Dark Matter

ASTR 333/433
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TR 11:30am–12:45pm
Sears 552

http://astroweb.case.edu/ssm/ASTR333/

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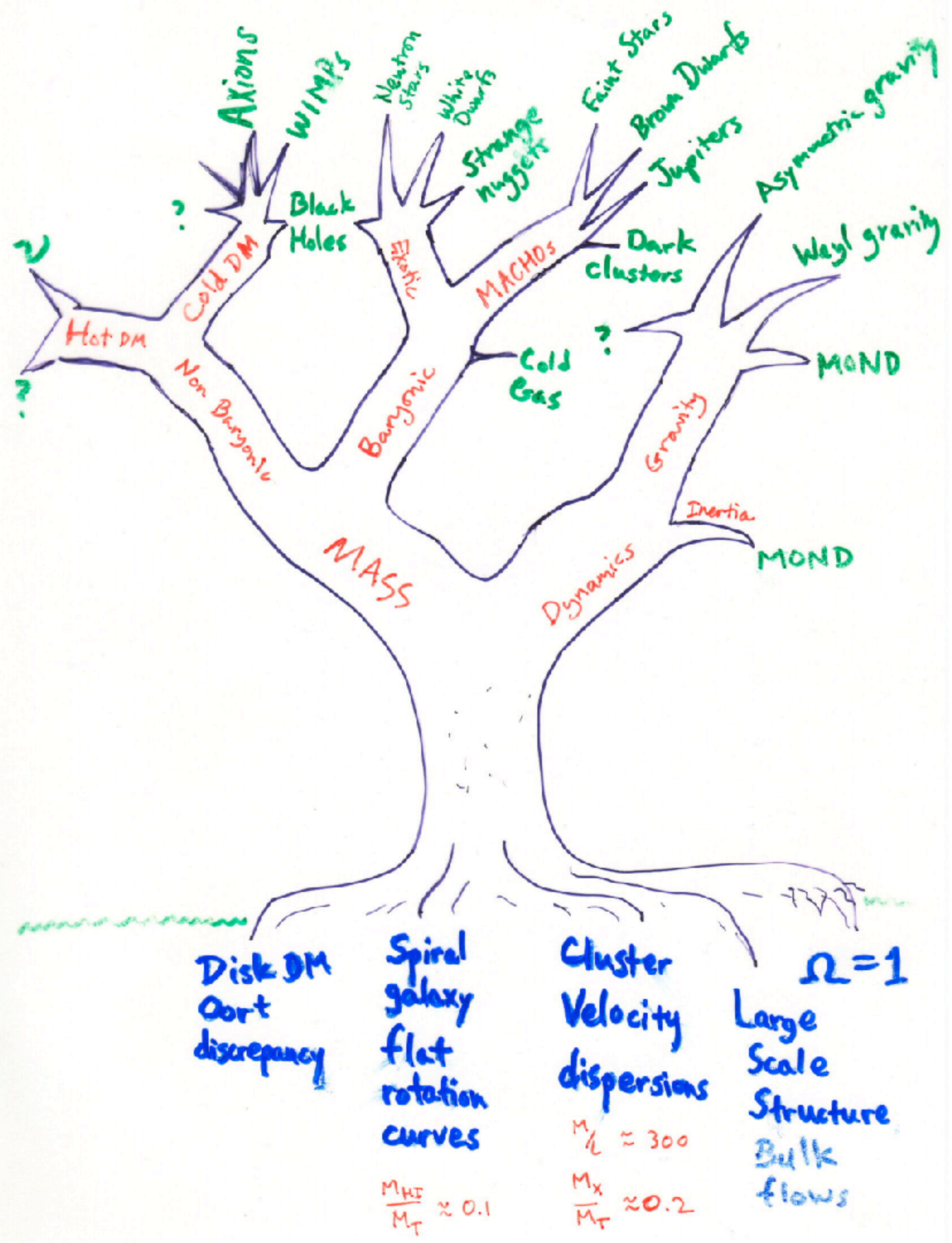
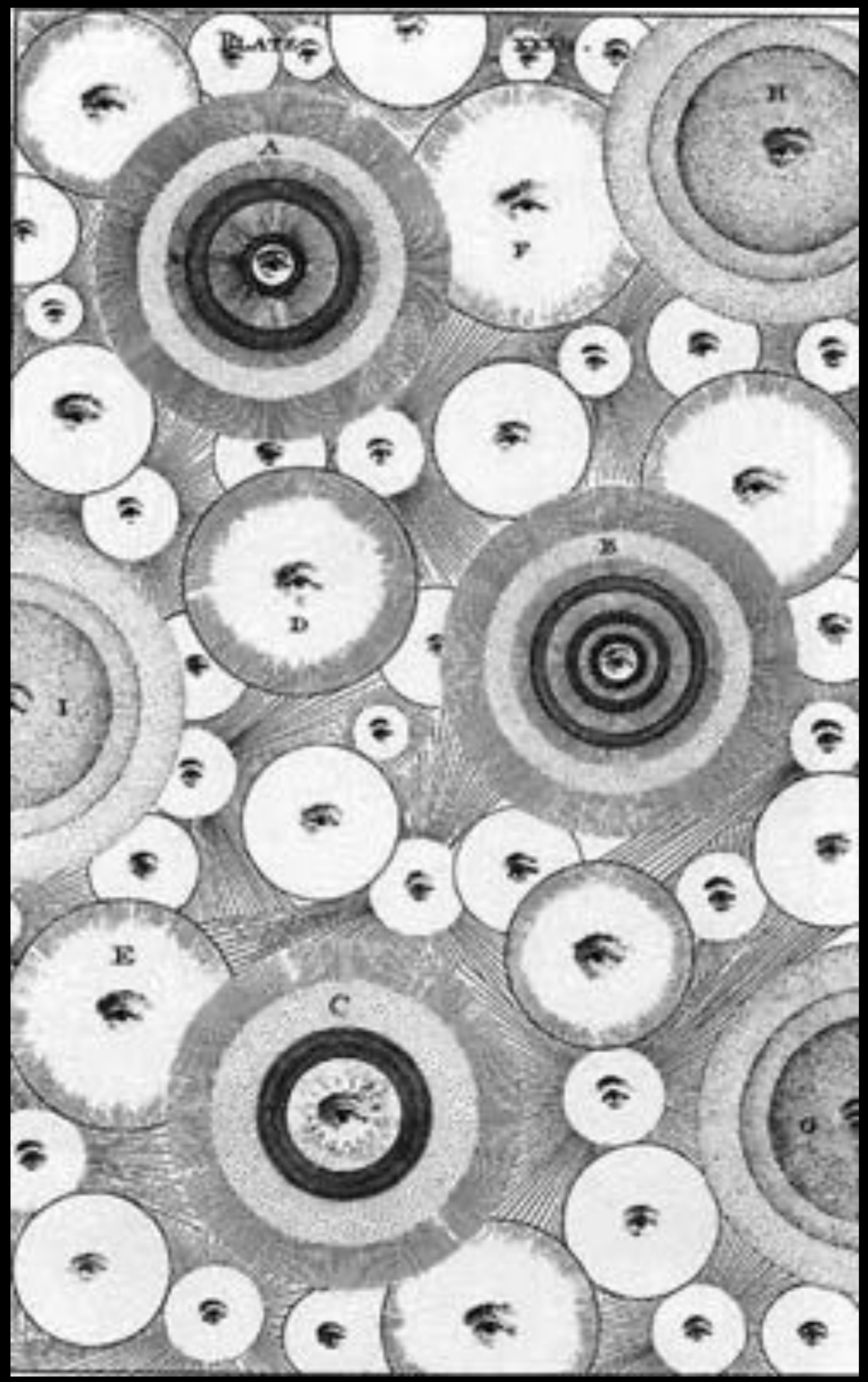




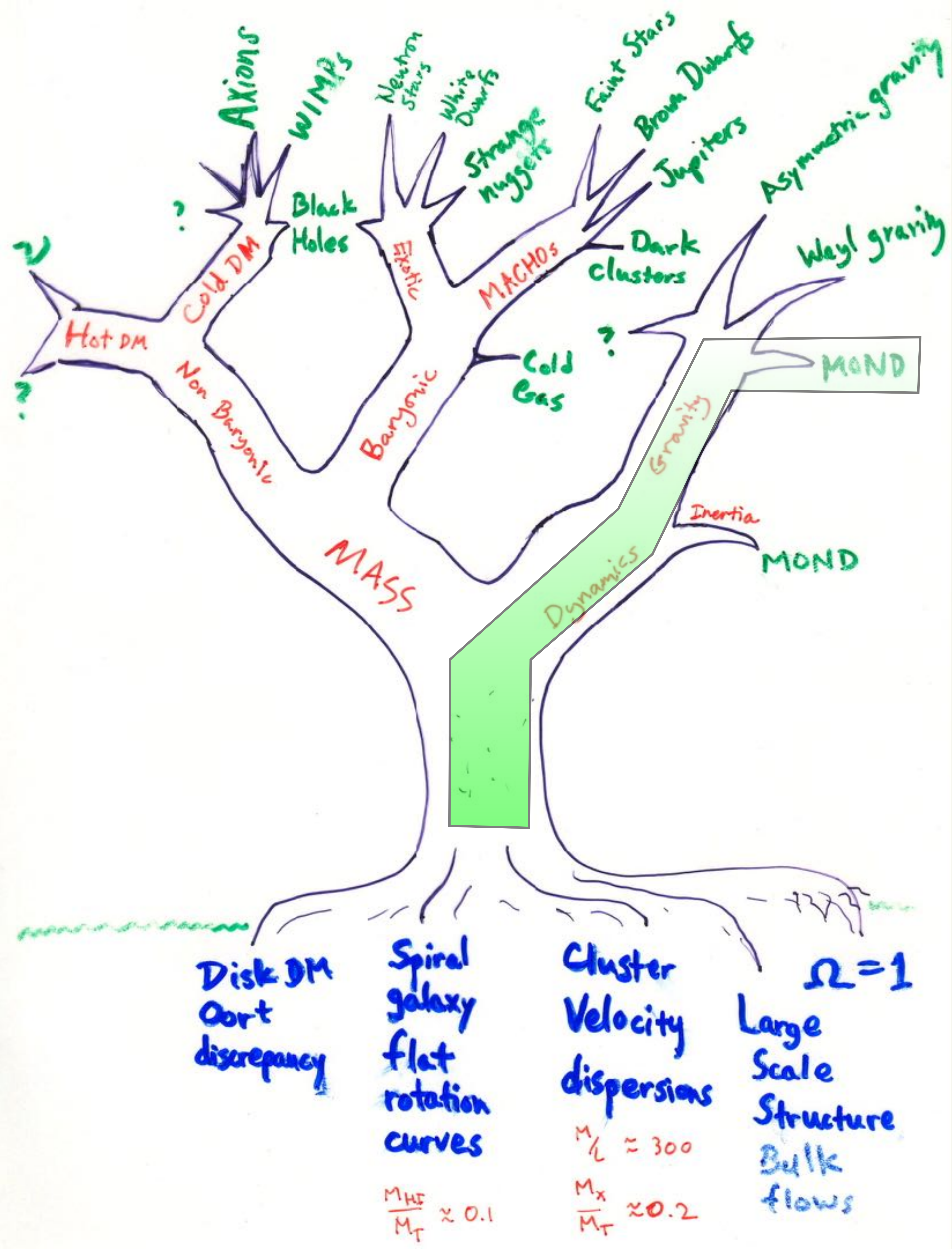
"No competent thinker, with the whole of the available evidence before him, can now, it is safe to say, maintain any single nebula to be a star system of coordinate rank with the Milky Way. A practical certainty has been attained that the entire contents, stellar and nebular, of the sphere belong to one mighty aggregation" [i.e., the Milky Way]

