

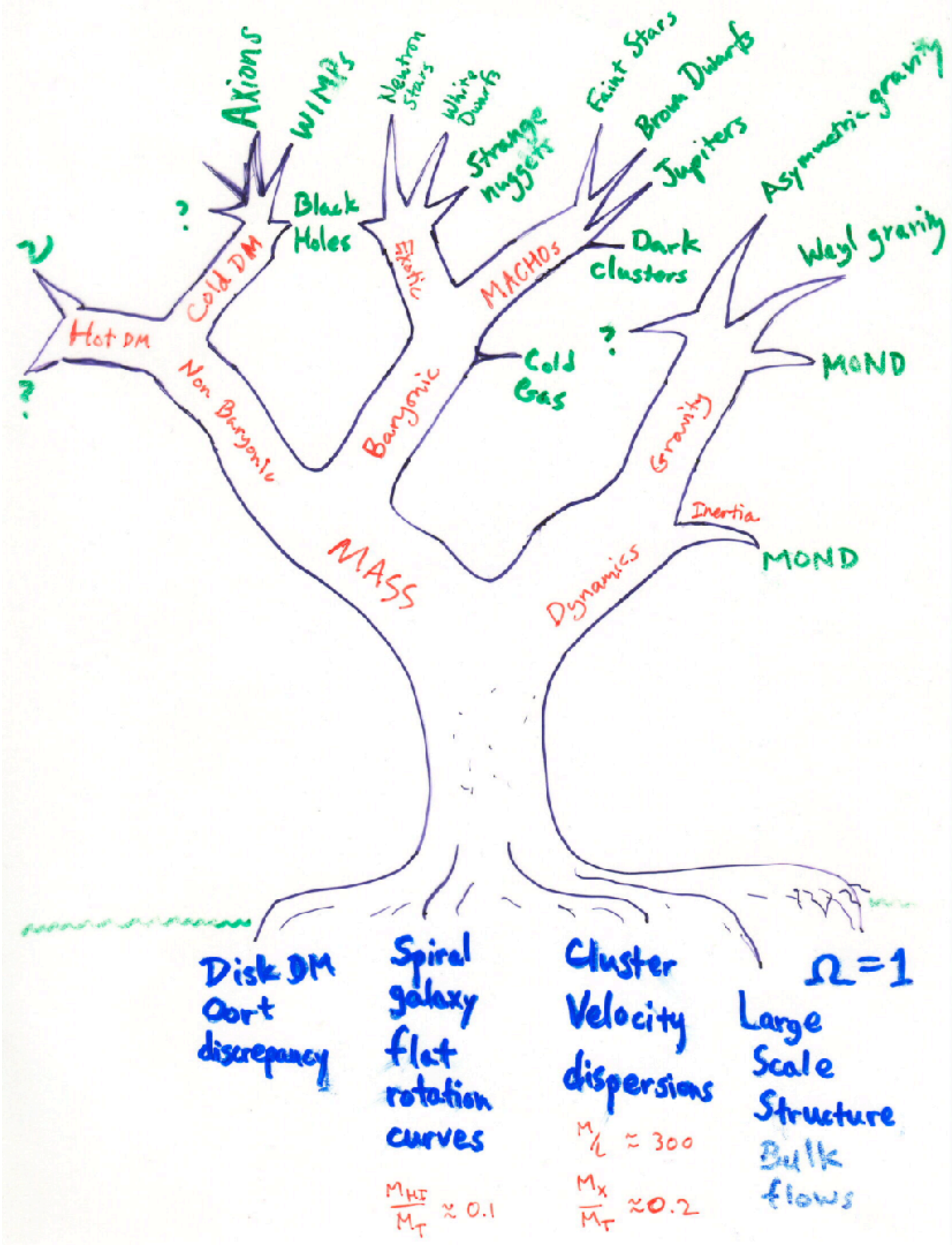
# DARK MATTER

ASTR 333/433  
SPRING 2026  
TR 11:30AM-12:45PM  
SEARS 552

<http://astroweb.case.edu/ssm/ASTR333/>

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A JWST First Release image of Cluster SMACS 0723. The image shows a dense field of galaxies, many of which are distorted and stretched into arcs and multiple images due to gravitational lensing. The galaxies are primarily yellow and orange, with some blue stars scattered throughout. A prominent bright star in the upper left and another in the lower right are marked with blue lines, indicating their positions. The background is dark, highlighting the individual galaxies and their distorted shapes.

JWST First Release image  
Gravitational lensing in  
Cluster SMACS 0723

Four distinct measures:  
velocity dispersion, hydrostatic equilibrium of X-ray gas, the Sunyaev-Zel'dovich effect, and gravitational lensing

# Mass estimators for Clusters of Galaxies

Four distinct measures:

velocity dispersion, hydrostatic equilibrium of X-ray gas, the Sunyaev-Zel'dovich effect, and gravitational lensing

- Virial mass estimator  $M = \frac{2.5}{G} \sigma^2 R_e$

- Hydrostatic equilibrium (X-ray)  $\frac{GM}{r} = -\frac{kT}{\mu m_p} \left( \frac{\partial \ln \rho}{\partial \ln r} + \frac{\partial \ln T}{\partial \ln r} \right)$

- S-Z effect  $M \propto D_A^2 \frac{\int \Delta T d\Omega}{\langle T \rangle}$

- gravitational lensing

$$\alpha_d = \frac{4GM}{c^2 b} \longrightarrow M(< \theta_l) = (1.1 \times 10^{14} M_\odot) \left( \frac{\theta_l}{30''} \right)^2 \left( \frac{D_L}{D_S} \right) \left( \frac{D_{LS}}{1 \text{ Gpc}} \right)$$

