

# DARK MATTER

ASTR 333/433  
SPRING 2026  
TR 11:30AM-12:45PM  
SEARS 552

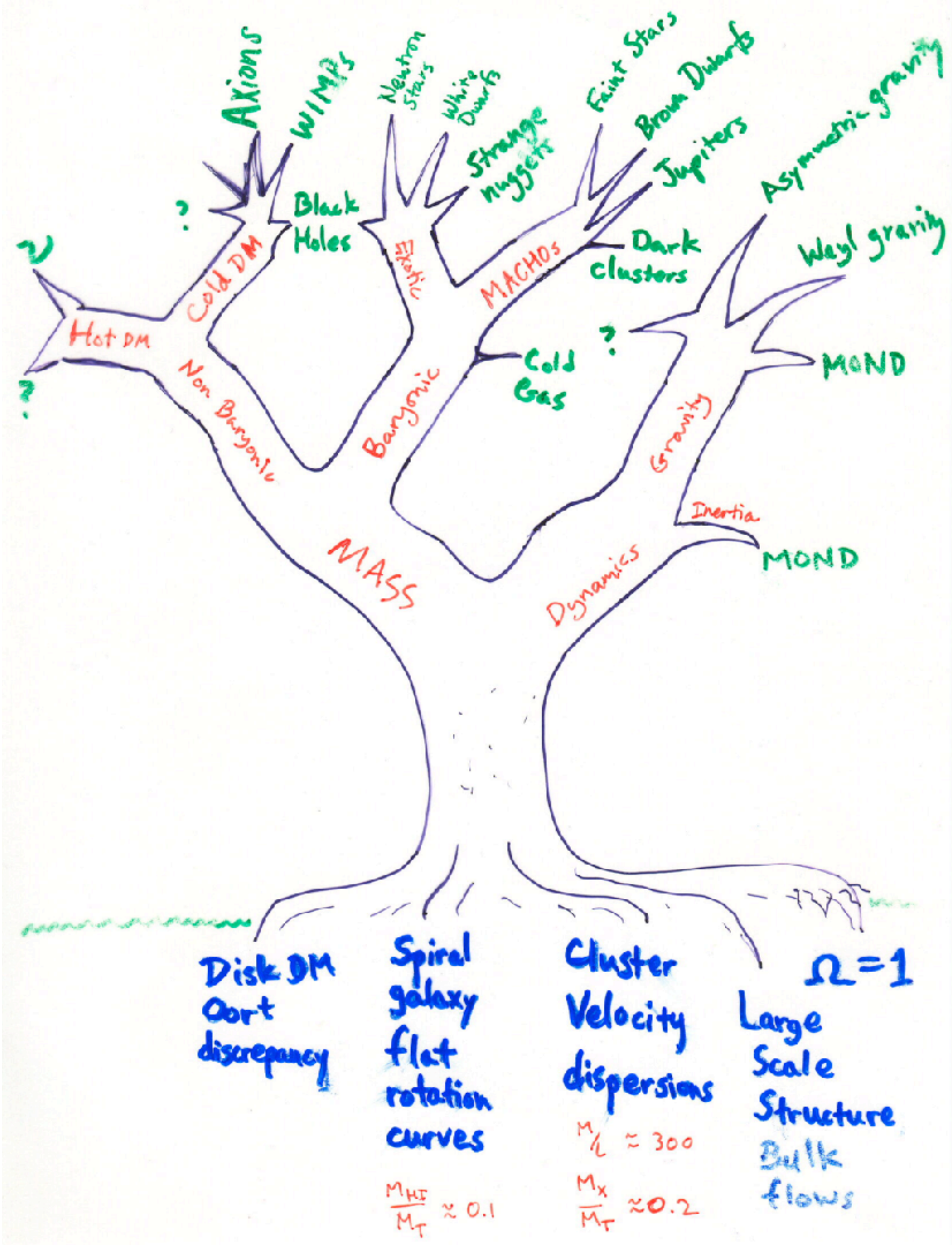
<http://astroweb.case.edu/ssm/ASTR333/>

PROF. STACY MCGAUGH  
SEARS 558  
368-1808

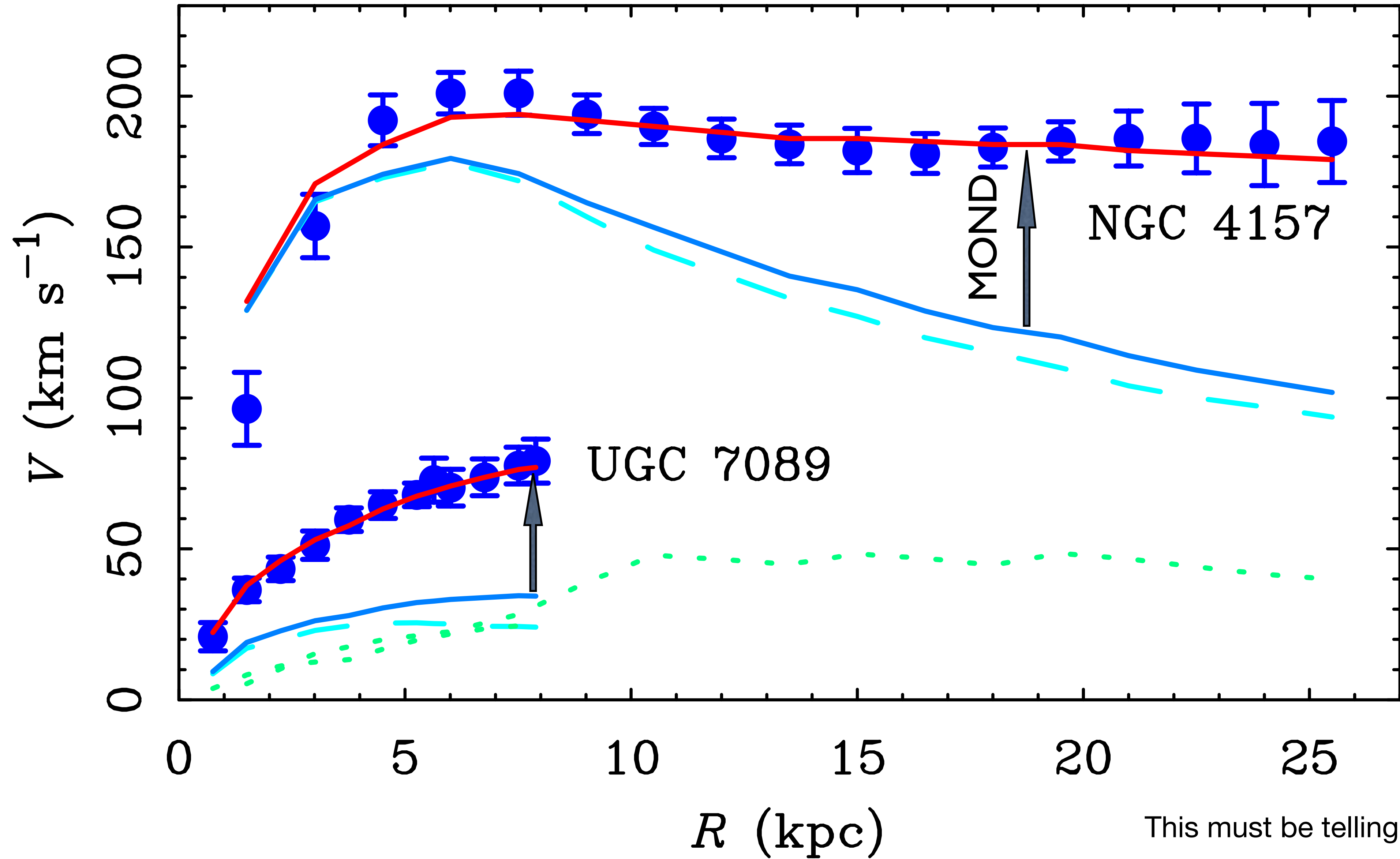
[stacy.mcgaugh@case.edu](mailto:stacy.mcgaugh@case.edu)

Homework Due next time

Final Thu 4/30 12 - 3 PM

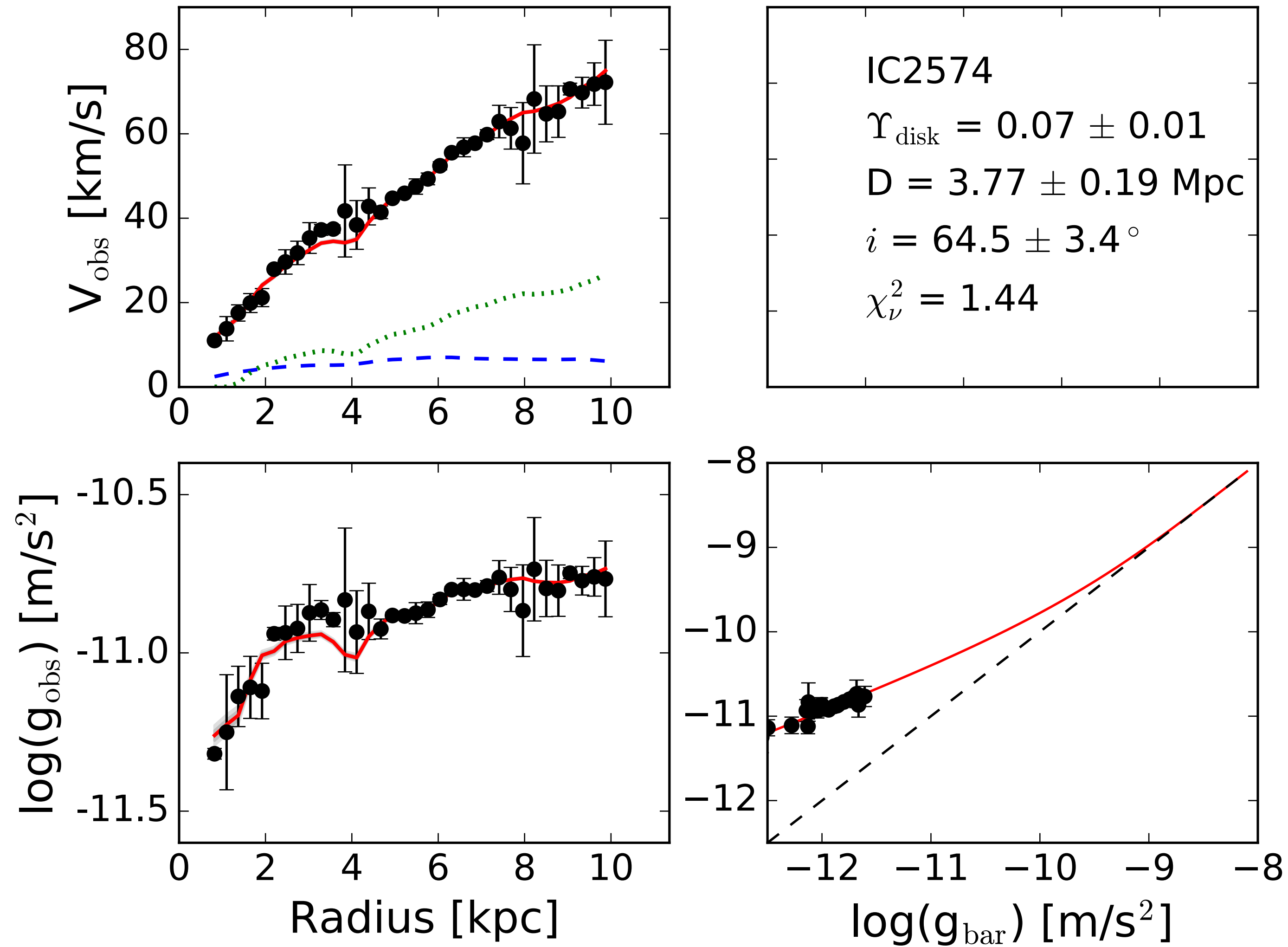


MOND is a surprisingly effective algorithm for mapping what you see to what you get