- Agnes Mary Clerke (1890)

Popular History of Astronomy during the Nineteenth Century

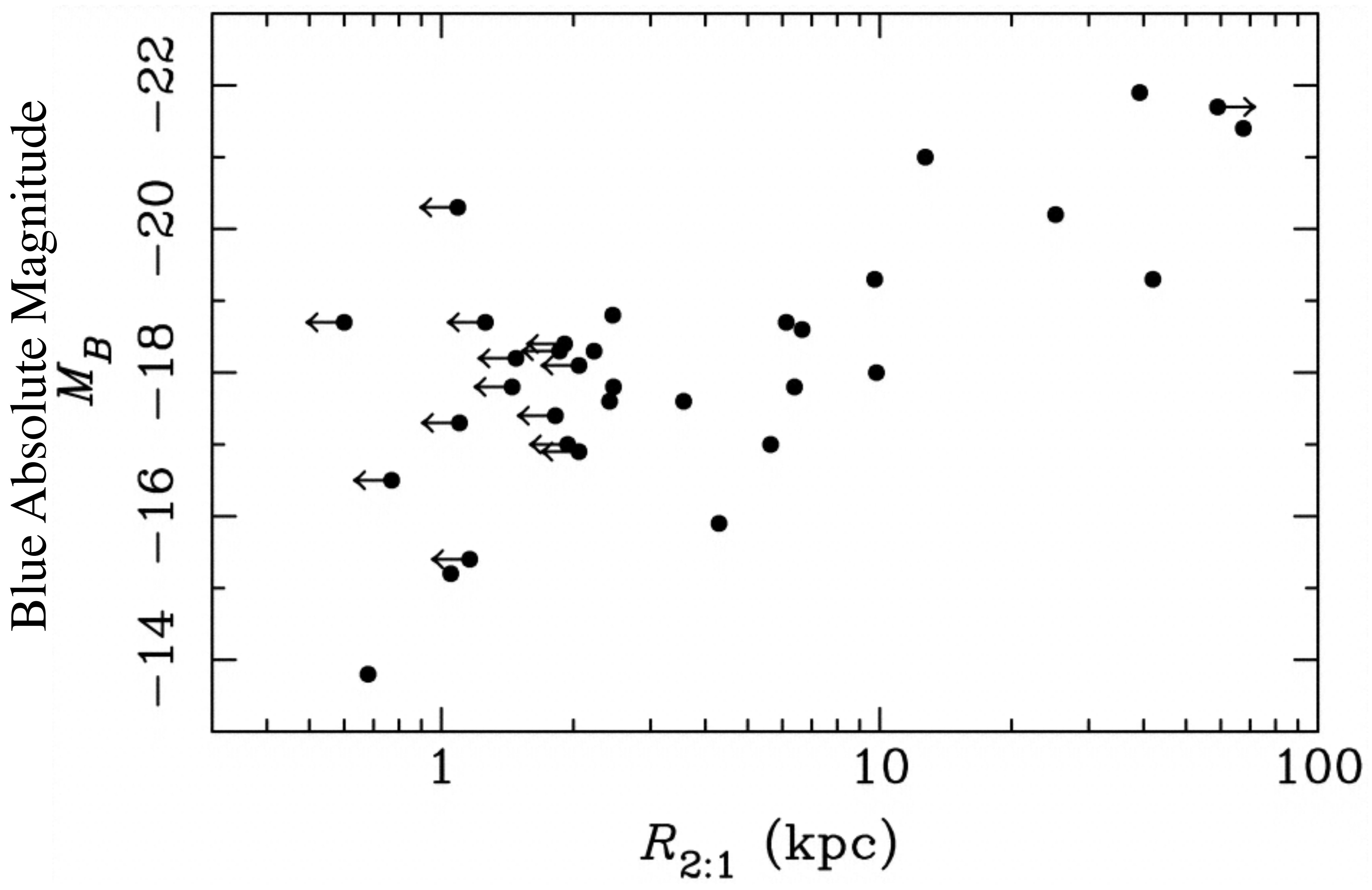
The existence of the acceleration discrepancy has been established beyond reasonable doubt. That this requires the existence of invisible mass has not.

The existence of dark matter has attained a practical certainty

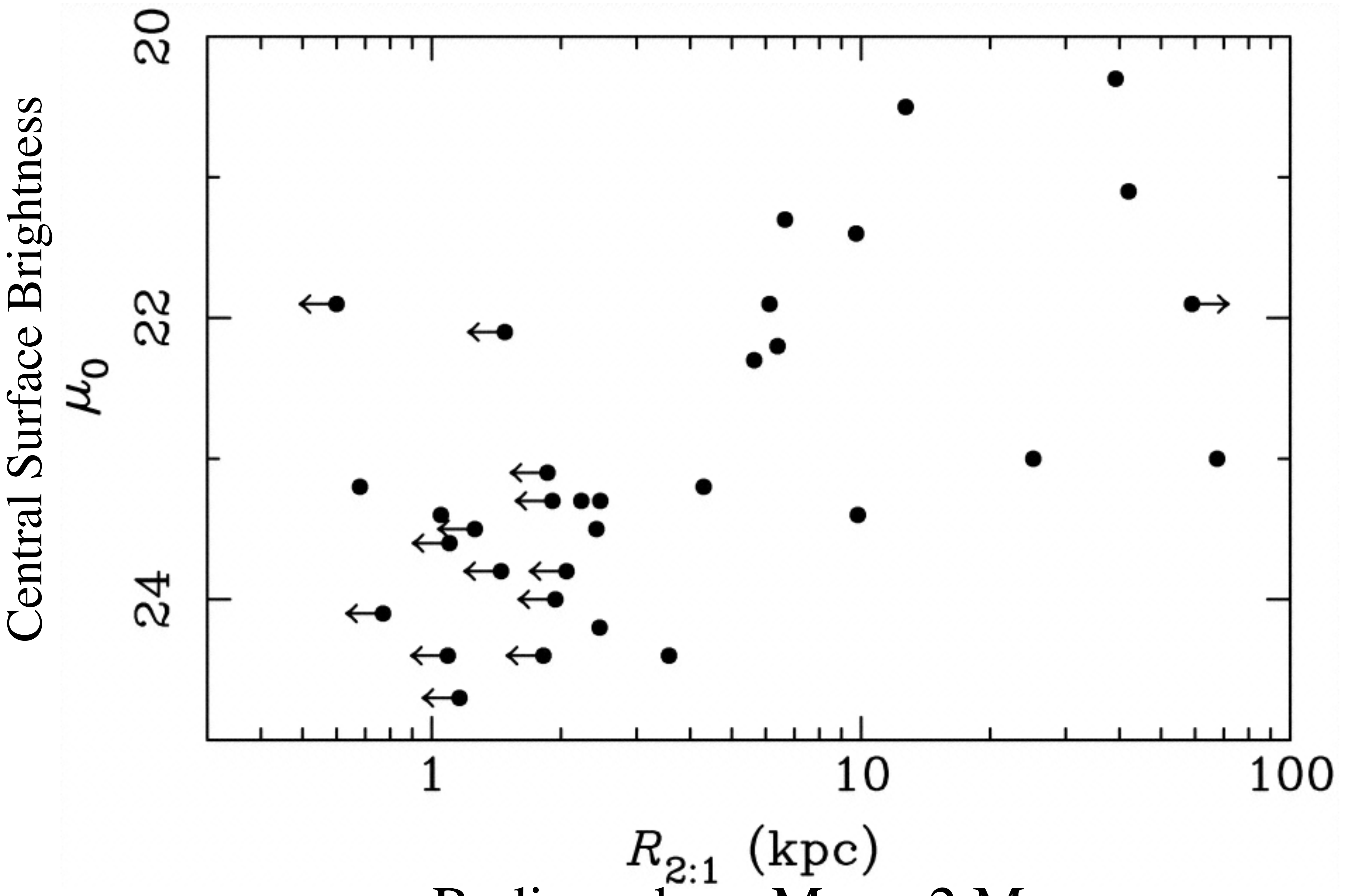


or maybe we've been using the wrong equation

Not any theory will do - length scale based modifications can be immediately excluded as the discrepancy does not appear at a particular length scale.



