Dark Matter

ASTR 333/433
Spring 2026
TR 11:30am-12:45pm
Sears 552

http://astroweb.case.edu/ssm/ASTR333/

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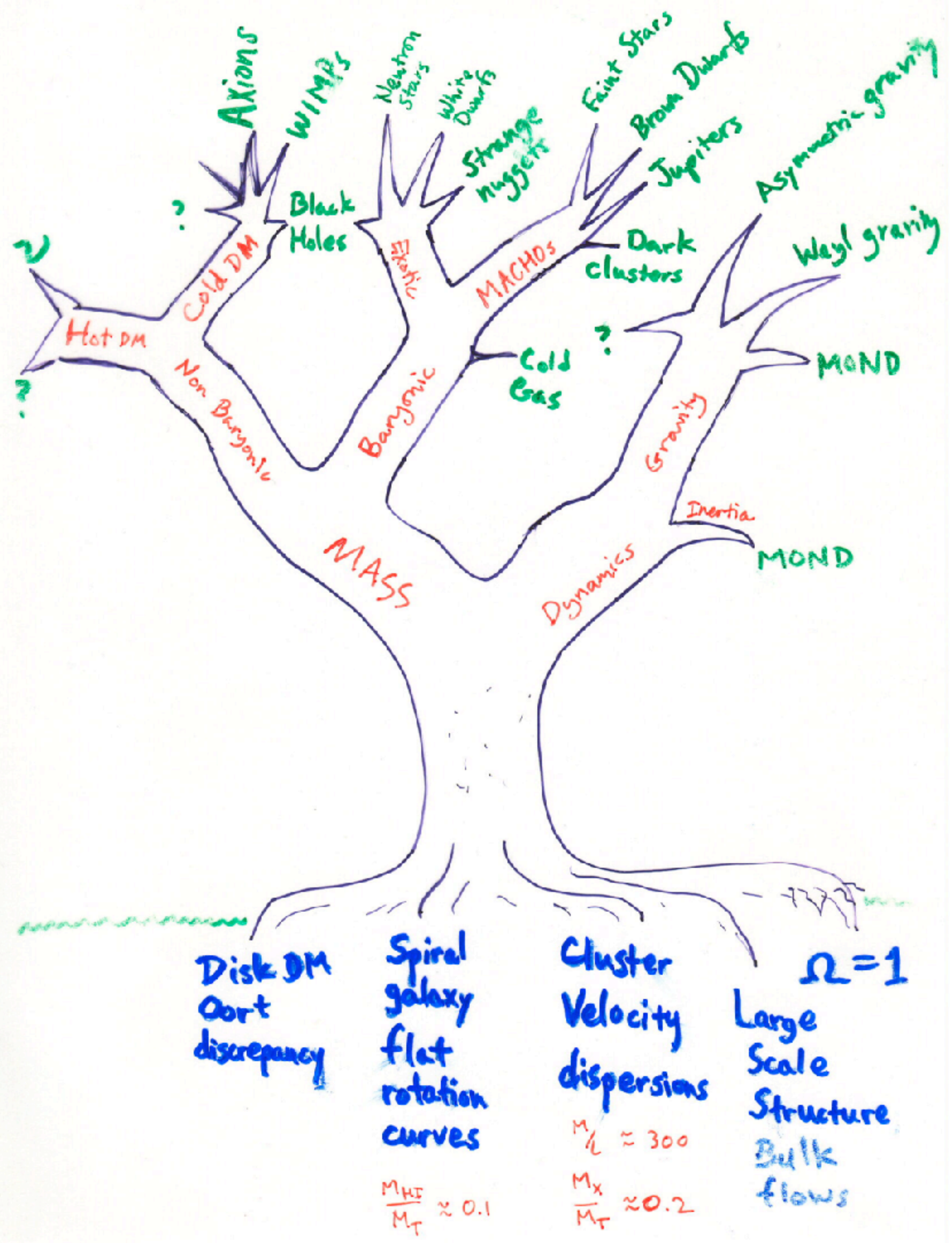
Homework due next time

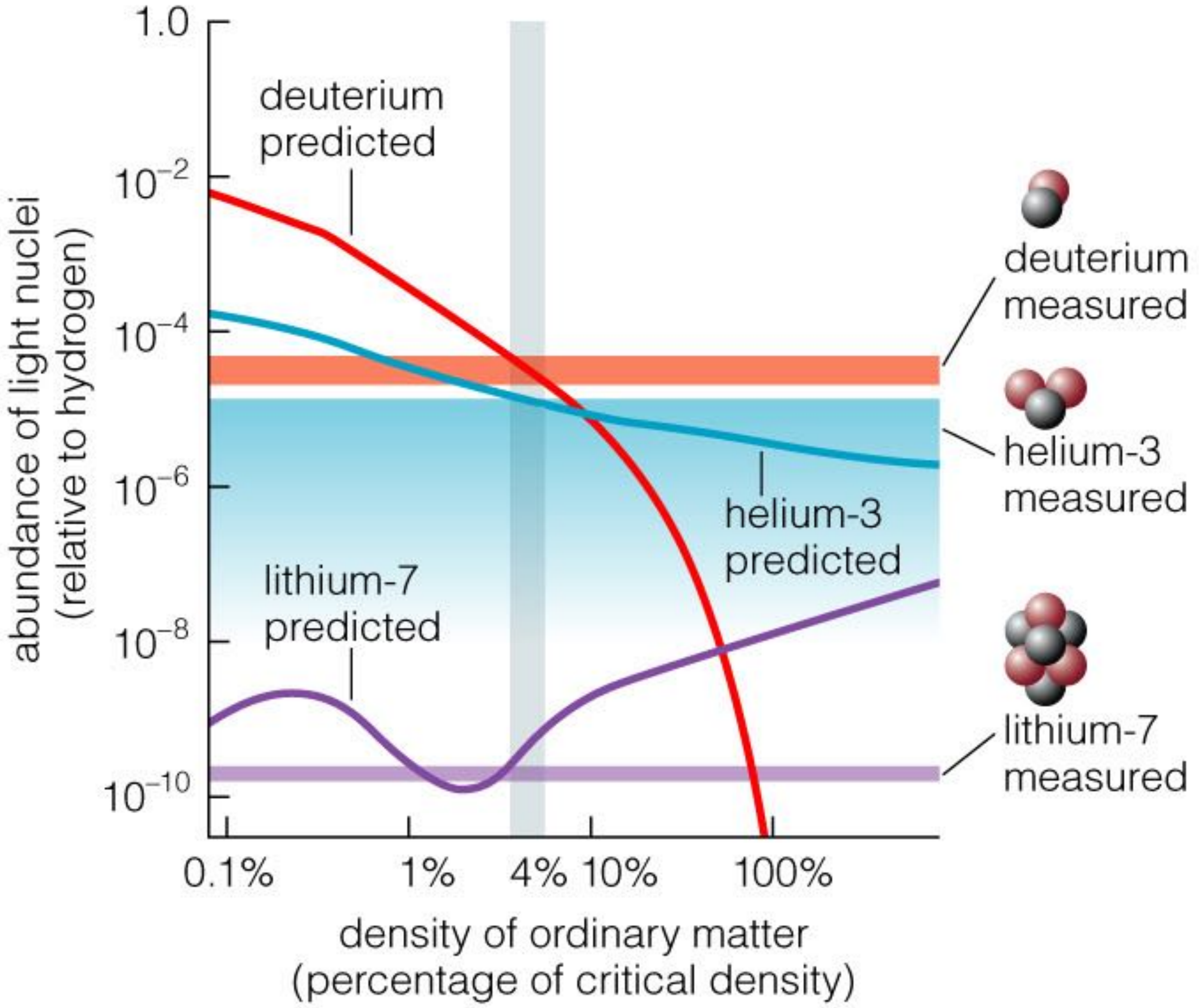
Case Western Reserve University EST. 1826



BBN products:

- 3/4 Hydrogen
- 1/4 Helium
- Traces of
  - deuterium
  - tritium
  - helium 3
  - lithium 6 & 7
  - berylium (decays to Li)

Abundances depend on the density of matter. The higher the density parameter (Ω_b), the more helium.



BBN products limited to light isotopes

no stable mass 5 or 8

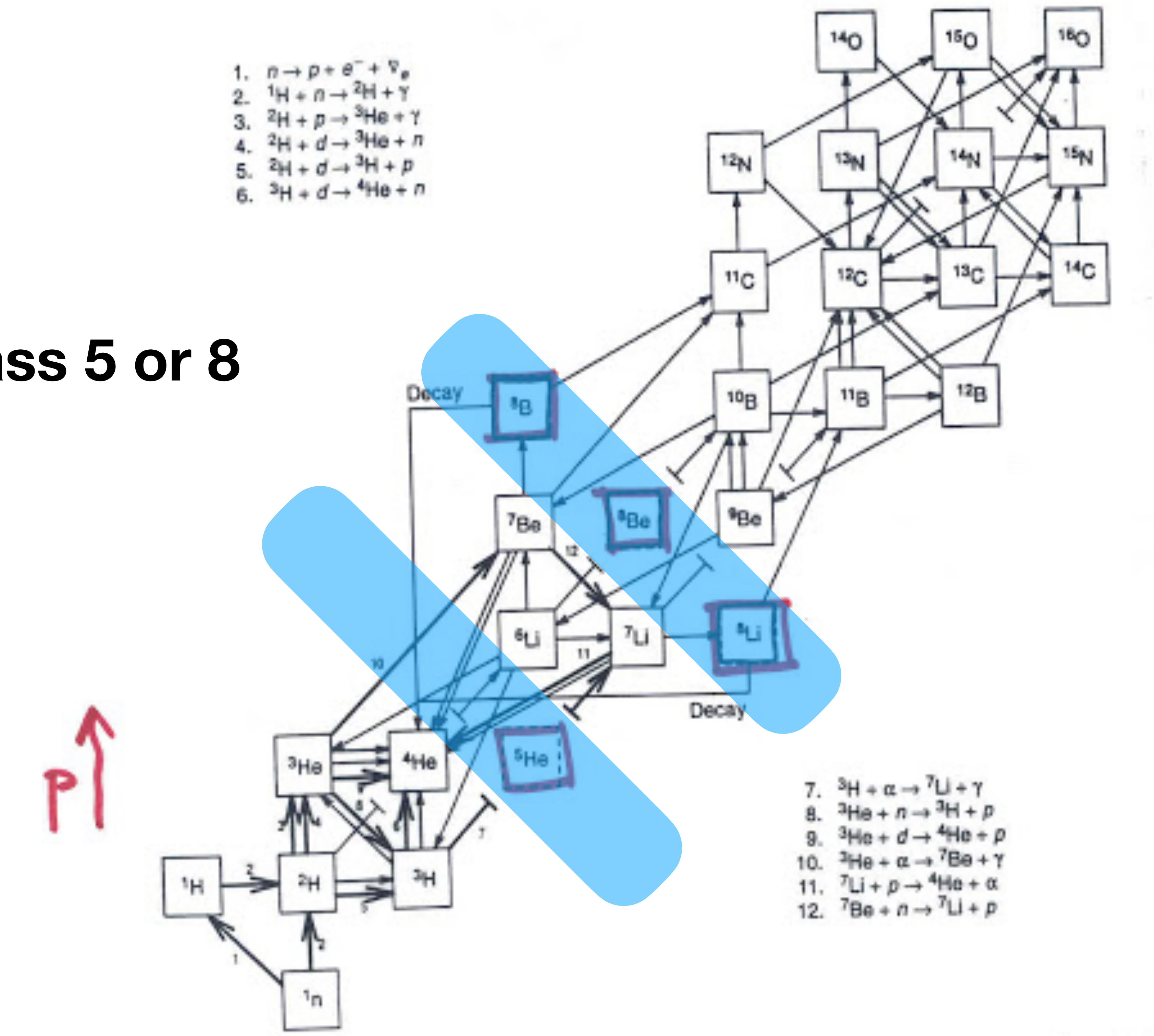


Fig. 2. The nuclear reaction network used for big-bang nucleosynthesis; the most important reactions are numbered and have bold arrows. The broken boxes for mass 5 and 8 indicate that all nuclides of this mass are very unstable.