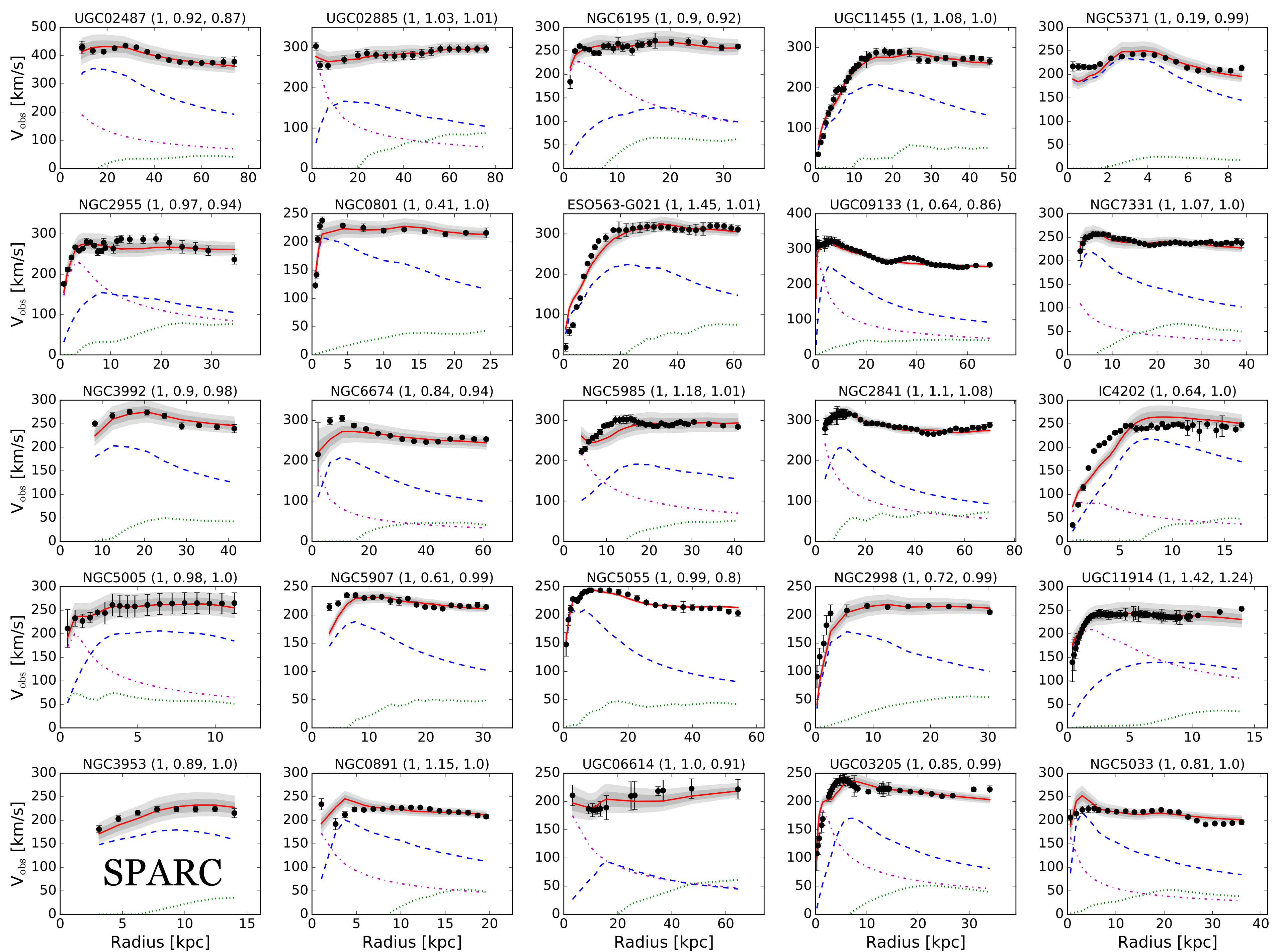
This must be telling us something

MOND fits are equivalent to choosing  $M^*/L$  to place galaxies on the RAR (Li et al. 2018)

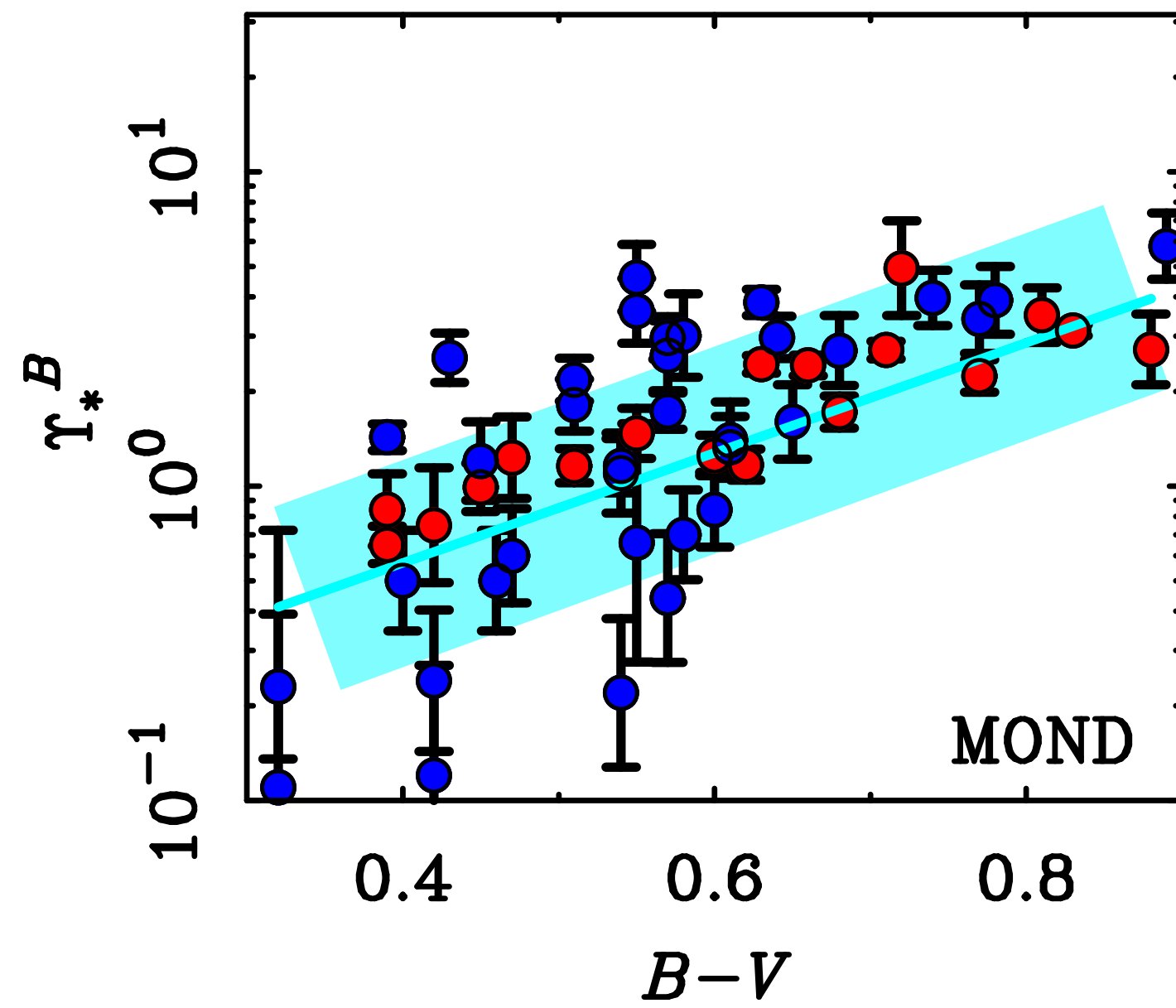


Distance and inclination are nuisance parameters in Bayesian fits.

Low surface density galaxies reside entirely in the low acceleration regime.



# MOND predictions



- The Tully-Fisher Relation

✓ Slope = 4

✓ Normalization =  $1/(a_0 G)$

✓ Fundamentally a relation between Disk Mass and  $V_{\text{flat}}$

✓ No Dependence on Surface Brightness

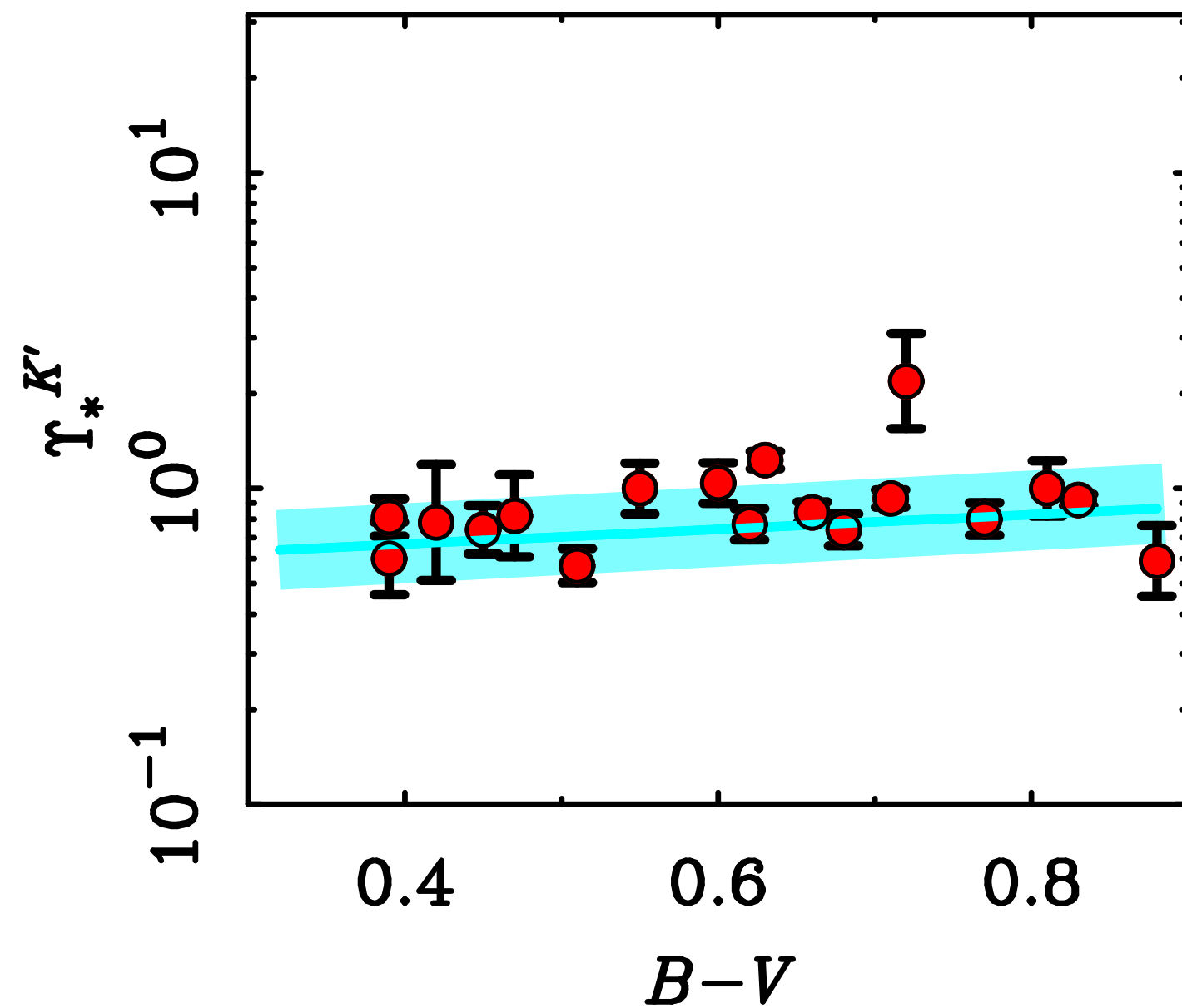
✓ Dependence of conventional M/L on radius and surface brightness