Radius where $M_{tot} = 2\ M_*$



Radius where $M_{tot} = 2\ M_*$

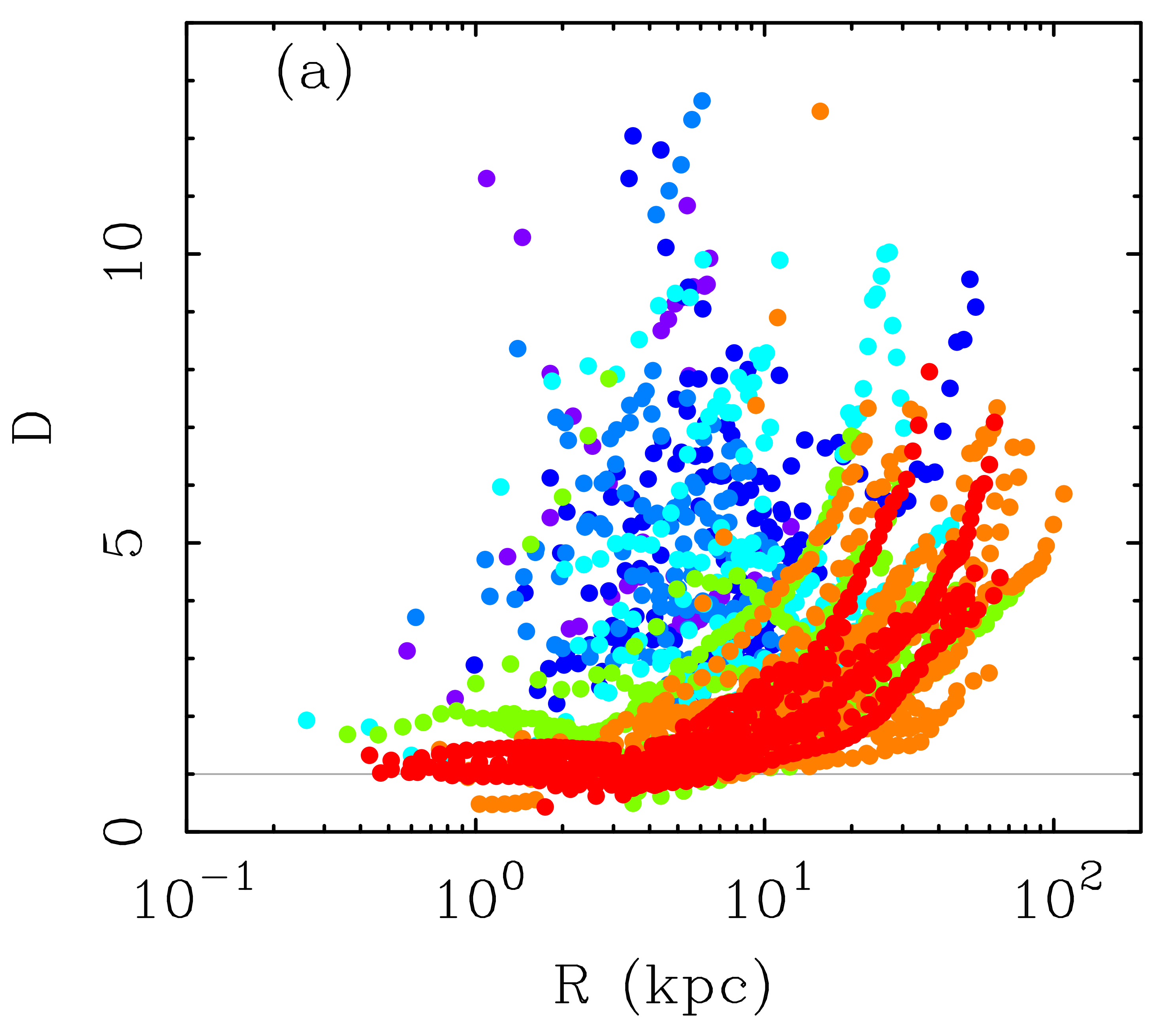
Bright galaxies do not require dark matter until far out while dim galaxies require it already at small radii.

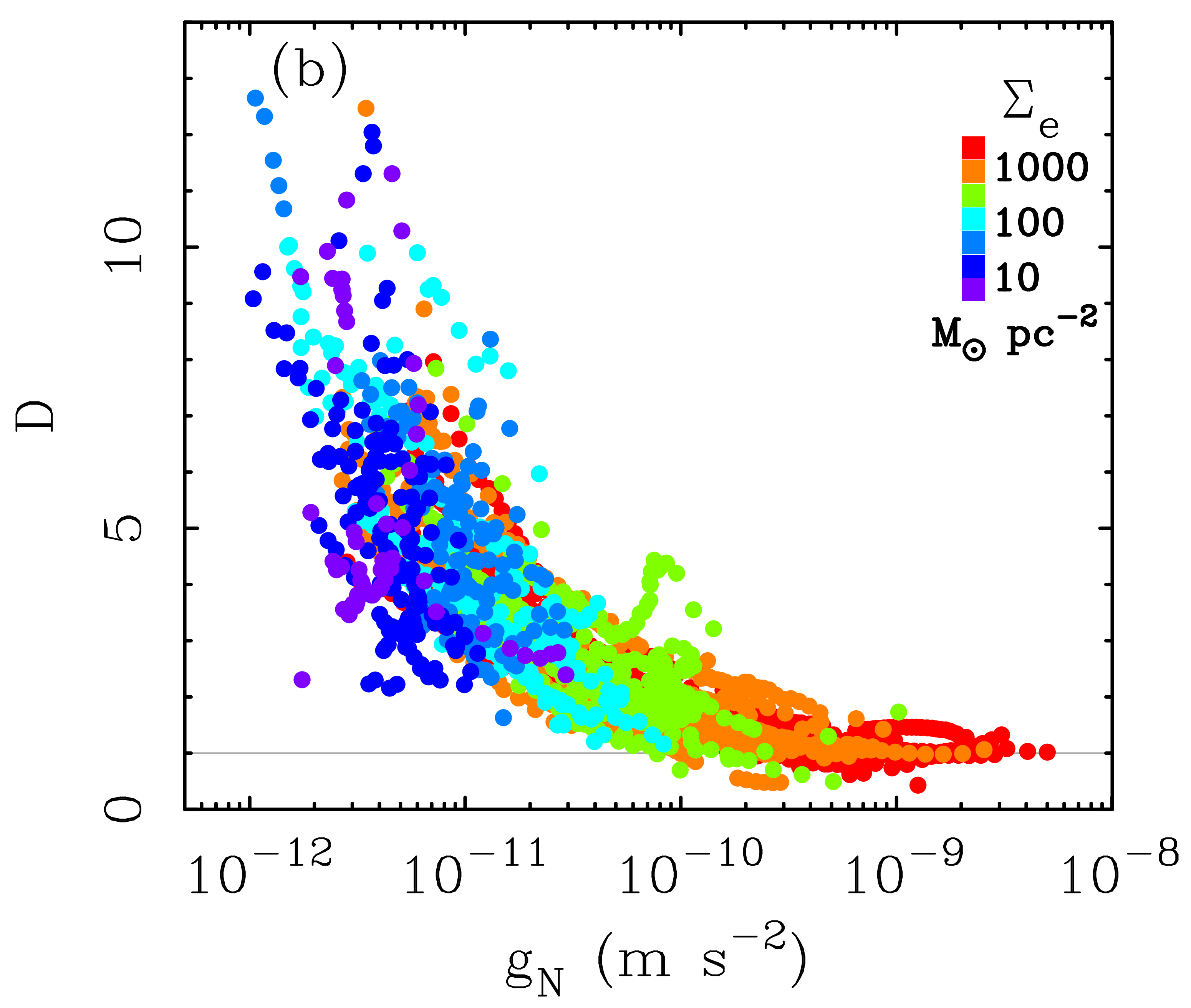
The mass discrepancy $D = \frac{a}{g_N}$ where $a = \frac{V^2}{R}$ and $g_N = -\frac{\partial \Phi_b}{\partial R}$

centripetal acceleration

gravitation due to baryons

Equivalent to the Radial Acceleration Relation

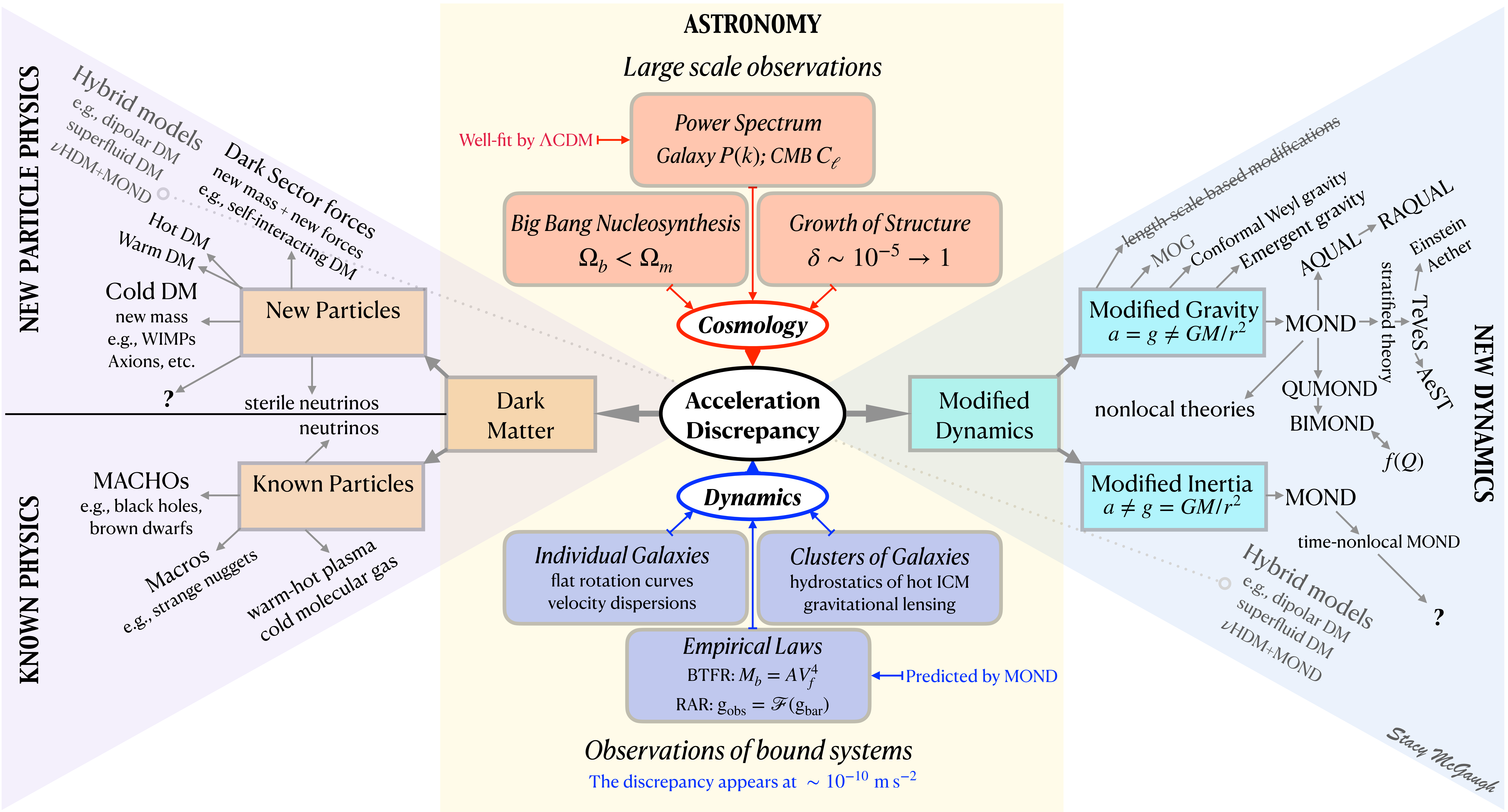




There is no unique size scale in the data. Can generically exclude any modification of gravity where a change in the force law appears at a specific length scale [e.g., f(R) gravity].