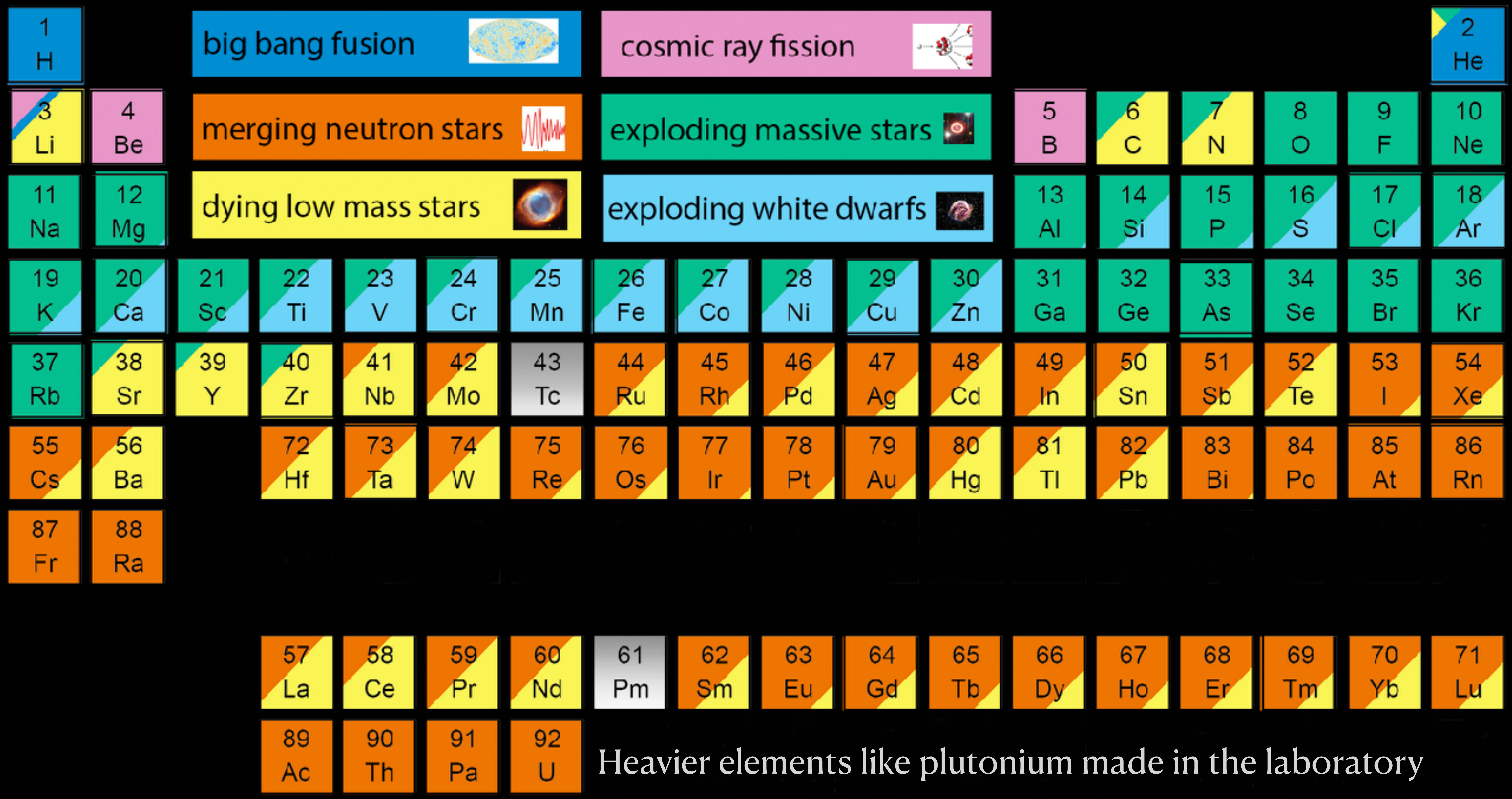
n

BBN restricted to isotopes of the light elements

Stars skip over the mass bottleneck via the triple alpha reaction

$3\ {}^4He \to {}^{12}C^*$

The Origin of the Solar System Elements

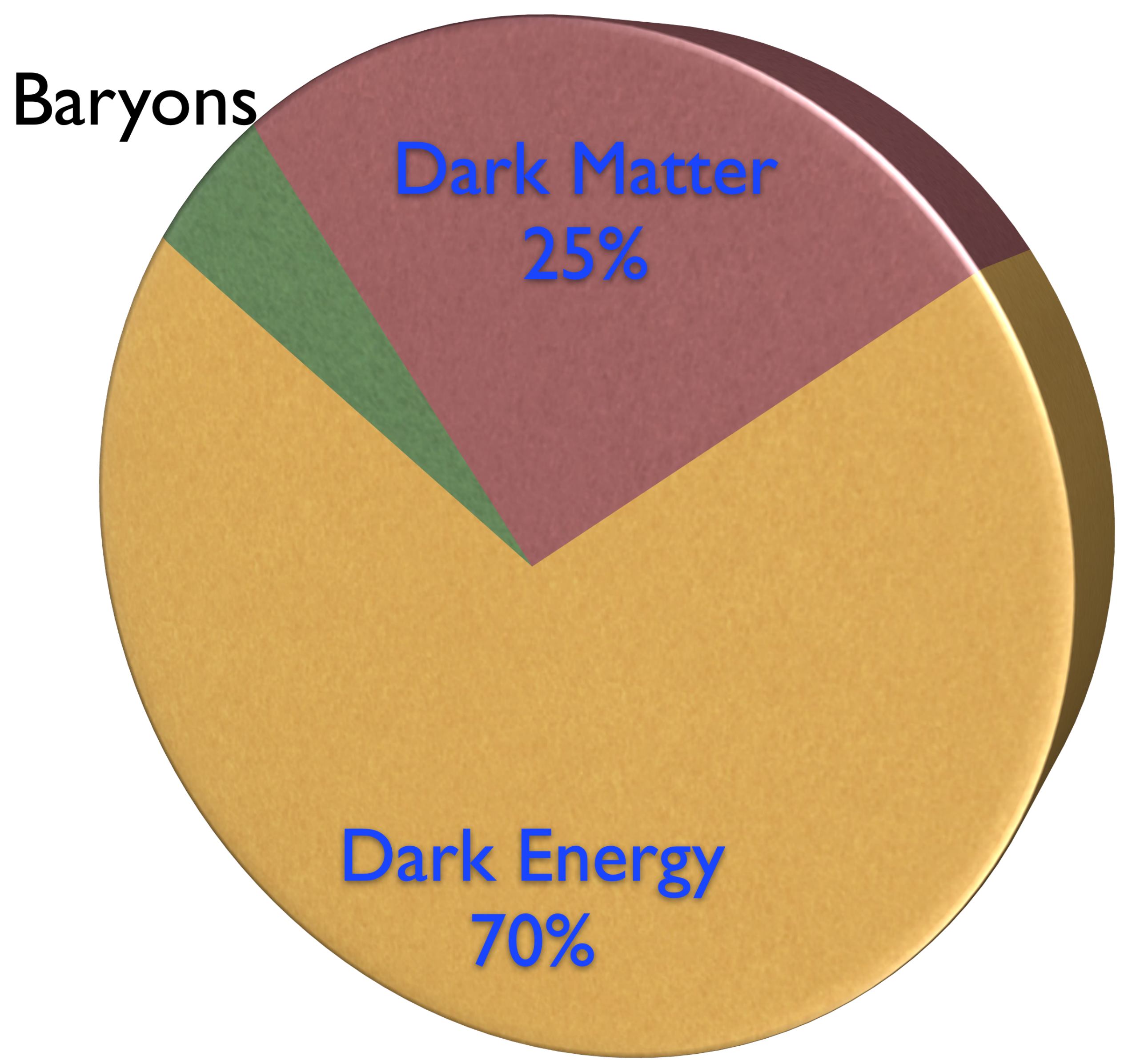


Heavier elements like plutonium made in the laboratory

Empirical Pillars of the Hot Big Bang

1. Hubble Expansion
2. Big Bang Nucleosynthesis Ω_b
3. Cosmic Microwave Background

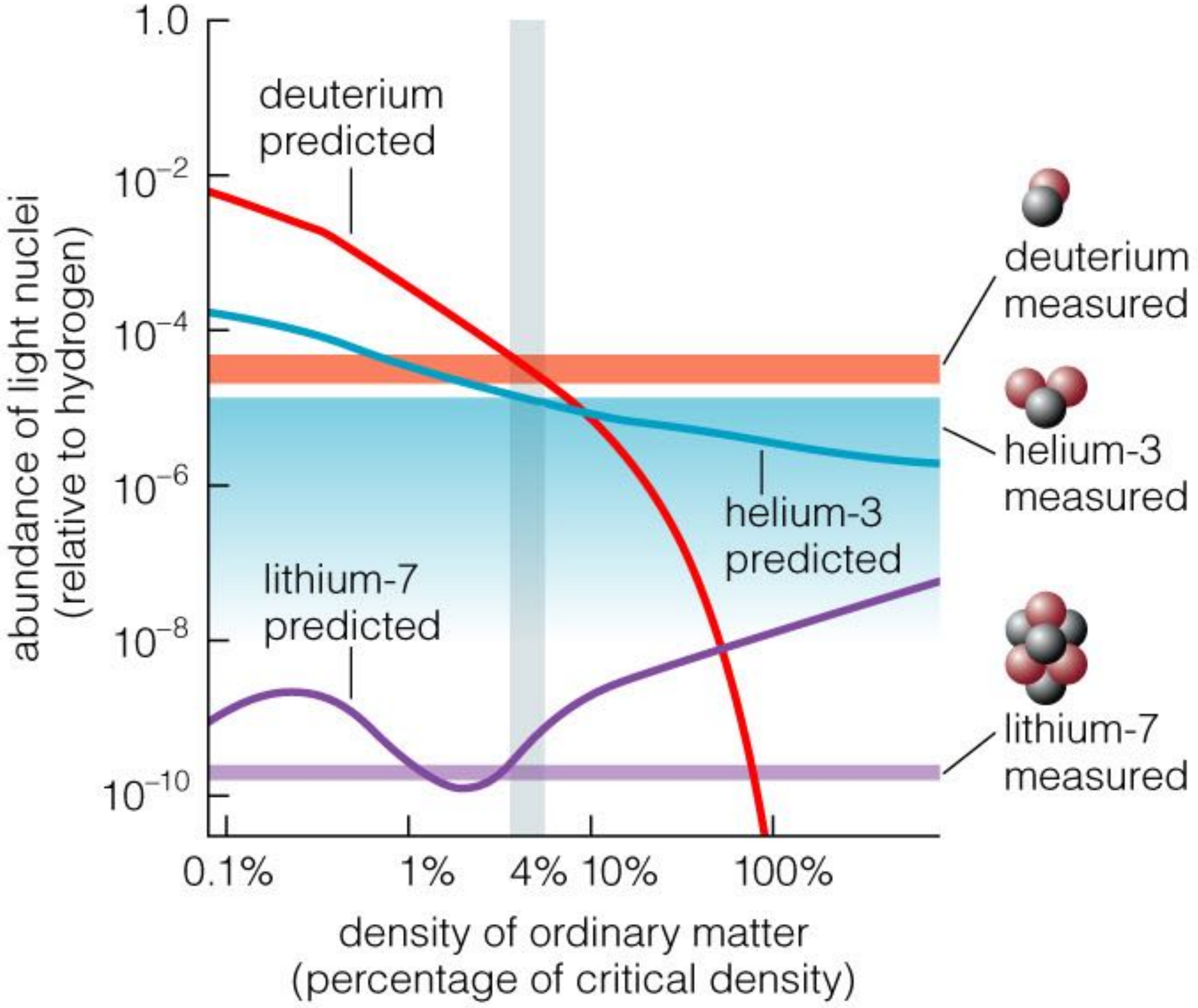
$\Omega_m = \Omega_b + \Omega_{DM}$



Auxiliary Hypotheses

- Dark matter Ω_{DM}
- Dark Energy Ω_Λ

Non-baryonic dark matter driven by $\Omega_m > \Omega_b$.
How do we know?