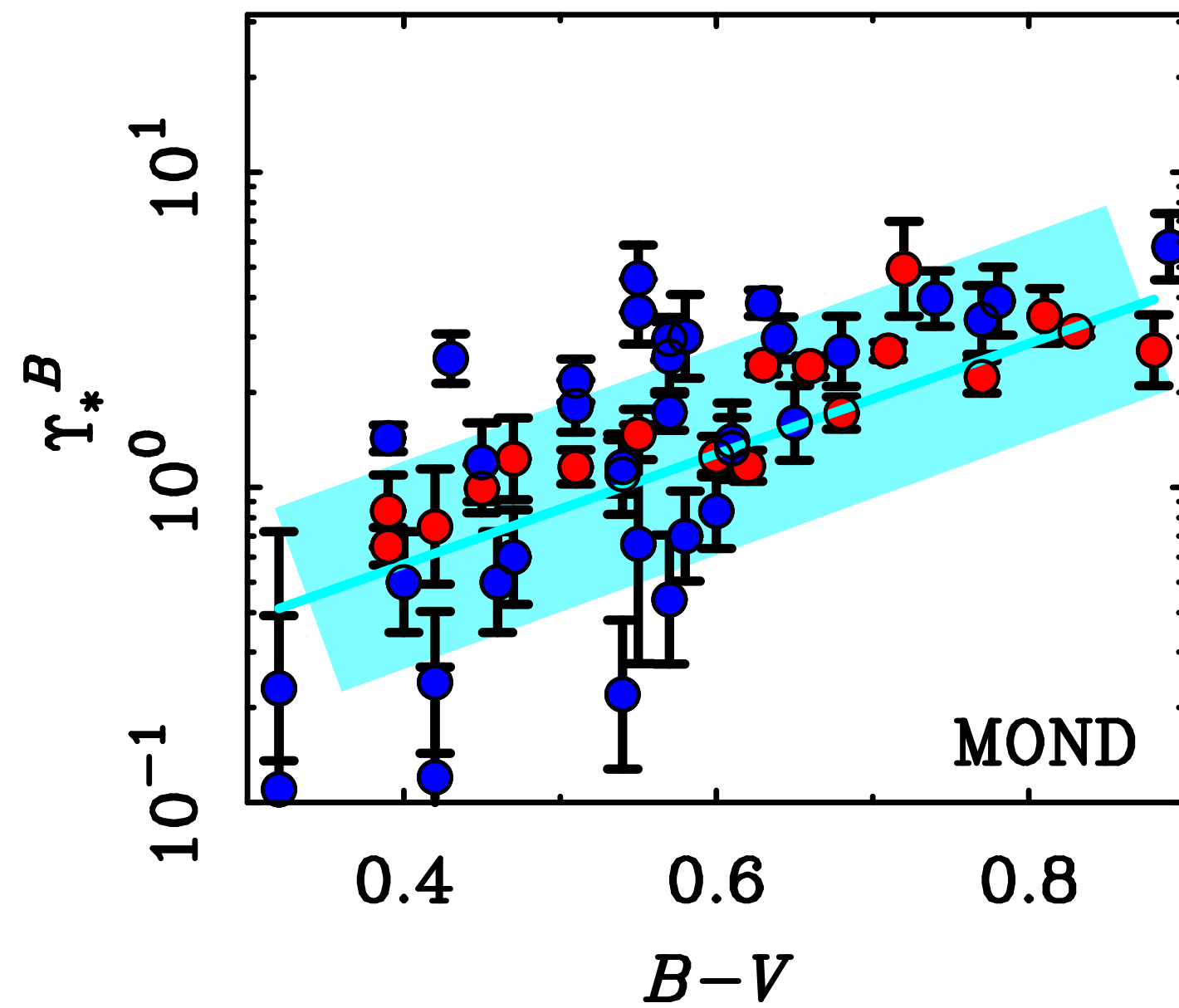
✓ Rotation Curve Shapes

✓ Surface Density  $\sim$  Surface Brightness

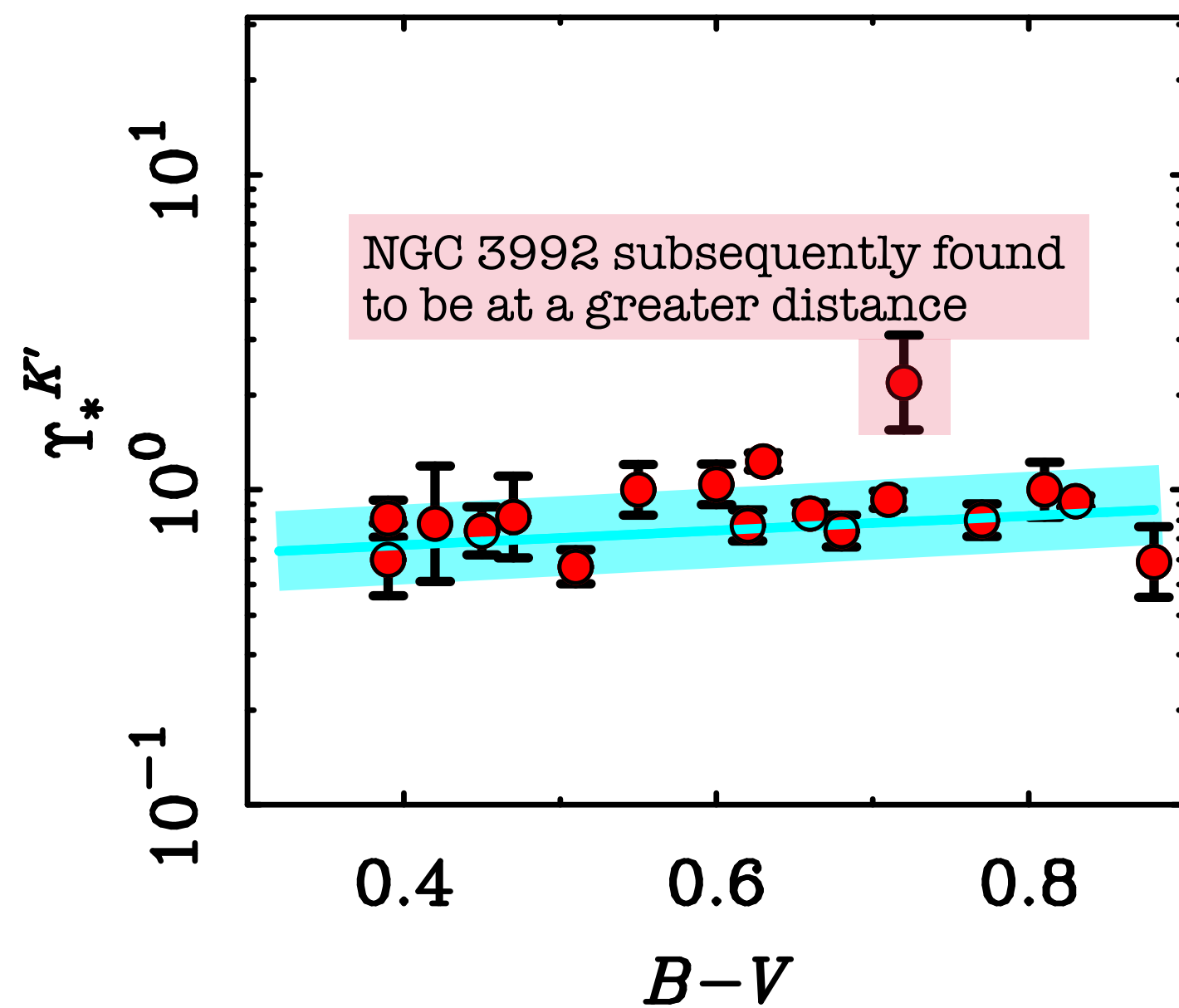
✓ Detailed Rotation Curve Fits

✓ Stellar Population Mass-to-Light Ratios





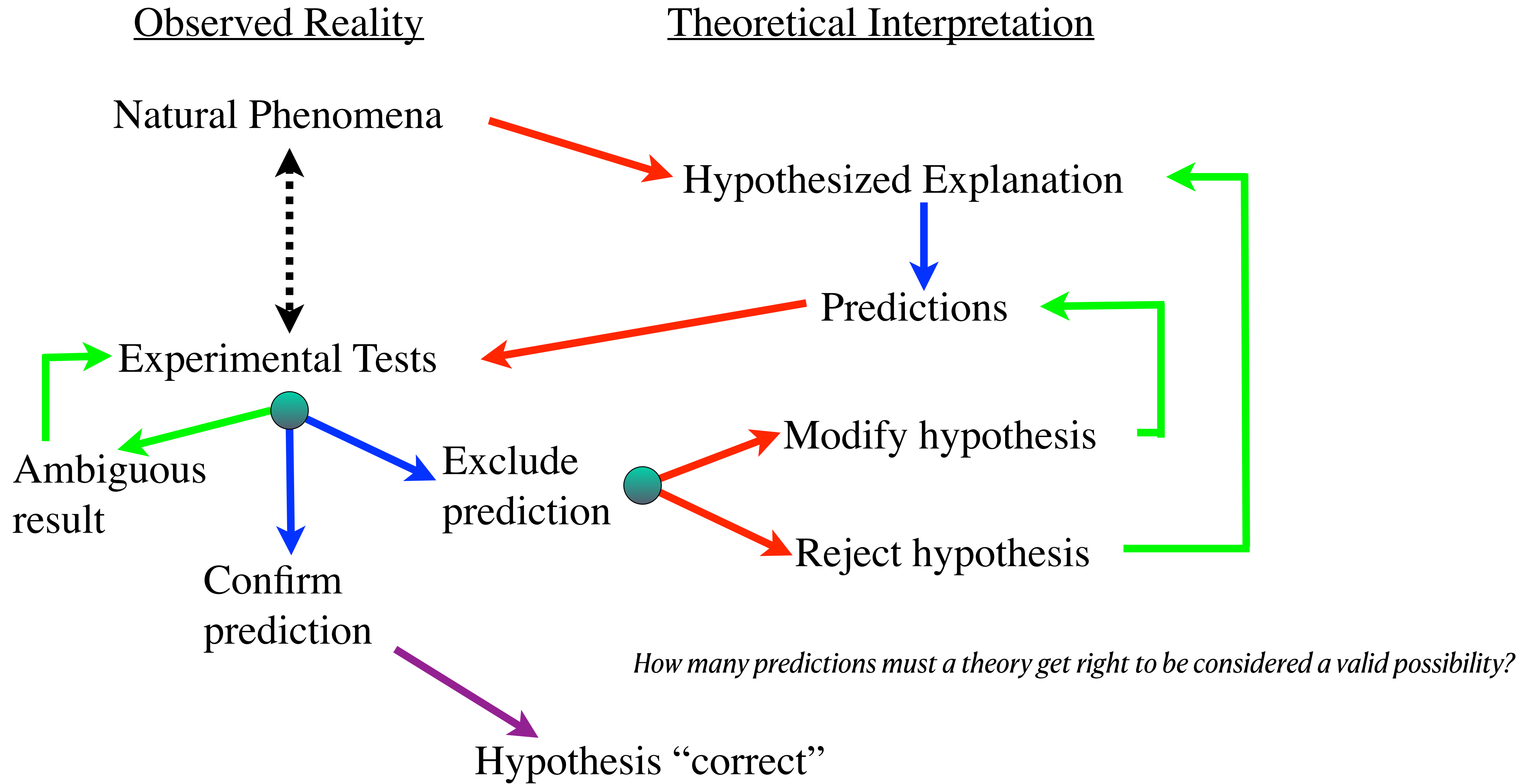
Best-fit mass-to-light ratio in good agreement with stellar population models in amplitude, color dependence, and scatter.