There is a characteristic acceleration scale in the data

*The **dark matter problem** might more appropriately be called the **acceleration discrepancy** (Bekenstein)*



A graphical representation of the Dark Matter tree

slide from second class

Recall our empirical laws - these are true in the data irrespective of their interpretation

- Flat rotation curves (Rubin-Bosma Law)

  Rotation curves tend asymptotically towards a constant rotation velocity that persists to indefinitely large radii: $V(R \to \infty) \to V_f$

- Tully-Fisher relation (Luminous, Stellar Mass, and Baryonic TF relations)

  The baryonic mass of galaxies scales as the fourth power of the flat rotation velocity: $M_b = AV_f^4$

- Central density relation (lower surface brightness galaxies exhibit larger mass discrepancies)

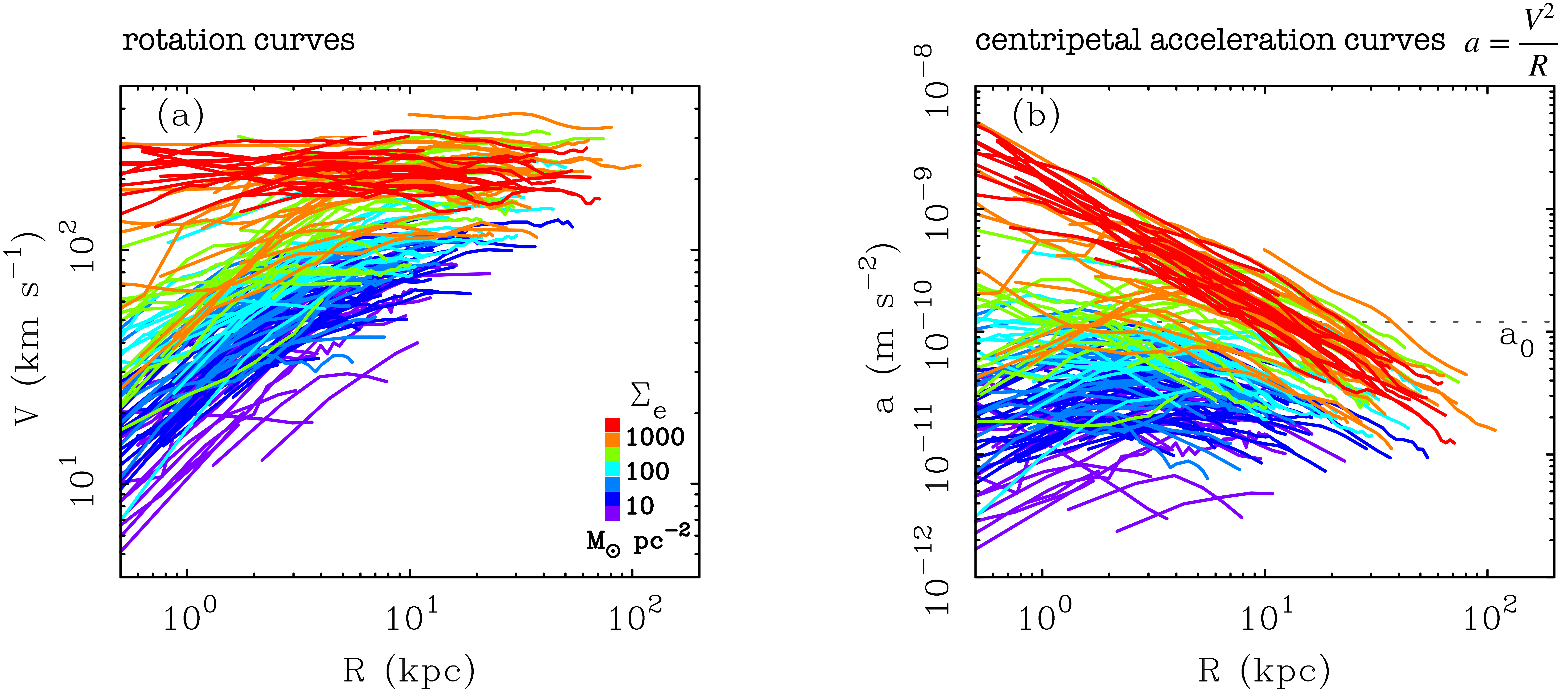
  The central dynamical surface densities of galaxies is related to their central surface brightnesses: $\Sigma_{dyn}(R \to 0) = f[\Sigma_*(R \to 0)]$

- Renzo's rule (Sancisi's Law)

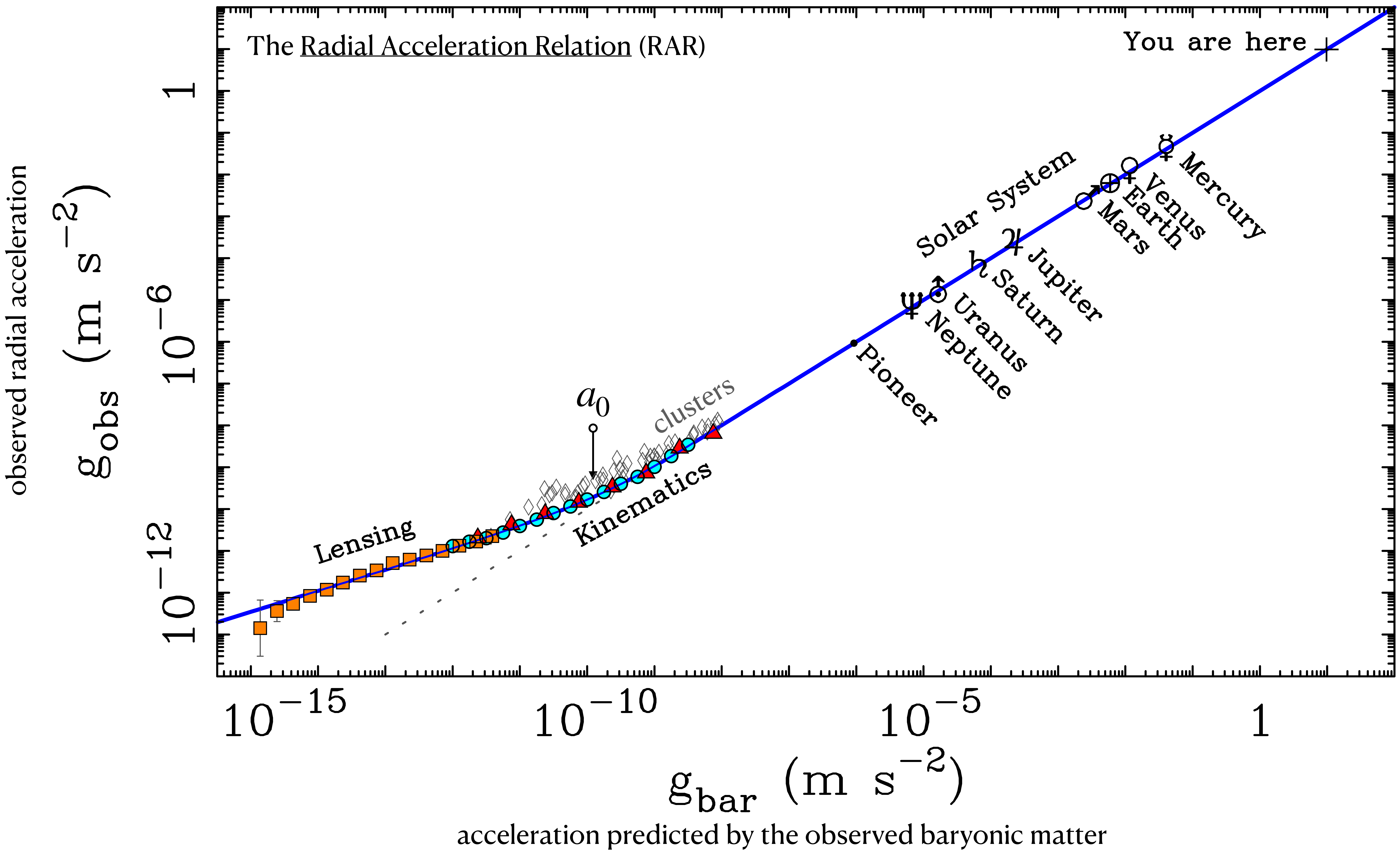
  "For any feature in the luminosity profile there is a corresponding feature in the rotation curve and vice versa." (Sancisi 2004).