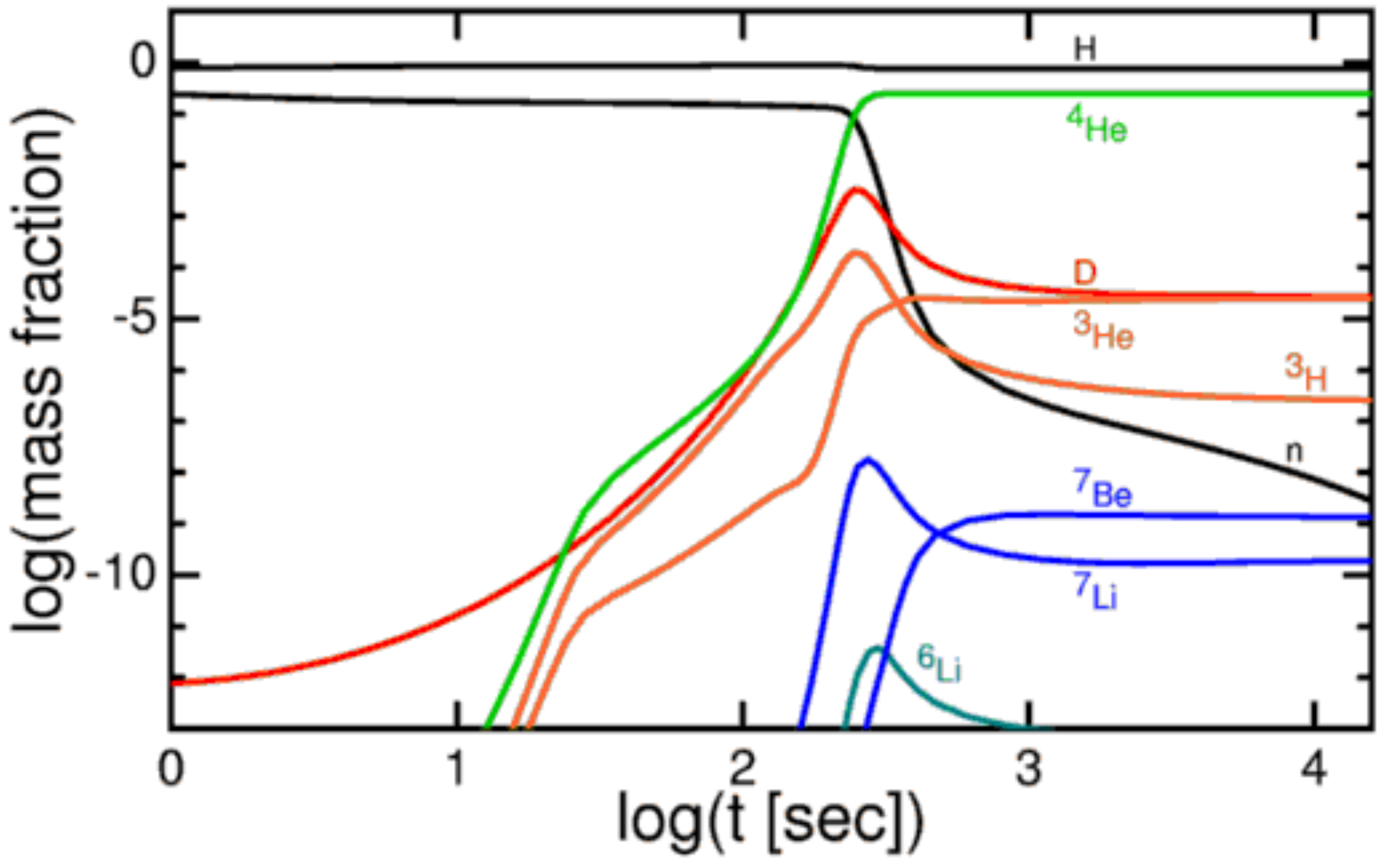


Ω_b

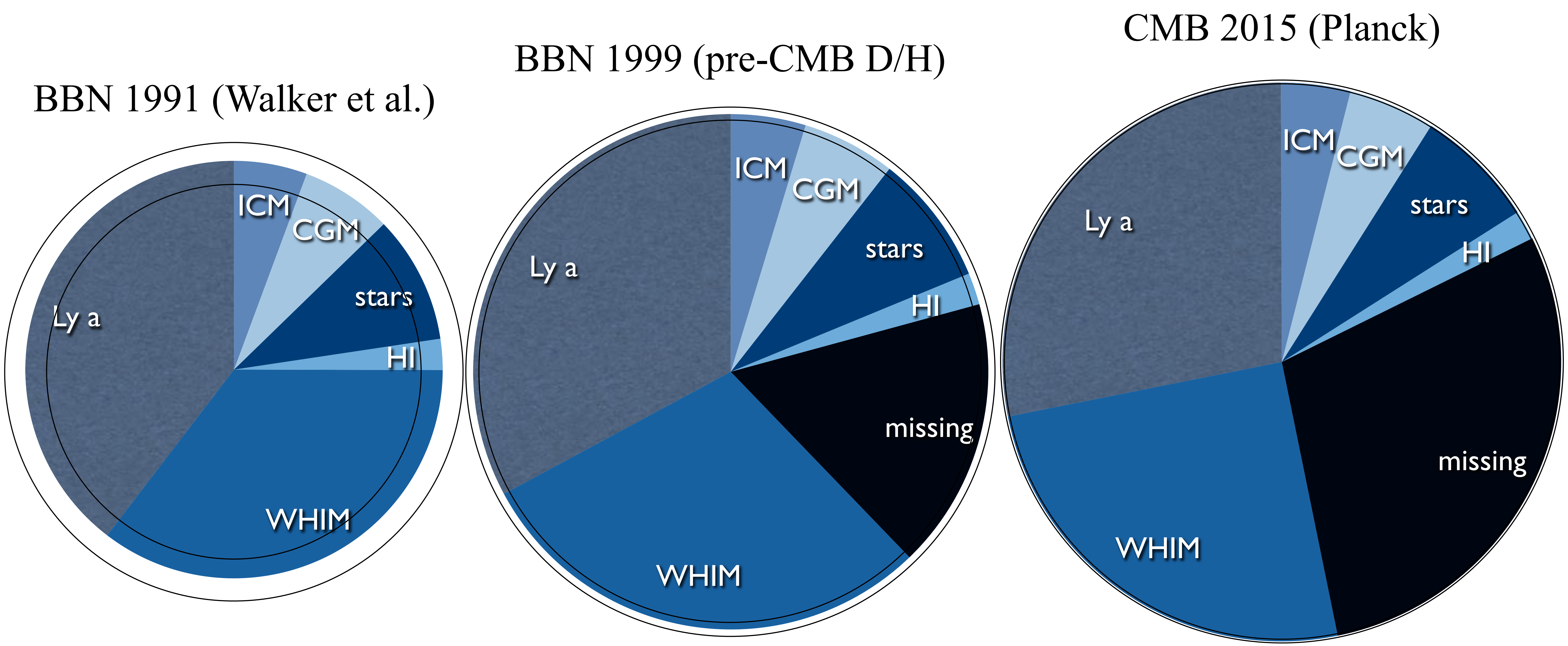
<u>How do we know?</u>

Theory: BBN in early universe

Observation: constraints on isotopic abundances of light elements



BBN estimates of Ω_b over time



$\Omega_b h^2 = 0.0125 \pm 0.0025$

$\Omega_b = 0.0255$

$\Omega_b h^2 = 0.019 \pm 0.001$

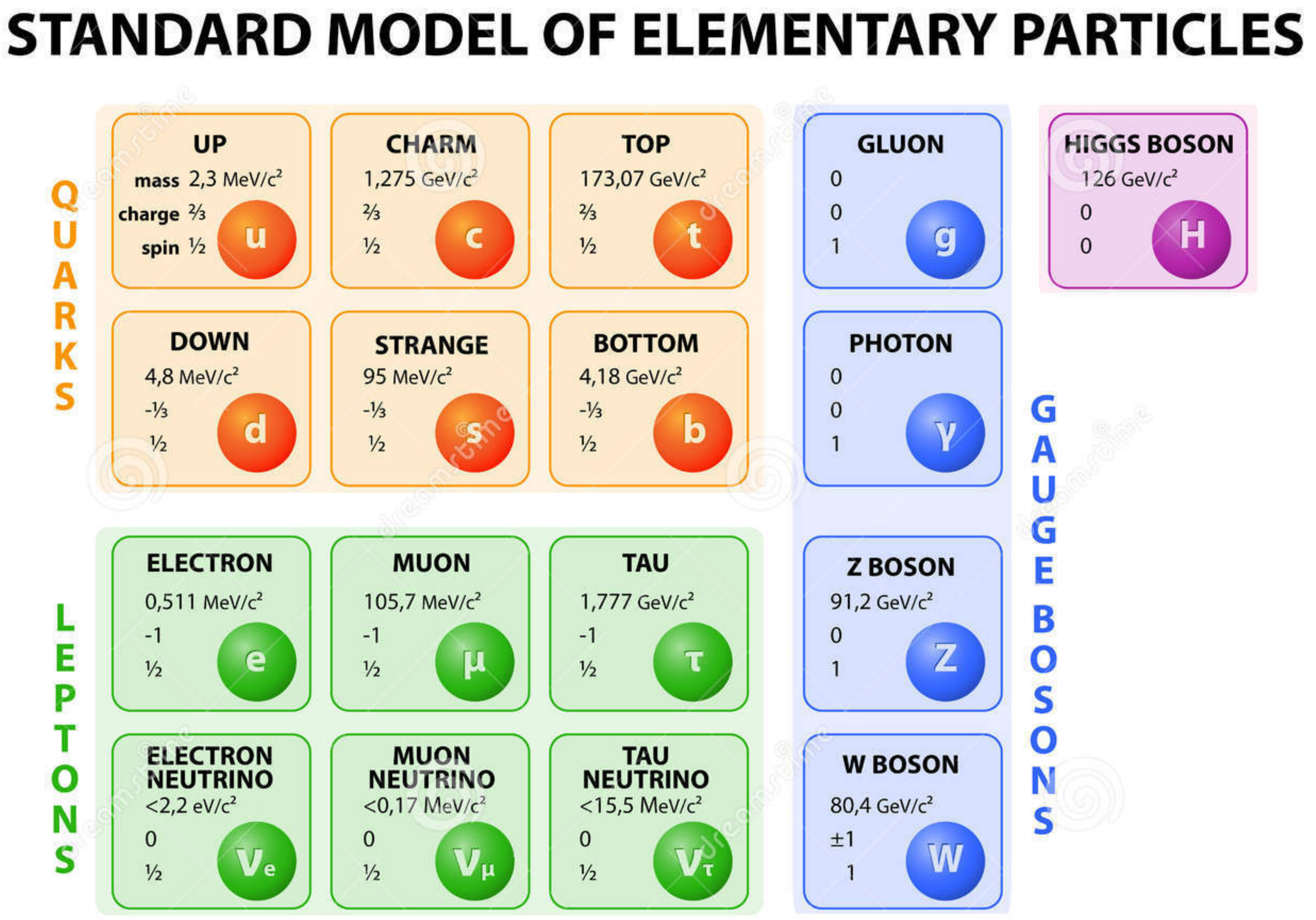
$\Omega_b = 0.0388$

$\Omega_b h^2 = 0.02230 \pm 0.00023$

$\Omega_b = 0.0455$ for $H_0 = 70$
$\Omega_b = 0.05$ for $H_0 = 66.8$
$\Omega_b = 0.04$ for $H_0 = 74.7$

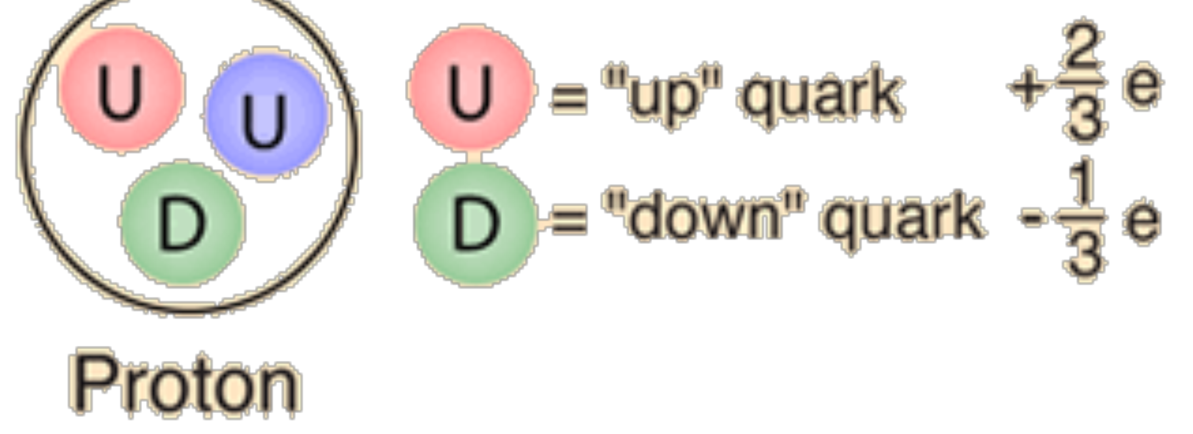
Our estimate of the baryon density Ω_b has grown over time.
The first step was in response to improved deuterium data;
the second was due to observation of the CMB acoustic power spectrum.

Whatever the non-baryonic dark matter is, it has to come from new physics beyond the standard model.

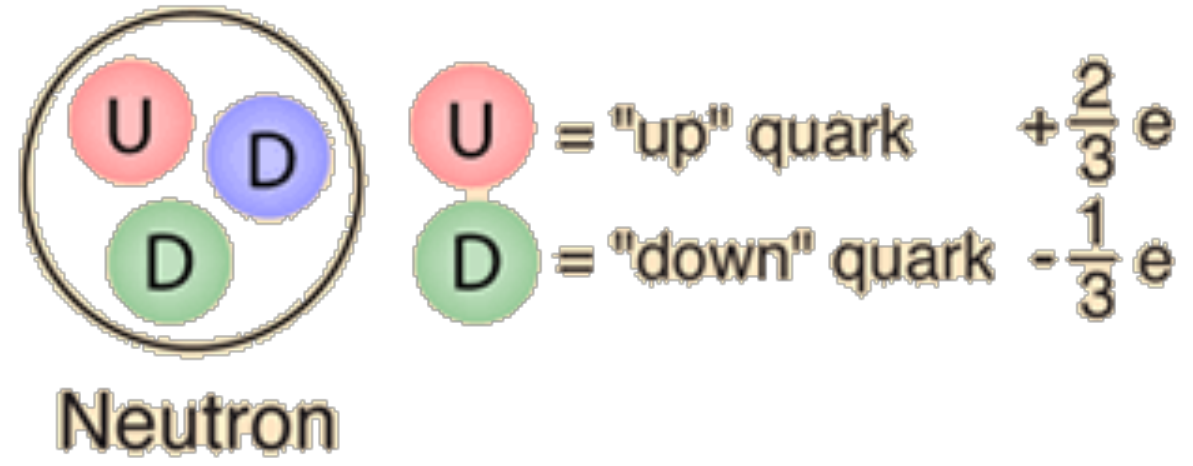


Baryons (3 quarks)