## MOND predictions

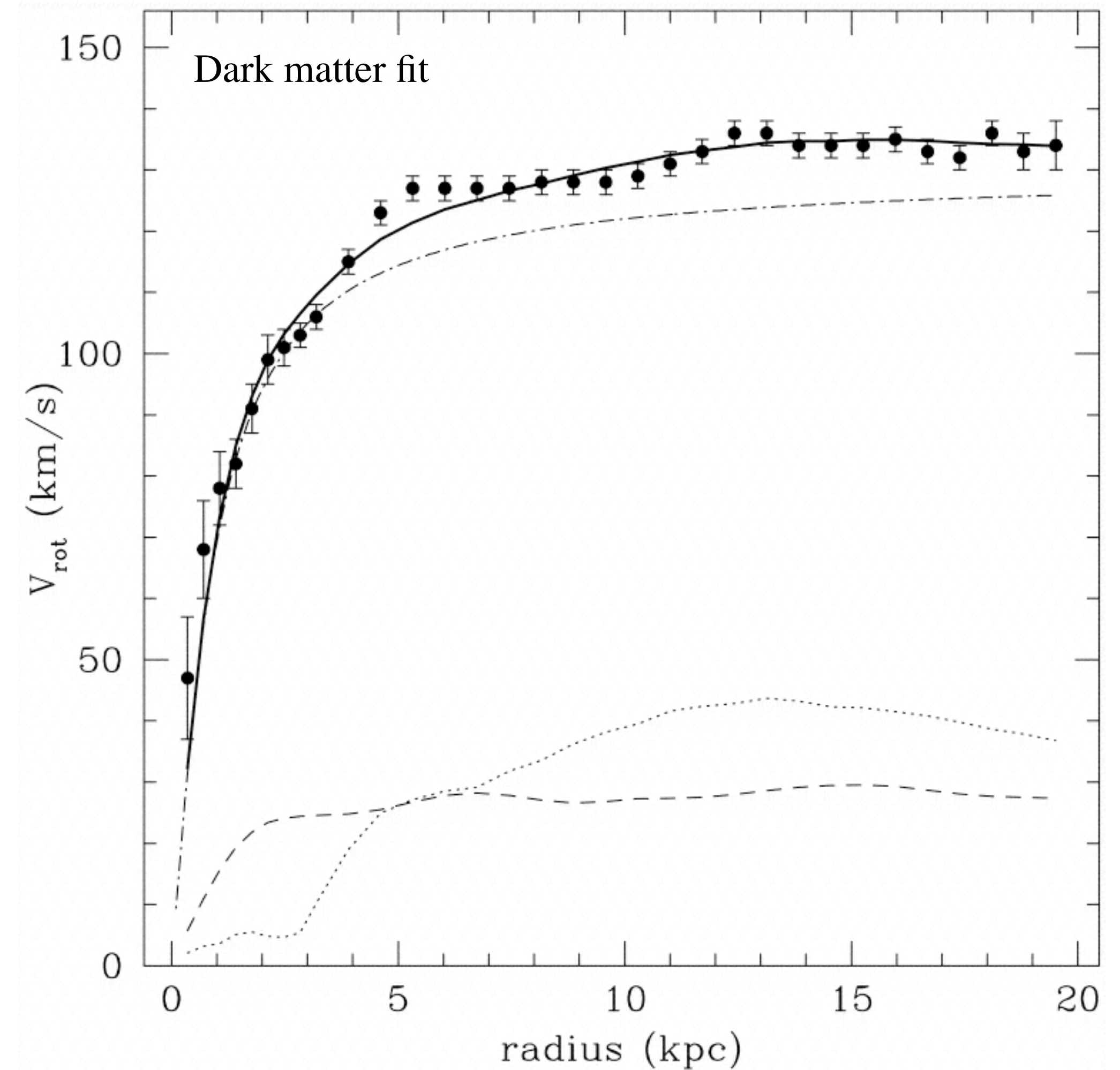
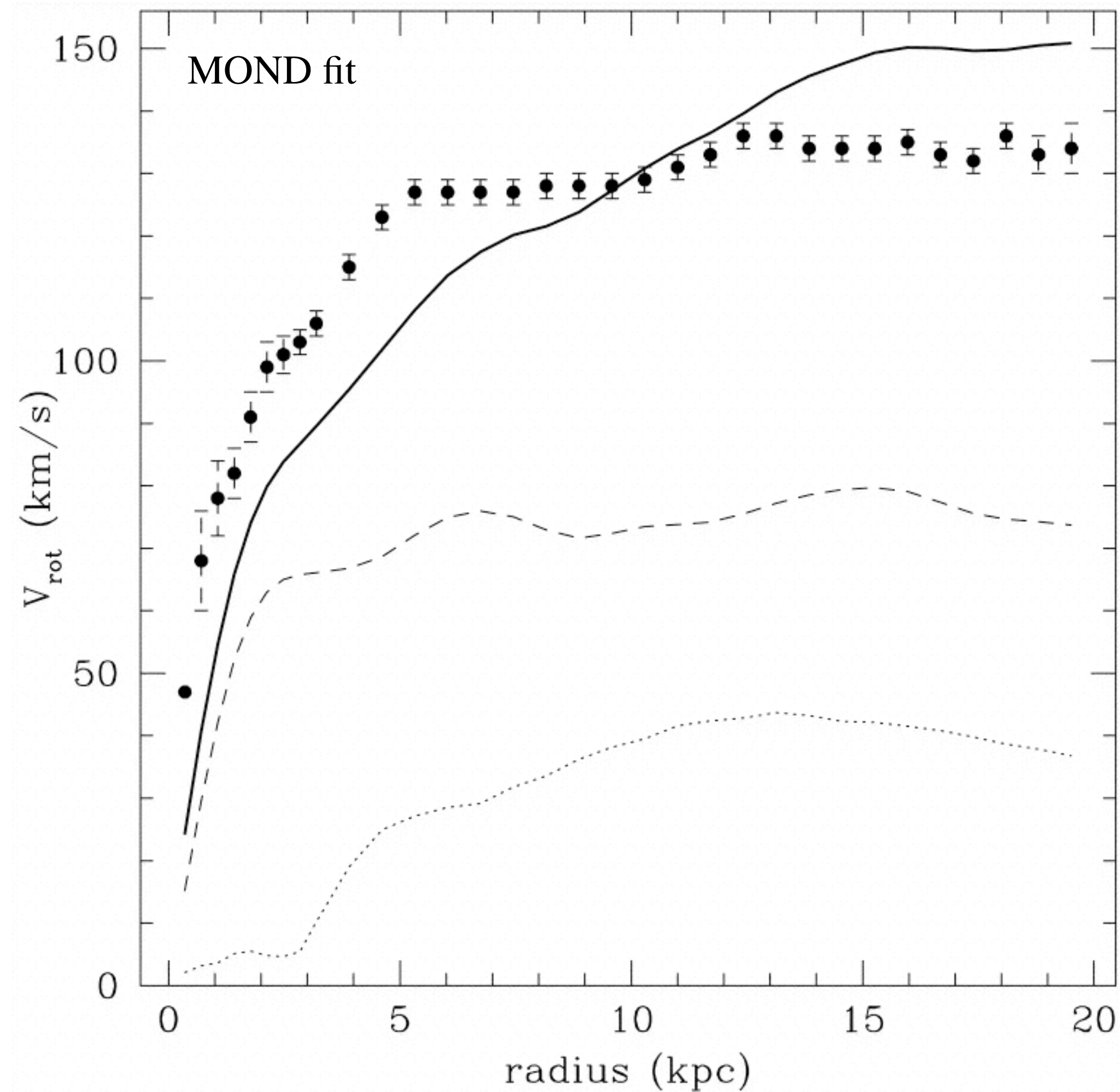
- The Tully-Fisher Relation
  - ✓ Slope = 4
  - ✓ Normalization =  $1/(a_0 G)$
  - ✓ Fundamentally a relation between Disk Mass and  $V_{\text{flat}}$
  - ✓ No Dependence on Surface Brightness
  