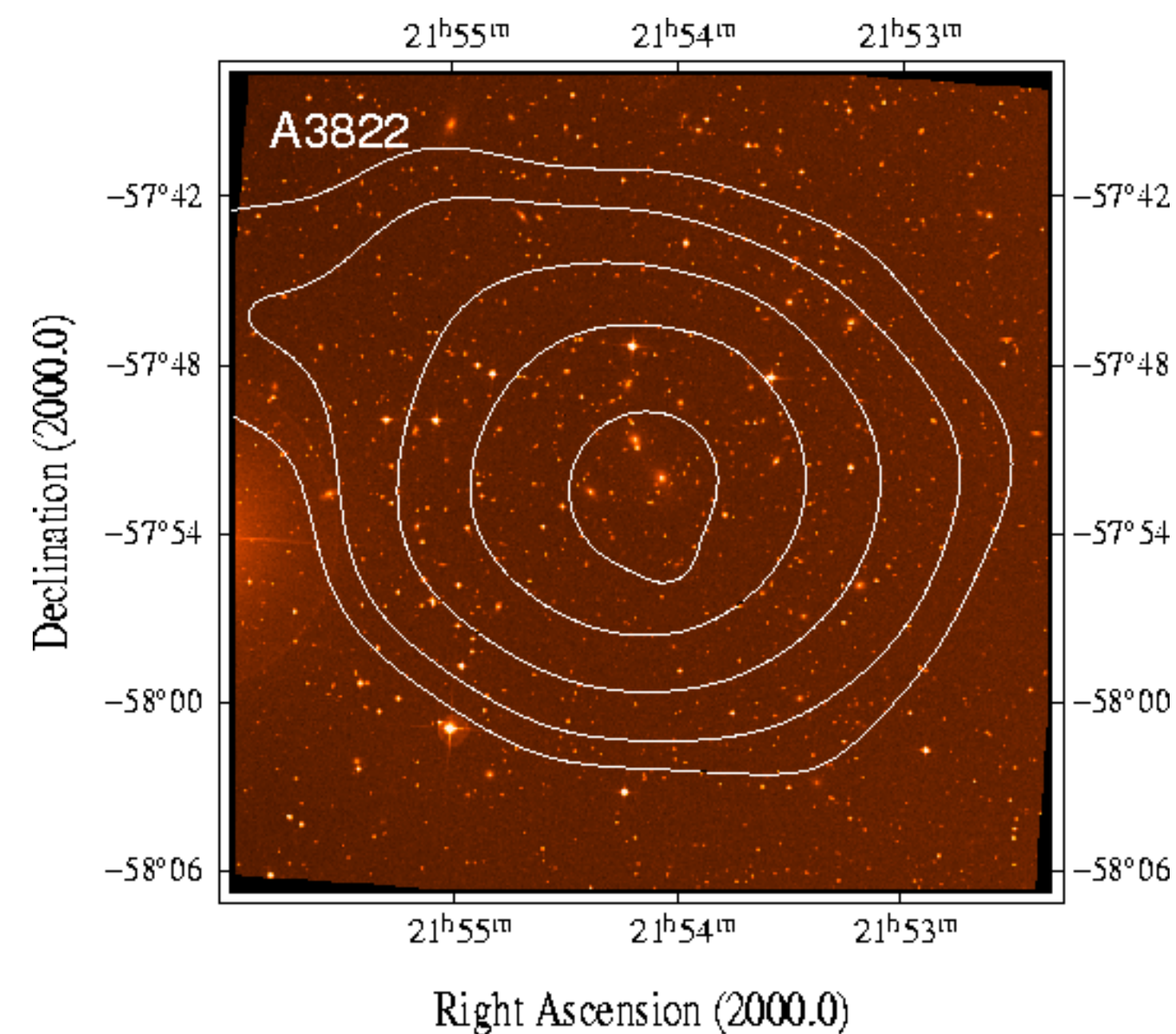
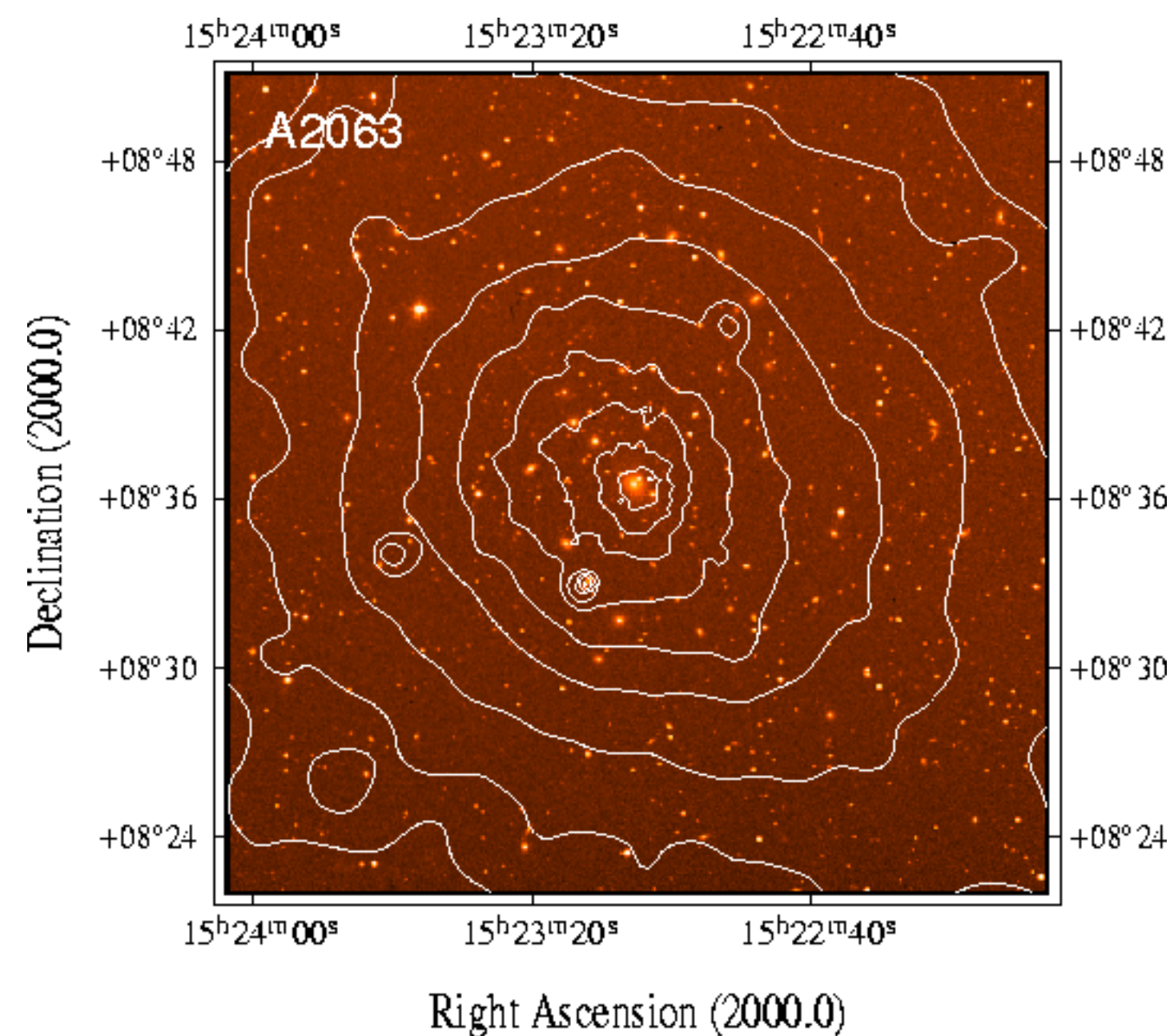
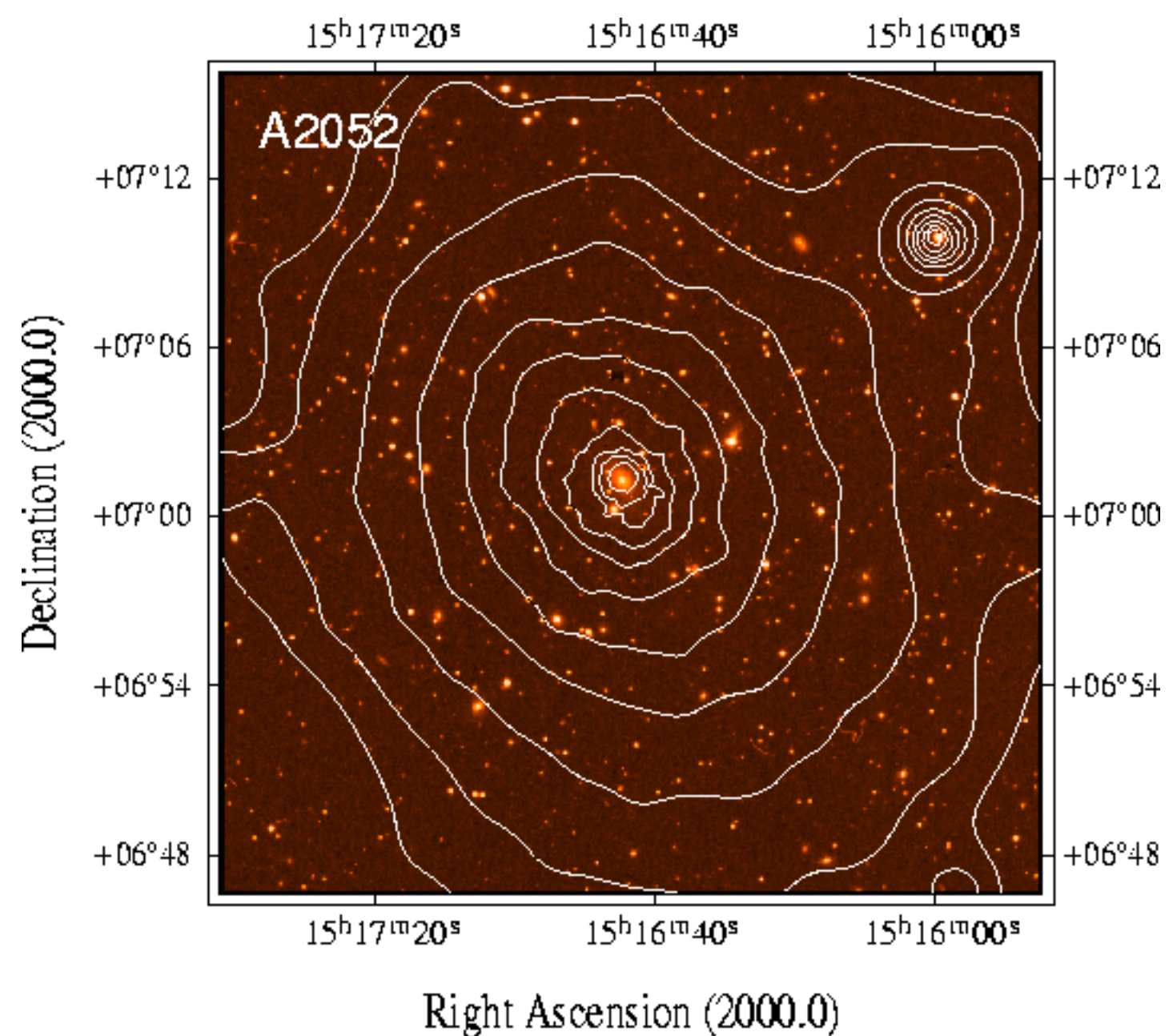
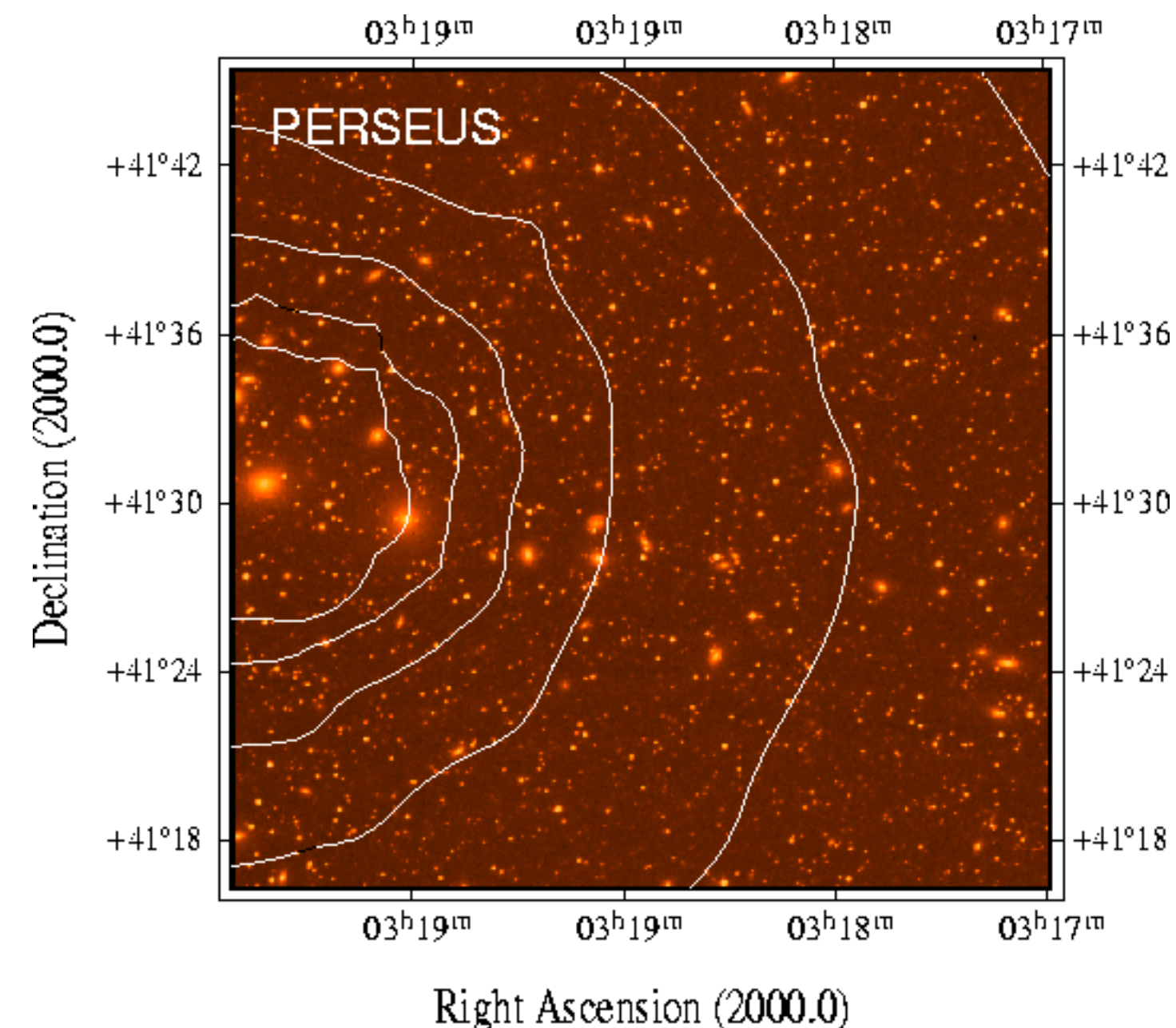
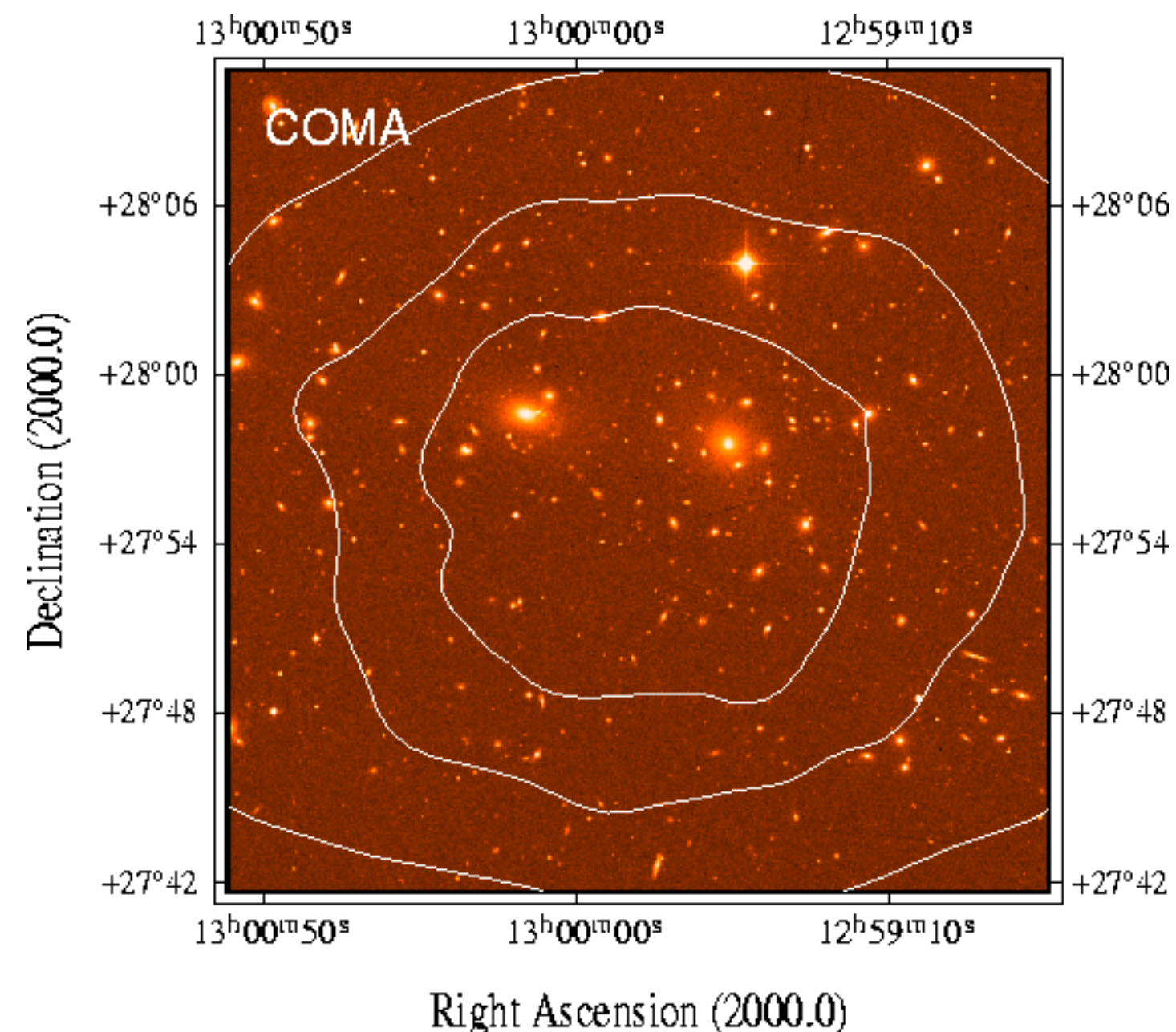
# Clusters in optical and X-ray (contours)