$m_p = 938.3$ eV

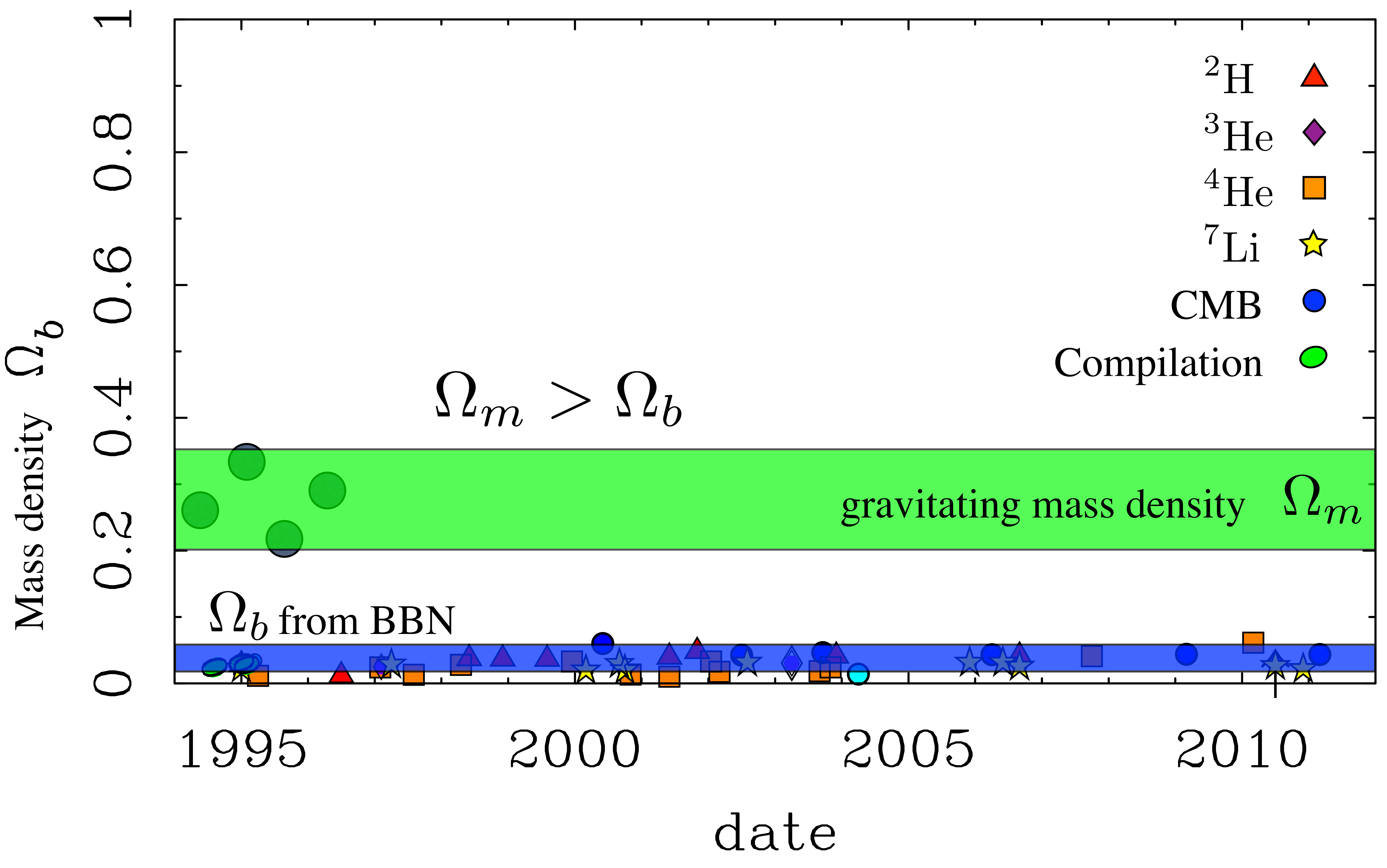


$m_n = 939.6$ eV



Up & down quarks are light: most of the rest mass is binding energy

There’s more mass than BBN allows in baryons



Measurements of the gravitating mass density

- Cluster M/L
  - – measure M/L of a cluster, combine with measured luminosity density of universe.
- Weak lensing
  - – measure shear over large scales
- Peculiar Velocity Field
  - – measure deviations from Hubble flow
- Power spectrum of galaxies
- CMB fits

Measurements of the gravitating mass density

- Cluster M/L
  - measure M/L of a cluster, combine with measured luminosity density of universe.
  - j from integrating the luminosity function of galaxies:

$$\rho_m = \left(\frac{M}{L}\right) j$$

  - Also, cluster baryon fractions:

$$f_b = \frac{M_b}{M_{tot}} = \frac{\Omega_b}{\Omega_m}$$

  - both assume clusters are representative of the whole.

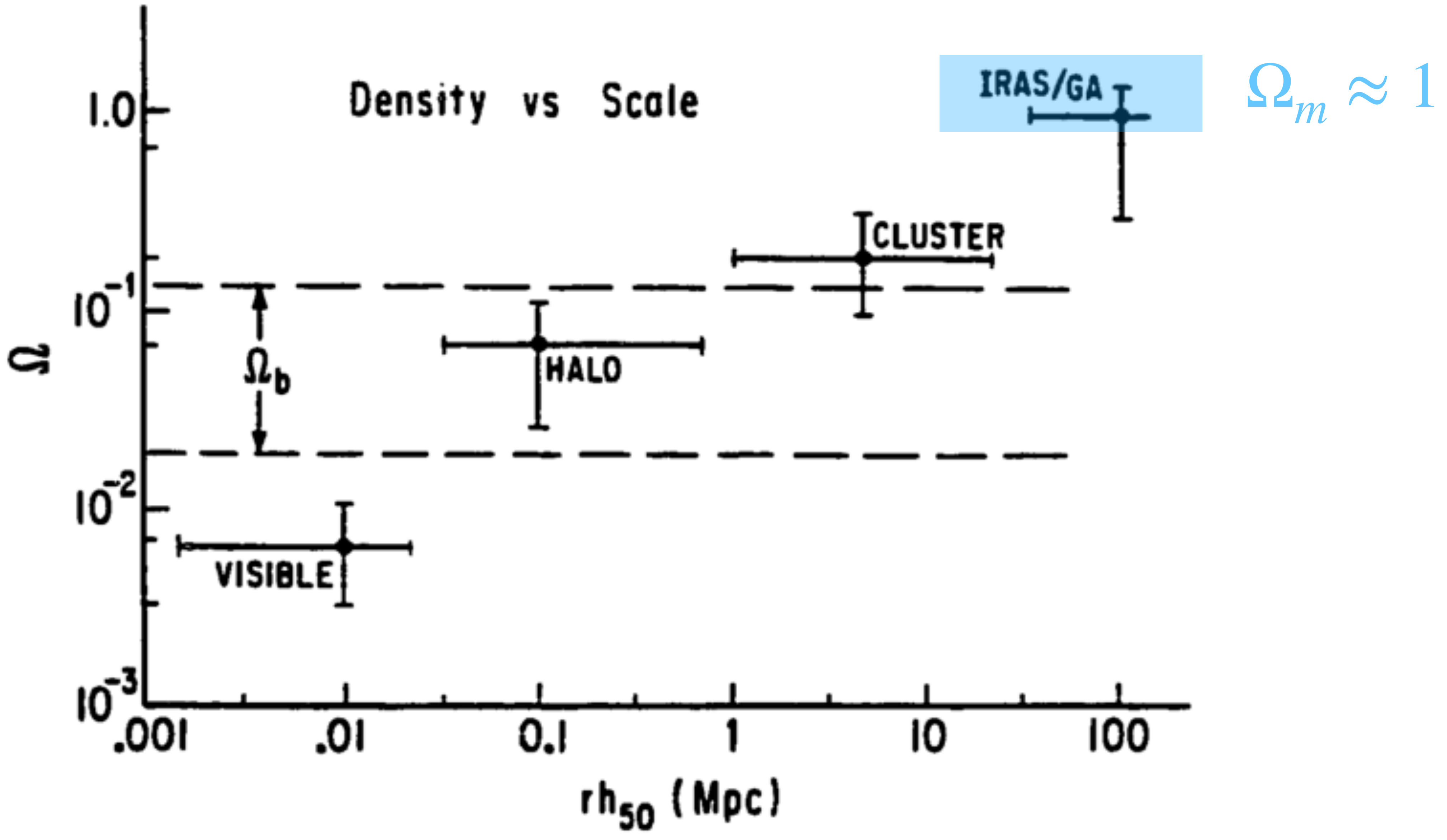


Figure 2. Implied densities versus the scale of the measurements.

Schramm (1992)

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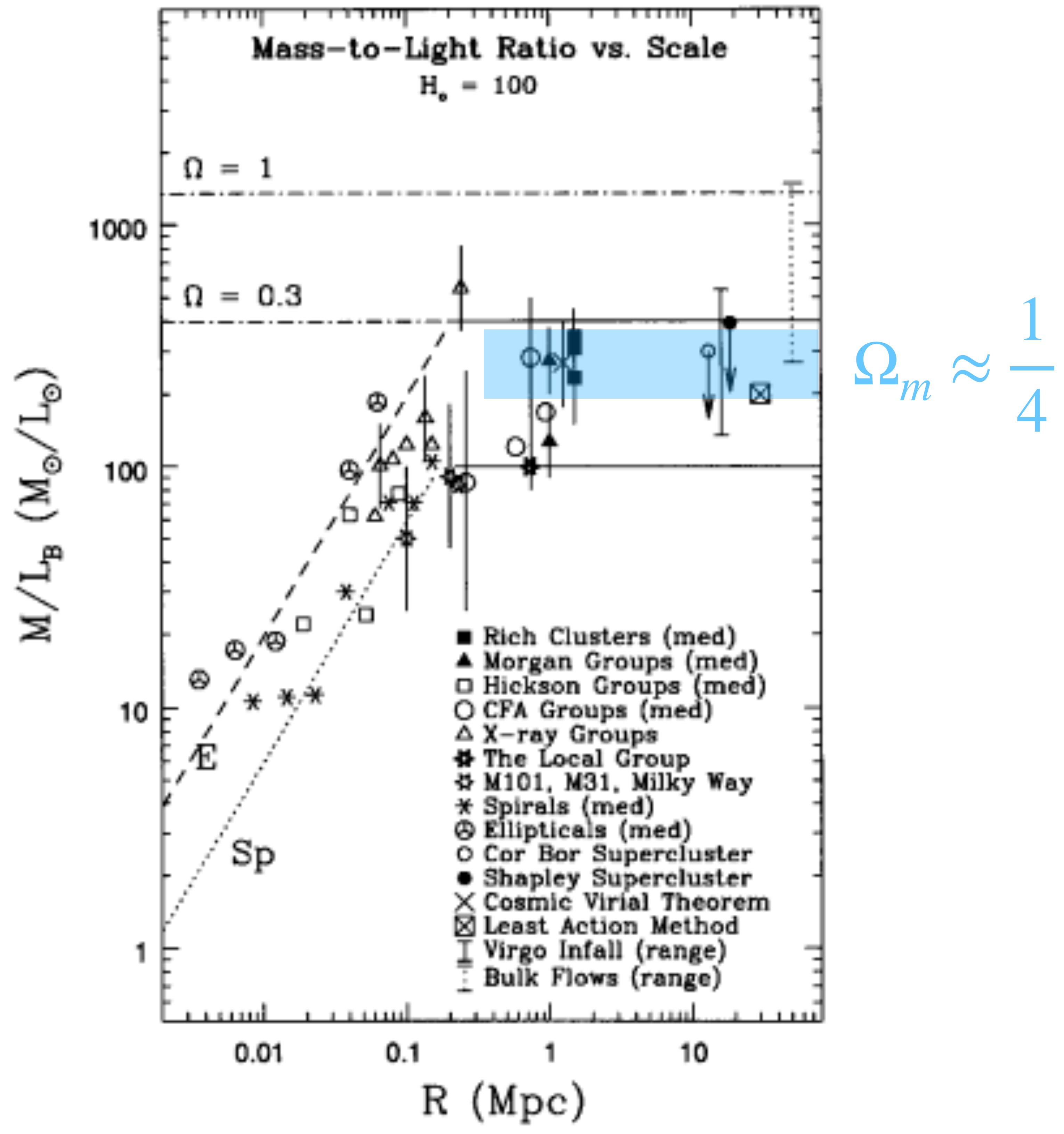
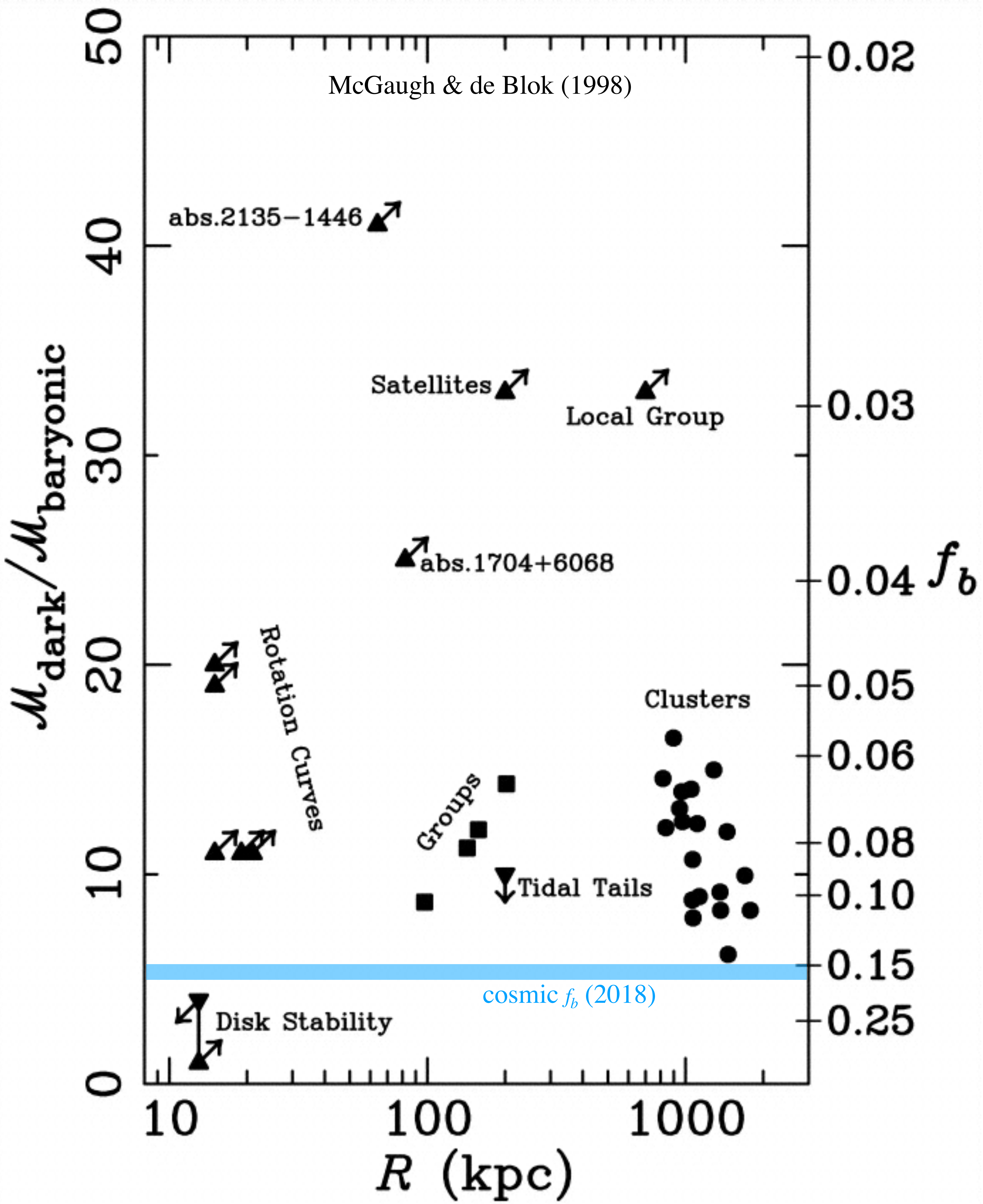


FIG. 2.—Composite mass-to-light ratio of different systems—galaxies, groups, clusters, and superclusters—as a function of scale. The best-fit $M/L_B \propto R$ lines for spirals and ellipticals (from Fig. 1) are shown. We present median values at different scales for the large samples of galaxies, groups and clusters, as well as specific values for some individual galaxies, X-ray groups, and superclusters. Typical 1 σ uncertainties and 1 σ scatter around median values are shown. Also presented, for comparison, are the M/L_B (or equivalently Ω) determinations from the cosmic virial theorem, the least action method, and the *range* of various reported results from the Virgocentric infall and large-scale bulk flows (assuming mass traces light). The M/L_B expected for Ω = 1 and Ω = 0.3 are indicated.