- ✓ Dependence of conventional M/L on radius and surface brightness
  
- ✓ Rotation Curve Shapes
  
- ✓ Surface Density  $\sim$  Surface Brightness
  
- ✓ Detailed Rotation Curve Fits
  
- ✓ Stellar Population Mass-to-Light Ratios

# Hypothesis Testing



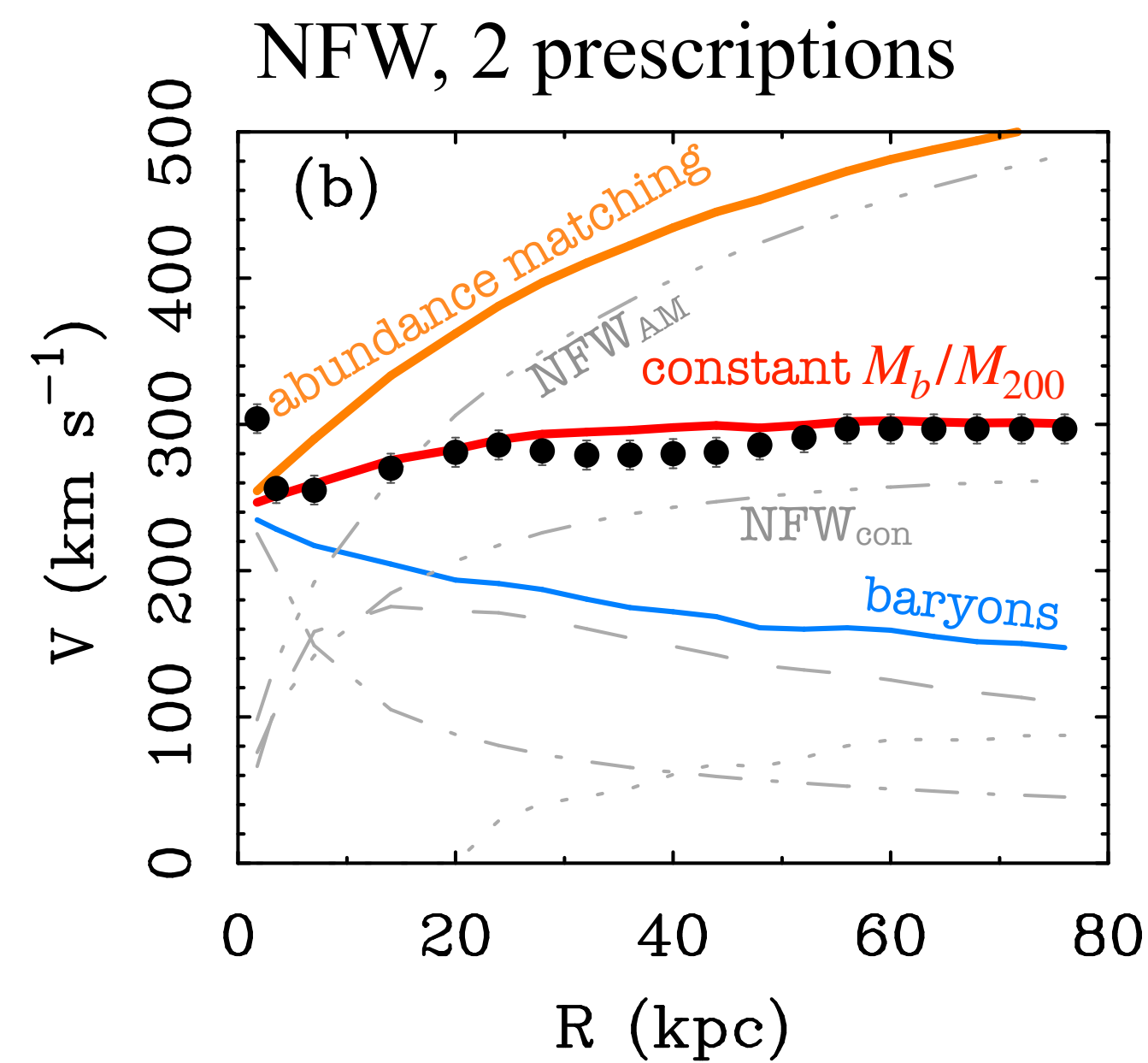
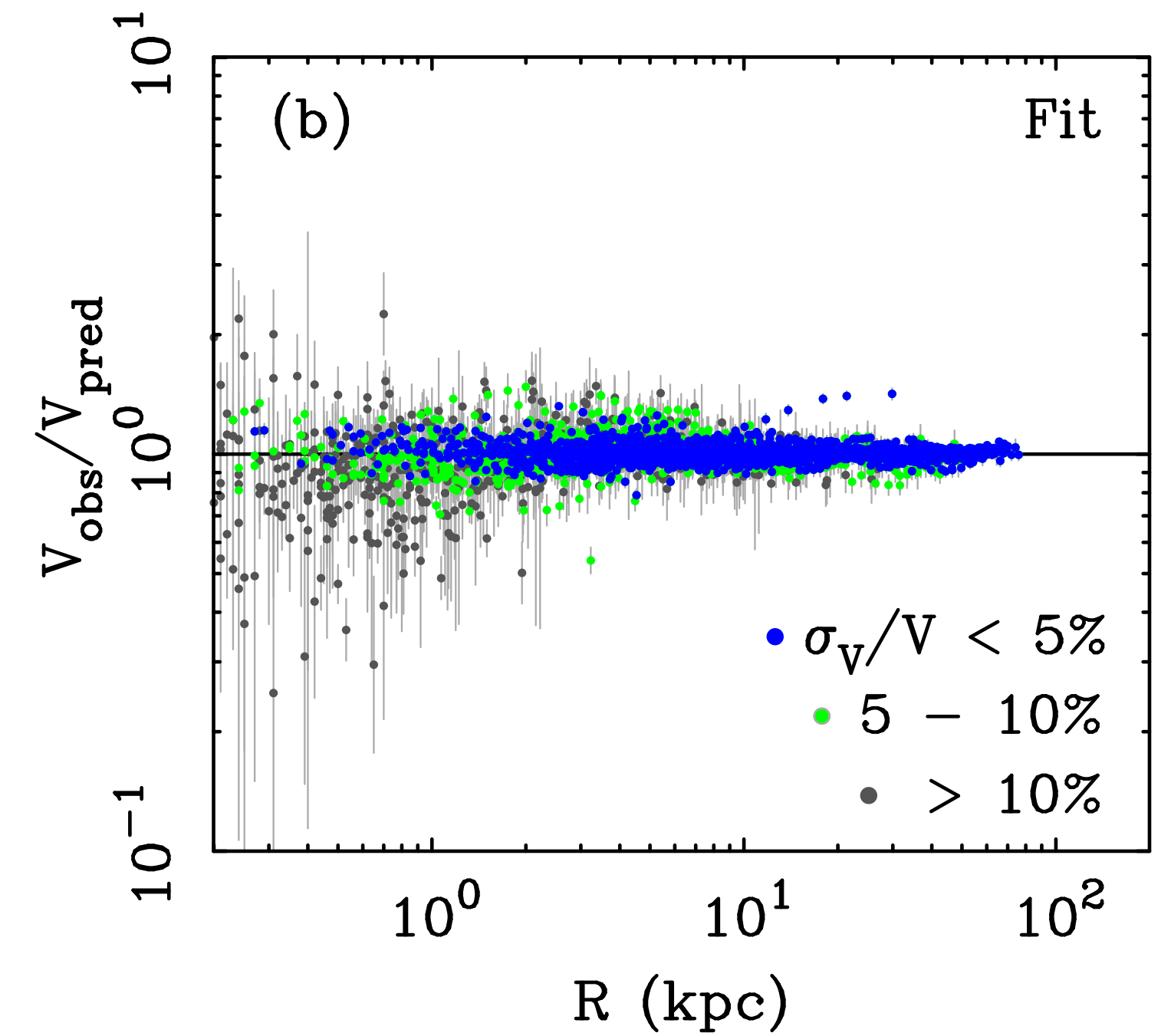
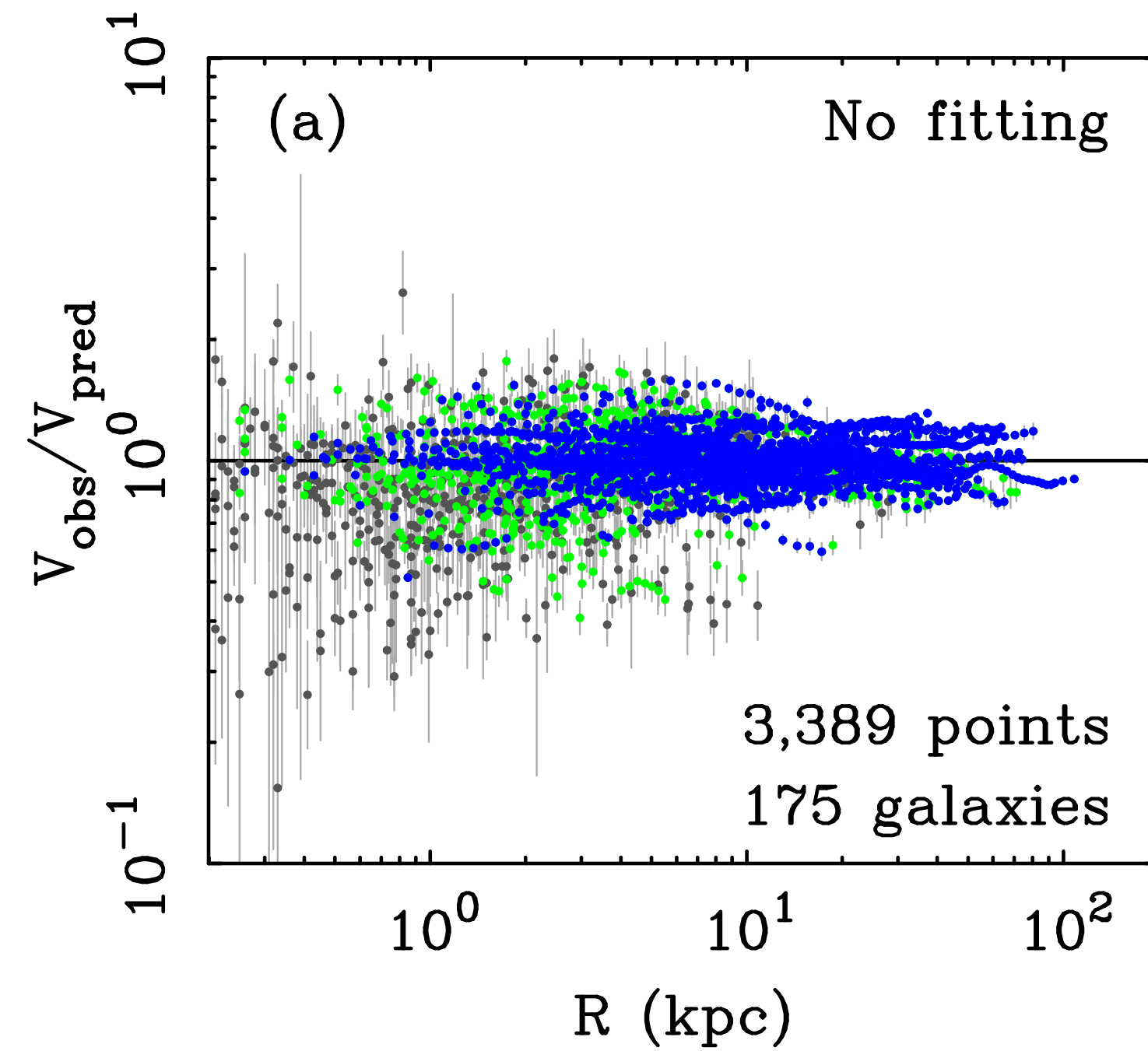
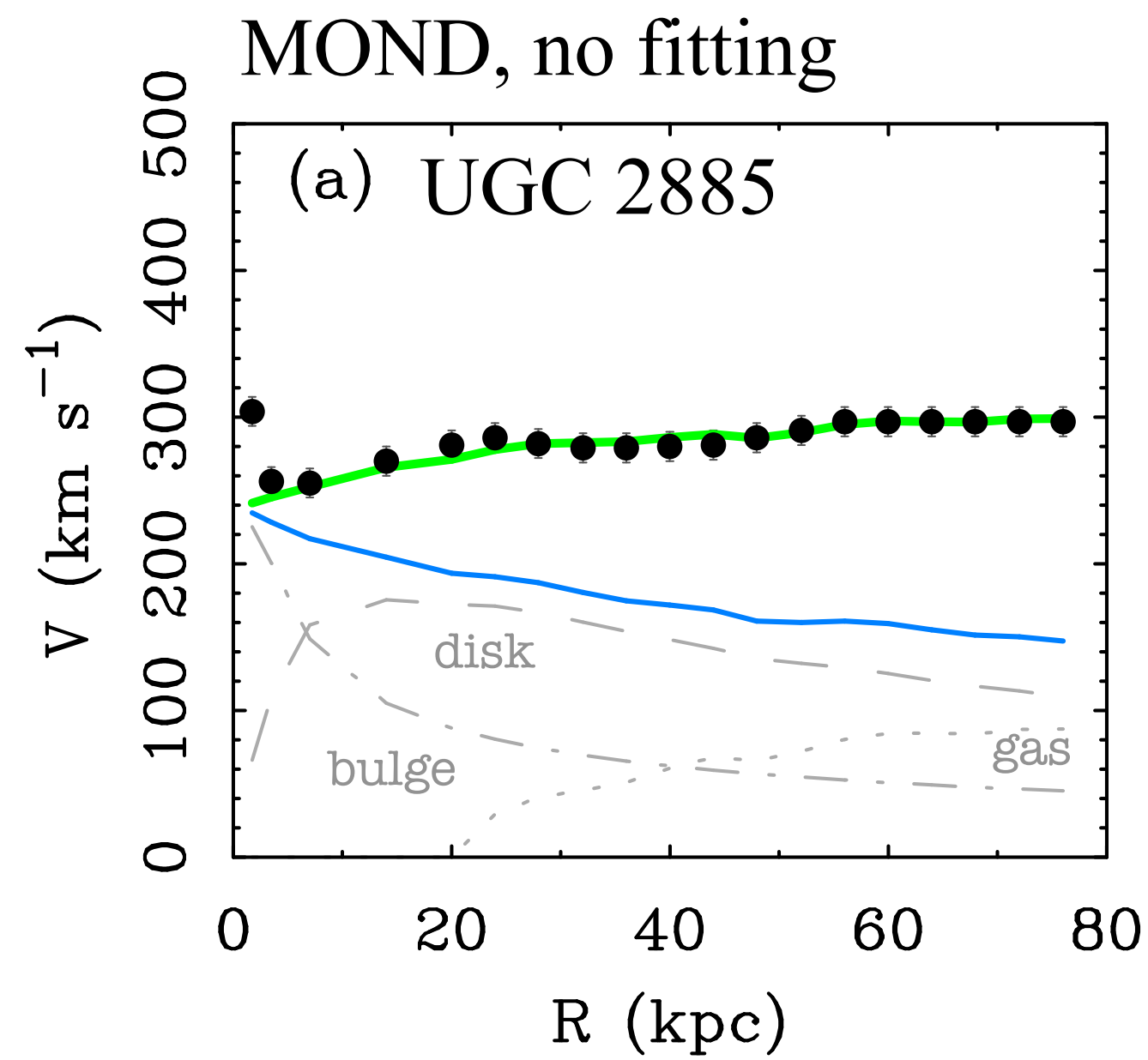
# Misconceptions Abound