Typically two types of X-ray sources

**Bremsstrahlung from hot ICM**  
(large scale, diffuse emission)

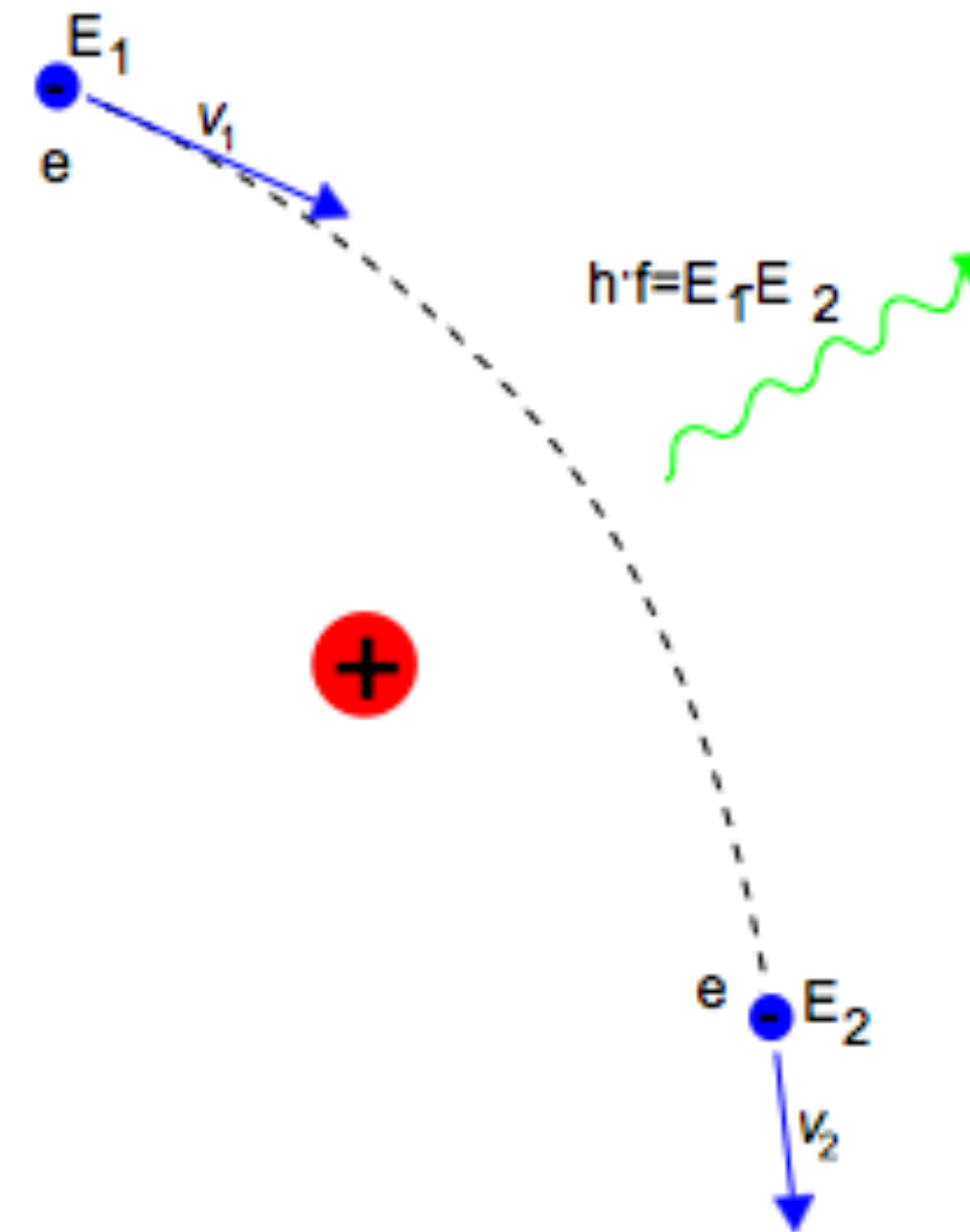
**AGN and other point sources**  
(point sources)

Reiprich ApJ, 567, 716-740 (2002)

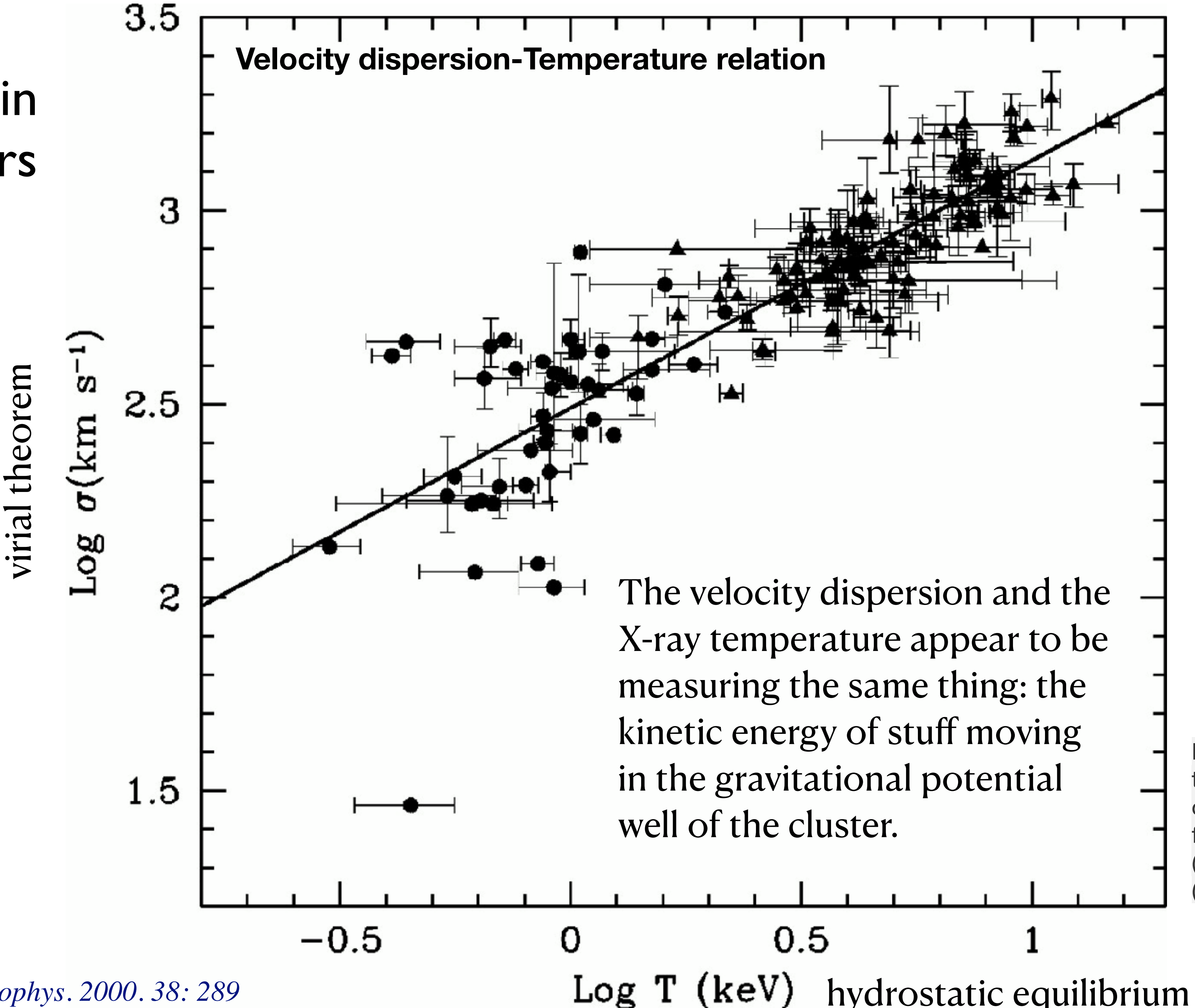


# Bremsstrahlung

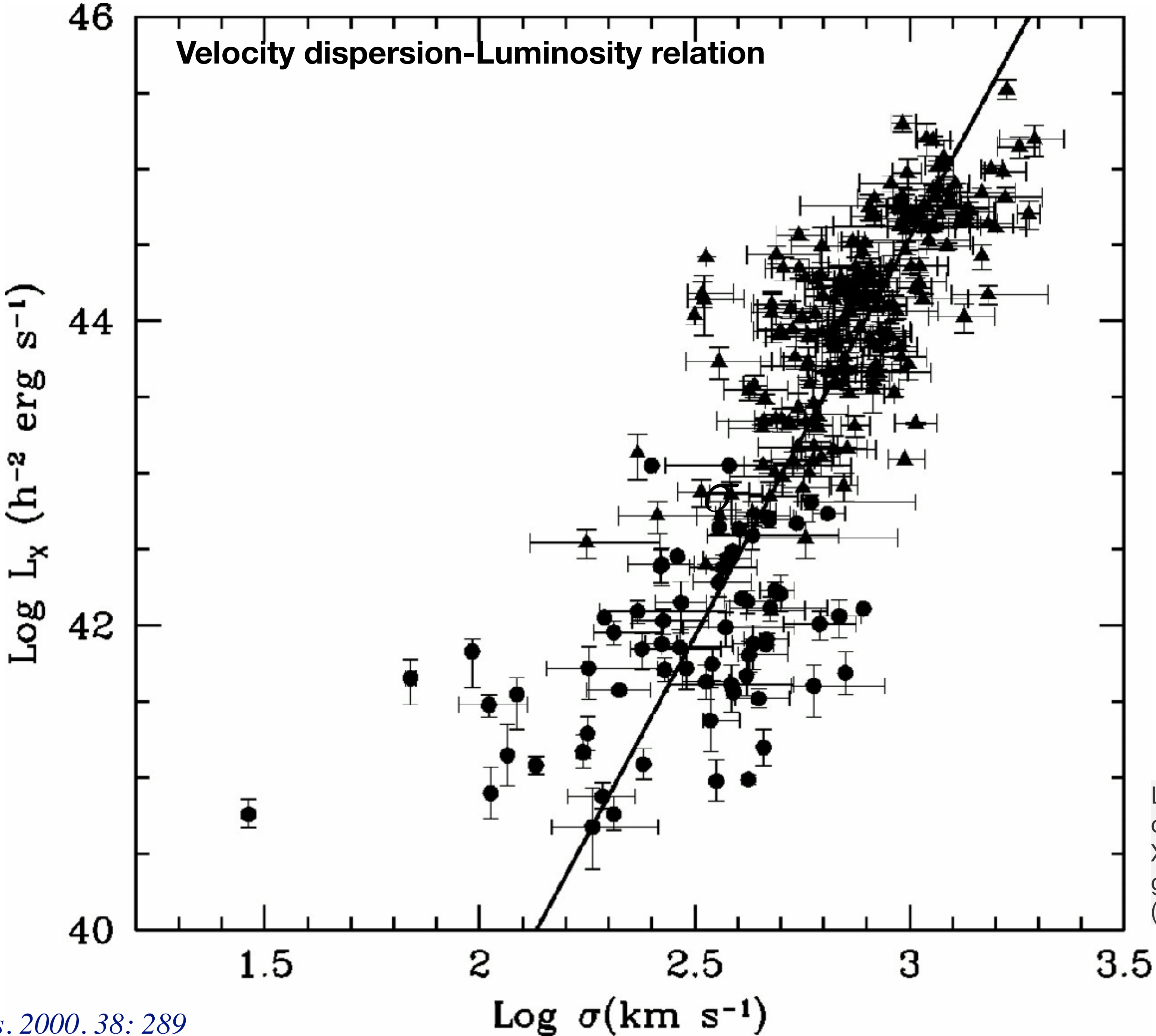
Gas falling into clusters shock heats to the virial temperature of the potential,  $kT \sim mV^2$  resulting in an intracluster medium (ICM) composed of hot plasma. This plasma radiates in X-rays via Bremsstrahlung (braking radiation).  
[Sometimes also called free-free radiation]  
Just classical radiation from accelerated charges.



# Global correlations in galaxy clusters

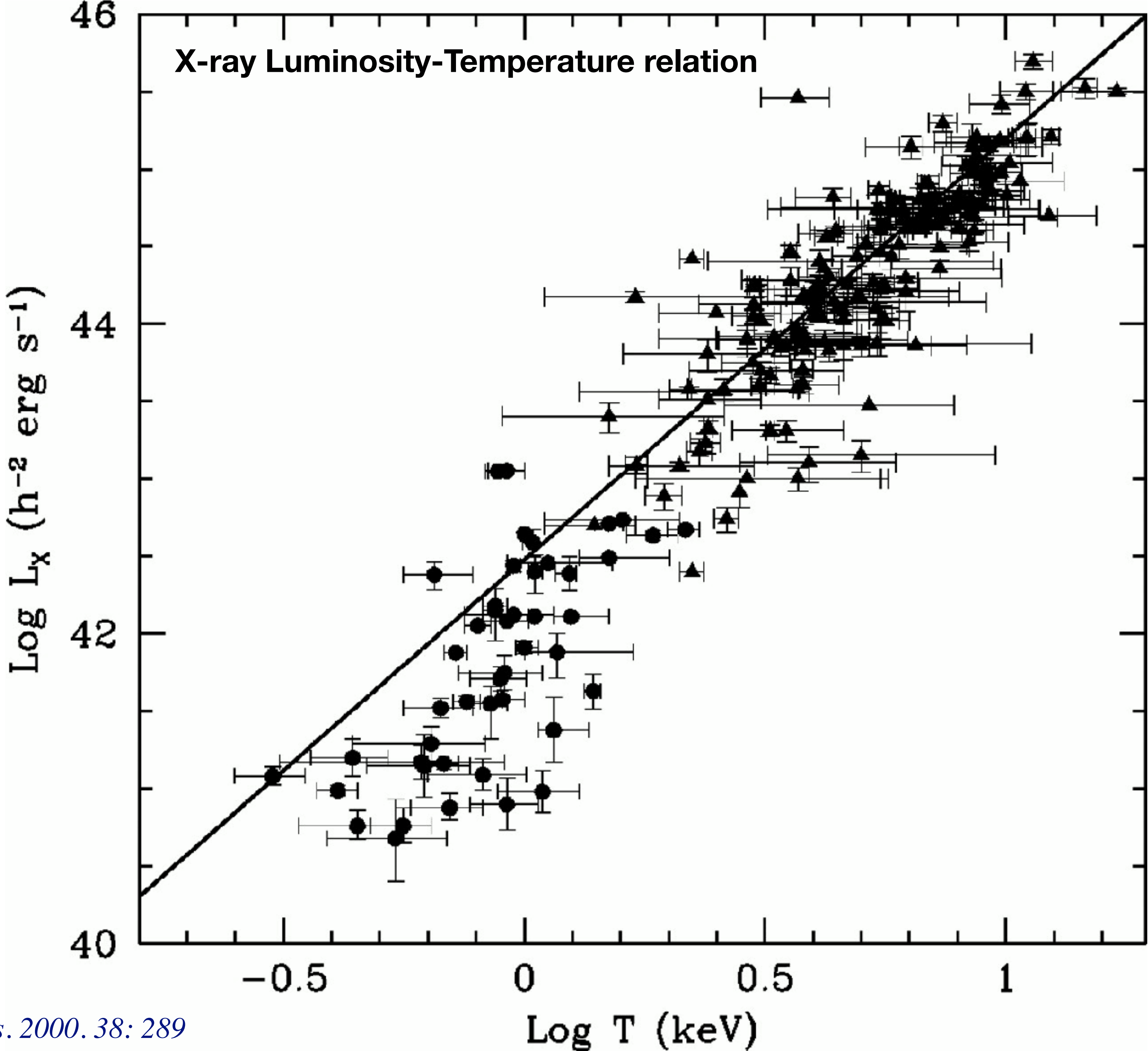


# Global correlations in galaxy clusters



Logarithm of optical velocity dispersion versus logarithm of X-ray luminosity for a sample of groups (circles) and clusters (triangles).

