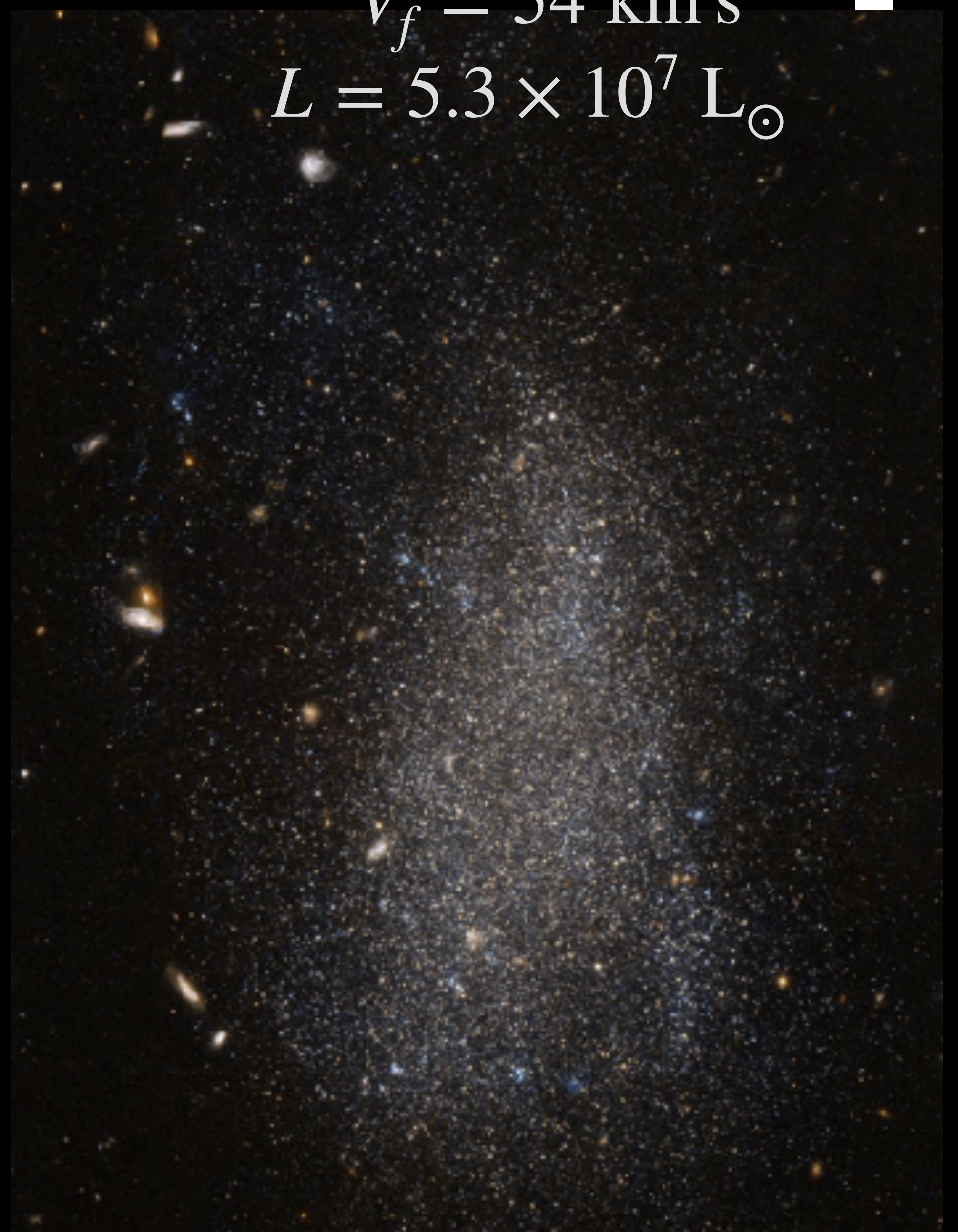
$L = 4.04 \times 10^{11}\ \mathrm{L_\odot}$



NGC 2403

$V_f = 131\ \mathrm{km\,s^{-1}}$

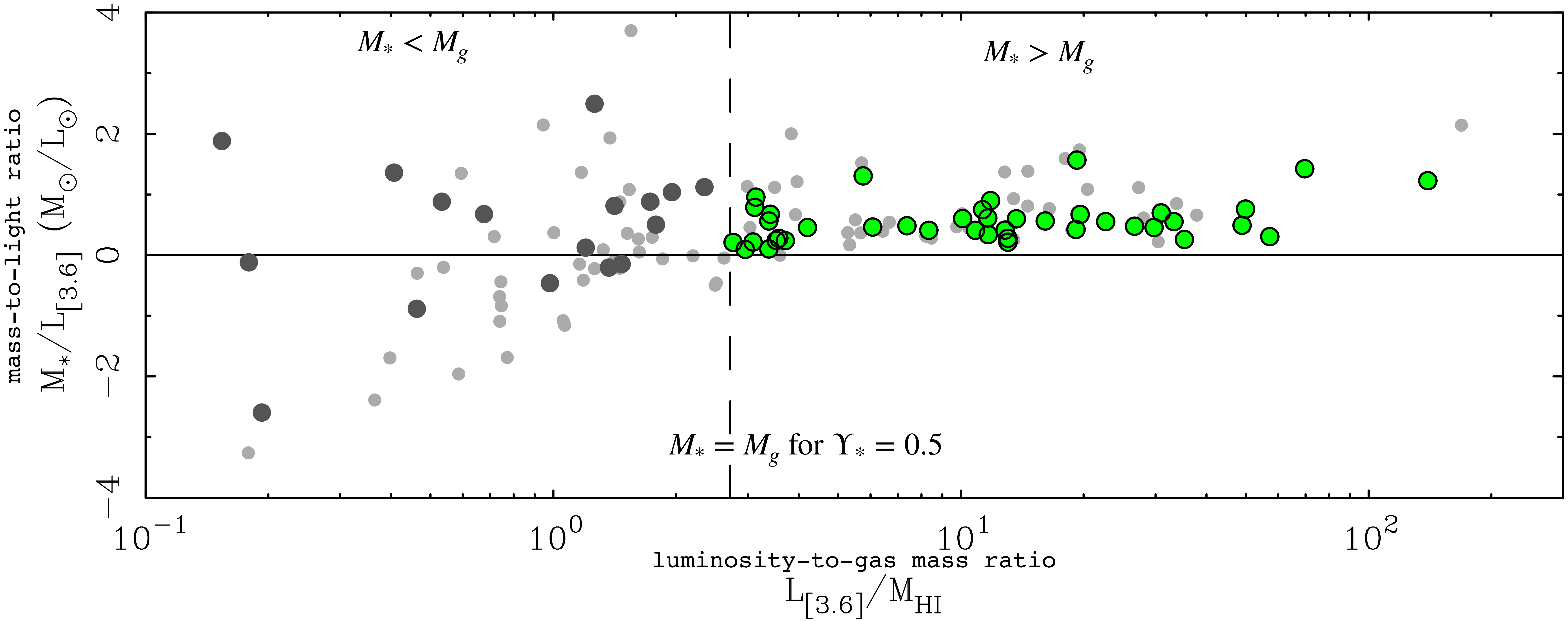
$L = 1.0 \times 10^{10}\ \mathrm{L_\odot}$



DDO 154

$V_f = 54\ \mathrm{km\,s^{-1}}$

$L = 5.3 \times 10^{7}\ \mathrm{L_\odot}$



Mass-to-light ratio from BTFR:

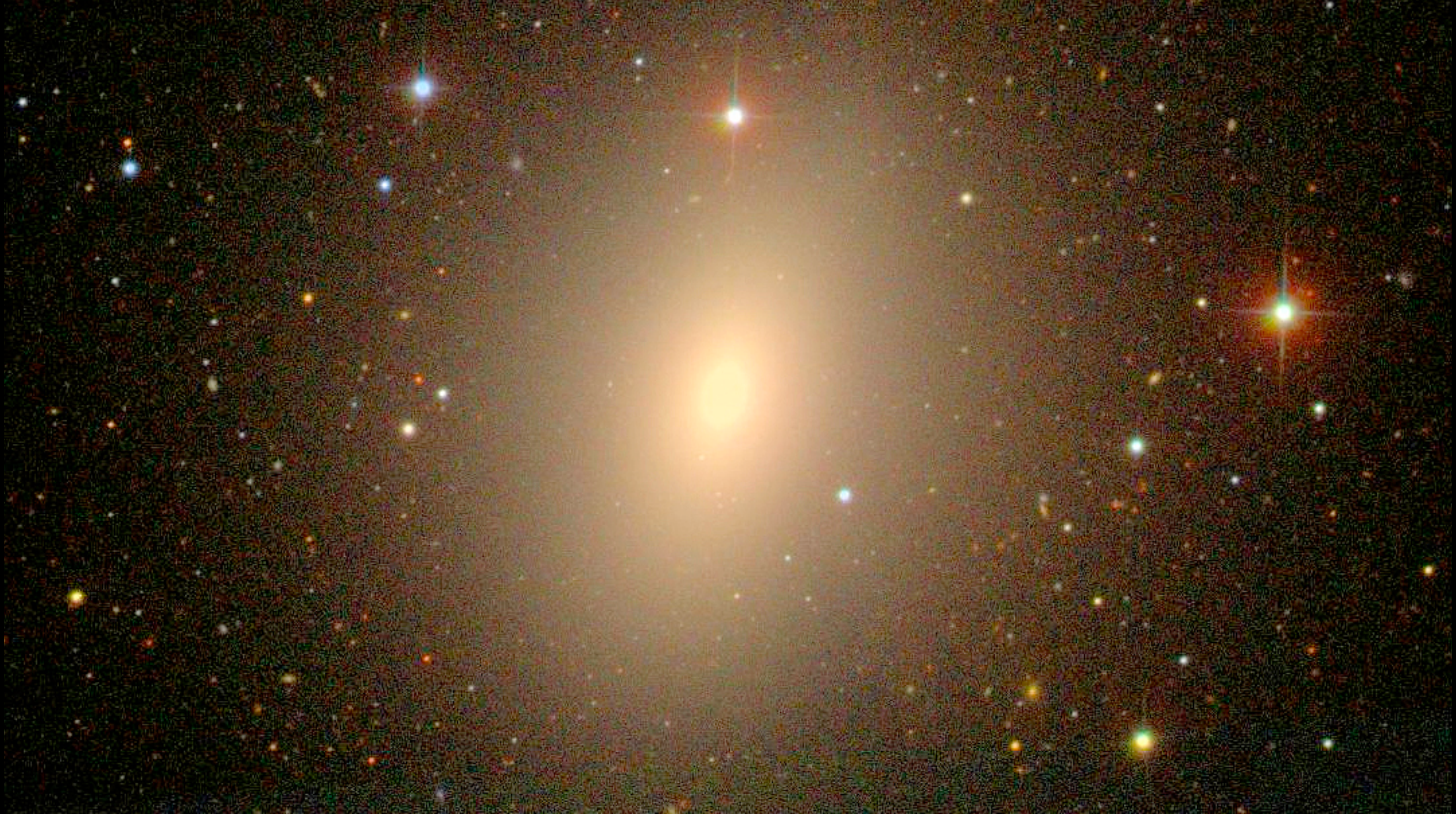
$$M_b = M_* + M_g = AV_f^4$$

$$M_* = AV_f^4 - M_g$$

$$\Upsilon_* = (AV_f^4 - M_g)/L$$

Elliptical Galaxies

Elliptical galaxies are presumed to reside in dark matter halos, but the evidence is less obvious than for spirals.