- Radial acceleration relation

  The observed centripetal acceleration is related to that predicted by the observed distribution of baryons: $\mathrm{g_{obs}} = \mathcal{F}(\mathrm{g_{bar}})$

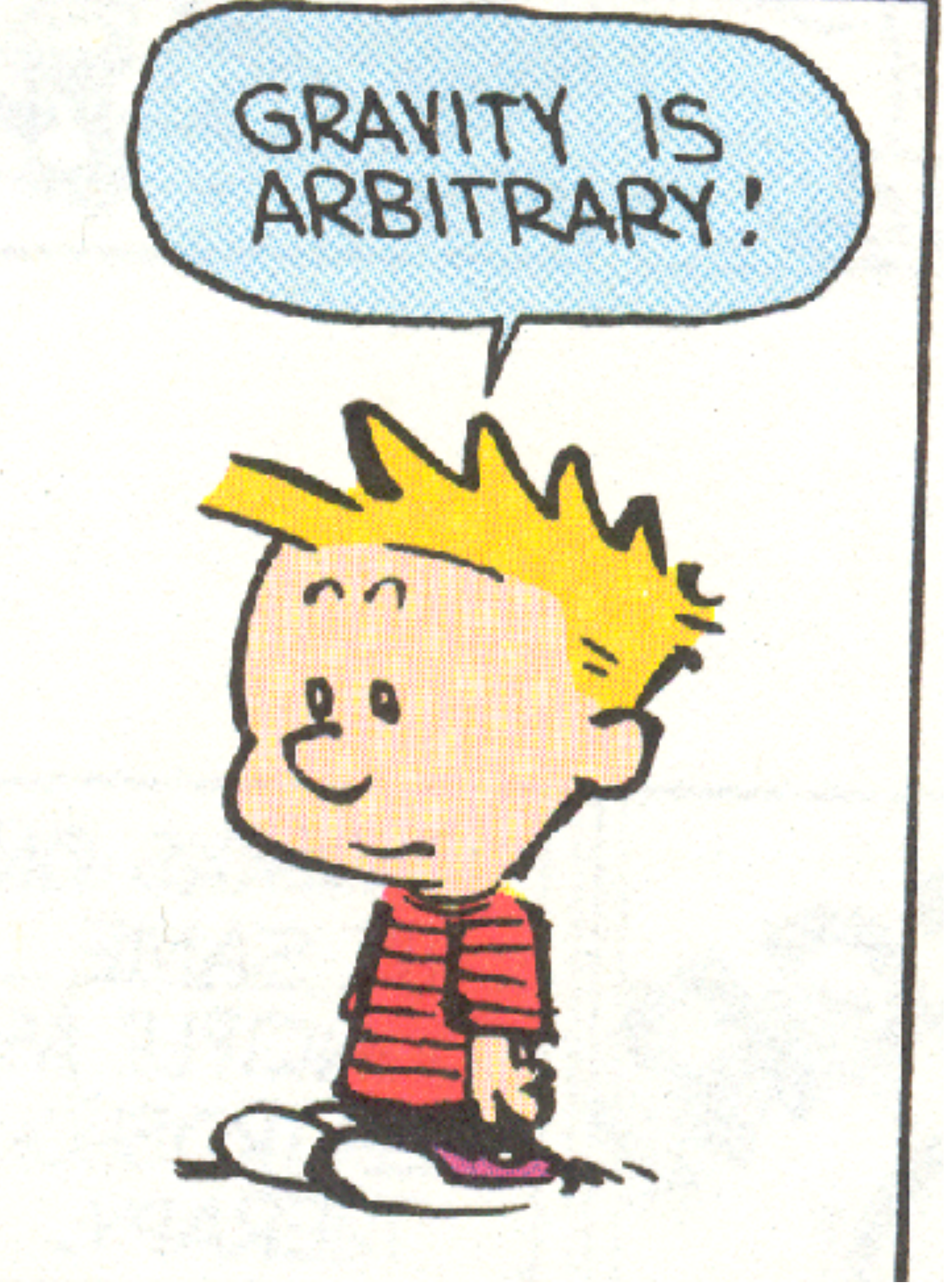

rotation curves
(a)
V (km s^{-1})
R (kpc)
Σ_e
1000
100
10
$M_\odot$ pc^{-2}
centripetal acceleration curves $a = \frac{V^2}{R}$
(b)
a (m s^{-2})
a_0
R (kpc)

There is a characteristic acceleration scale in the DM problem



MOND

Modified Newtonian Dynamics (Milgrom 1983)



Instead of invoking dark matter, modify gravity (or inertia). Milgrom suggested a modification at a particular acceleration scale a_0

Newtonian regime

$$a = g_N \text{ for } a \gg a_0$$

MOND regime

$$a = \sqrt{g_N a_0} \text{ for } a \ll a_0$$

MOND regime invariant under transformations $(t, \mathbf{x}) \to \lambda(t, \mathbf{x})$

http://www.scholarpedia.org/article/The_MOND_paradigm_of_modified_dynamics

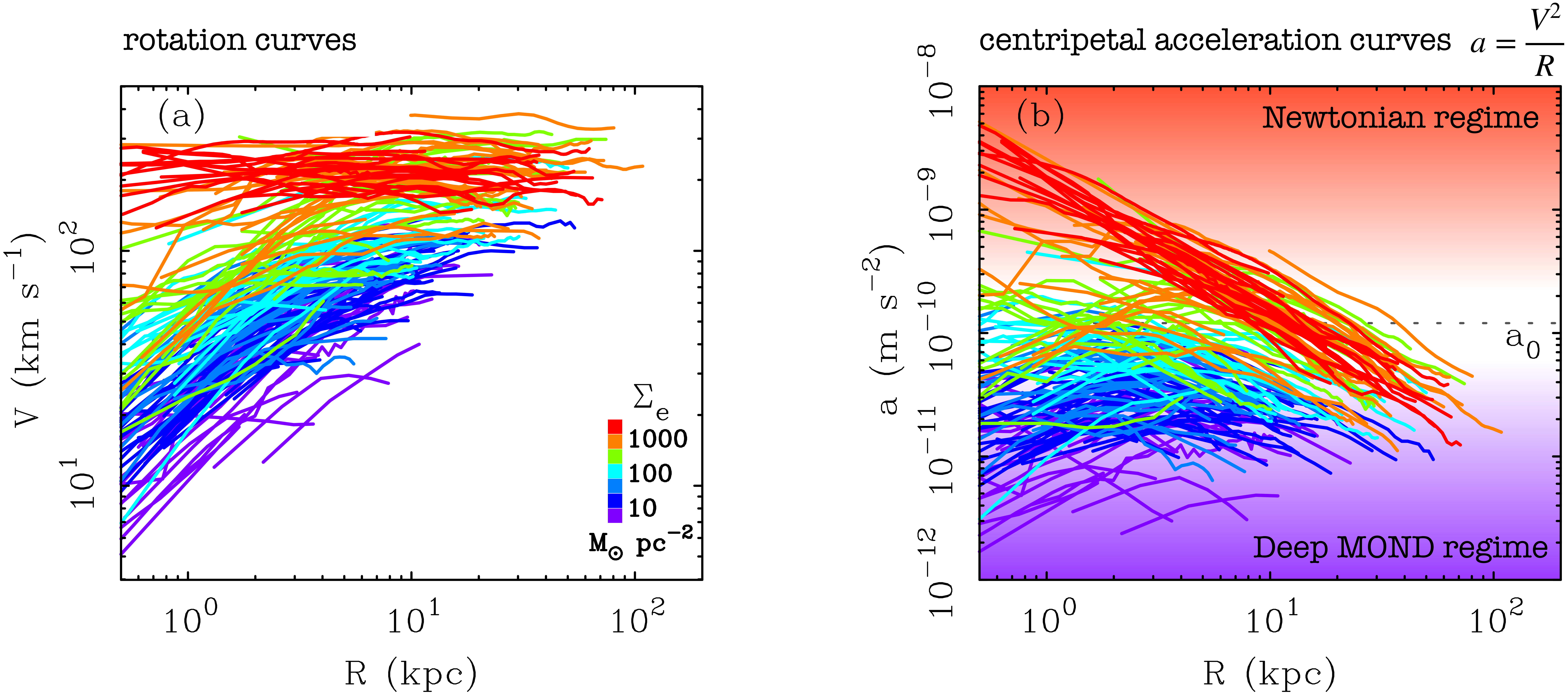
Regimes smoothly joined by $$\mu\left(\frac{a}{a_0}\right) a = g_N$$

$$\mu(x) \to 1 \text{ for } x \gg 1 \qquad \mu(x) \to x \text{ for } x \ll 1 \qquad x = \frac{a}{a_0}$$

Modified Poisson equation

$$\nabla \left[\mu\left(\frac{\nabla\Phi}{a_0}\right) \nabla\Phi\right] = 4\pi G\rho$$

Derived from aquadratic Lagrangian of Bekenstein & Milgrom (1984) to satisfy energy conservation.



rotation curves
(a)
V (km s^{-1})
10^1
10^2
R (kpc)
10^0
10^1
10^2
Σ_e
1000
100
10
$M_\odot$ pc^{-2}
centripetal acceleration curves $a = \frac{V^2}{R}$
(b)
Newtonian regime
a_0
Deep MOND regime
a (m s^{-2})
10^{-8}
10^{-9}
10^{-10}
10^{-11}
10^{-12}
R (kpc)
10^0
10^1
10^2

ApJ, 270, 381

Milgrom 1983

A major step in understanding ellipticals can be made if we can identify them, at least approximately, with idealized structures such as the FRCL spheres discussed above. I have also studied isotropic and nonisotropic isothermal spheres, in the modified dynamics, as such possible structures. I found that they have properties which very much resemble those of ellipticals and galactic bulges. I describe these in Milgrom (1983*c*).

VIII. PREDICTIONS

The main predictions concerning galaxies are as follows.

1. Velocity curves calculated with the modified dynamics on the basis of the observed mass in galaxies should agree with the observed curves. Elliptical and S0 galaxies may be the best for this purpose since (*a*) practically no uncertainty due to obscuration is involved and (*b*) there is not much uncertainty due to the possible presence of molecular hydrogen.

2. The relation between the asymptotic velocity (V_∞) and the mass of the galaxy (M) ($V_\infty^4 = MGa_0$) is an absolute one.

3. Analysis of the z-dynamics in disk galaxies using the modified dynamics should yield surface densities which agree with the observed ones. Accordingly, the same analysis using the conventional dynamics should yield a discrepancy which increases with radius in a predictable manner.

4. Effects of the modified dynamics are predicted to be particularly strong in dwarf elliptical galaxies (for review of properties see, e.g., Hodge 1971 and Zinn 1980). For example, those dwarfs believed to be bound to our Galaxy would have internal accelerations typically of order $a_{in} \sim a_0/30$. Their (modified) acceleration, g, in the field of the Galaxy is larger than the internal ones but still much smaller than a_0, $g = (8\ \text{kpc}/d)a_0$, based on a value of $V_\infty = 220\ \text{km s}^{-1}$ for the Galaxy, and where d is the distance from the dwarf galaxy to the center of the Milky Way ($d \sim 70$–220 kpc). Whichever way the external acceleration turns out to affect the internal dynamics (see the discussion at the end of § II, the section on small groups in Paper III, and Paper I), we predict that when velocity dispersion data is available for the dwarfs, a large mass discrepancy will result when the conventional dynamics is used to determine the masses. The dynamically determined mass is predicted to be larger by a factor of order 10 or more than that which can be accounted for by stars. In case the internal dynamics is determined by the external acceleration, we predict this factor to increase with d and be of order (d/8 kpc) (as long as $a_{in} \ll g$, $h_{50} = 1$).

Prediction 1 is a very general one. It is worthwhile listing some of its consequences as separate predictions, numbered 5–7 below (note that, in fact, even prediction 2 is already contained in prediction 1).