“MOND was designed to fit rotation curves, so it is guaranteed to fit.”

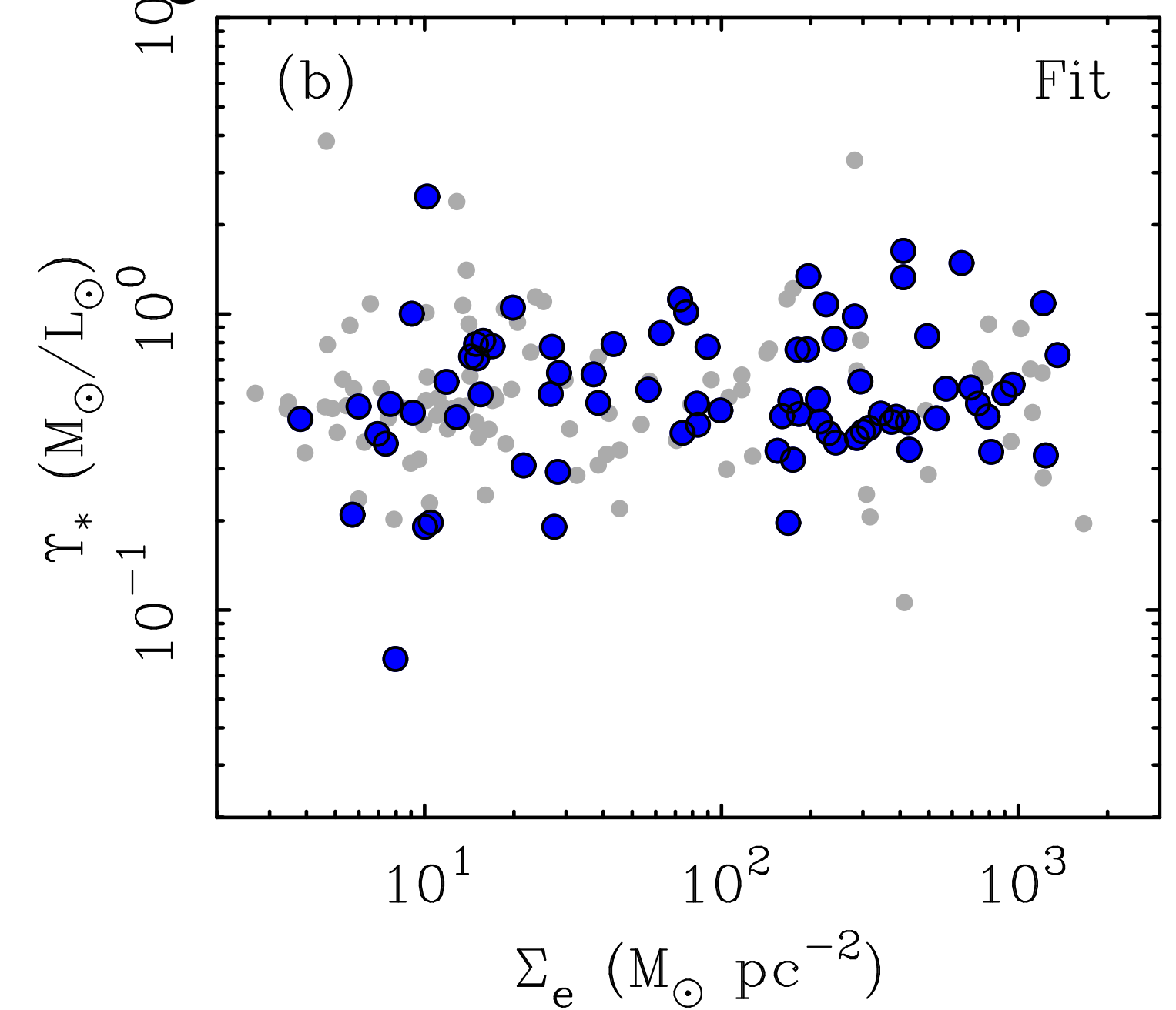
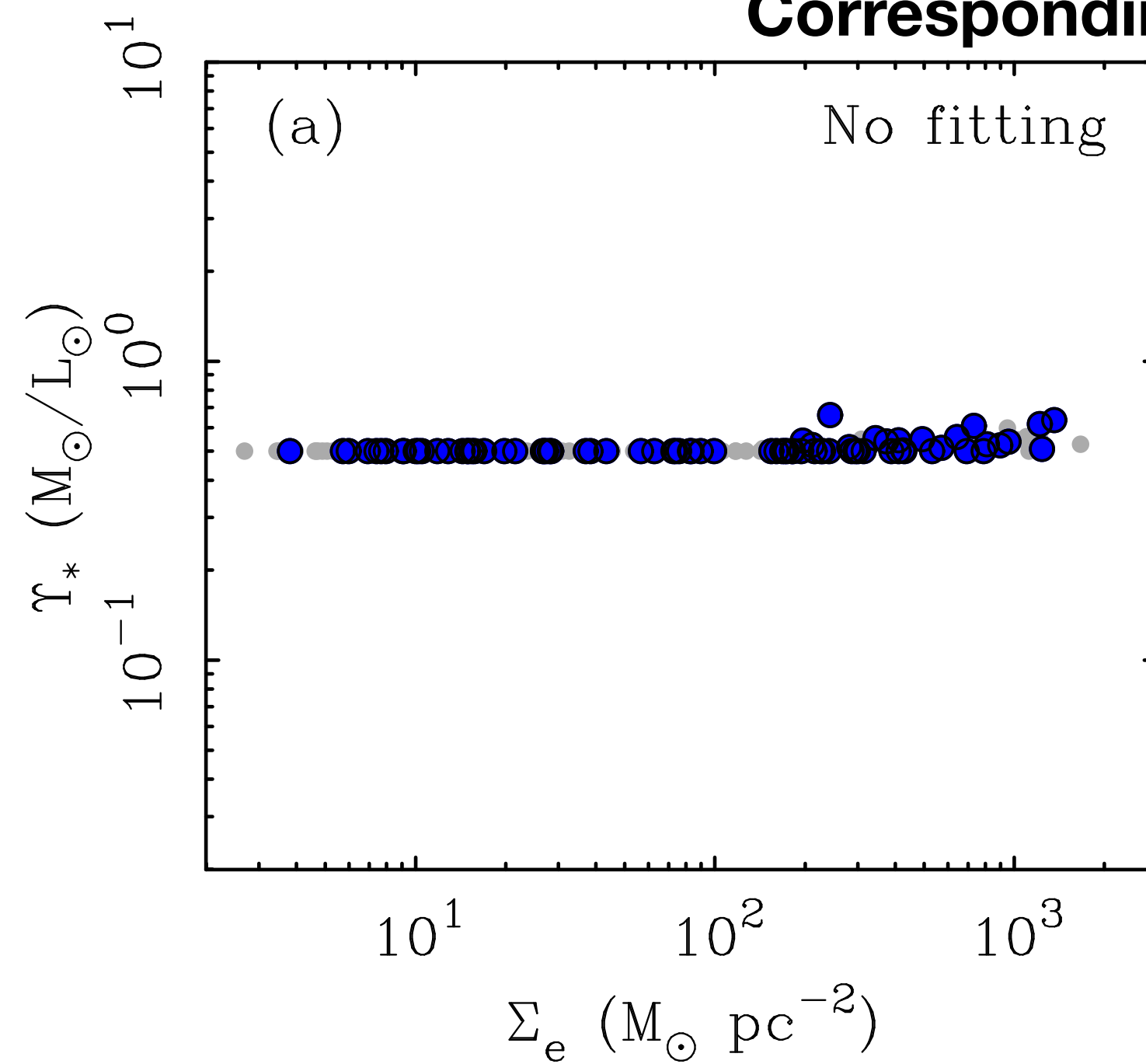