Bahcall, Lubin, & Dorman (1995)

– cluster baryon fractions

$$f_b = \frac{M_b}{M_{tot}} = \frac{\Omega_b}{\Omega_m}$$



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Measurements of the gravitating mass density

- Weak lensing
  - – measure shear over large scales

Dark Energy Survey
arxiv:2002.11124

$$\Omega_m \approx 0.18 \pm 0.04$$

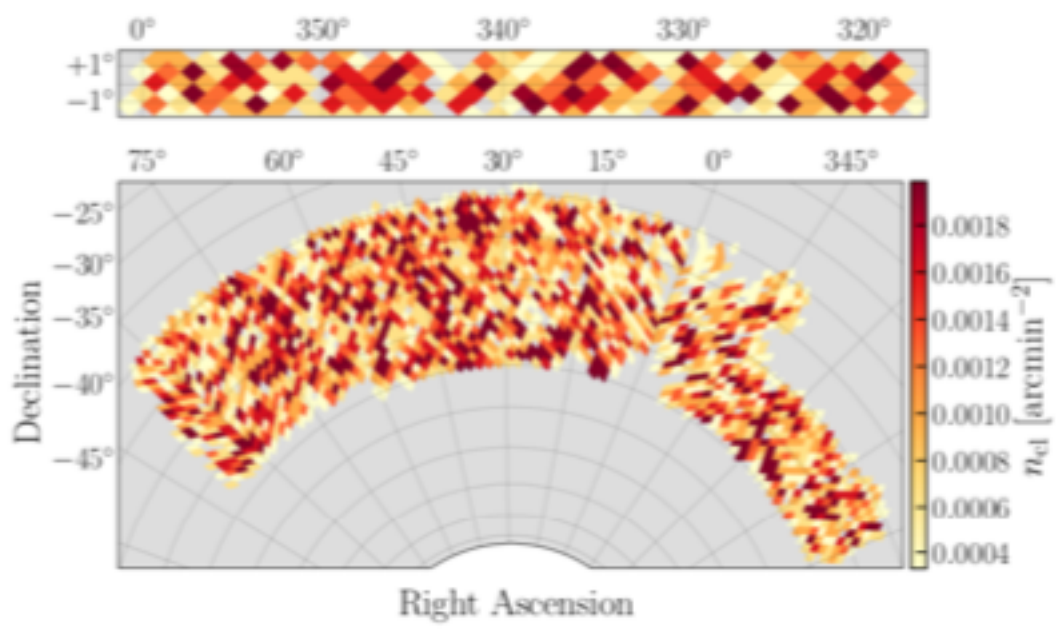

FIG. 1. The DES Y1 redMaPPer cluster density over the two non-contiguous regions of the Y1 footprint: the Stripe 82 region (116 deg^2; *upper* panel) and the SPT region (1321 deg^2; *lower* panel).

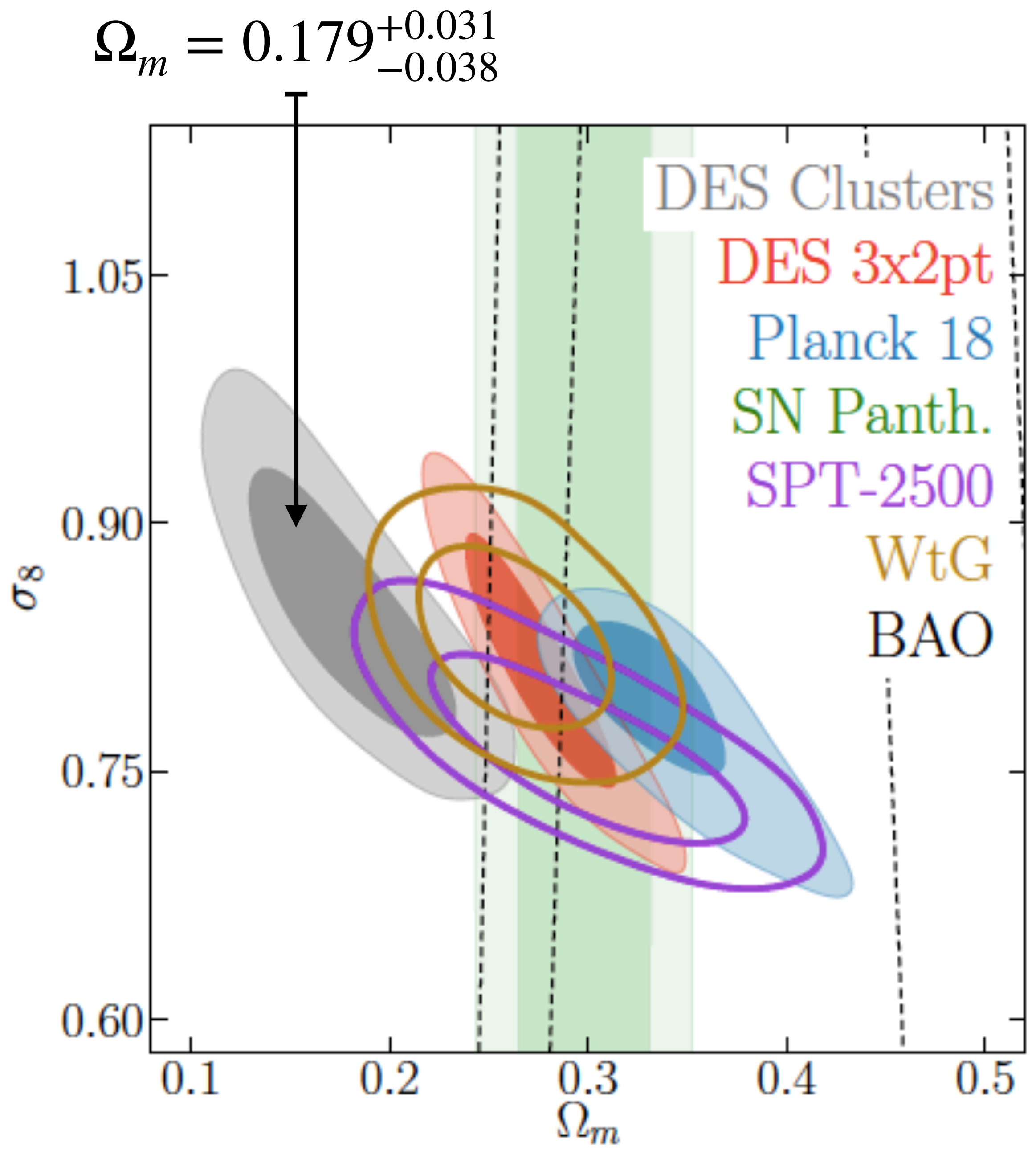

FIG. 6. Comparison of the 68% and 95% confidence contours in the σ_8-Ω_m plane derived from DES Y1 cluster counts and weak-lensing mass calibration (*gray* contours) with other constraints from the literature: BAO from the combination of data from Six Degree Field Galaxy Survey [6dF [62]], the SDSS DR 7 Main galaxy sample [63], and the Baryon Oscillation Spectroscopic Survey [BOSS [64]] (*black dashed* lines); Supernovae Pantheon [65] (*green* contours); DES-Y1 3x2 from [20] (*red* contours); *Planck* CMB from [2] (*blue* contours); SPT-2500 from [9] (*violet* contours); WtG from [7] (*gold* contours).

The EUCLID satellite will update this

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Virgocentric infall

The Virgo cluster is the largest nearby over-density.
Its gravity distorts the Hubble flow.
We fall towards it so it appears to recede less than
it should by an amount that depends on its mass

682 TONRY AND DAVIS (1981)

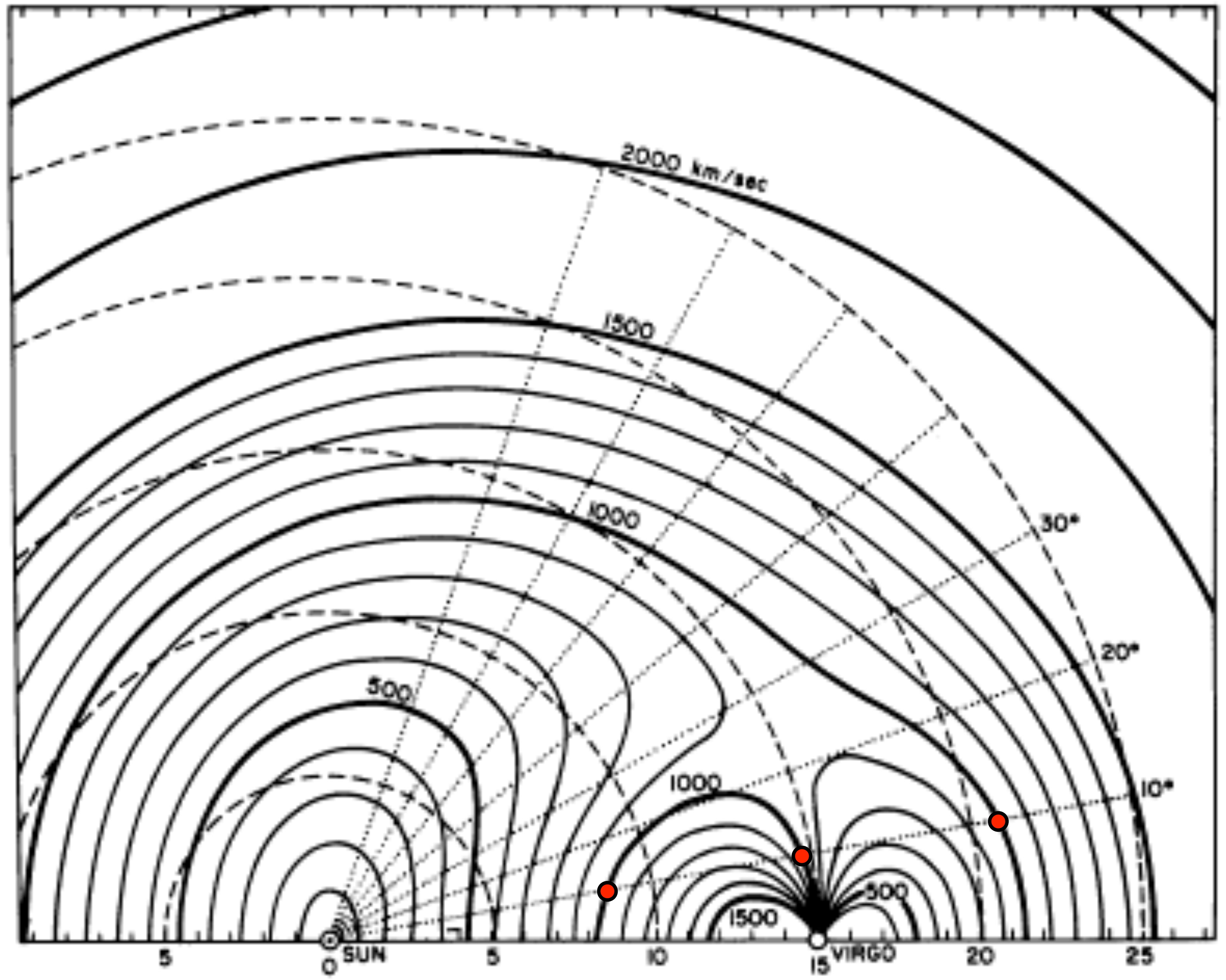


FIG. 1.—On a two-dimensional grid with the Earth and the Virgo cluster on the x axis, redshift contours are plotted for a Hubble flow perturbed by a Virgocentric flow. An infall velocity of 400 km s^{-1} at our position is assumed. A pure Hubble flow would be concentric circles.

Dashed lines: smooth Hubble flow

Solid lines: uniform expansion distorted by the gravity of the Virgo cluster

$V = H_0 D$ along lines of sight near Virgo can be triple valued!