5. Measuring *local* M/L values in disk galaxies (assuming conventional dynamics) should give the following results: In regions of the galaxy where $V^2/r \gg a_0$ the local M/L values should show no indication of hidden mass. At a certain transition radius, local M/L should start to increase rapidly. The transition radius should occur where $V^2/r \approx a_0$. This test has the following advantages: (*a*) It does not require an absolute calibration of M/L as we are concerned only with variations of this quantity; (*b*) Effects of the modified dynamics manifest themselves more clearly in local mass determination than in the integrated masses; and (*c*) In many cases this test requires information on local behavior in the disk only while the spheroid can be neglected. This makes the determination of mass from velocity more certain.

6. Disk galaxies with low surface brightness provide particularly strong tests (a study of a sample of such galaxies is described by Strom 1982 and by Romanishin *et al.* 1982). As low surface brightness means small accelerations, the effects of the modification should be more noticeable in such galaxies. We predict, for example, that the proportionality factor in the $M \propto V_\infty^4$ relation for these galaxies is the same as for the high surface density galaxies. In contrast, if one wants to obtain a correlation $M \propto V_\infty^4$ in the conventional dynamics (with additional assumptions), one is led to the relation $M \propto \Sigma^{-1} V_\infty^4$ (see, for example, Aaronson, Huchra, and Mould 1979), where Σ is the average surface brightness. This implies that low surface density galaxies, of a given velocity, have a mass higher than predicted by the M-V relation derived for normal surface density galaxies.

We also predict that the lower the average surface density of a galaxy is, the smaller is the transition radius, defined in prediction 5, in units of the galaxy's scale length. In fact, if the average surface density is very small we may have a galaxy in which $V^2/r < a_0$ everywhere, and analysis with conventional dynamics should yield local M/L values starting to increase from very small radii.

7. As the study of model rotation curves shows, we predict a correlation between the value of the average surface density (or brightness) of a galaxy and the steepness with which the rotational velocity rises to its asymptotic value (as measured, for example, by the radius at which $V = V_\infty/2$ in units of the scale length of the disk). Small surface densities imply slow rise of V.

IX. DISCUSSION

The main results of this paper can be summarized by the statement that the modified dynamics eliminates the need to assume hidden mass in galaxies. The effects in galaxies which I have considered, and which are commonly attributed to such hidden mass, are readily explained by the modification. More specifically:

MOND predictions

- The Tully-Fisher Relation
  - Slope = 4
  - Normalization = $1/(a_0 G)$
  - Fundamentally a relation between Disk Mass and V_{flat}
  - No Dependence on Surface Brightness
- Dependence of conventional M/L on radius and surface brightness
- Rotation Curve Shapes
- Surface Density ~ Surface Brightness
- Detailed Rotation Curve Fits
- Stellar Population Mass-to-Light Ratios

"Disk Galaxies with low surface brightness provide particularly strong tests"

In MOND limit of low acceleration

$$a = \sqrt{g_N a_0}$$

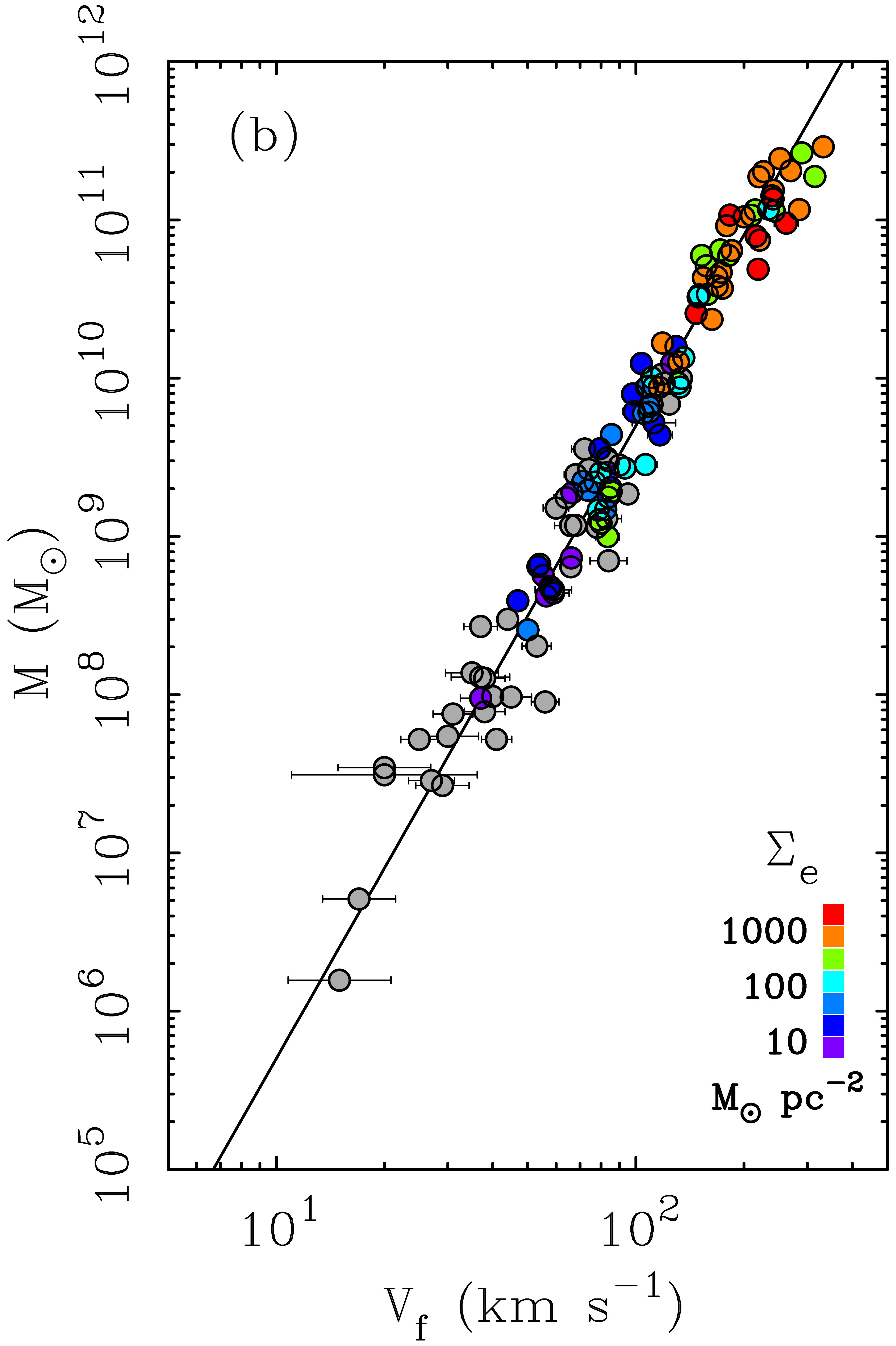
$$\frac{V^2}{R} = \sqrt{\frac{GM}{R^2} a_0}$$

Simple (point mass) example: note that radial dependence cancels; squaring both sides gives TF relation:

$$V^4 = a_0 GM$$

In general (not a point mass): $M_b = AV_f^4$ with $A = \frac{\zeta}{a_0 G}$

where $\zeta \approx 0.8$ for disk galaxies of finite thickness
(recall that flattened mass distributions rotate faster than their spherical equivalent)



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Why?
Physics of the BTFR scaling relation

dark matter

halos: $V_{DM} = \sqrt{\frac{GM_{DM}}{r}}$

baryons: $V_b = \sqrt{\frac{GM_b}{r}}$

$$V(r) = \sqrt{\frac{G[M_b(r) + M_{DM}(r)]}{r}}$$

TF Slope should be around 3

$$M_b \propto M_{200} \propto V_{200}^3$$

Should depend on baryon distribution, unless all disks are submaximal

Should work as long as object not tidally disrupted

MOND

$$M_{tot} = M_b = \frac{V^4}{a_0 G}$$

an absolute consequence of the force law for a << a_0:

$$g_N = \mu\left(\frac{g}{a_0}\right) g$$

Newtonian regime:

$\mu \rightarrow 1$ for $g \gg a_0$ so $g = g_N$

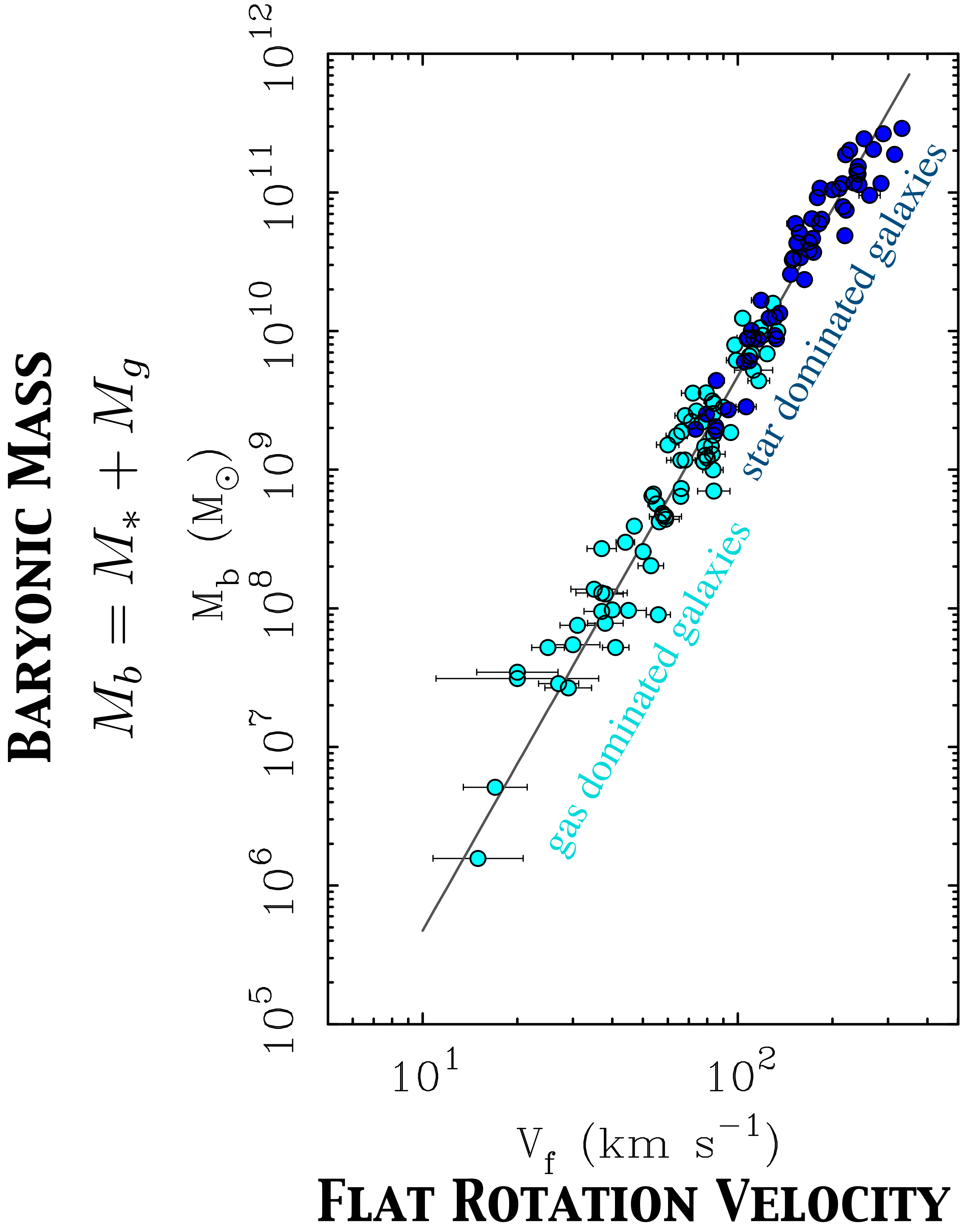
MOND regime:

$\mu \rightarrow g/a_0$ for $g \ll a_0$ so $g = \sqrt{g_N a_0}$

Should only work for objects in MOND regime

Baryonic Tully-Fisher relation

The amplitude of the rotation speed correlates with mass

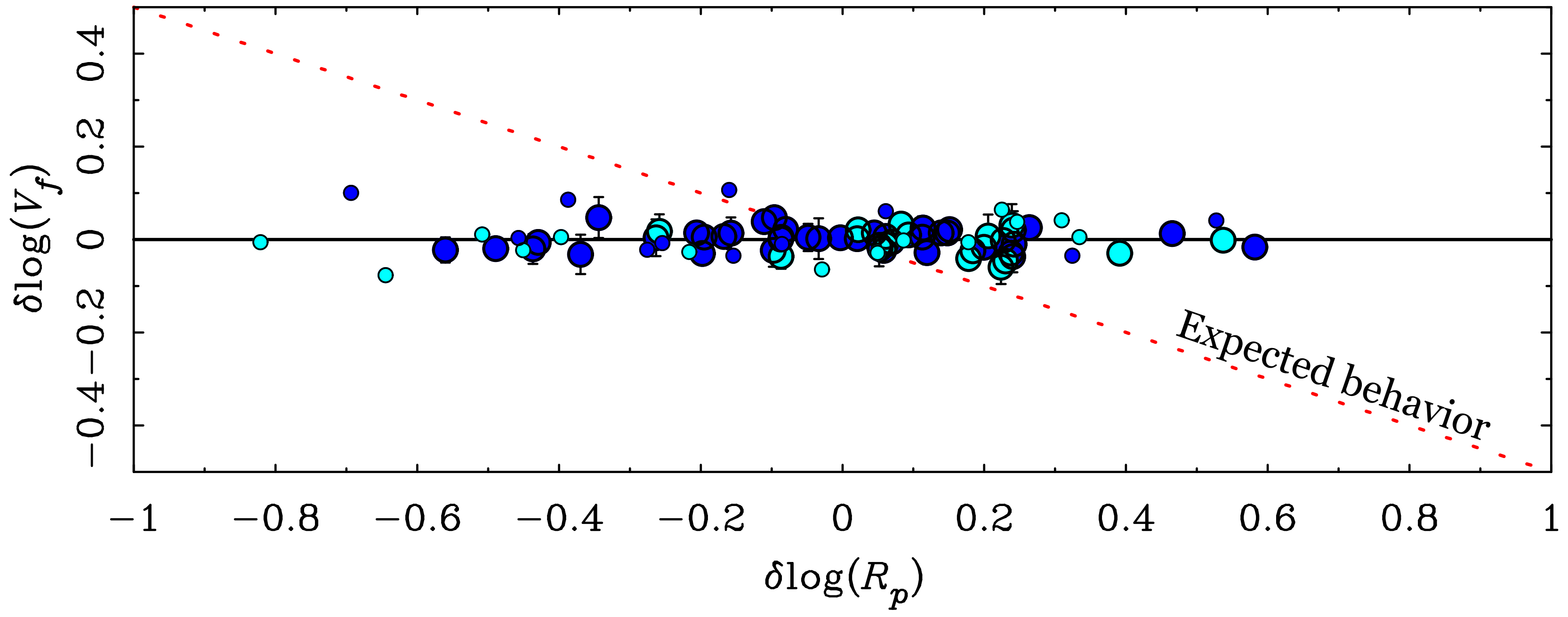


Keplerian prediction: $V = \sqrt{\frac{G[M_b(r) + M_{DM}(r)]}{r}}$

Observed: $V = \sqrt[4]{a_0 G M_b}$

depends only on mass - no residuals with galaxy size!

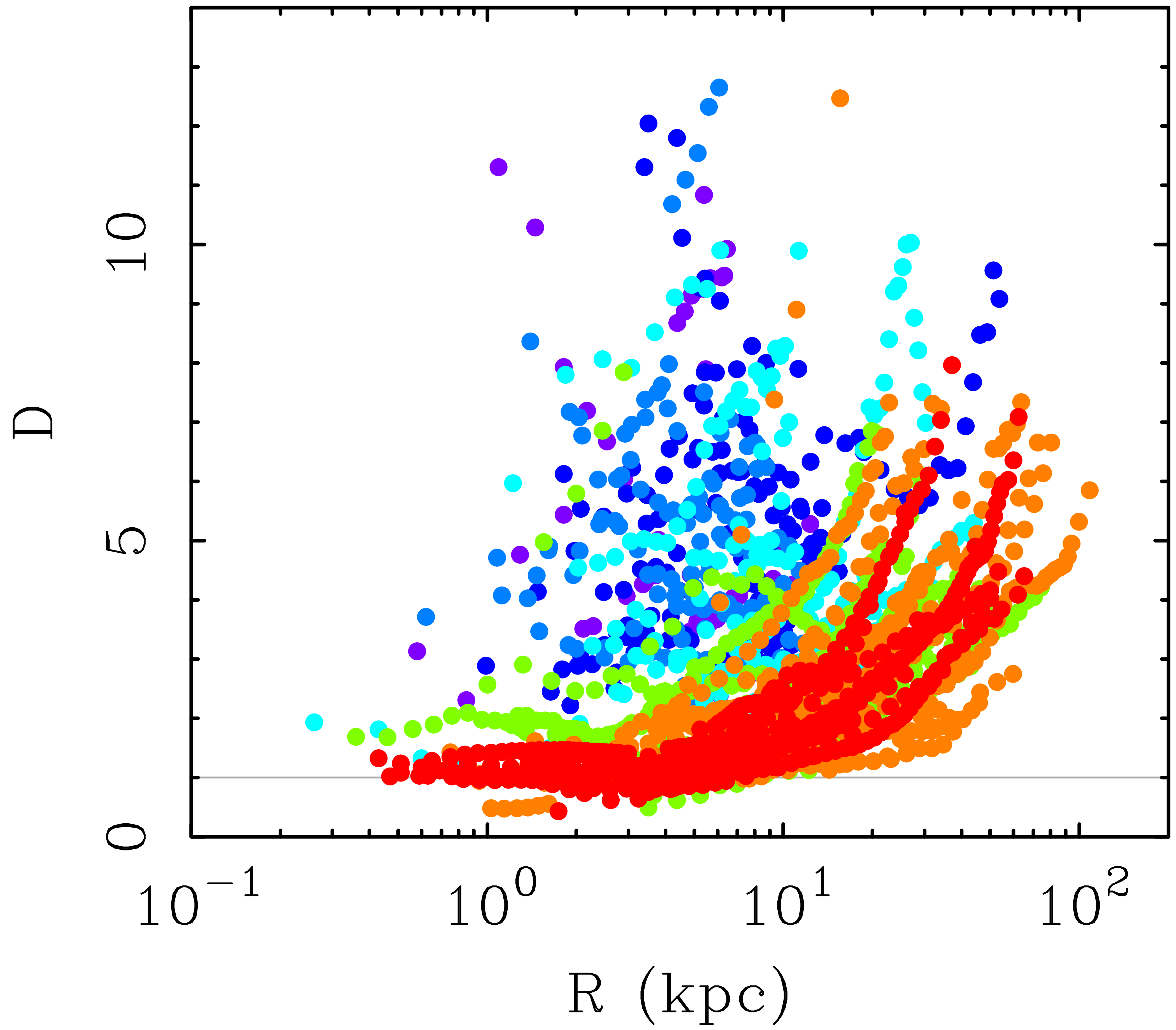
Residuals



"This result surprised the bejeepers out of us, too"
McGaugh & de Blok (astro-ph/9801102)