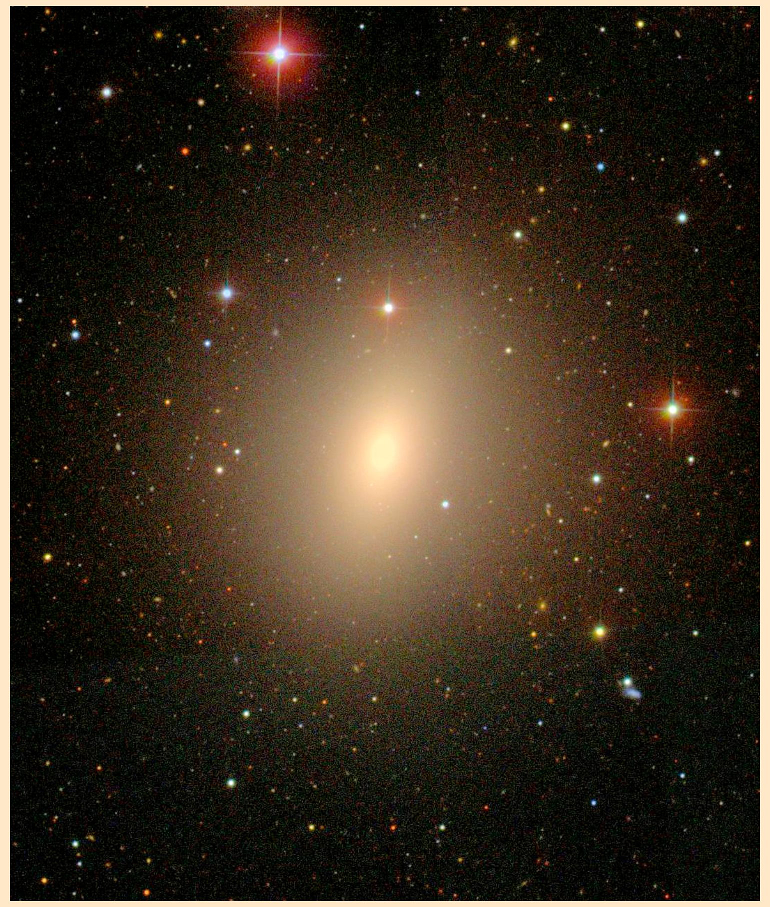


Elliptical Galaxies

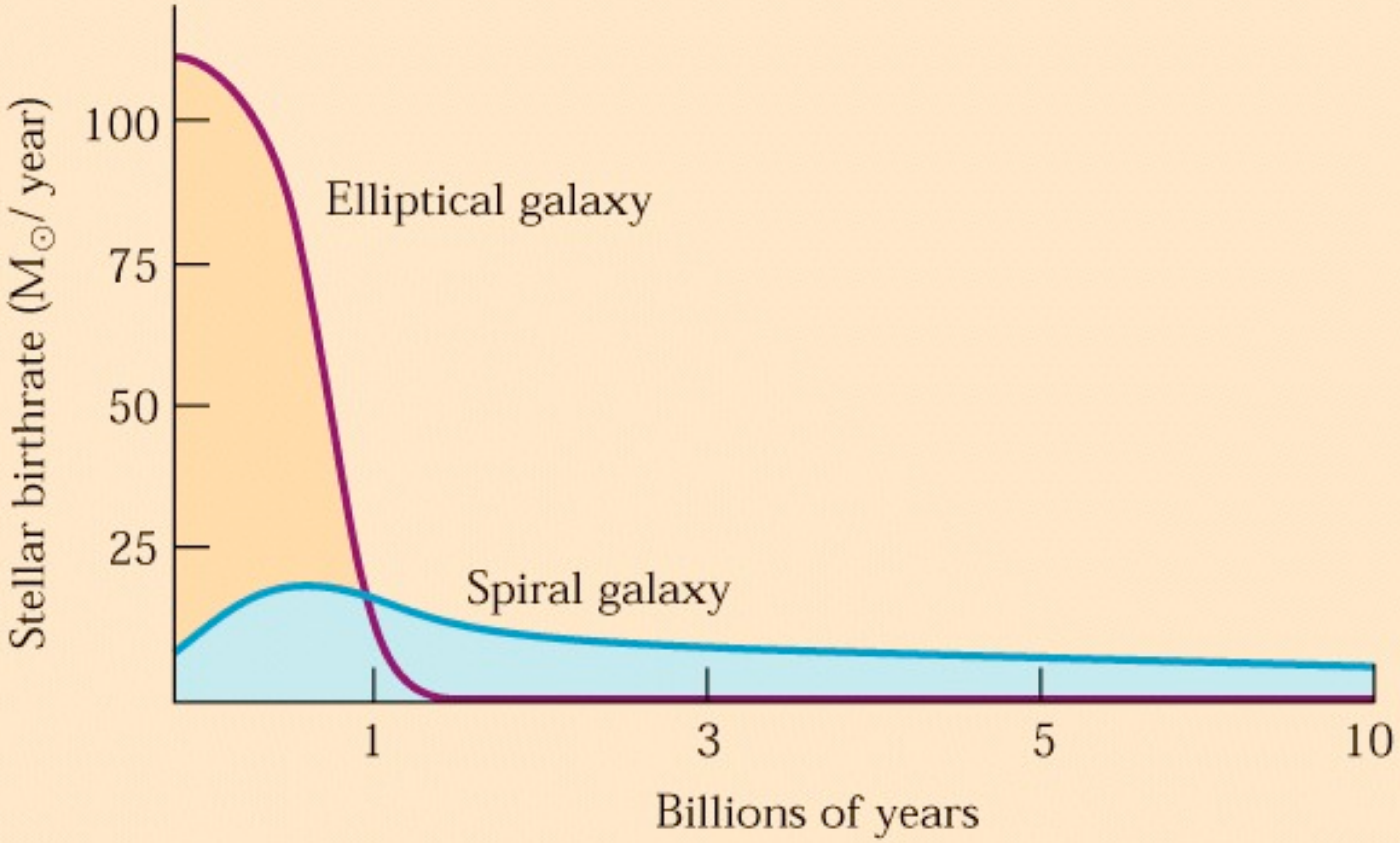
composed mostly of older stars

Generic Star Formation History

Elliptical



old stars

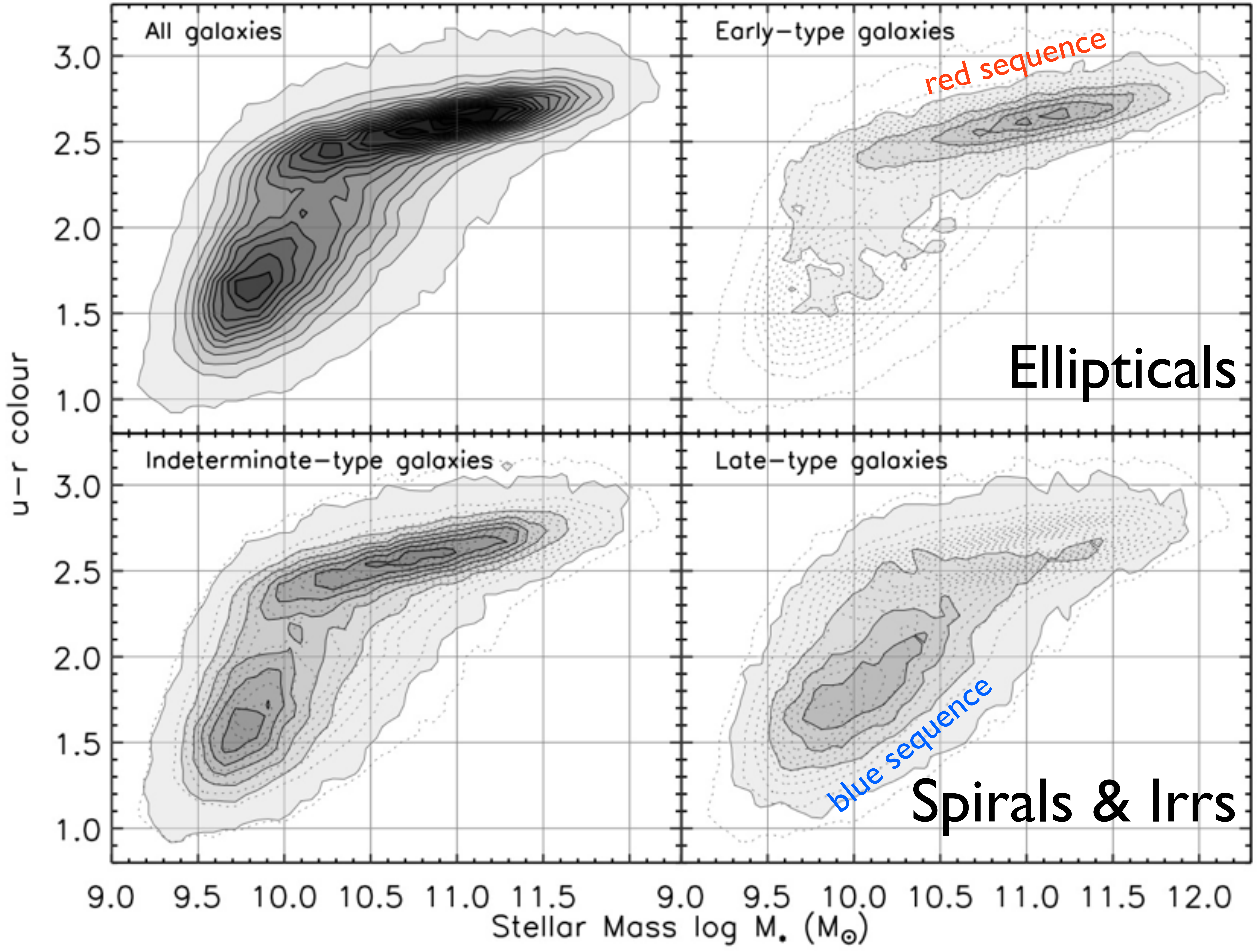


Spiral

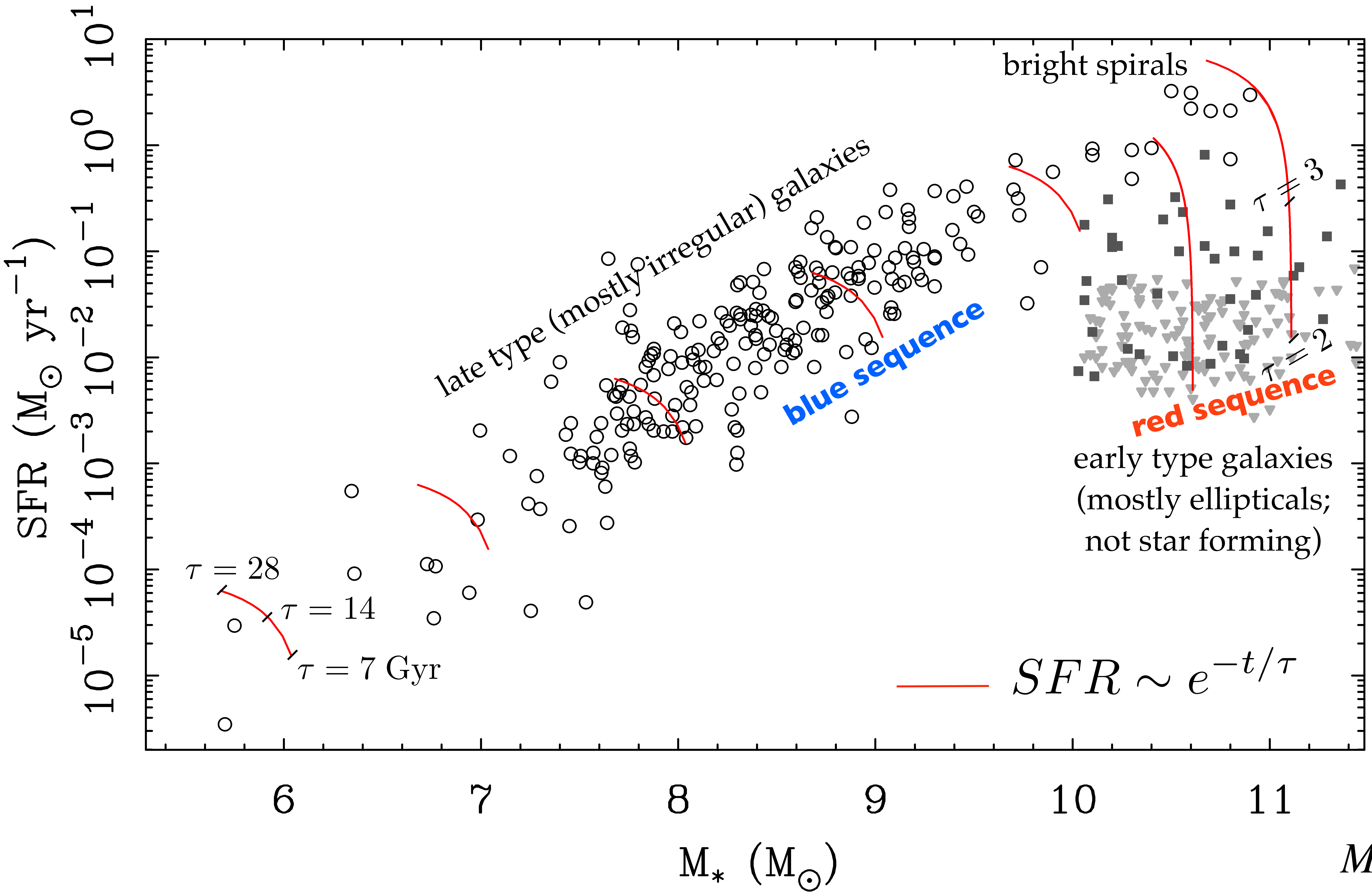
old stars
young stars
cold gas

Expect the older stars in elliptical galaxies to have higher mass-to-light ratios than in spirals.