Modern version: Cosmic Flows
https://edd.ifa.hawaii.edu/CF4calculator/

Measurements of the gravitating mass density

- Peculiar Velocity Field
  - measure deviations from Hubble flow

in linear regime $\frac{\delta\rho}{\rho} \ll 1$

$$\frac{\delta v}{v} \approx \frac{d \ln H}{d \ln \rho} \frac{\delta\rho}{\rho} \approx -\frac{1}{3} \frac{\Omega_m^{0.6}}{b} \frac{\delta\rho_g}{\rho_g}$$

peculiar velocity

distortion in Hubble flow induced by

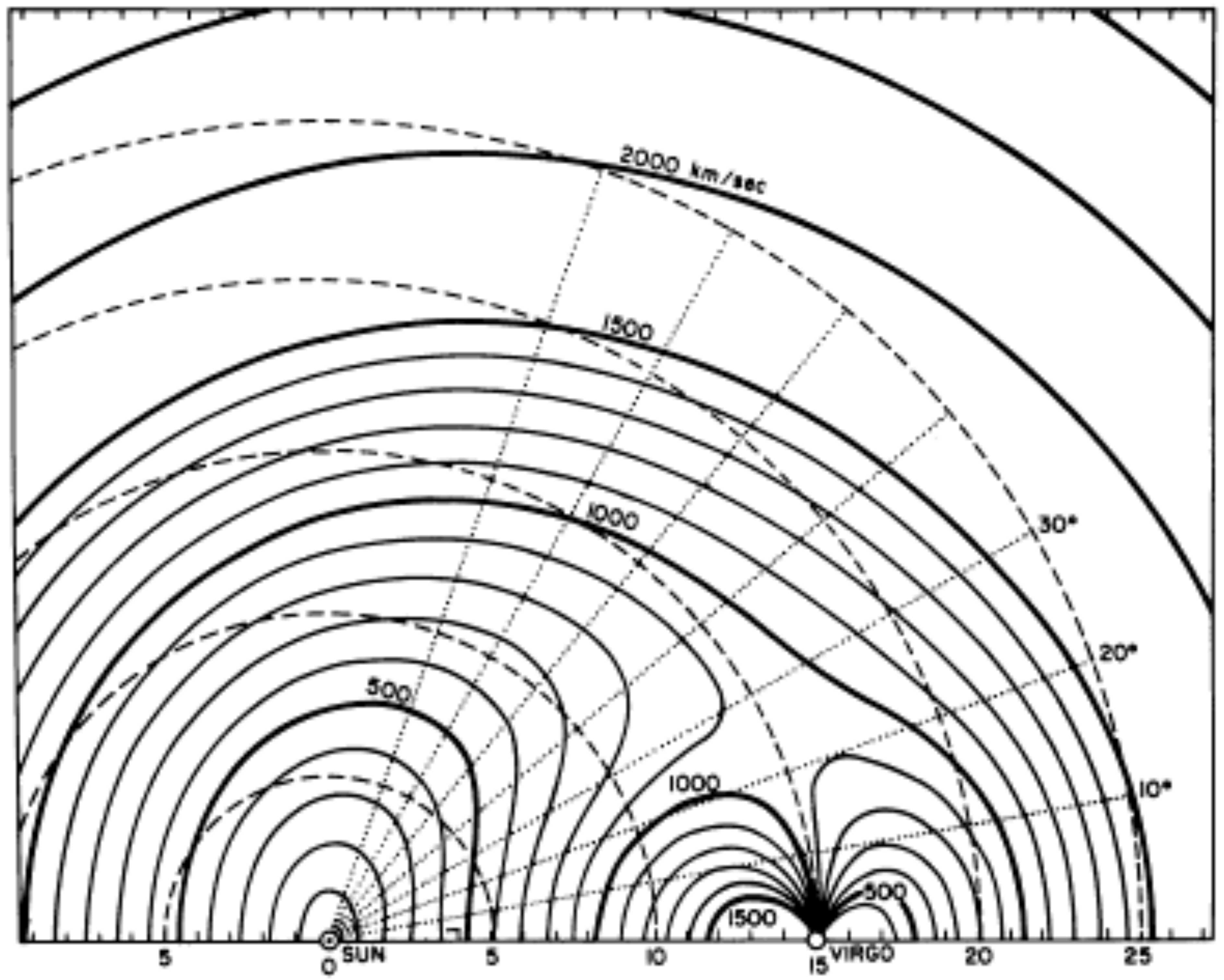
mass over-density

bias

BIAS ***b***
relates
galaxy over-densities
to mass over-densities

$$\Omega_m = 0.25 \pm 0.05$$

TONRY AND DAVIS



BIAS ***b***
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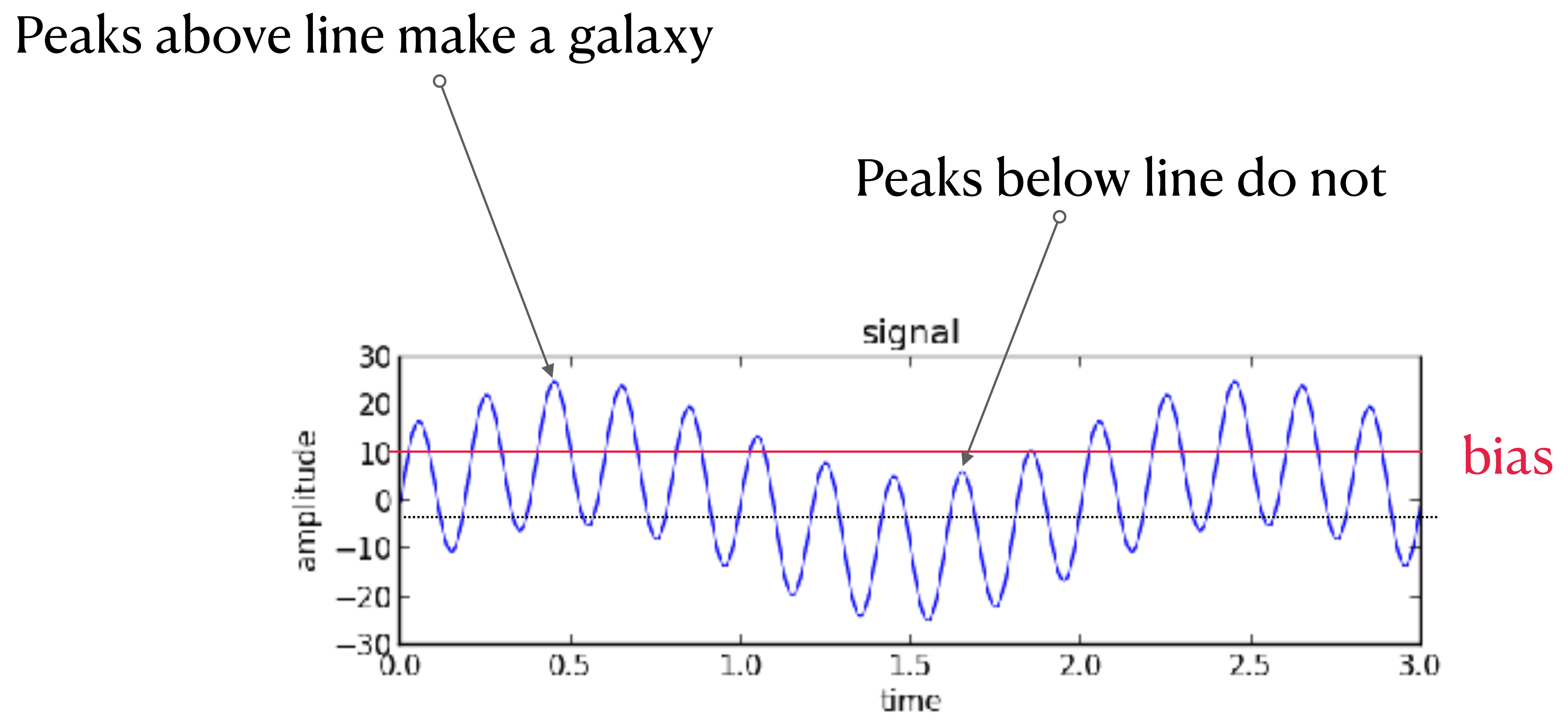


$$\frac{\delta\rho_g}{\rho_g} = b\frac{\delta\rho_m}{\rho_m}$$

$$\sigma_8 = \frac{1}{b}$$ in a sphere of radius 8 Mpc

Davis et al. (1980) found

$$\Omega_m = 0.4 \pm 0.1$$

with modern distances this becomes

$$\Omega_m = 0.25 \pm 0.05$$

basically unchanged for nearly 40 years

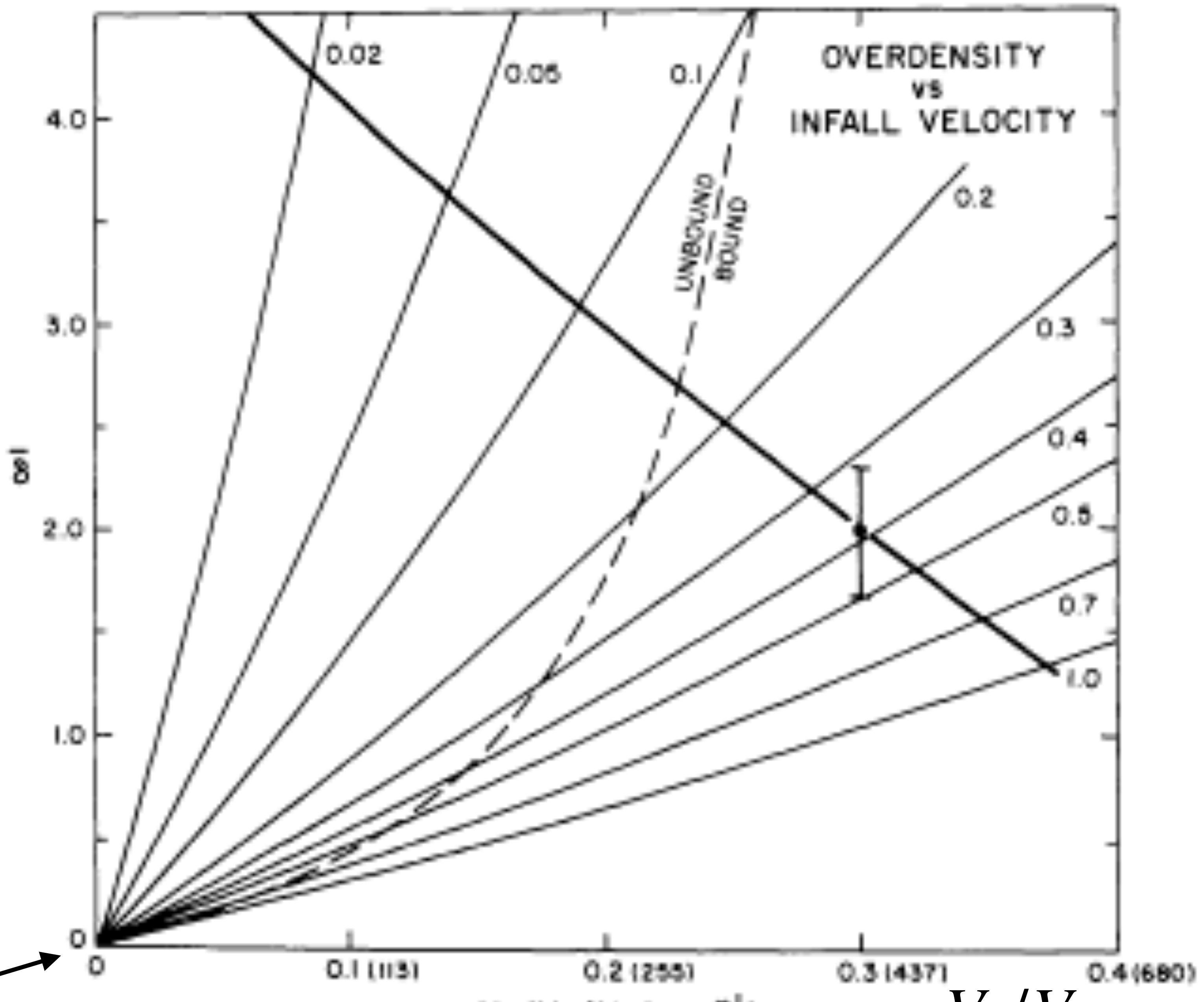
Lines are lines of constant Ω_m

Estimates of v_p

Velocity	Source
380 ± 75	Smoot and Lubin 1979
480 ± 75	Aaronson *et al.* 1980
350 ± 50	de Vaucouleurs and Bollinger 1979
290 ± 30[a]	Yahil 1980
190 ± 130	Schechter 1968

[a] Calculated with respect to the centroid at the local group as defined by Yahil *et al.* 1977.

$\frac{\delta\rho}{\rho}$



V_p/V_H

FIG. 1.—The mean overdensity of Virgo vs. v_p/v_H for various values of Ω. The x-axis is also labeled with v_p, using a recessional velocity to Virgo of 1020 km s^{-1}. The measured overdensity is prescribed by the heavy line, and is marked at the favored position as given by the anisotropy of the Hubble flow and microwave background radiation. The error bar is an estimate of the 90% confidence limit of our determination of $\bar{\delta}$. Models to the right of the dotted line are bound to Virgo.

mated, roughly by
density of galaxies
as r^{-2}; if the mas
duced peculiar vel
this model will app
which have not yet
Virgo core and wit
km s^{-1} are assume
scale peculiar mot
effect on the com
100 km s^{-1} Mpc^{-1},
300 km s^{-1} galacti
Aaronson, and Hu
sional velocity of
Table 2 lists the c
dom sample when b
assumed here v_p/v_H
The mean overd
not at all sensitive

VIRG

Distance[a] (Mpc)
0–5. . . . . . .
5–10. . . . . .
10–15. . . . .
15–20. . . . .
20–25. . . . .
25–30. . . . .
30–35. . . . .
$\bar{\delta}$. . . . .