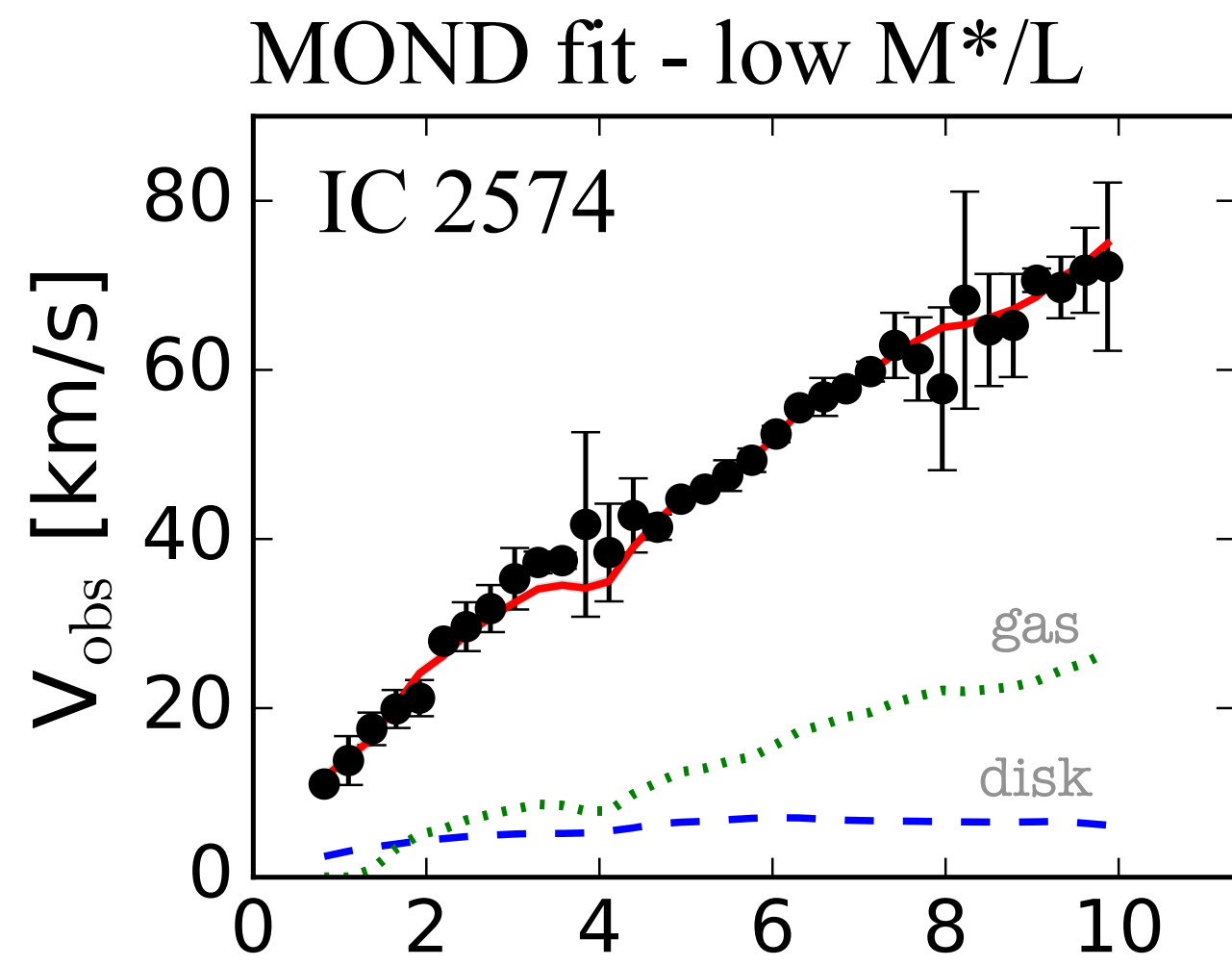
Fits of a hybrid NGC 2403/UGC 128 galaxy. This galaxy was constructed by using the NGC 2403 rotation curve data, but replacing its surface photometry by that of UGC 128.