color-magnitude relation for galaxies

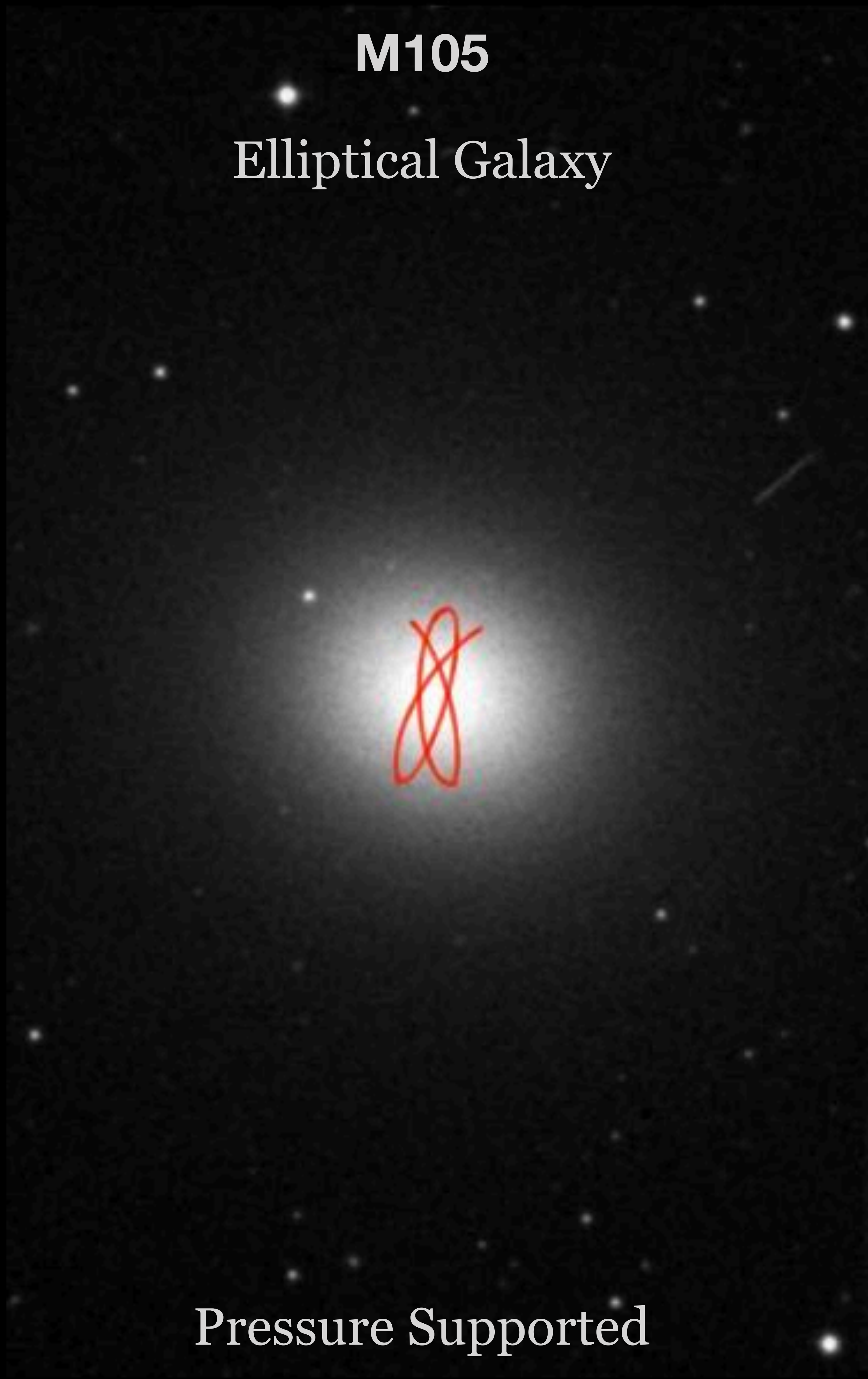


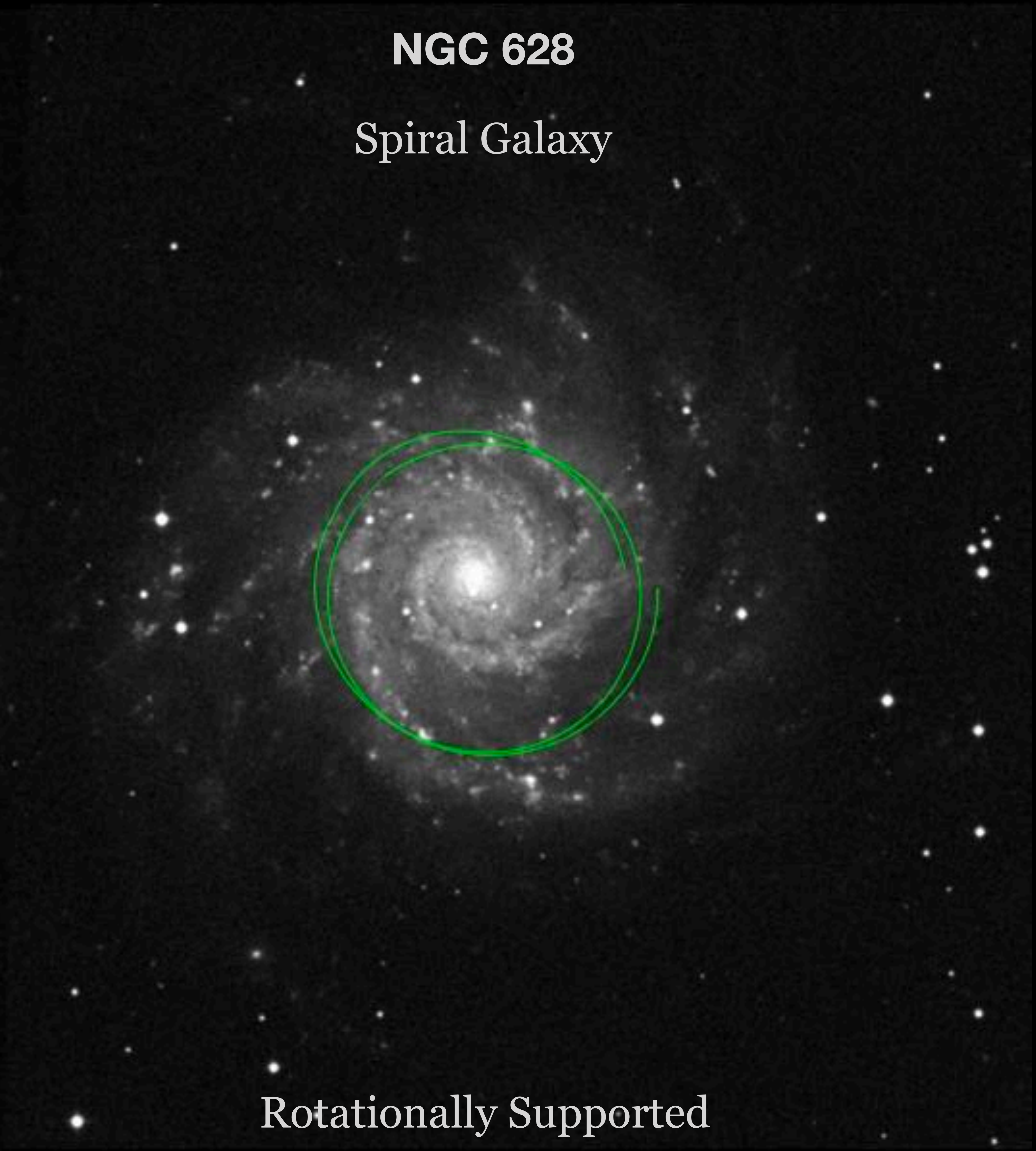
"Main Sequence of Star Forming Galaxies"