[a] Assumes
tance and H
[b] 600 obje
[c] Within l

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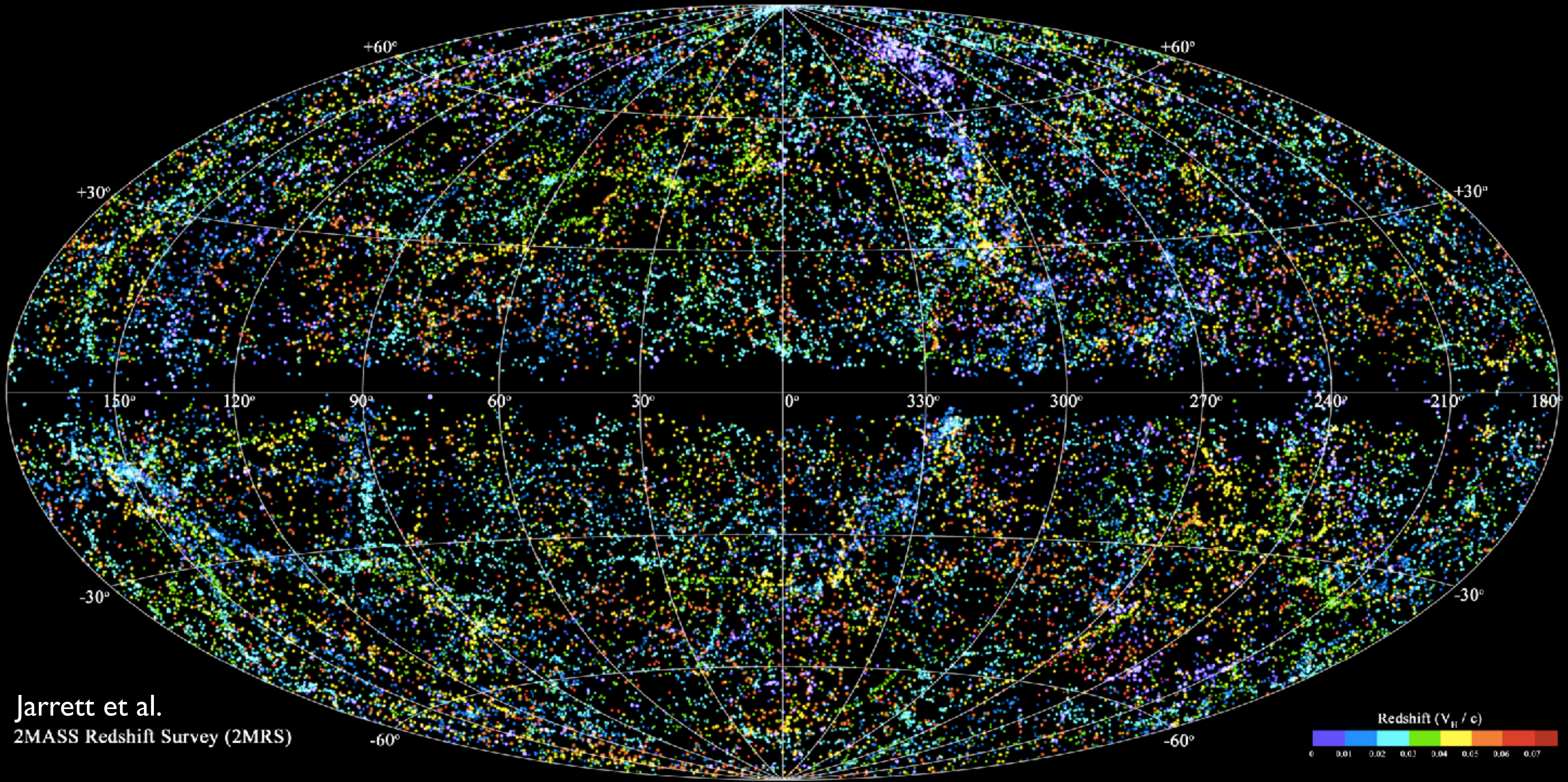
$z = 0$

$z = 1090$

Measurements of the gravitating mass density

- Power spectrum of galaxies



Jarrett et al.

2MASS Redshift Survey (2MRS)

- # Power spectrum of galaxies

$$\delta \equiv \frac{\delta\rho}{\rho}$$

The power spectrum is commonly used to quantify large scale structure.
It is the related to the 2 point correlation function via Fourier transform.

2 point correlation function: $$\xi(\vec{r}) = \langle \delta(\vec{x})\delta(\vec{x}+\vec{r}) \rangle$$

The 2 point correlation function is the probability of finding one galaxy near another in excess over a random distribution.

Power spectrum: $$P(k) = \langle |\delta_k|^2 \rangle$$ where $$k = \frac{2\pi}{\lambda}$$

where k is the wavenumber corresponding to the scale λ

Fourier transform:

$$\xi(\vec{r}) = \frac{V}{(2\pi)^3} \int |\delta_k|^2 e^{-i\vec{k}\cdot\vec{r}} d^3k$$ averaged over volume V

$|\delta_k|^2 \rightarrow P(k)$

Large Scale Structure

Quantified with the **correlation function** $\xi(r)$ which is the Fourier transform of the **power spectrum** $P(k)$.

The correlation function is the excess probability of finding a galaxy near another galaxy over that in a random distribution.

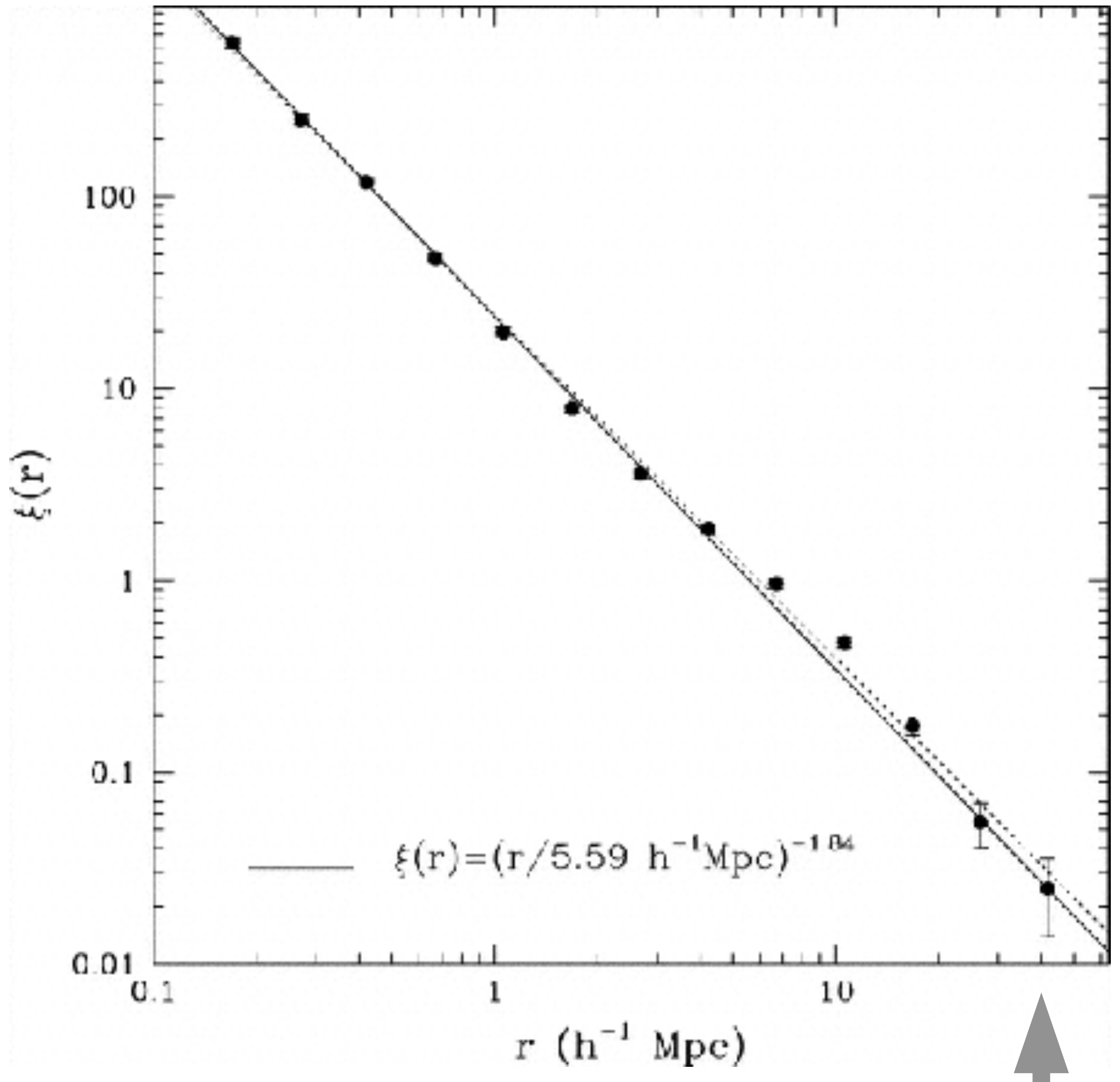
$$\frac{dN}{N} = [1 + \xi(r)]dV \qquad \xi(r) = \frac{V}{(2\pi)^3}\int P(k)e^{-\vec{k}\cdot\vec{r}}\, d^3k$$

$$P(k) \propto |\delta(k)|^2 \propto k^n \qquad \xi(r) \propto r^{-(n+3)} \qquad \delta \equiv \frac{\delta\rho}{\rho}$$

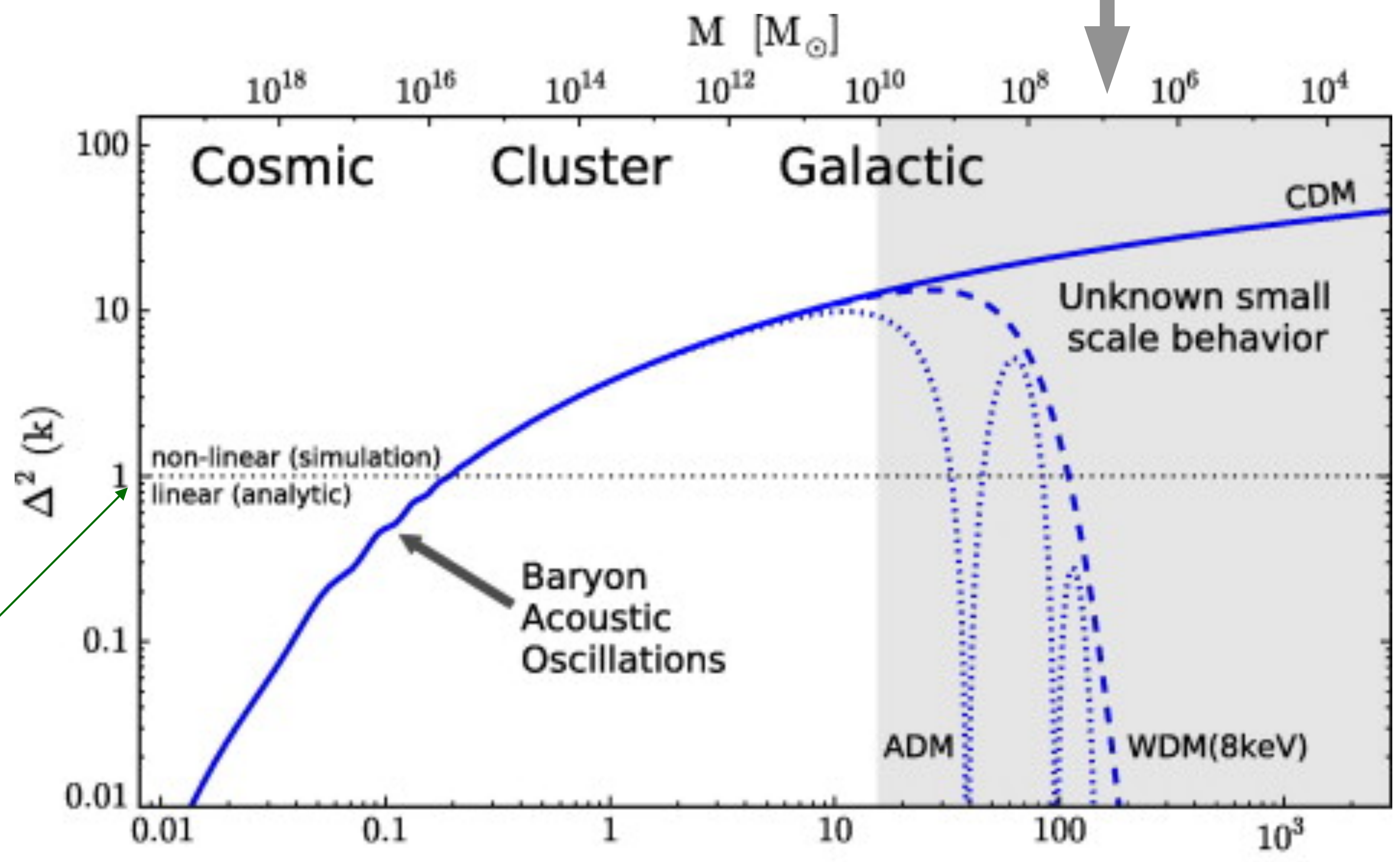
Harrison-Zeldovich spectrum has $n = 1$, which is a Gaussian random field. Inflation predicts $n \approx 1$, but different flavors of Inflationary theory predict slightly different values depending on the shape of the Inflationary potential (the Inflaton). Planck measures $n = 0.965 \pm 0.004$

$\delta > 1$ marks the transition to the non-linear regime where perturbation theory no longer applies.