$$D = \frac{g_{obs}}{g_{bar}} = \frac{M_{tot}}{M_{bar}} \approx \frac{M}{L}$$

MOND predictions



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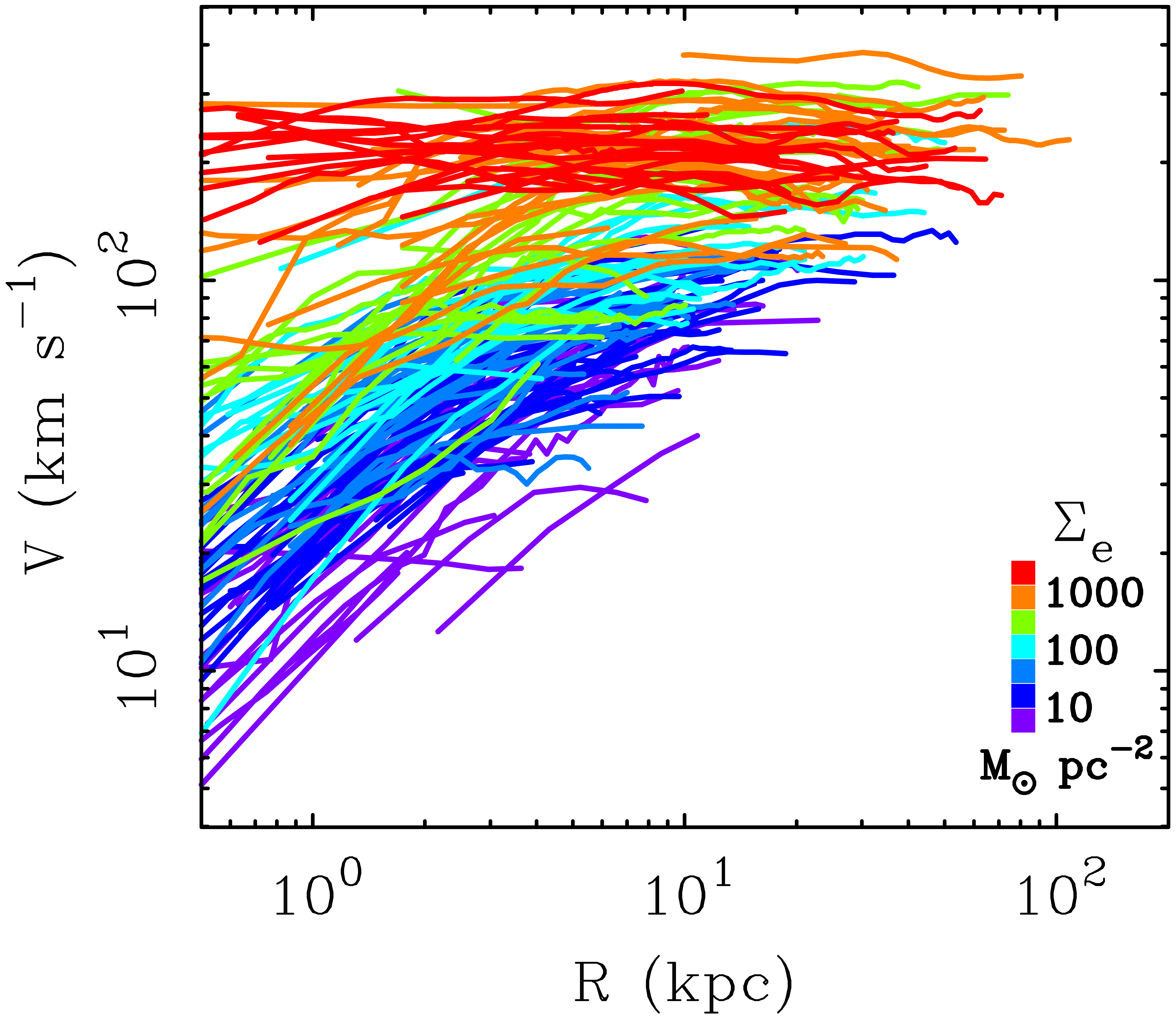

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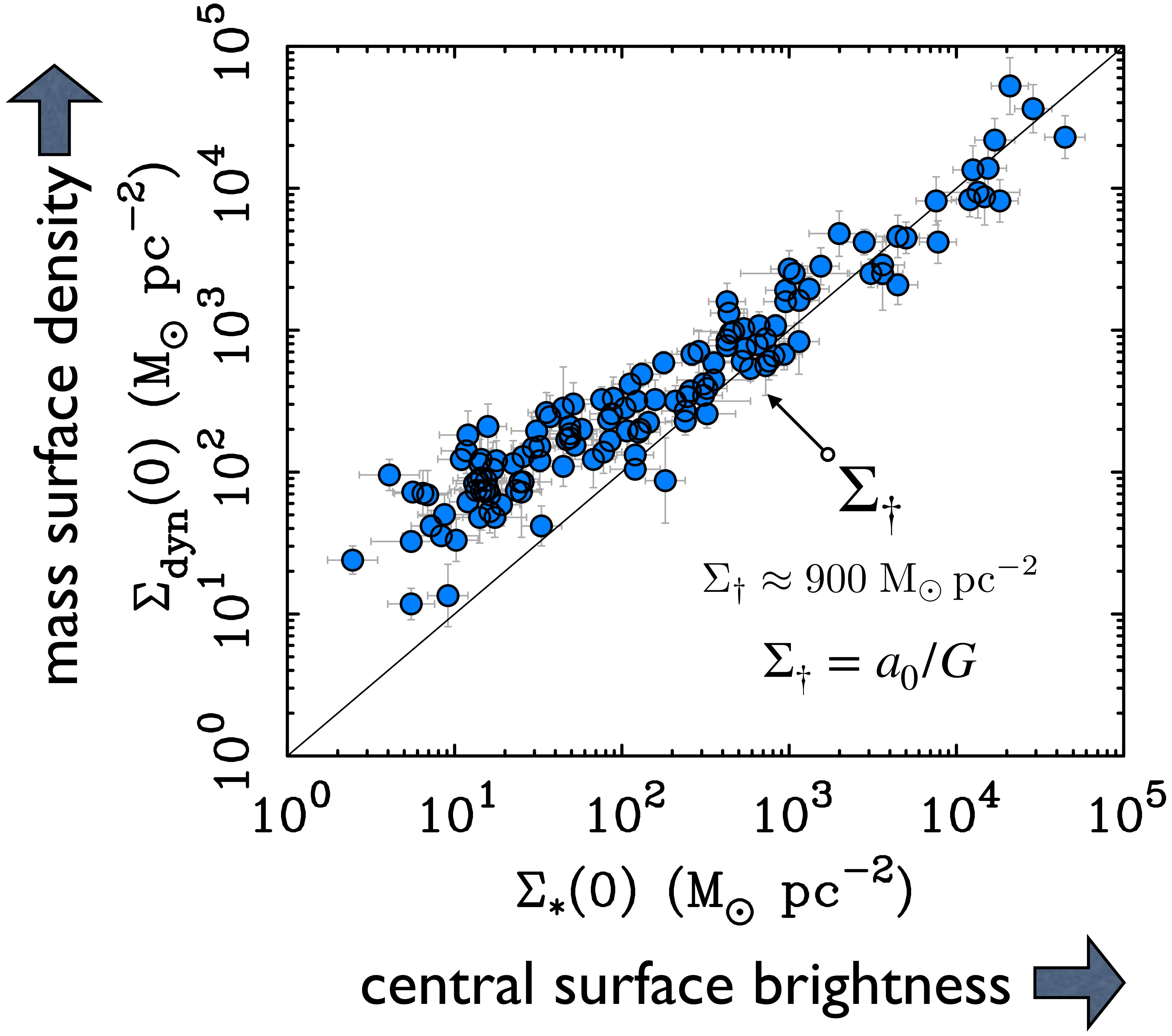
MOND predictions

$$\Sigma_{\rm dyn} \sim \Sigma_* \text{ for } \Sigma \gg \Sigma_\dagger$$

$$\Sigma_{\rm dyn} \sim \Sigma_*^{1/2} \text{ for } \Sigma \ll \Sigma_\dagger$$



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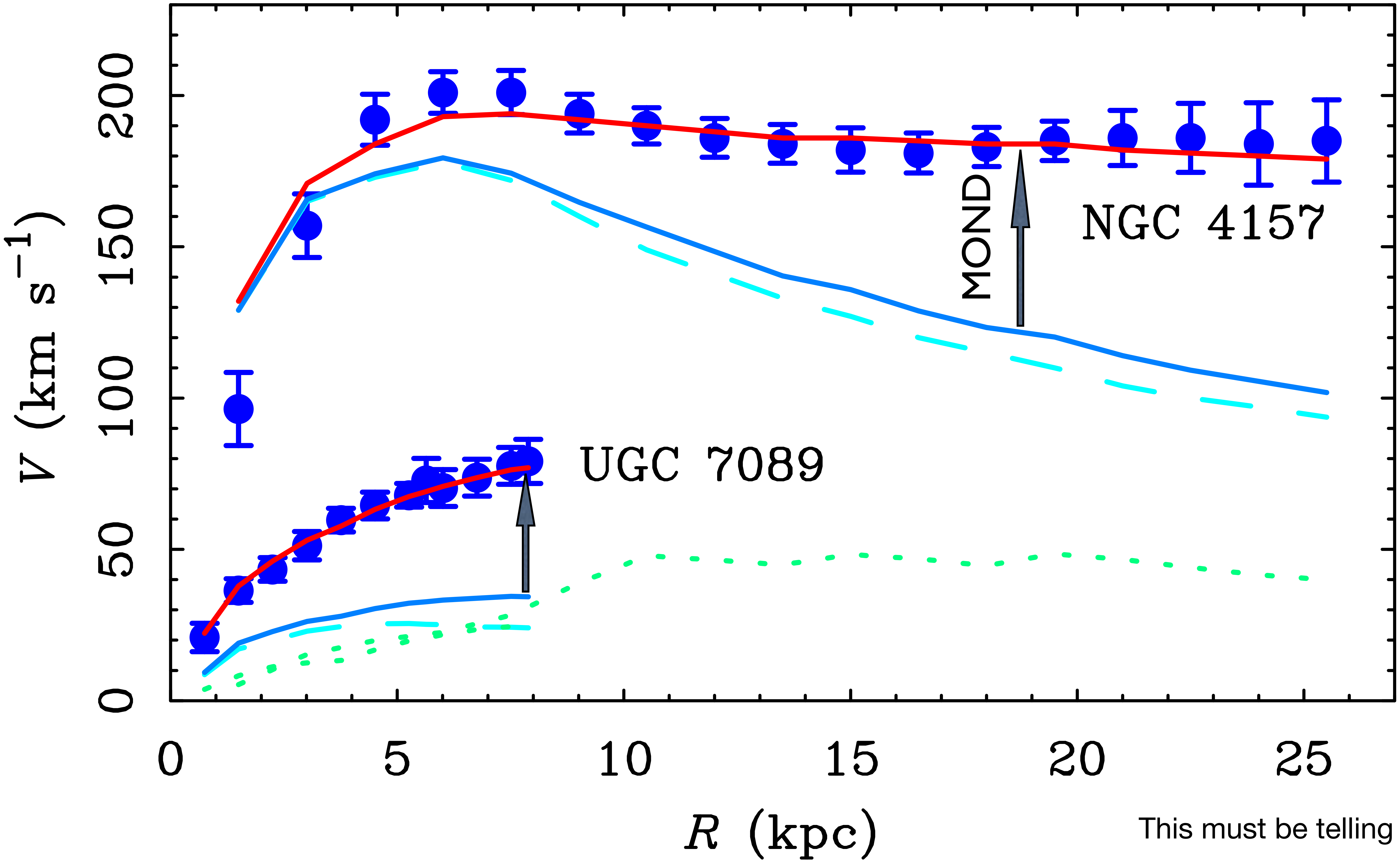
bend depends on shape of the interpolation function (arxiv:1607.05103)

MOND is a surprisingly effective algorithm for mapping what you see to what you get



This must be telling us something