$$M_* = \int SFR\,dt$$

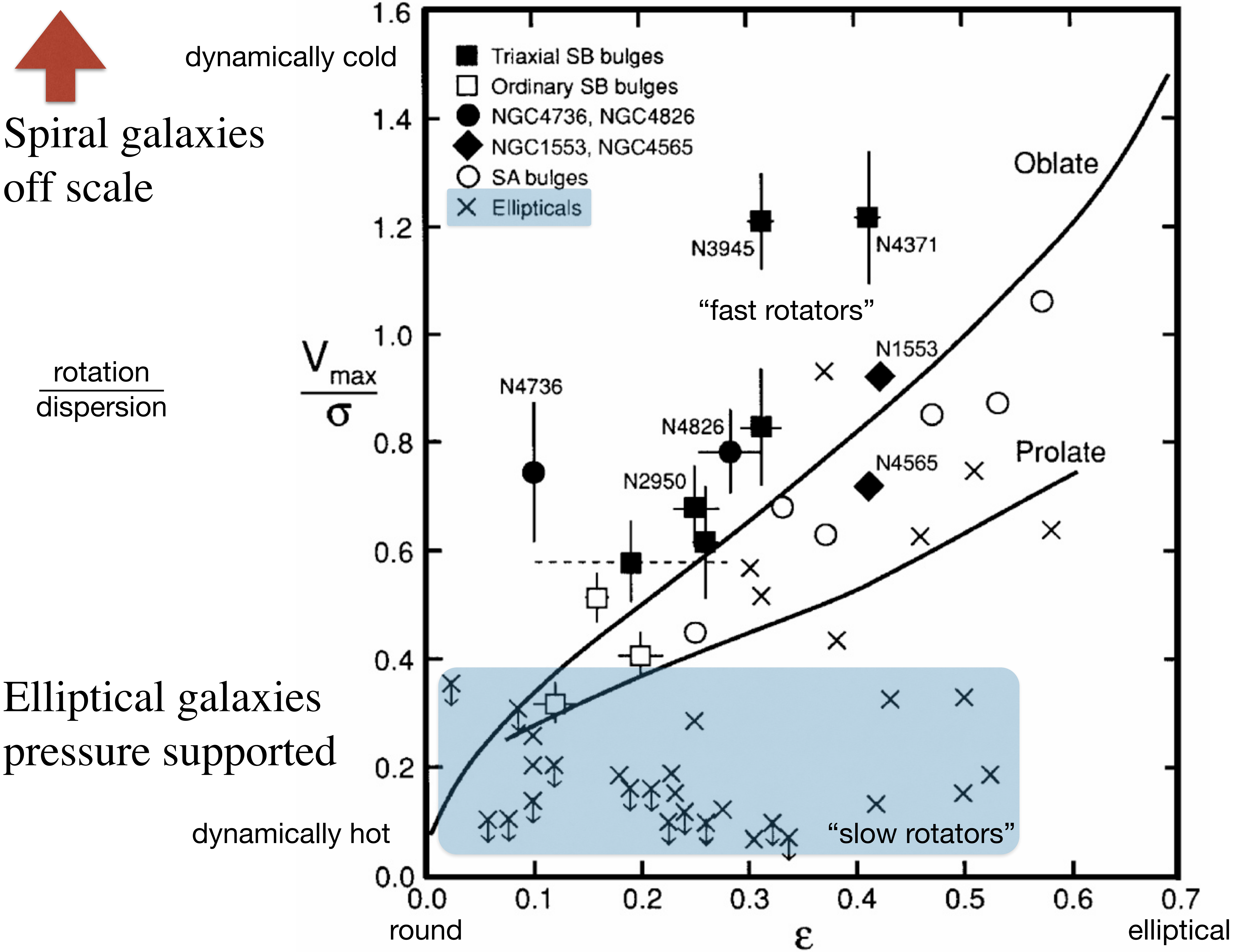
Stellar orbits in galaxies





Eccentric radial orbits
Random orientations

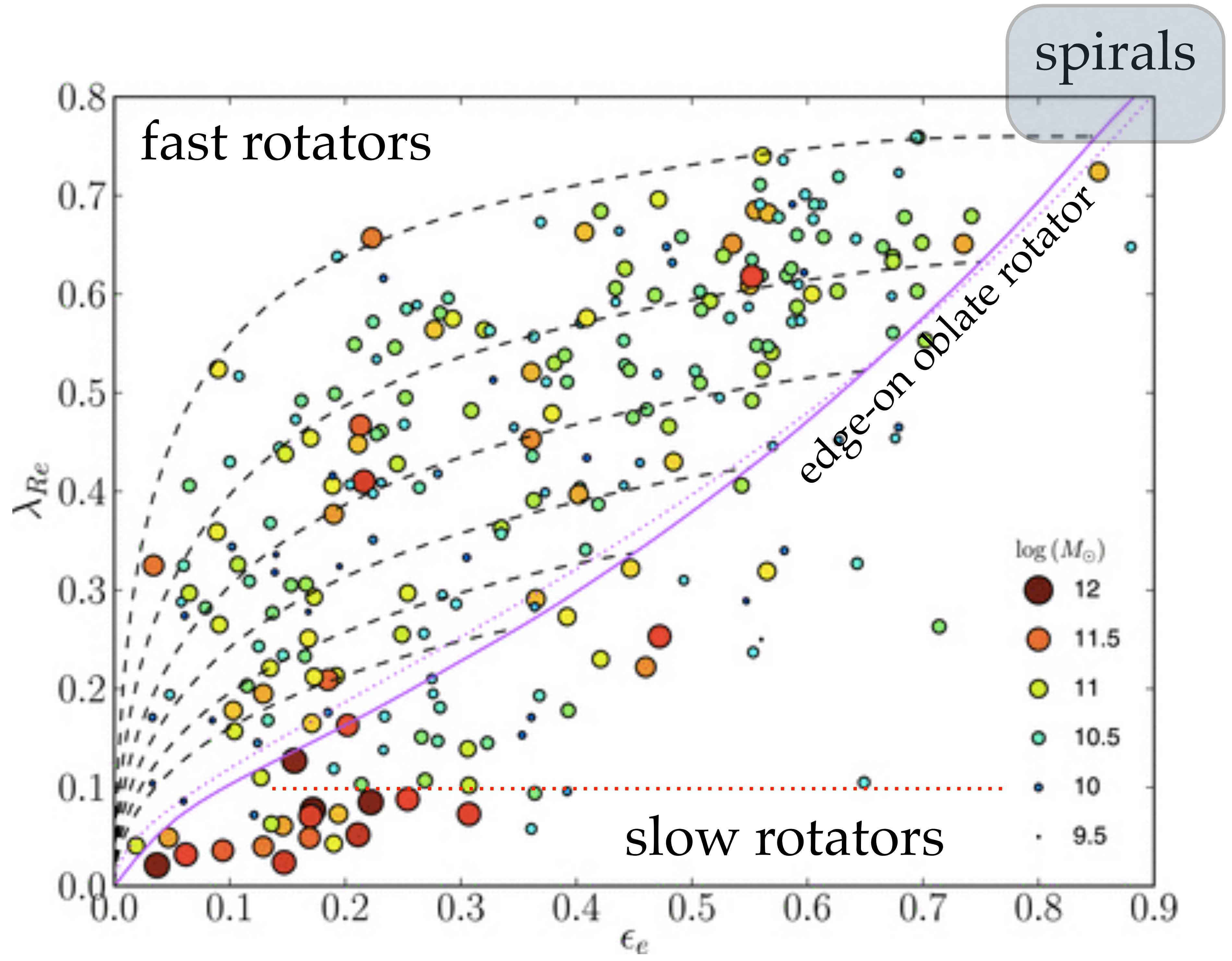
Nearly circular orbits
Same direction, same plane



Spiral galaxies
off scale
dynamically cold
rotation
dispersion
V_{max}/σ
Elliptical galaxies
pressure supported
dynamically hot
Triaxial SB bulges
Ordinary SB bulges
NGC4736, NGC4826
NGC1553, NGC4565
SA bulges
Ellipticals
N3945
N4371
"fast rotators"
N1553
N4736
N4826
N2950
N4565
Oblate
Prolate
"slow rotators"
1.6
1.4
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
round
ε
elliptical

$$\lambda_R = \frac{\langle R|V|\rangle}{\langle R\sqrt{V^2+\sigma^2}\rangle}$$

specific angular momentum



ellipticity

Massive ellipticals mostly pressure supported (slow rotators) while many (not all) lower mass ellipticals are fast rotators. These are often S0 galaxies.

Dashed lines represent different inclinations for different intrinsic ellipticities

Schwarzschild's method

Combine possible orbit families to best match the data

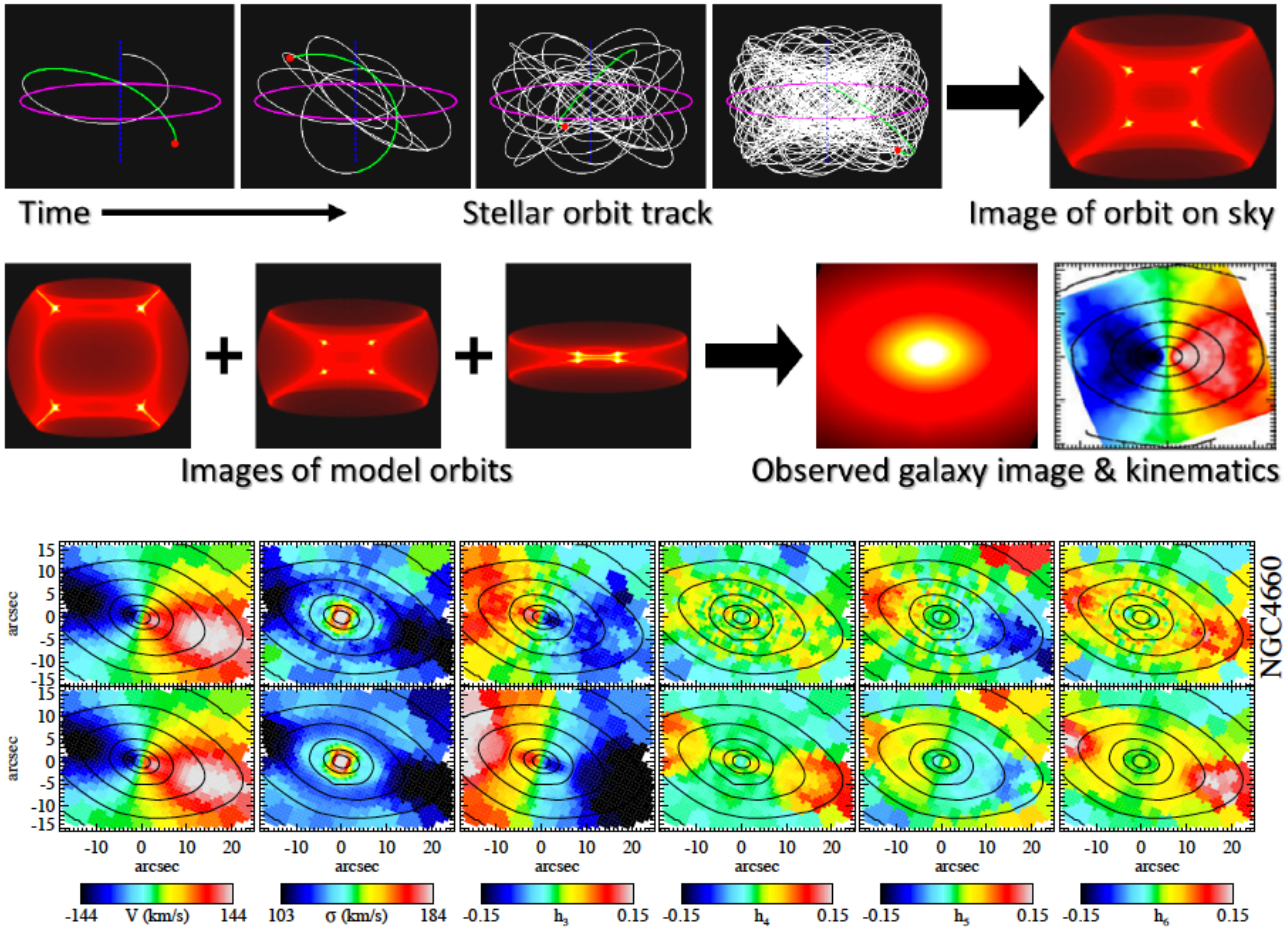


Figure 3. Schwarzschild's orbit-superposition method. *Top Row:* numerical integration of a single orbit in the adopted gravitational potential. After a sufficiently long time the density (of regular orbits) converges to a fixed distribution. *Middle Row:* the method finds the linear combination of thousands of orbits (three representative are shown here) which best fits the galaxy image and stellar kinematics. *Bottom two rows:* data (top) versus model (bottom) comparison. The model can fit the full stellar line-of-sight velocity distribution, here parametrized by the first six Gauss-Hermite moments (from Cappellari et al. 2007).