# Global correlations in galaxy clusters



Logarithm of the X-ray temperature versus logarithm of X-ray luminosity for a sample of groups (circles) and clusters (triangles).

# Beta models

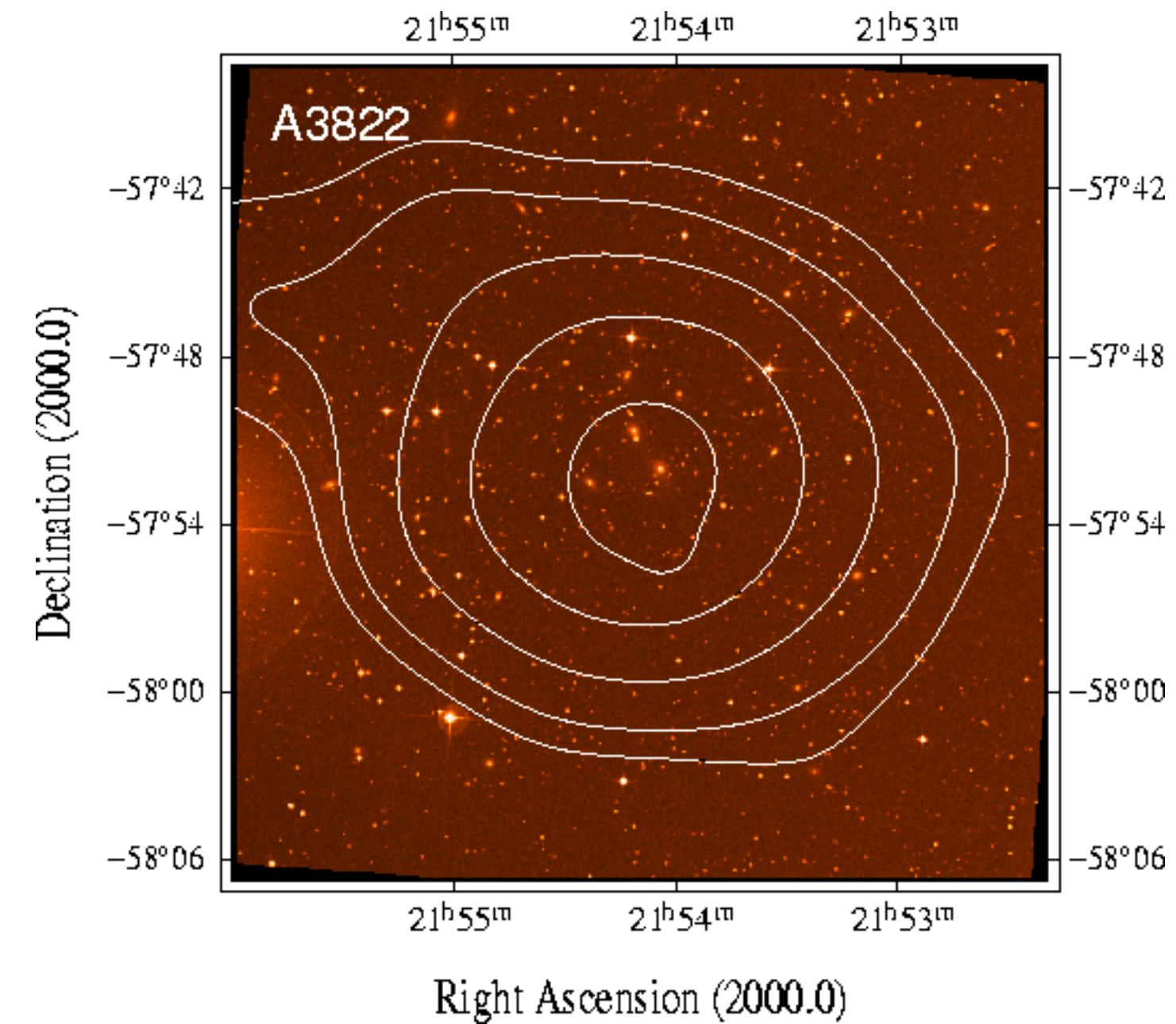
The X-ray surface brightness at a projected radius  $R$   
for an isothermal sphere is given by:

$$S(R) = S_0 [1 + (R/r_c)^2]^{-3\beta + 1/2}$$

$S_0$             central surface brightness

$r_c$             core radius of gas distribution

$$\beta \equiv \frac{\mu m_p \sigma^2}{kT_g} = \frac{\text{specific energy in galaxies}}{\text{specific energy in the hot gas}}$$



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$\mu$  is the mean molecular weight

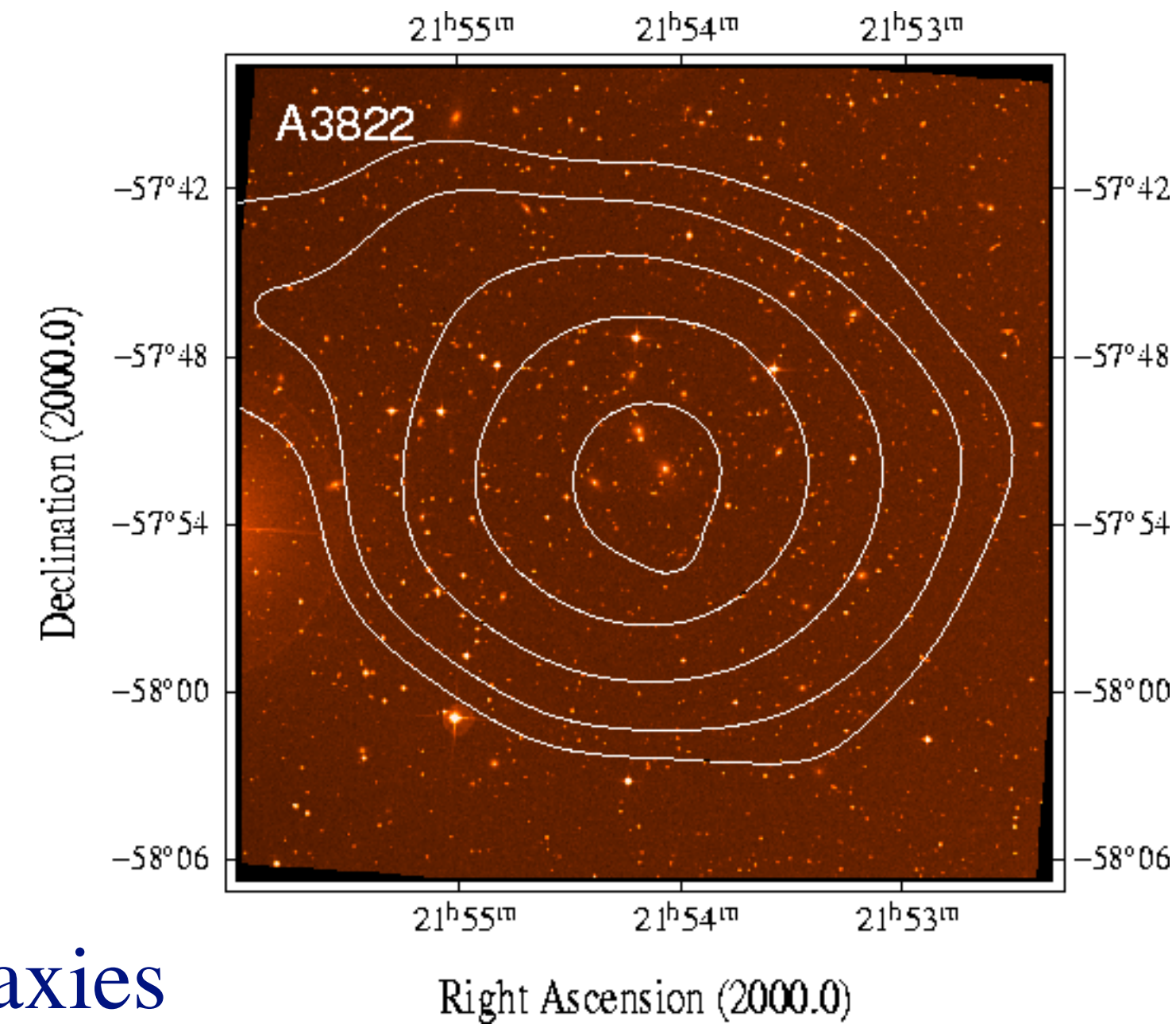
$m_p$  is the mass of the proton

$\sigma$  is the one-dimensional velocity dispersion of the galaxies

$T_g$  is the temperature of the ICM

Typically the gas is assumed to be isothermal

$\beta$  treated as fit parameter; typically  $\sim 2/3$   
 BUT often higher when sigma well measured;  
 and often lower in groups



# Mass Estimator

$$M(< r) = - \frac{r}{G} \frac{kT}{\mu m_p} \left( \frac{\partial \ln \rho}{\partial \ln r} + \frac{\partial \ln T}{\partial \ln r} \right)$$



the 3D gas density profile  $\rho(r)$  is obtained by fitting the standard beta model to the surface brightness profile.