SDSS correlation function (Zehavi et al 2005)

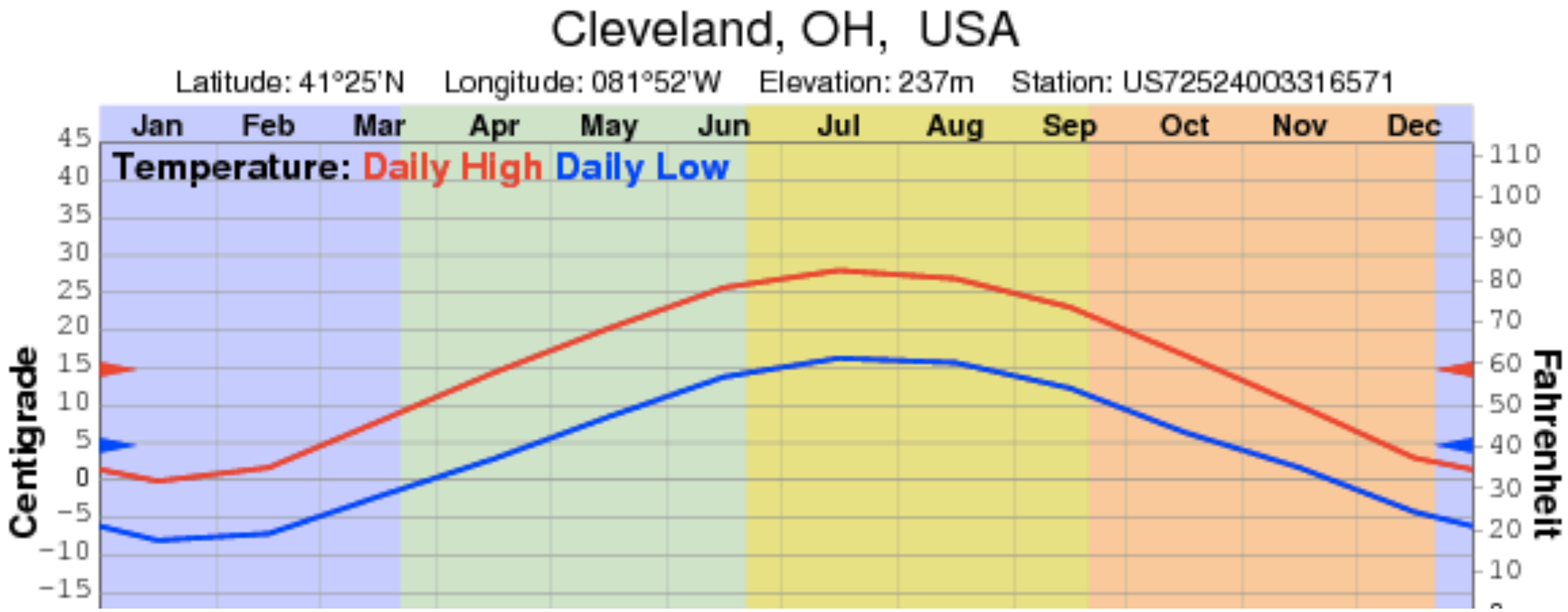


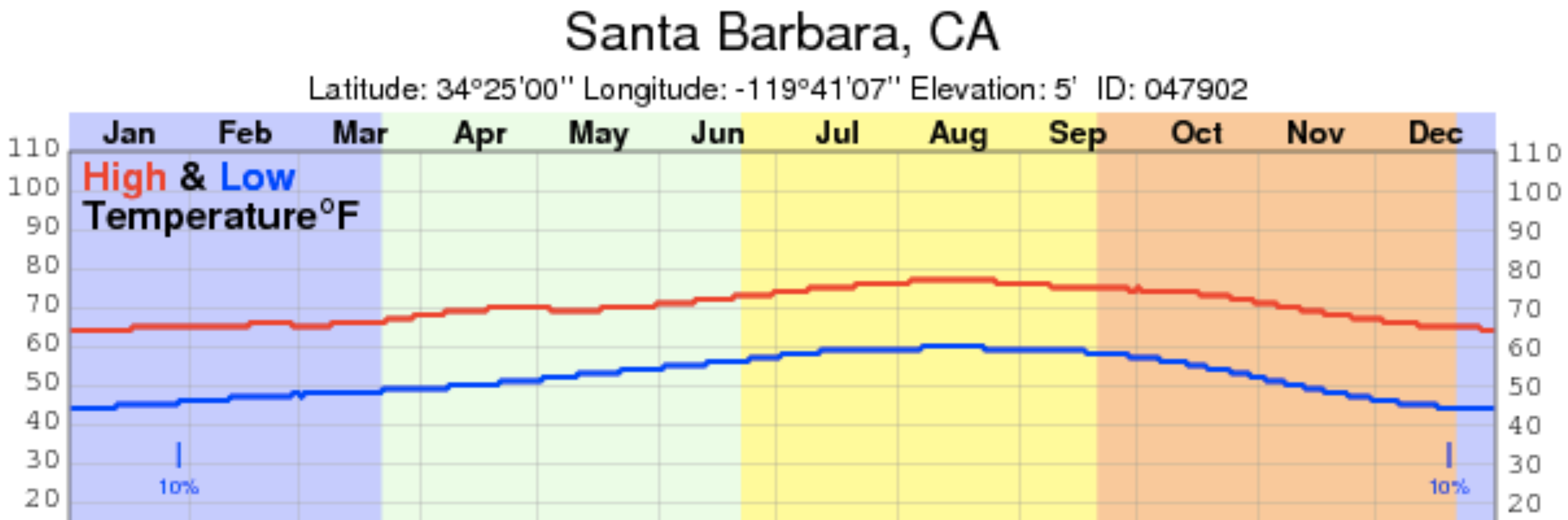
Power Spectrum



Power Spectrum

Example: weather in Cleveland and Santa Barbara
More power on long time scales in Cleveland (seasonal variation)

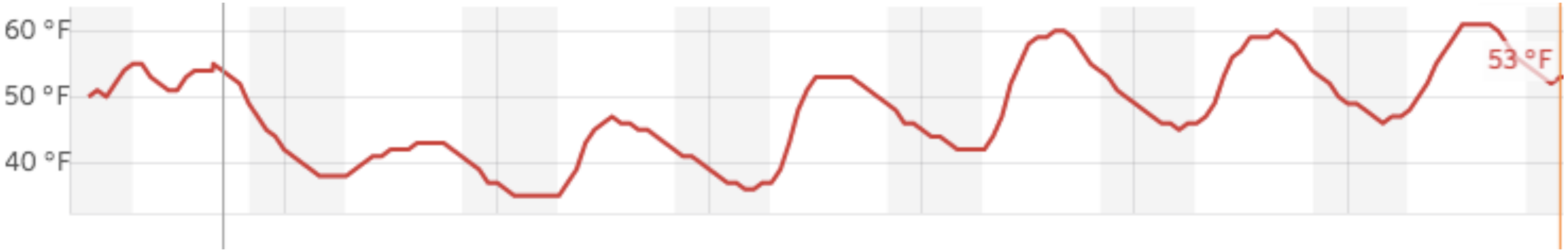




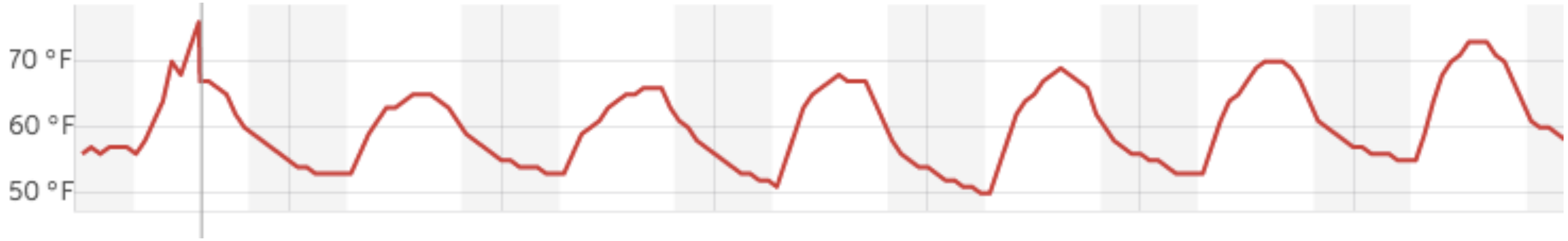
Power Spectrum

Example: weather in Cleveland and Santa Barbara
Similar power on short time scales in Santa Barbara (diurnal variation)

Cleveland forecast



Santa Barbara forecast



A power spectrum is a Fourier transform that quantifies
the relative variability on different scales

- # Power spectrum of galaxies

$\delta \equiv \frac{\delta\rho}{\rho}$ $\qquad k = \frac{2\pi}{\lambda}$

Power law power spectrum: $P(k) = \langle |\delta_k|^2 \rangle \propto k^n$

where n = 1 is scale free, with the same power on all scales.
This is observed to be nearly the case on large scales that have not yet collapsed. It is modulated on small scales by structure formation.

One way to think of it is the rms variation at each scale λ

$$M \sim \lambda^3 \qquad \delta_{\rm rms} \propto M^{-(n+3)/6}$$

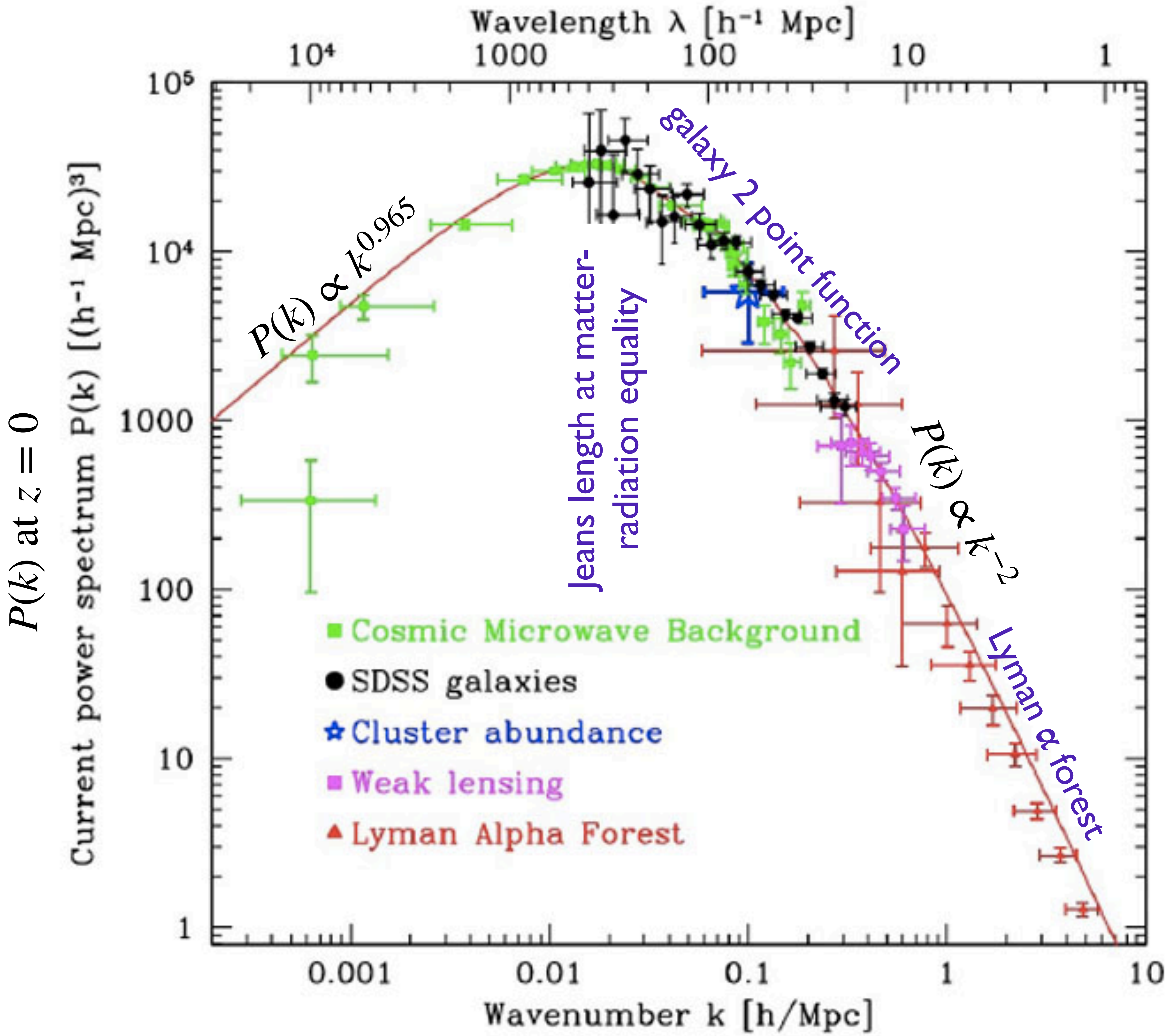
There is more rms variance on small scales, so more power there.
[On very large scales, the universe is homogeneous, so no variance.]

By convention, the normalization is set on a scale of 8 Mpc, where

$\frac{\delta N_{gal}}{N_{gal}} = 1$ with corresponding mass variance σ_8

Planck estimates: $n = 0.965 \pm 0.004$

$$\sigma_8 = 0.811 \pm 0.006$$



Jeans length at matter-radiation equality

$$\lambda_J = c_s \sqrt{\frac{\pi}{G\rho}}$$

sound speed

$$c_s^2 = \frac{\partial P}{\partial \rho} = \frac{1}{3} c^2$$

at smaller scales, things go non-linear from gravitational collapse, pressure, dissipation, feedback, etc. Described by a Transfer function

$$T(k) \equiv \frac{\delta_k(z=0)}{D(z)\delta_k(z)}$$

where D(z) is the linear growth factor - what it would have been without all these nasty non-linear effects.