anisotropy

Orbital Anisotropy

The anisotropy parameter measures how radial or circular orbits are

anisotropy $$\beta = 1 - \frac{\sigma_t^2}{\sigma_r^2}$$

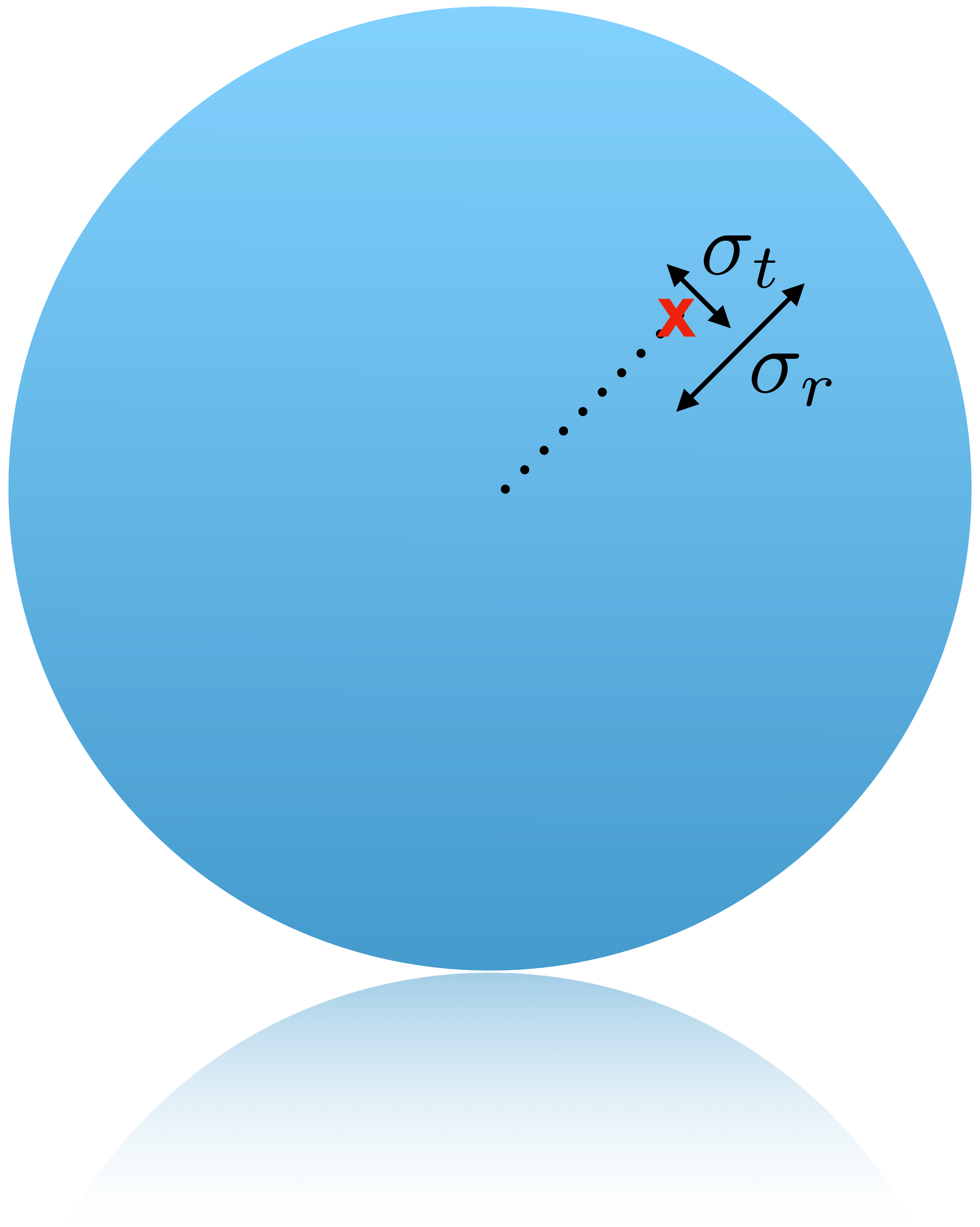
σ_t^2: tangential velocity dispersion

σ_r^2: radial velocity dispersion

Jean's equation (one form)

$$GM(r) = r\sigma_r^2\left(-\frac{\partial \ln \nu_*}{\partial r} - \frac{\partial \ln \sigma_r^2}{\partial r} - 2\beta\right)$$

logarithmic gradients
(just the power law slope!)



$\nu_*(r)$ is the density distribution of tracer particles
e.g., exponential disk, $r^{1/4}$ law

Phase Space

More generally, can define a combination of configuration and momentum as 6D phase space

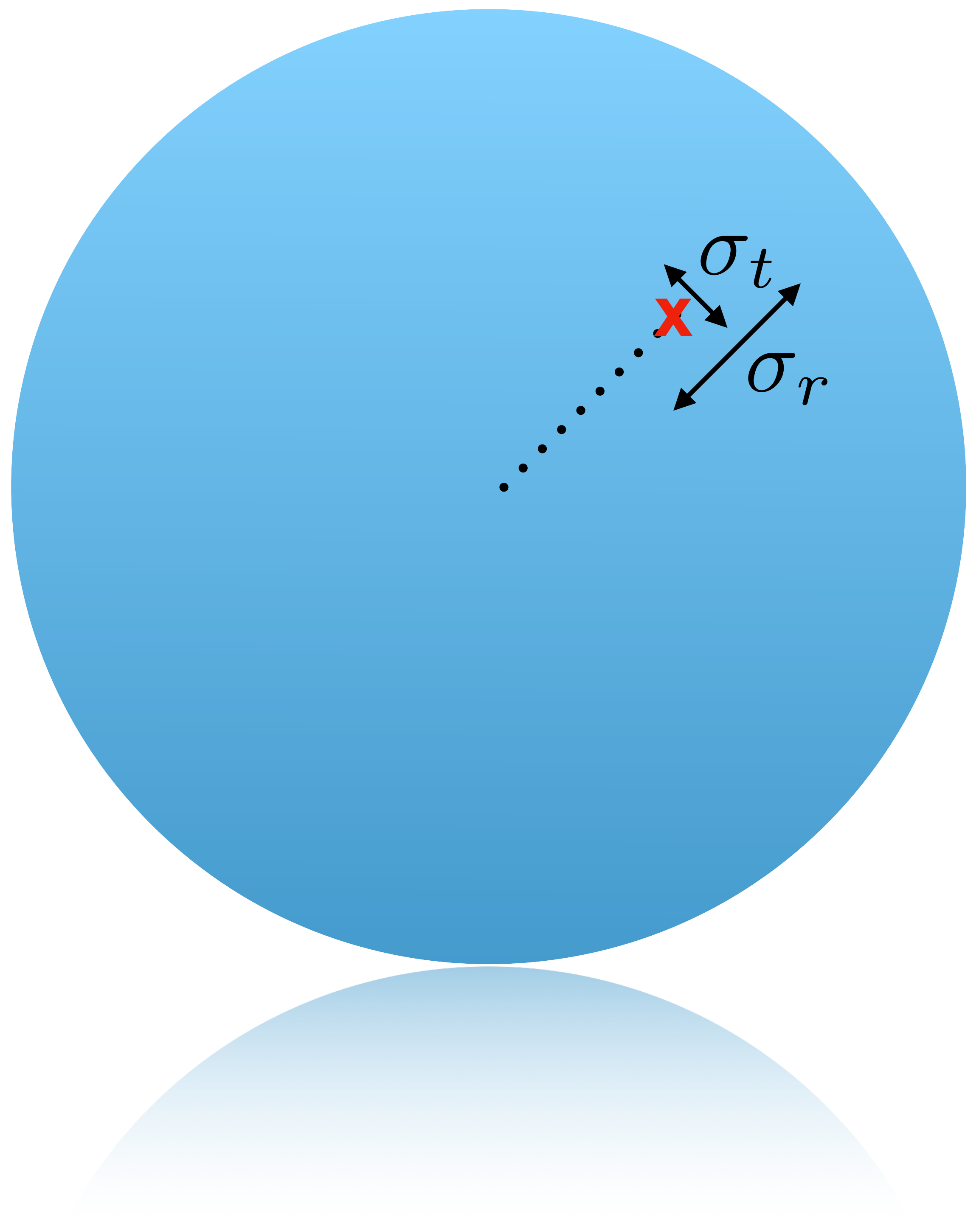
phase space $f(x_i, \dot{x}_i)$ configuration space $x_i = x, y, z$ momentum space $\dot{x}_i = \dot{x}, \dot{y}, \dot{z}$

Collisionless Boltzmann equation

$$\frac{\partial f}{\partial t} + \overrightarrow{V} \cdot \overrightarrow{\nabla} f - \overrightarrow{\nabla}\Phi \cdot \frac{\partial f}{\partial \overrightarrow{V}} = 0$$

Works for equilibrium systems; relying on the fact that the gravitational potential depends only on position, not momentum.