the gas temperature is measured from the X-ray spectrum

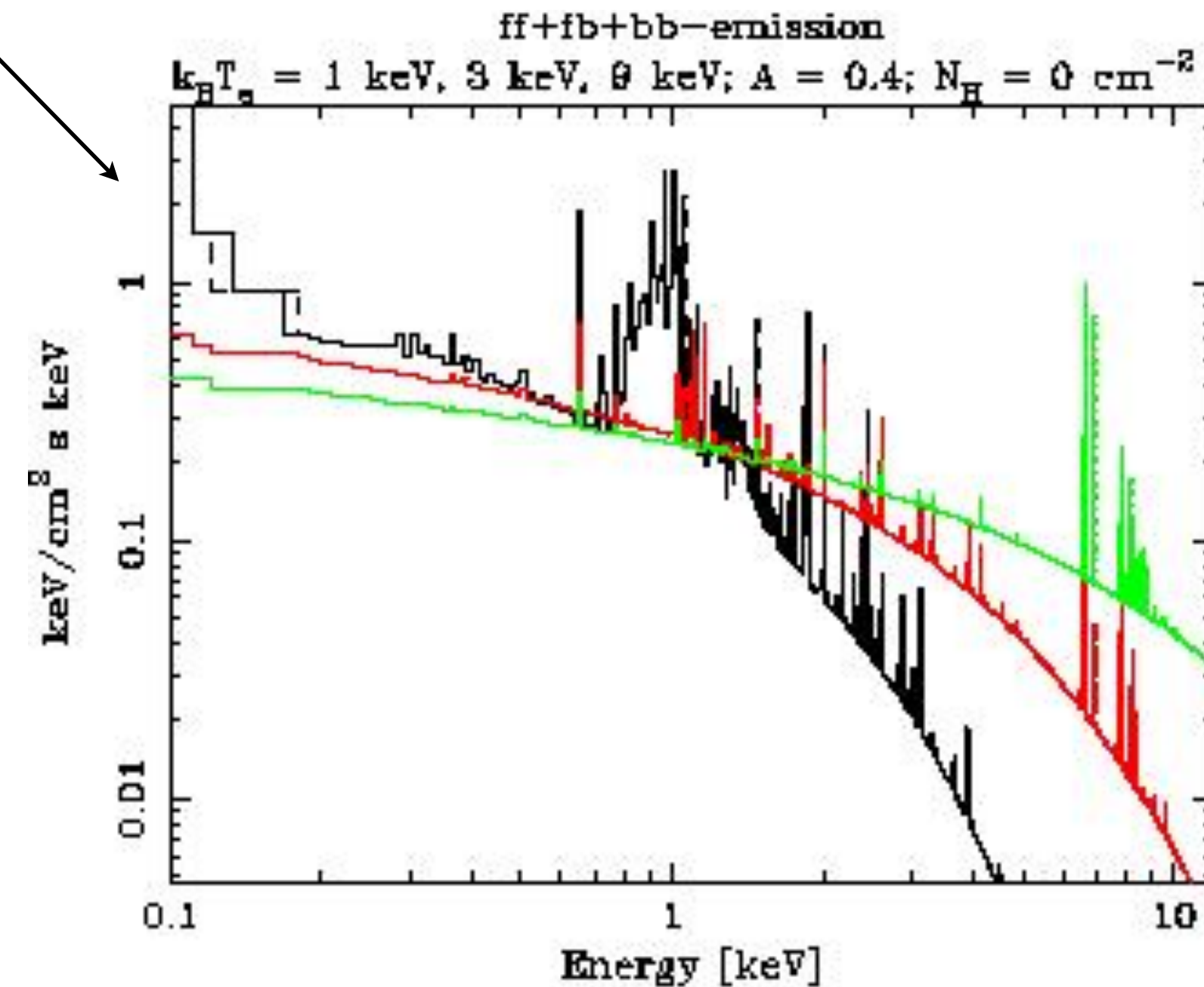
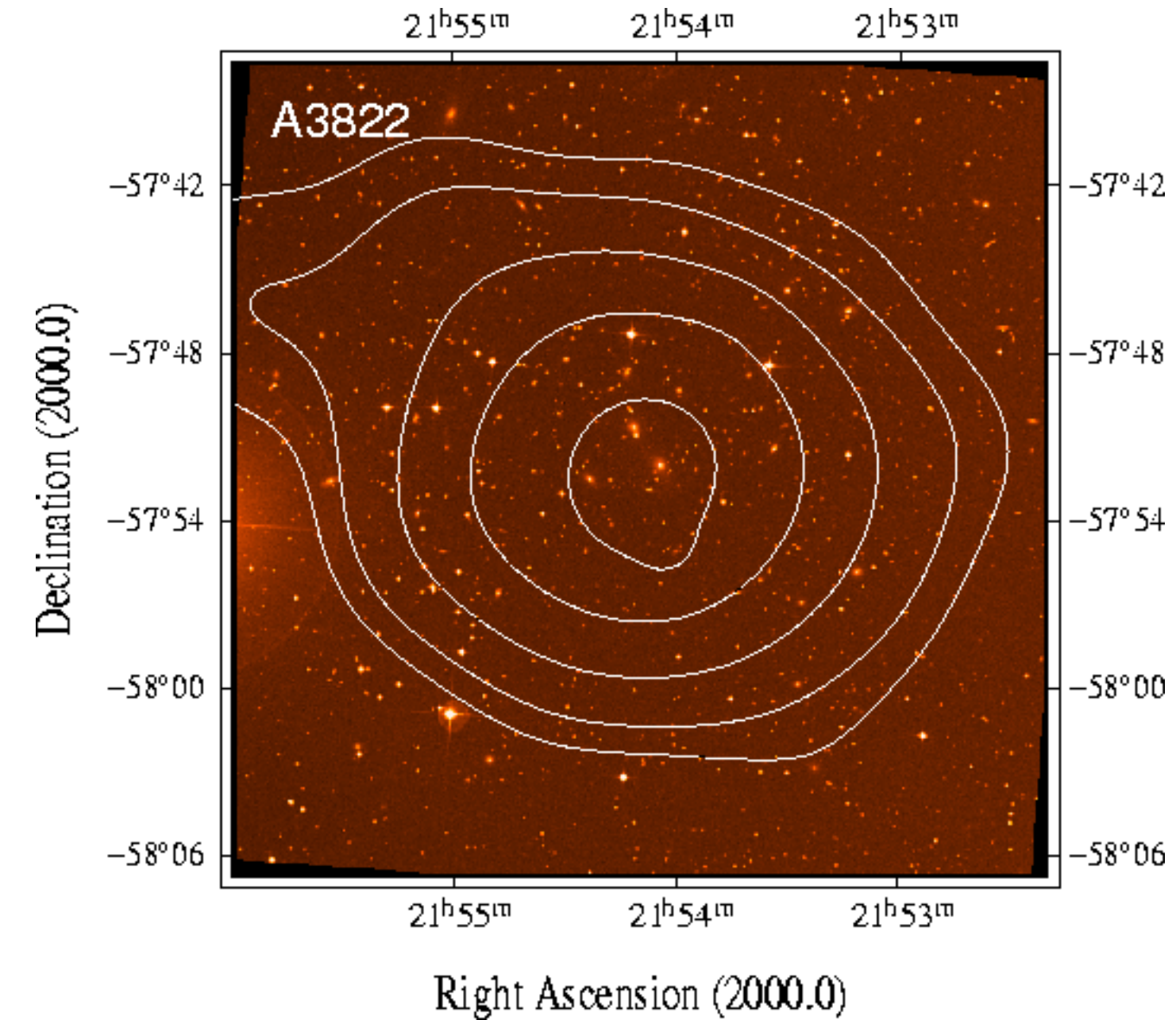
## Assumes

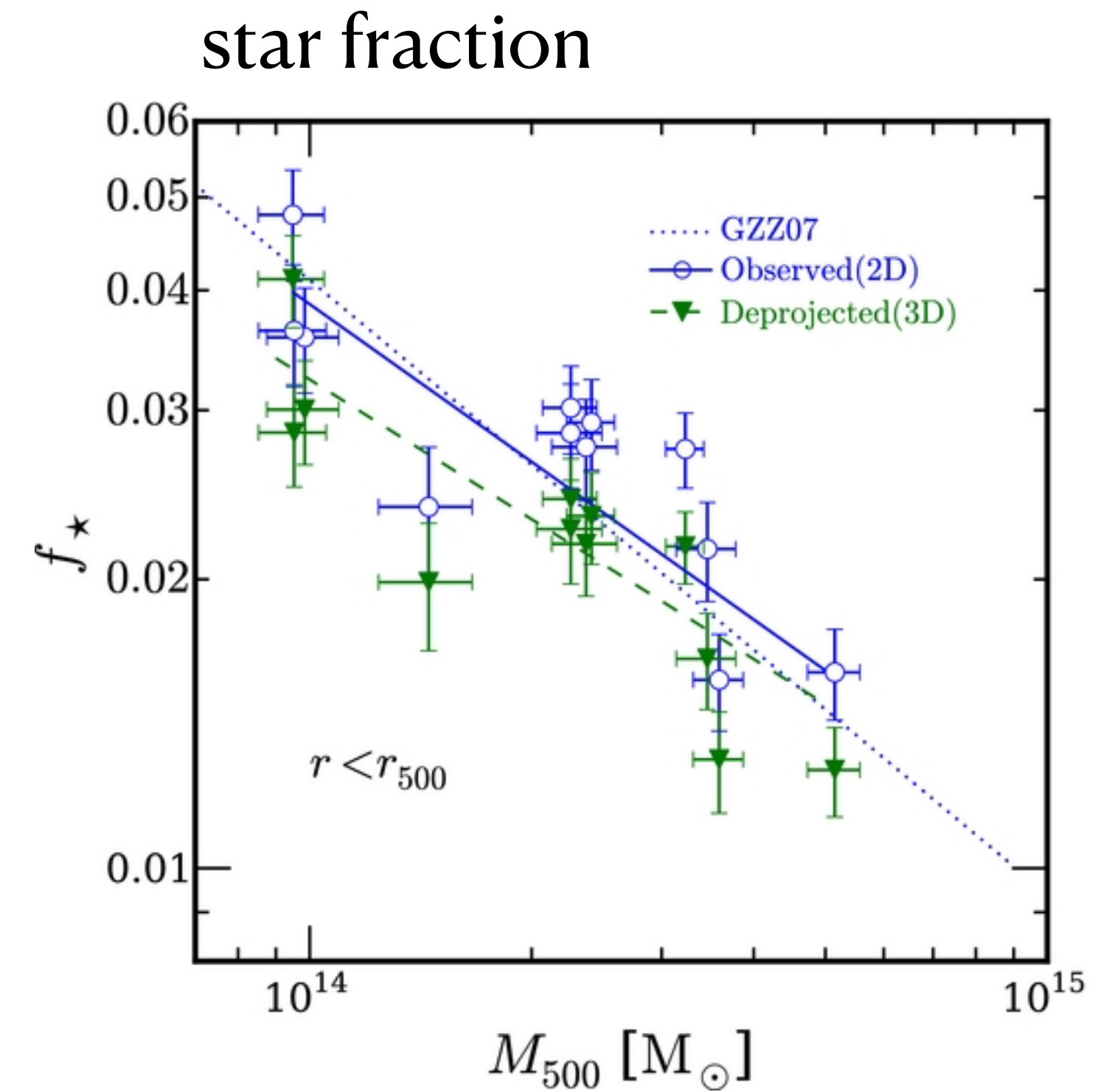
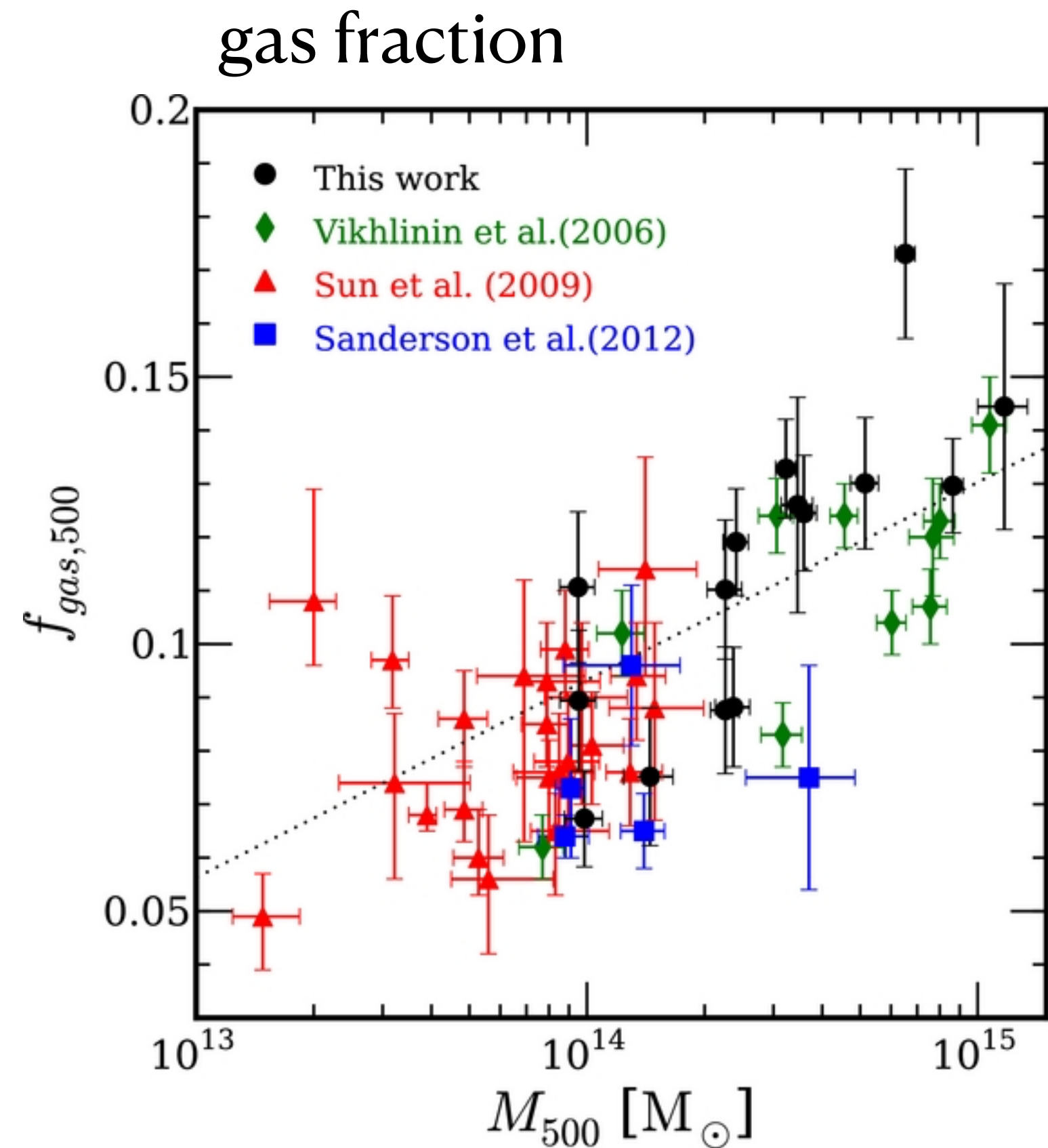
- hydrostatic equilibrium
- sphericity

often assumes

- isothermality  $\longrightarrow \frac{\partial \ln T}{\partial \ln r} = 0$

basically, temperature traces the kinetic energy:  $T \sim V^2$  so  $M \sim TR$

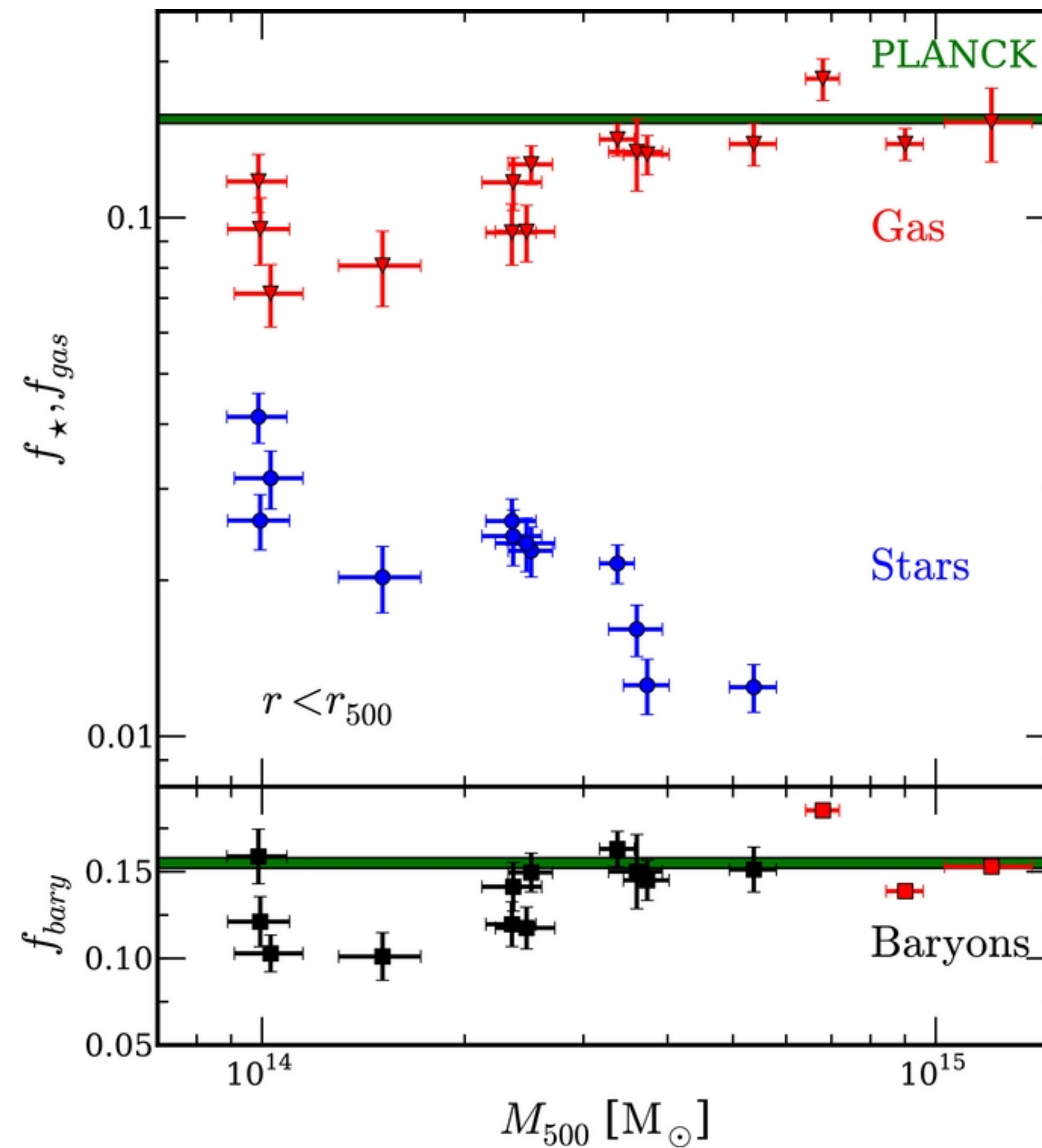




Typical result:

ICM gas outweighs the stars by factor of  $\sim 6$ ;  
 outweighed by dark matter by the same factor

# gas, star, and baryon fraction



cluster baryon fraction comparable to the cosmic baryon fraction, but only for high mass clusters

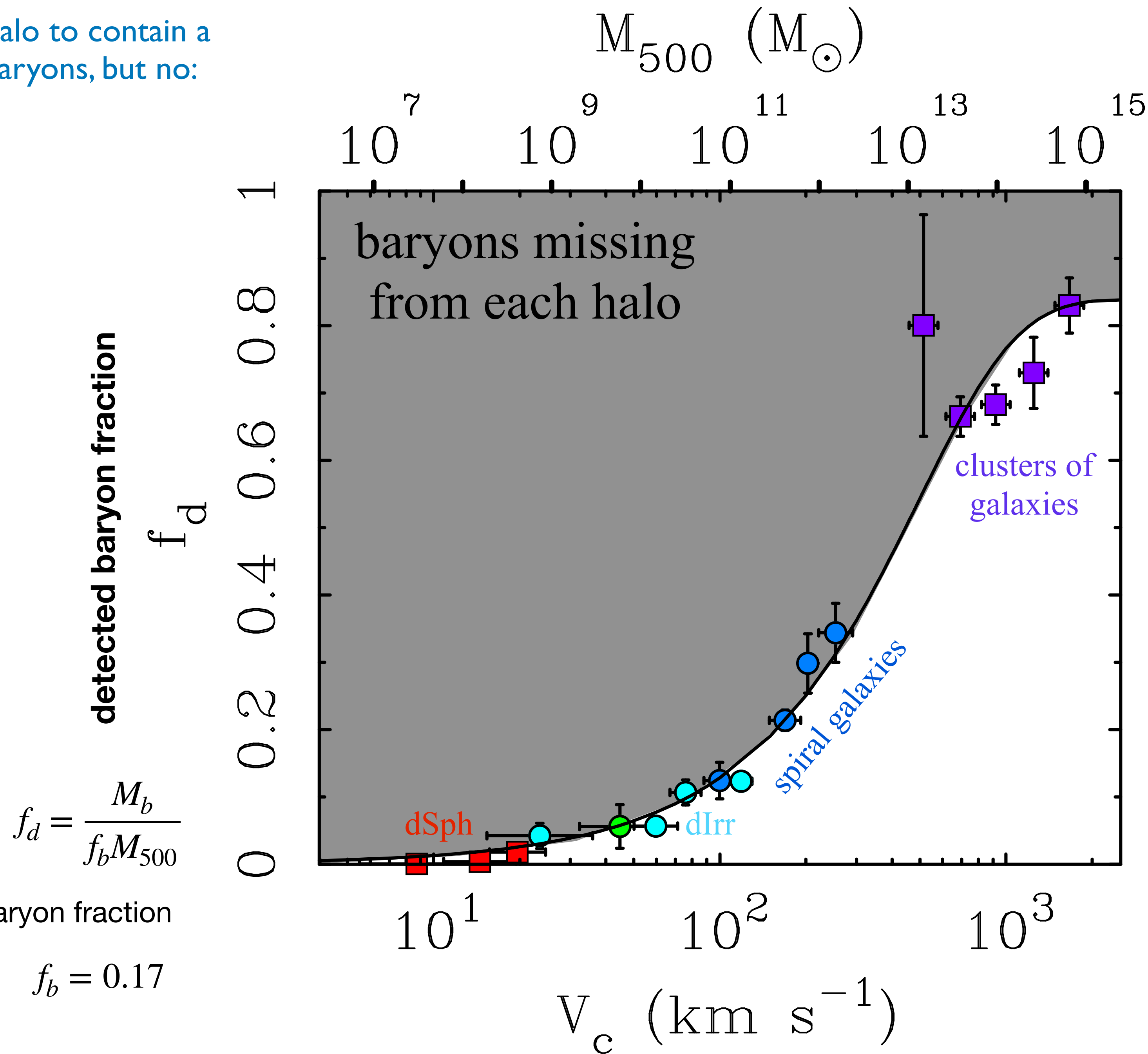
Typical result:

ICM gas outweighs the stars by factor of  $\sim 6$ ;  
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# The halo missing baryon problem

Expect each halo to contain a fair share of baryons, but no:

$$M_b < f_b M_{200}$$

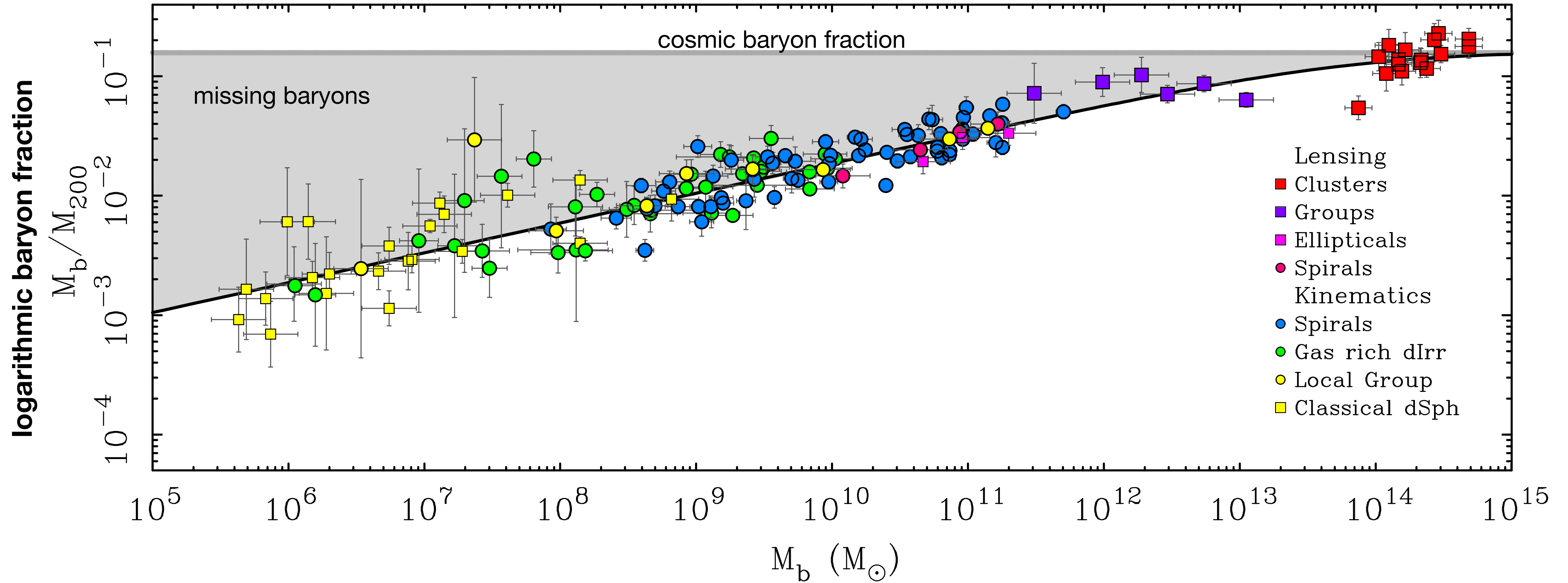


$$f_d = \frac{M_b}{f_b M_{500}}$$

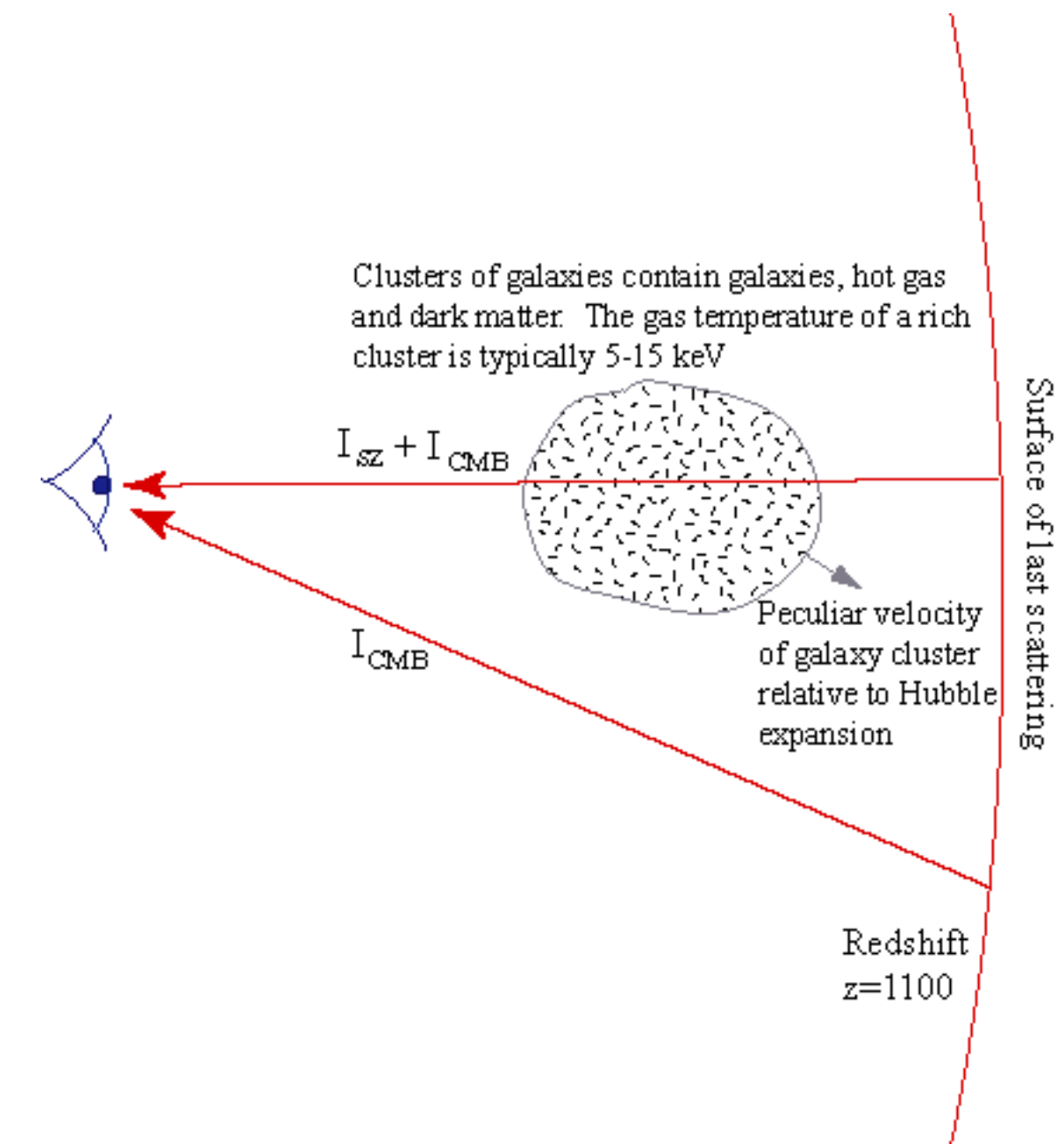
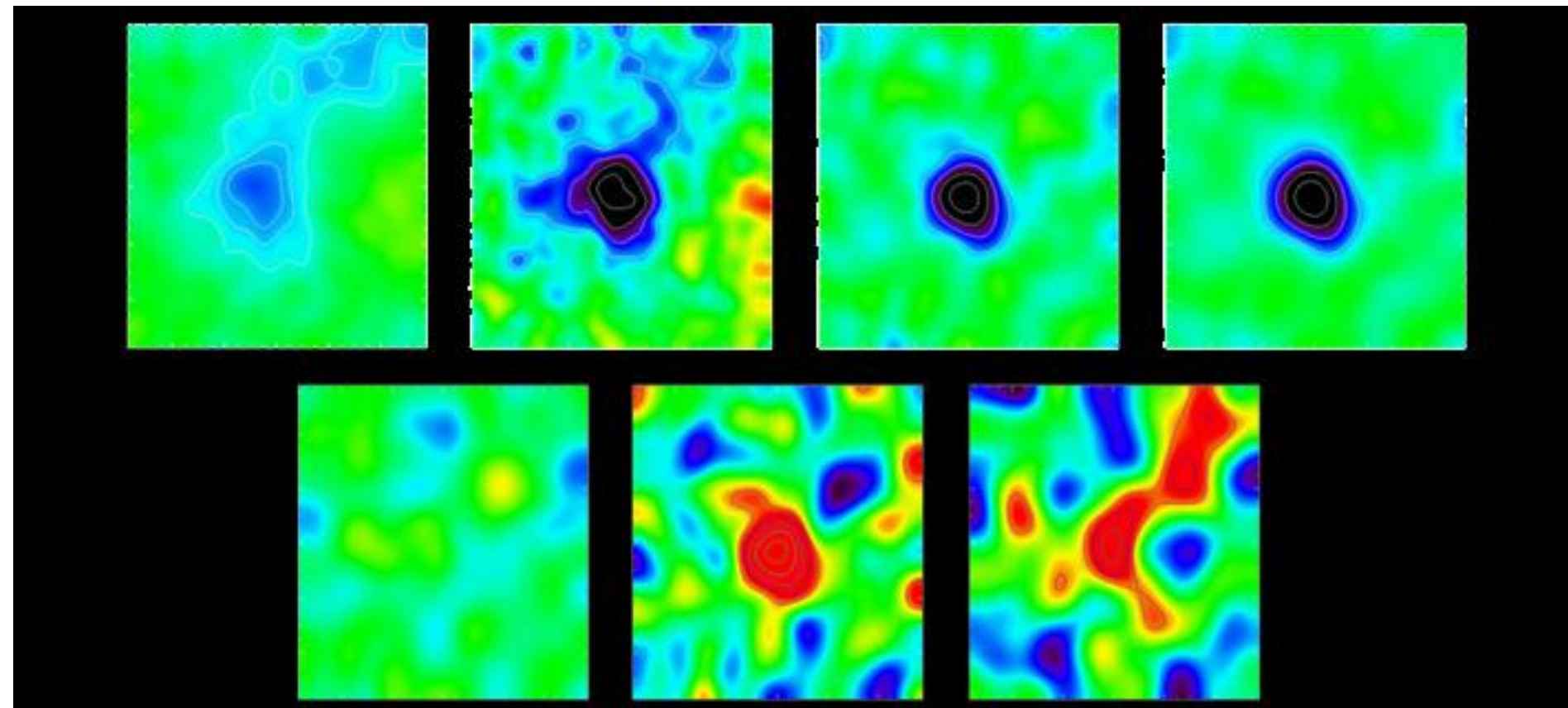
cosmic baryon fraction

$$f_b = 0.17$$

# The halo missing baryon problem



# SUNYAEV-ZEL'DOVICH (SZ) EFFECT



Compton scattering of CMB photons by hot ICM plasma

# Frequency dependent change in intensity

$$\frac{\delta I_\nu}{I_\nu} = -y \frac{x e^x}{e^x - 1} \left[ 4 - x \coth \left( \frac{x}{2} \right) \right]$$

where  $x = \frac{h\nu}{kT_{rad}}$  and  $y = \int \sigma_T n_e \frac{kT_g}{m_e c^2} dl$

↑  
CMB

$y$  is the Compton  $y$ -parameter which quantifies how much effect the plasma has

↑  
electron density

↑  
Thomson scattering cross-section

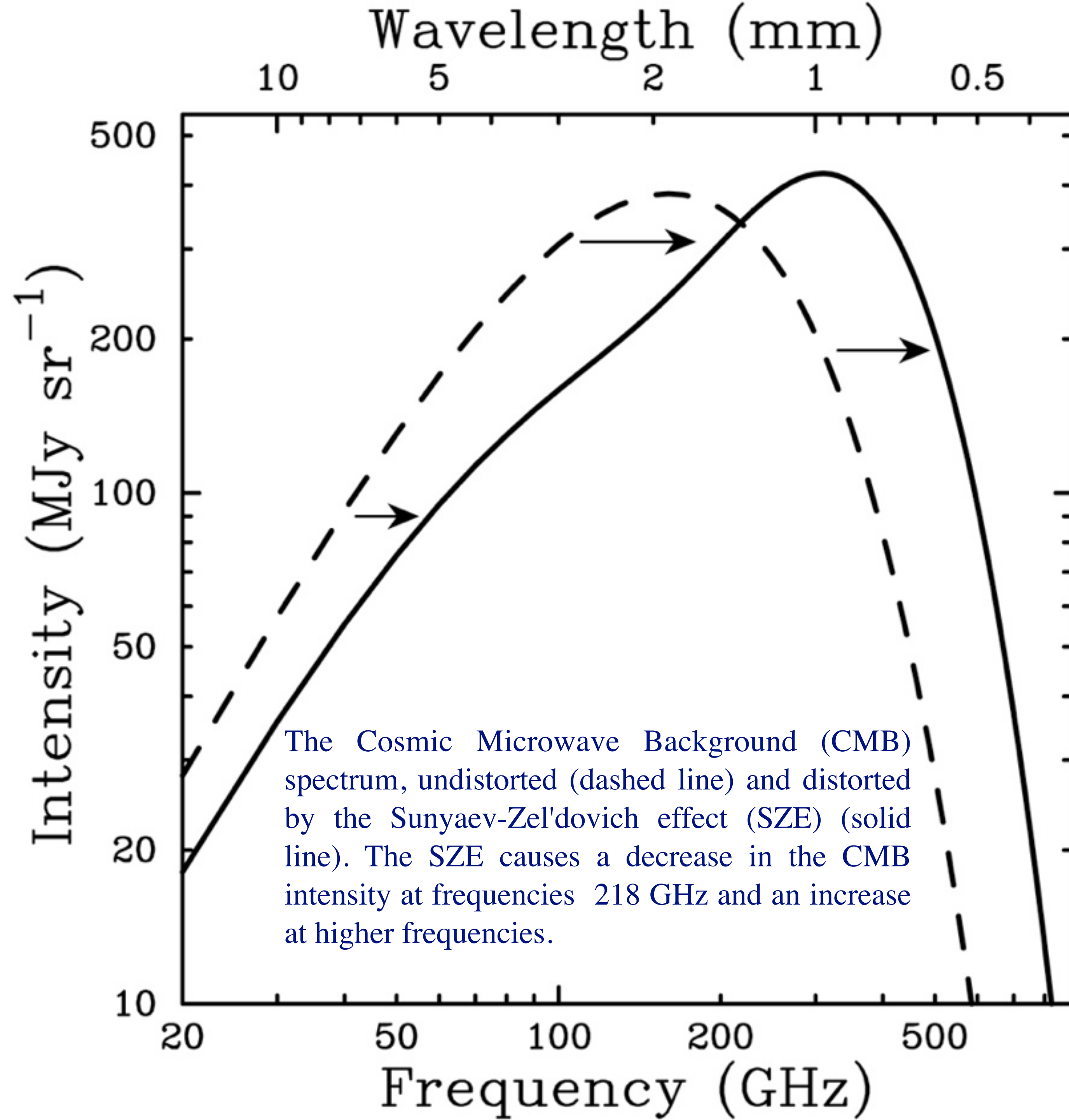
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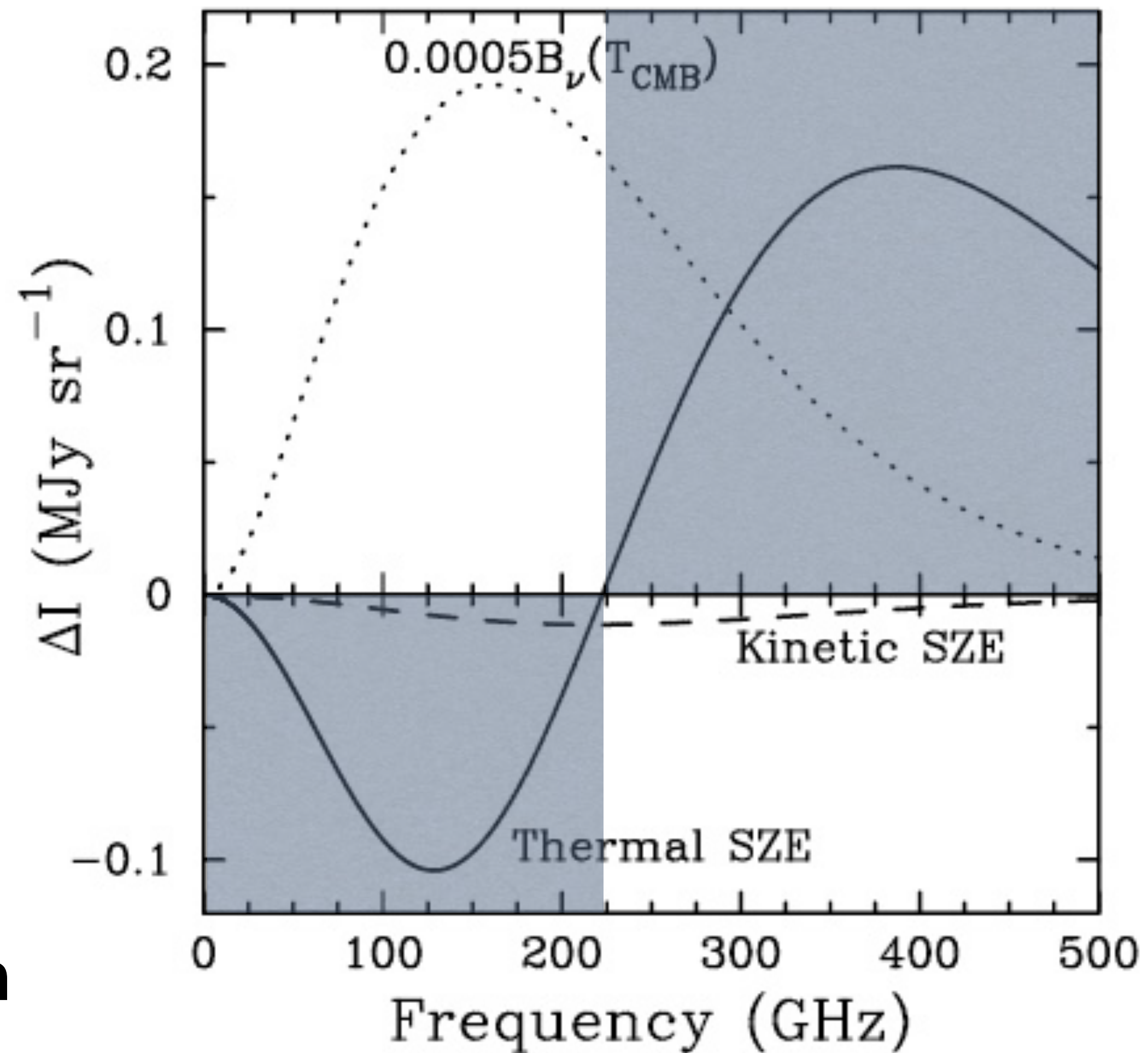
at low frequency in the Rayleigh-Jeans tail,