PHASE SPACE IS CONSERVED
Can mix in empty space (spread things out), but cannot compress.



$$\nu(x_i) = \int f(x_i, \dot{x}_i)\, d^3\dot{x}_i$$ density is the integral over momenta

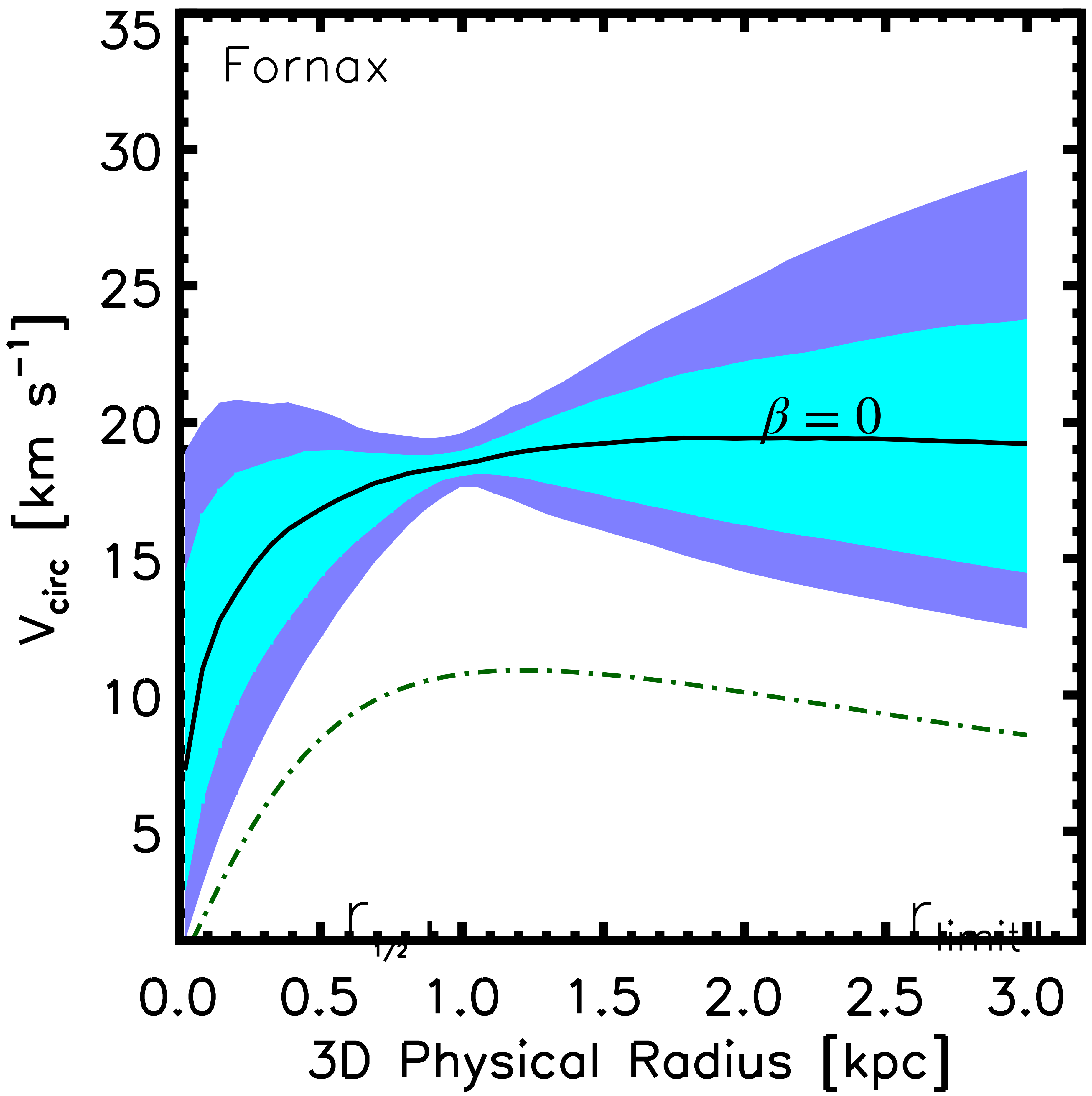
Fornax

dwarf spheroidal (dSph)
satellite of the Milky Way

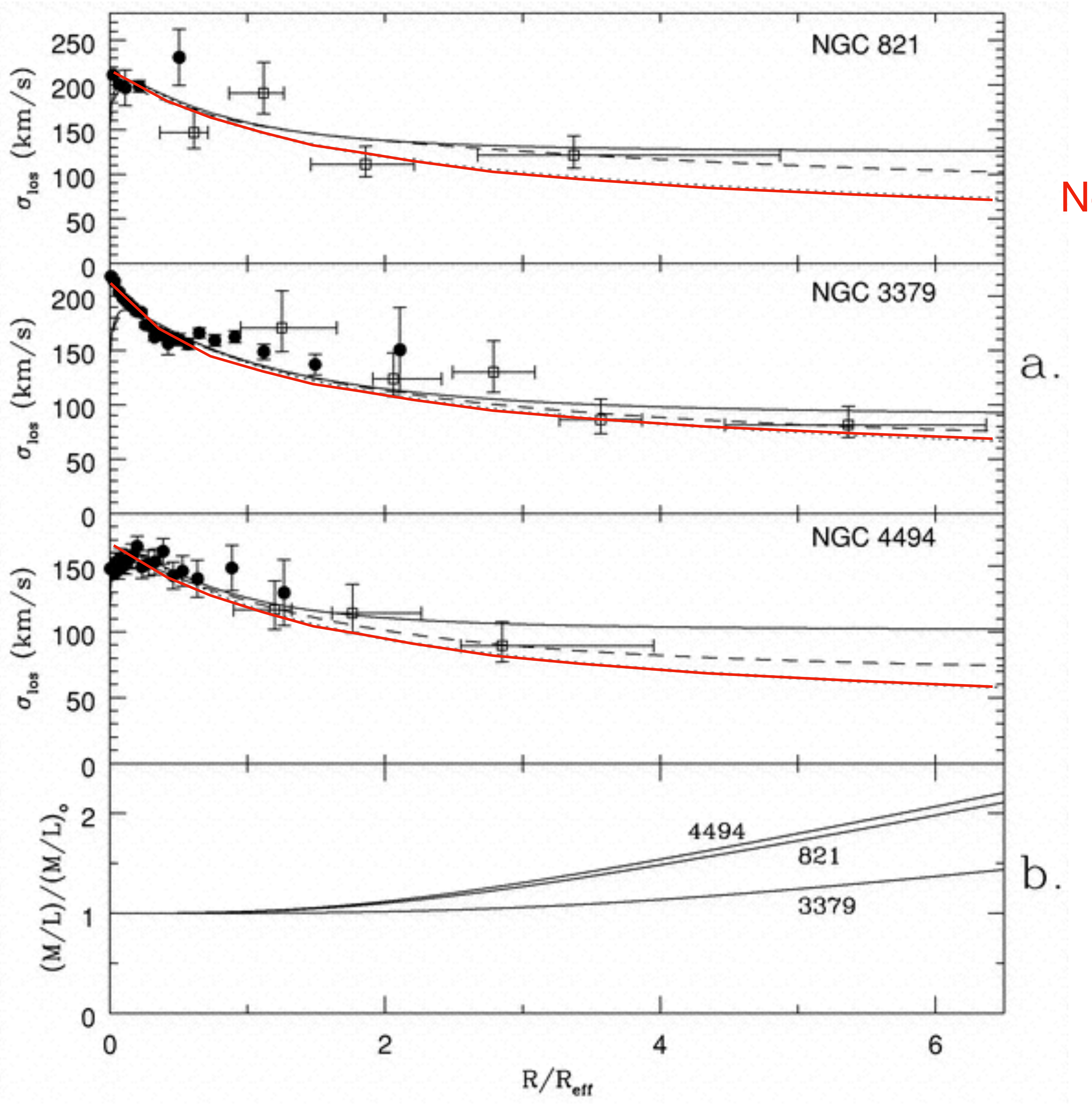
Jean's equation $$GM(r) = r\sigma_r^2 \left(-\frac{\partial \ln \nu_*}{\partial r} - \frac{\partial \ln \sigma_r^2}{\partial r} - 2\beta \right)$$

Mass-anisotropy degeneracy

$M(r)$ degenerate with $\beta(r)$



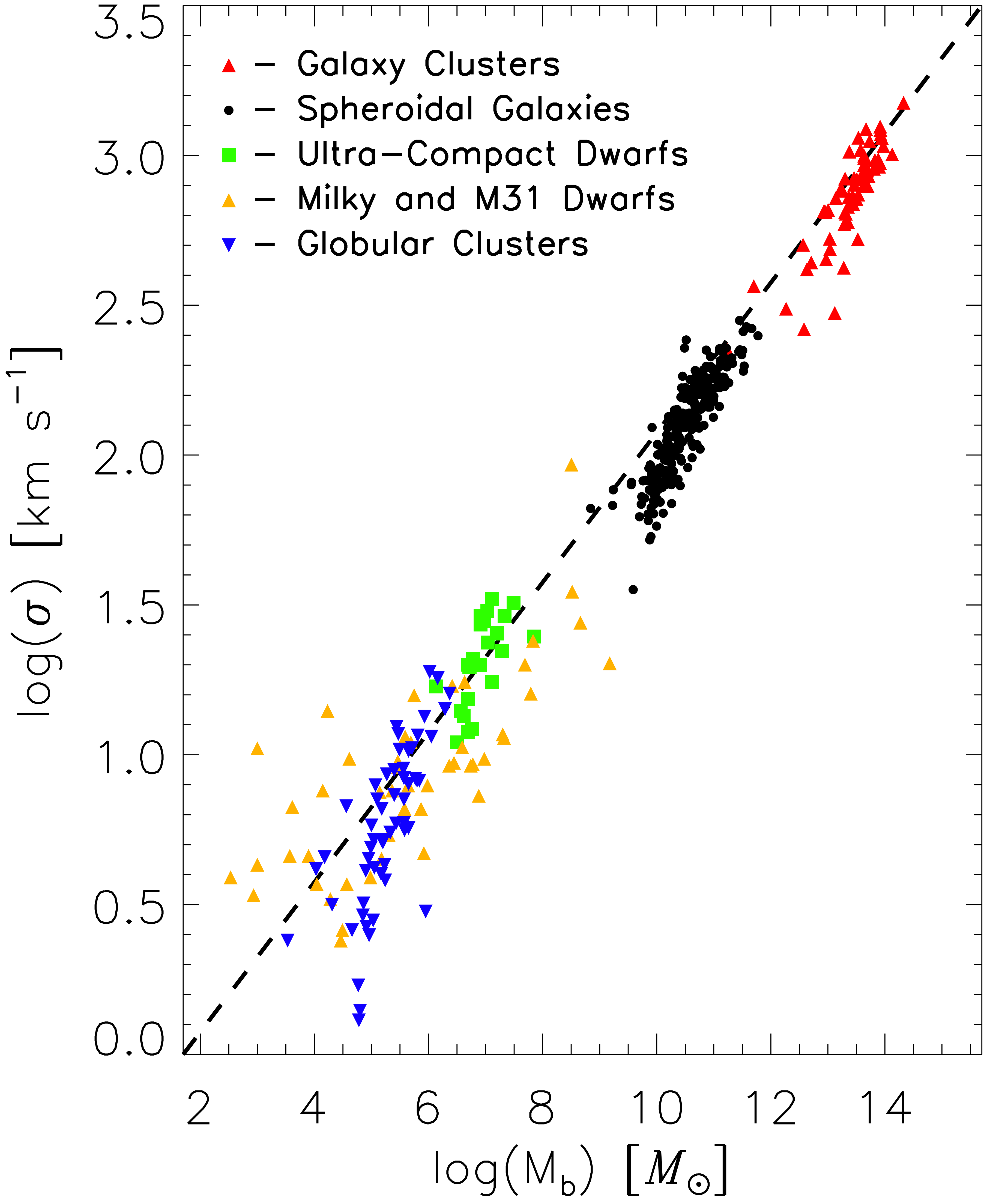
Velocity dispersion profiles for 3 ETGs
measured from stars at small R, PN at large R