$$\frac{\delta I}{I} = \frac{\delta T}{T} = -2y$$



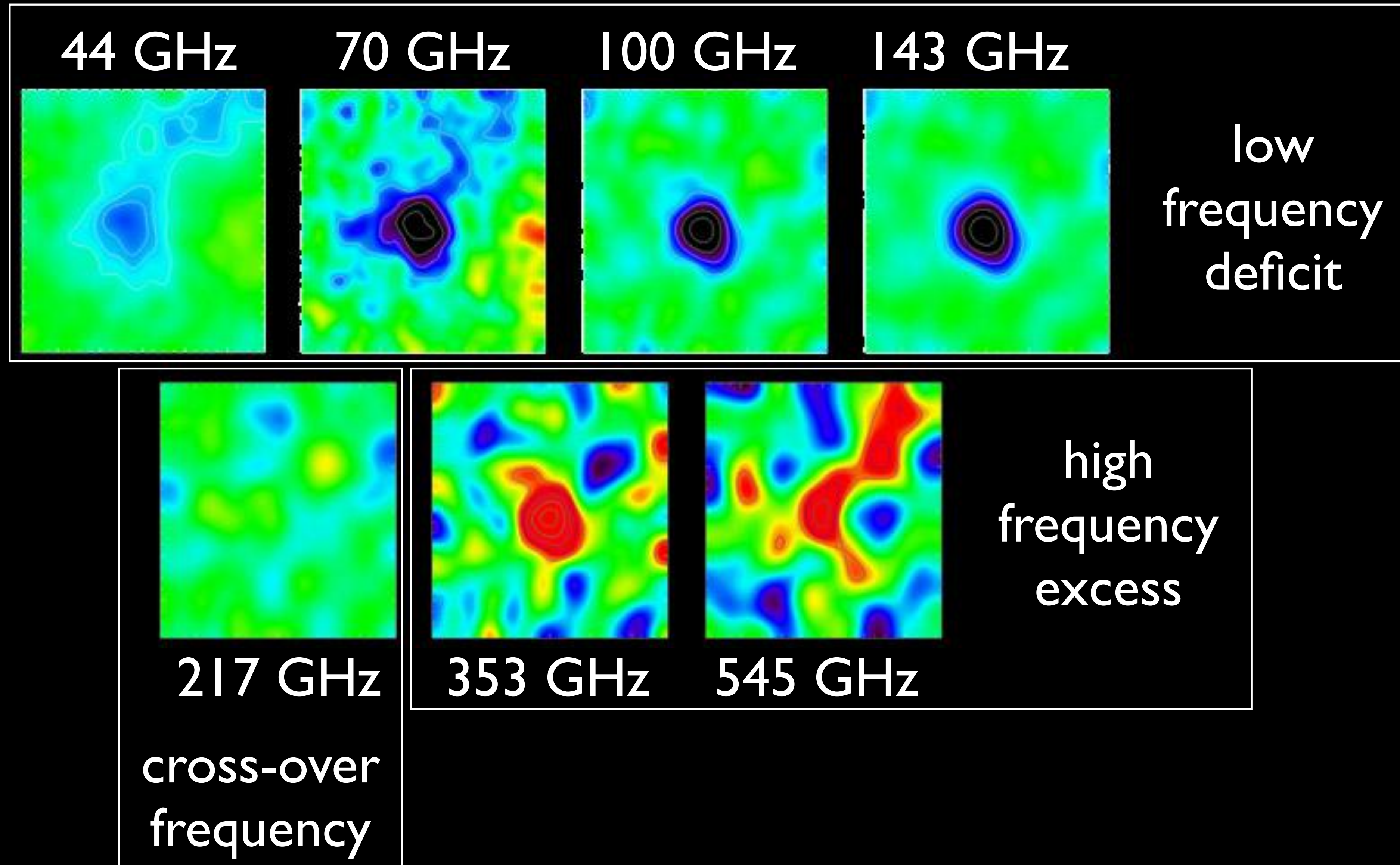
Thermal SZ effect  
from Compton  
scattering of CMB  
photons by cluster  
plasma

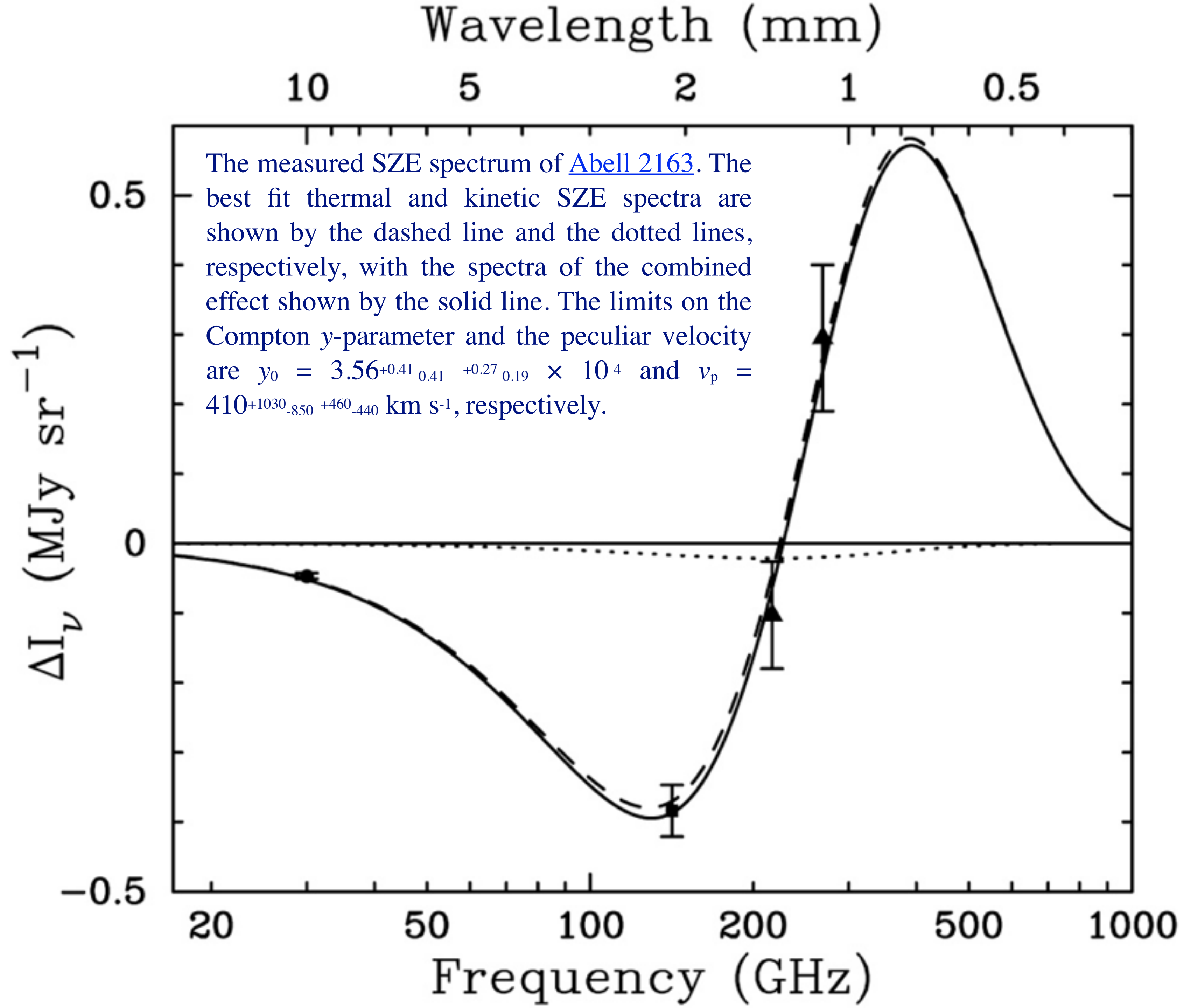
Kinematic SZ effect from  
peculiar velocity of  
cluster wrt CMB frame

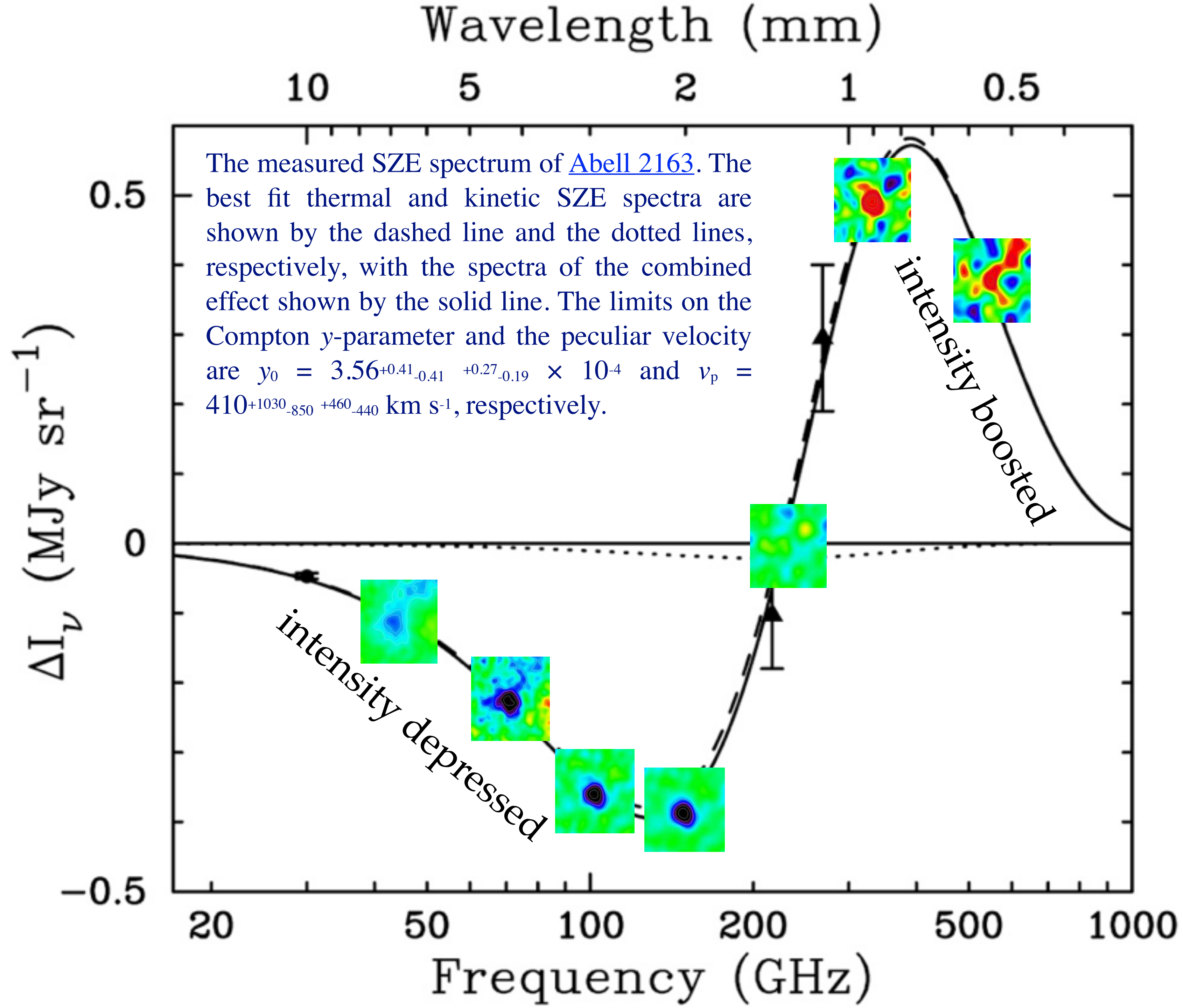


# SUNYAEV-ZEL'DOVICH EFFECT

detected by Planck







integrated change in CMB temperature

$$\int \Delta T d\Omega \propto \frac{N_e \langle T_e \rangle}{D_A^2} \propto \frac{M \langle T_e \rangle}{D_A^2}$$

depends on the total number of electrons, their temperature, and the area they subtend on the sky.

In effect measures Pressure, or Mass if T known.

$D_A$  is the angular diameter distance.

At high  $z$ , it varies slowly, while the density increases as  $(1+z)^3$

... SZ effect weak, but nearly independent of redshift!