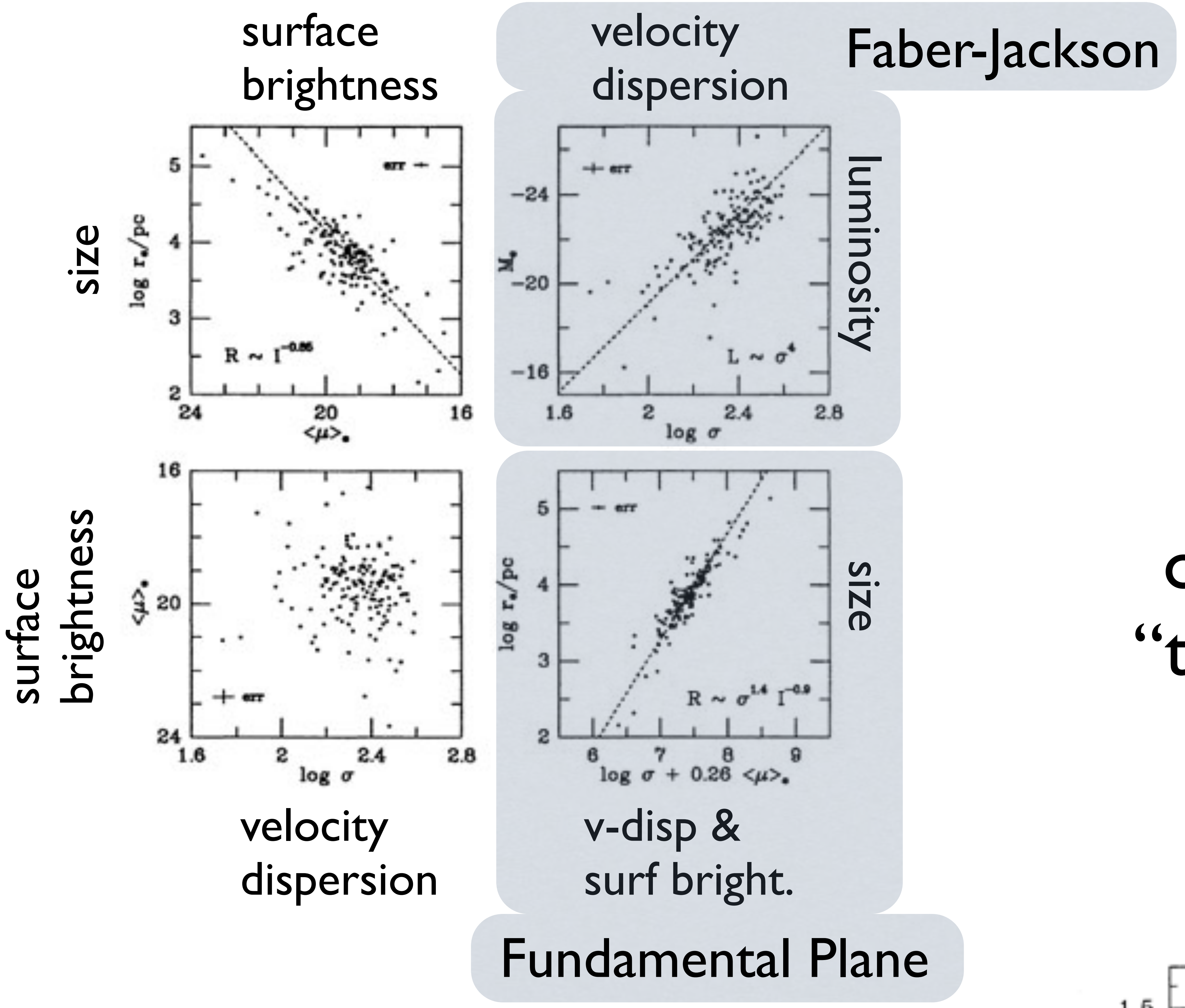
No dark matter

Faber-Jackson (pressure supported)

Tully-Fisher for Ellipticals



Fundamental Plane (pressure supported)



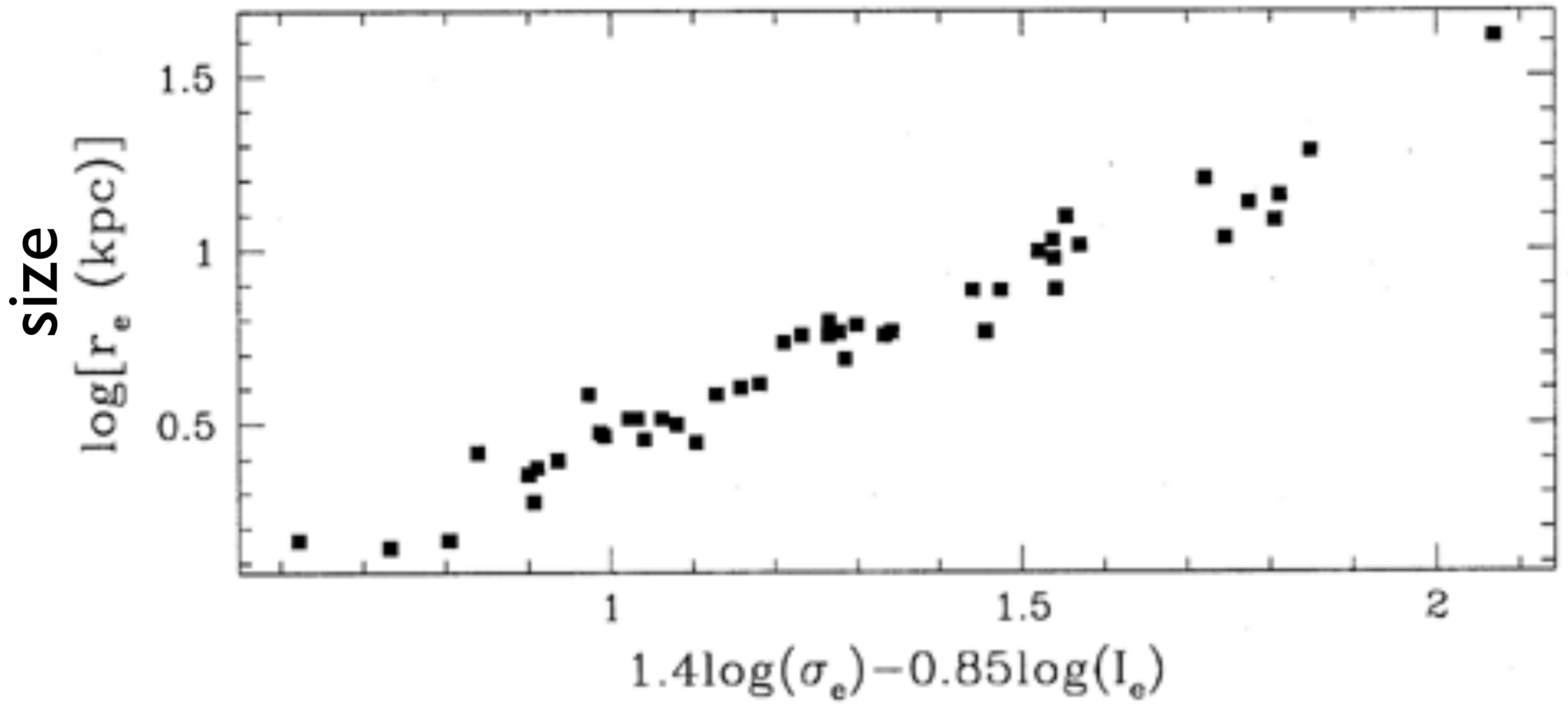
"Viral" fundamental plane

$$R_e \propto \sigma^2 I_e^{-1}$$

observed fundamental plane
"tilted" wrt virial expectation:

$$R_e \propto \sigma^{1.4} I_e^{-0.85}$$

velocity dispersion & surface brightness



$$M \propto \sigma^2 R_e$$ virial theorom

$$L \propto I_e R_e^2$$ luminosity, surface brightness size