## Residuals from rotation curves

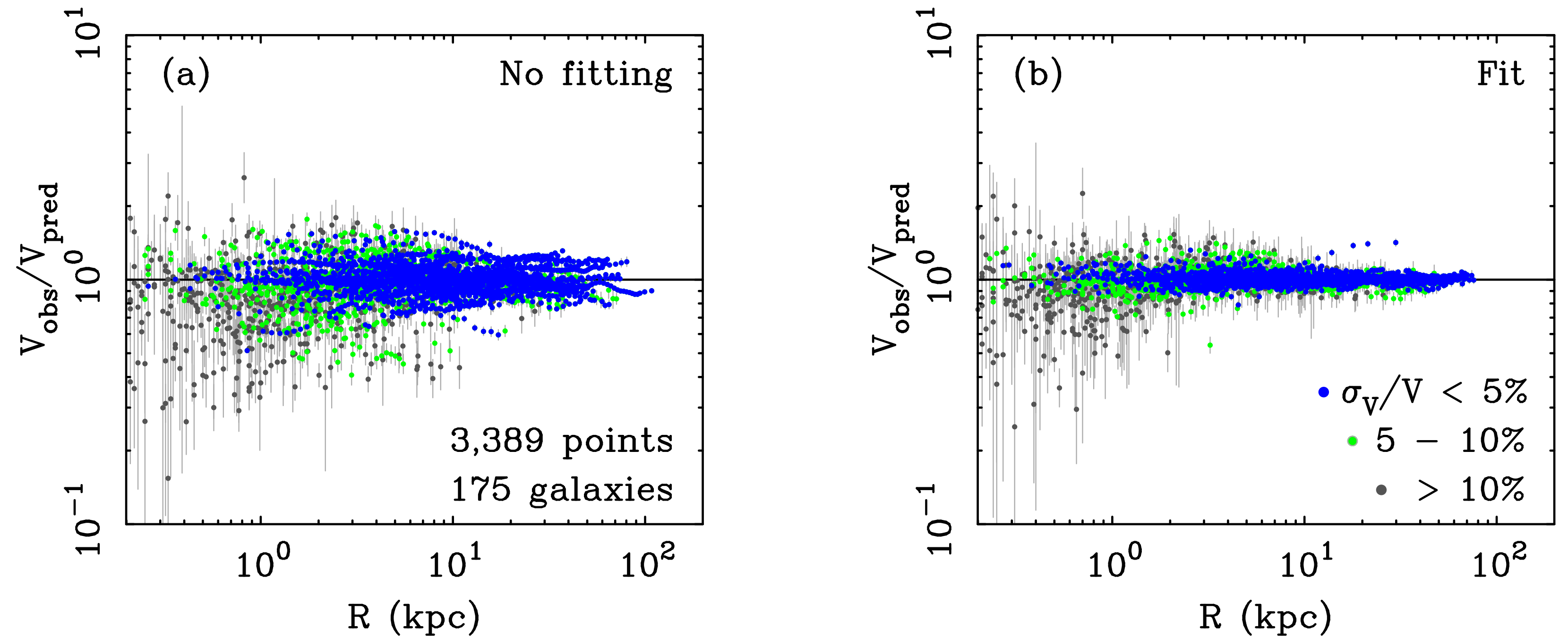


## Corresponding mass-to-light ratios

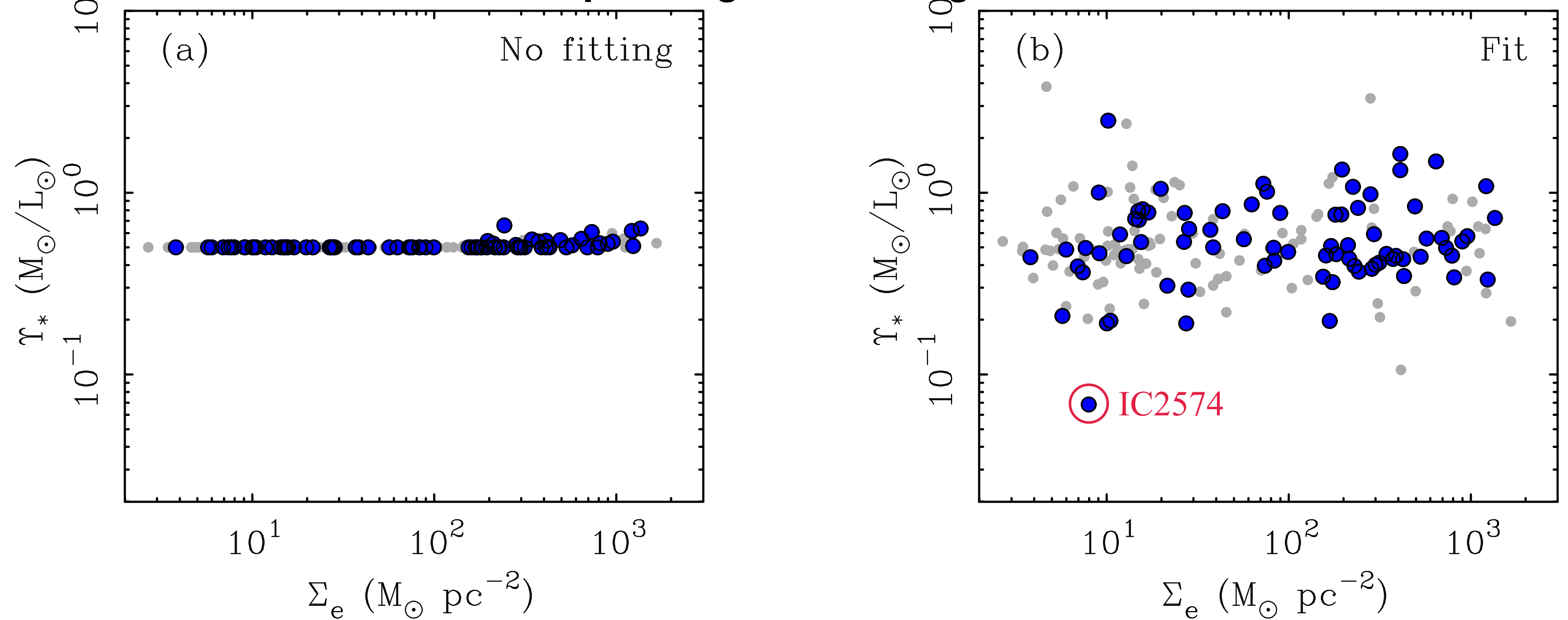




## Residuals from rotation curves



## Corresponding mass-to-light ratios



# Distance

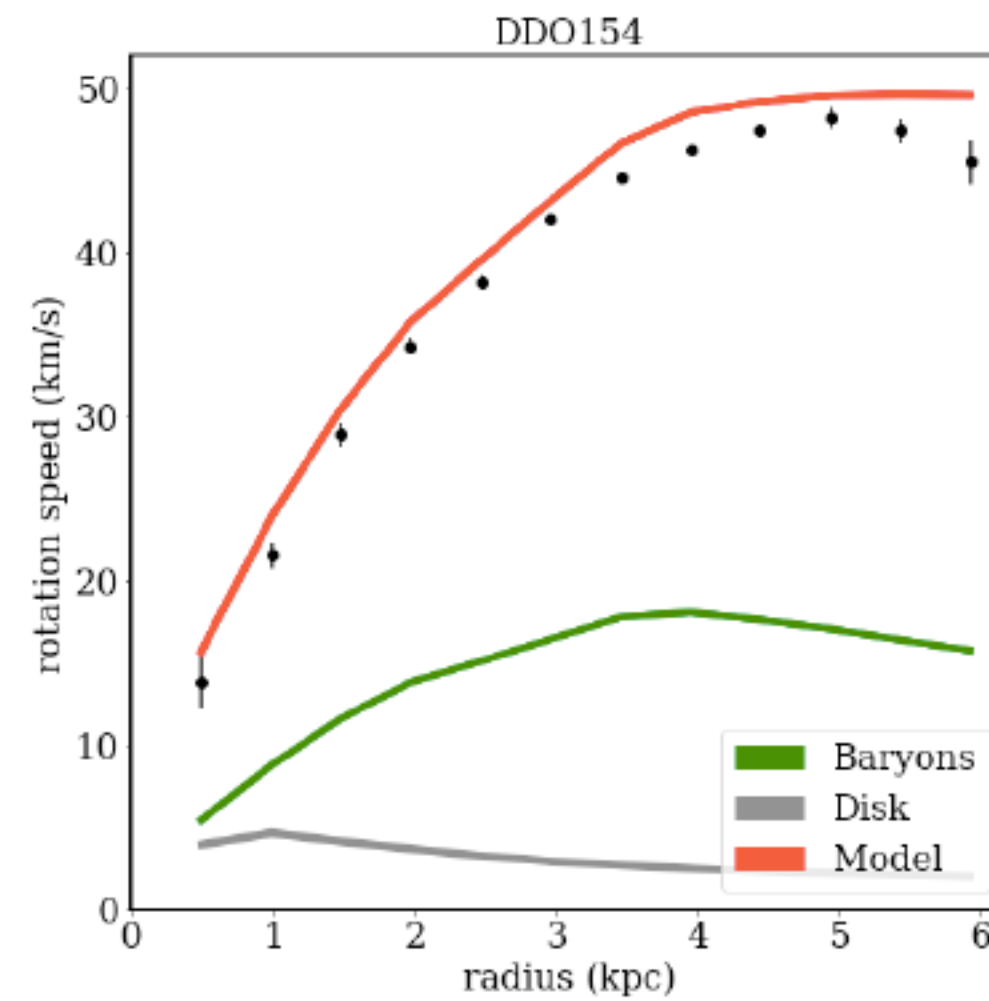
$$a \sim \frac{V^2}{R}$$

Distance and inclination play subtle but important roles in MOND fits. Make a good application of Bayesian statistics with the mass-to-light ratio as the one physical fit parameter and distance and inclination as nuisance parameters.

## Fixed distance

TRGB distance

$$D = 4.04 \pm 0.08 \text{ Mpc}$$

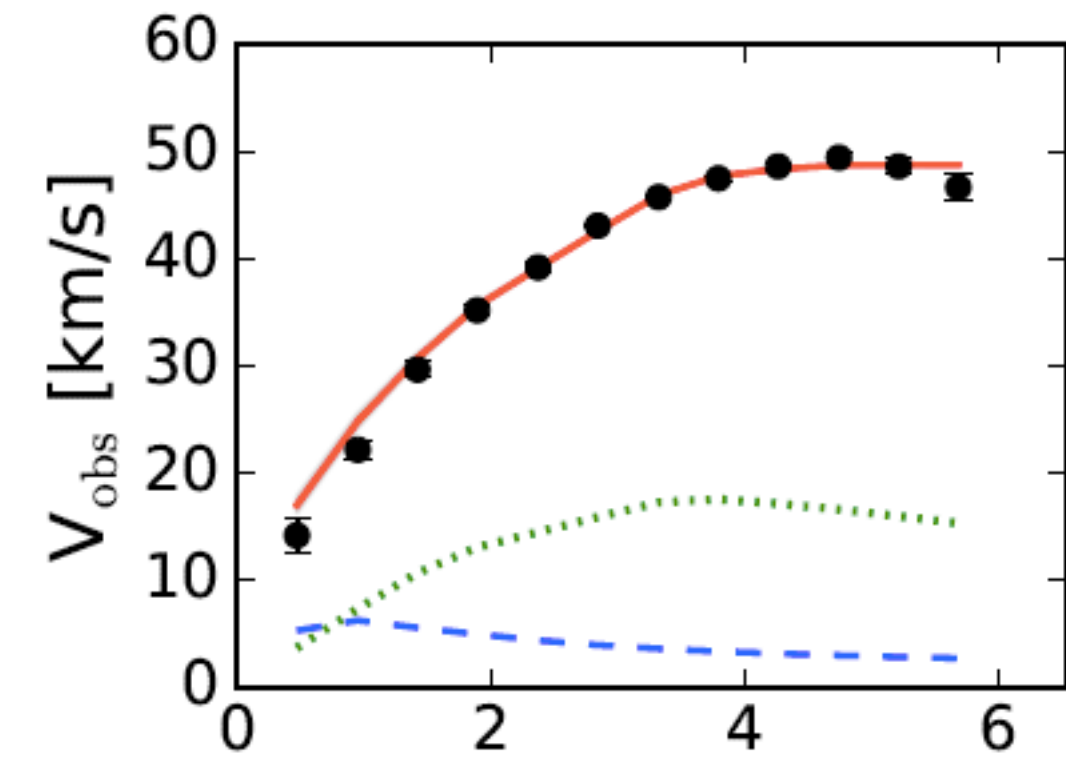


The MOND fit to DDO 154 from Ren et al. (2018).

## Distance treated as nuisance parameter

Bayesian fitted distance

$$D = 3.87 \pm 0.16 \text{ Mpc}$$

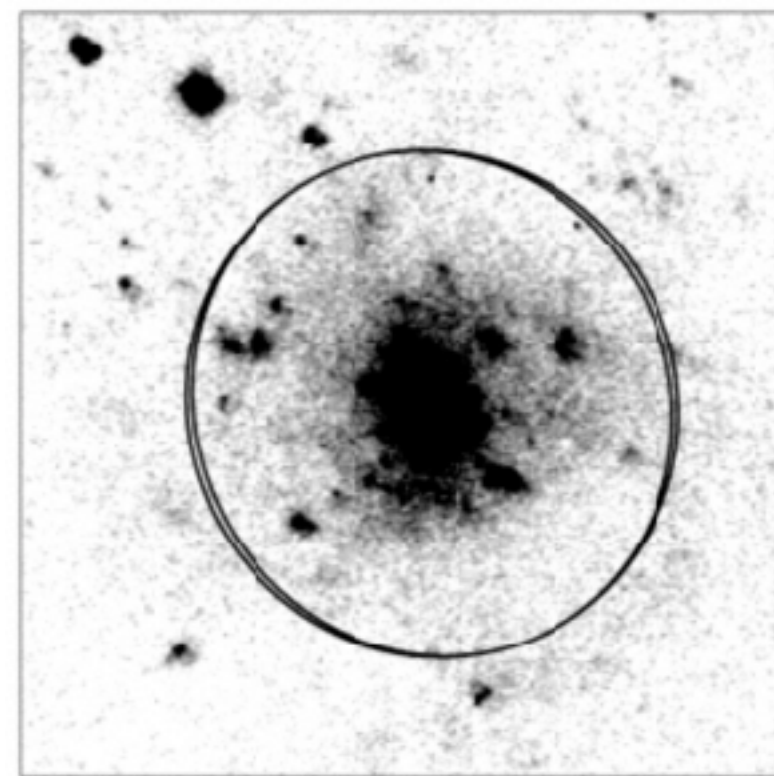


The MOND fit to DDO 154 from Li et al. (2018) using the same data.

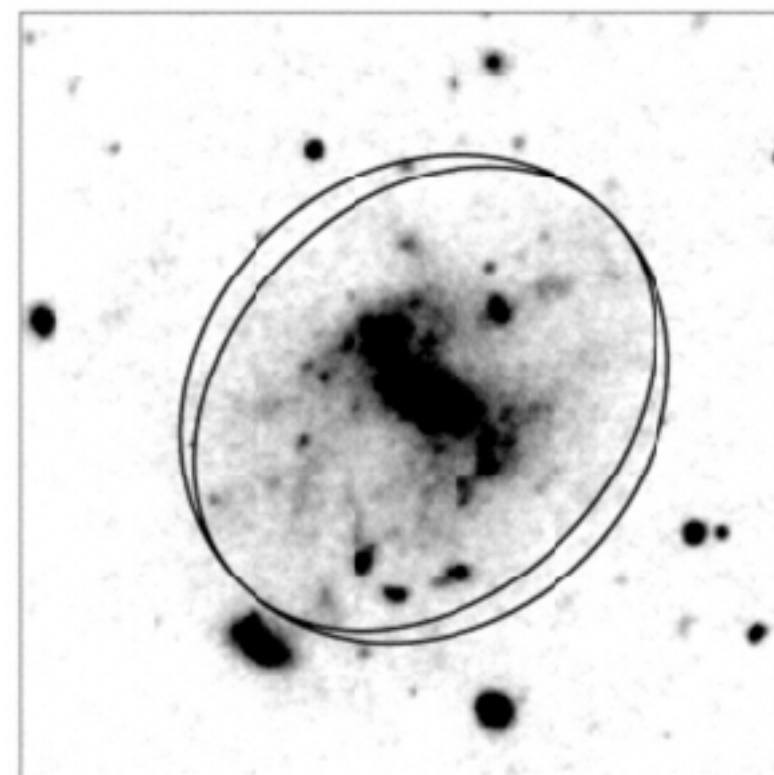
# Inclination

Distance and inclination play subtle but important roles in MOND fits. Make a good application of Bayesian statistics with the mass-to-light ratio as the one physical fit parameter and distance and inclination as nuisance parameters.

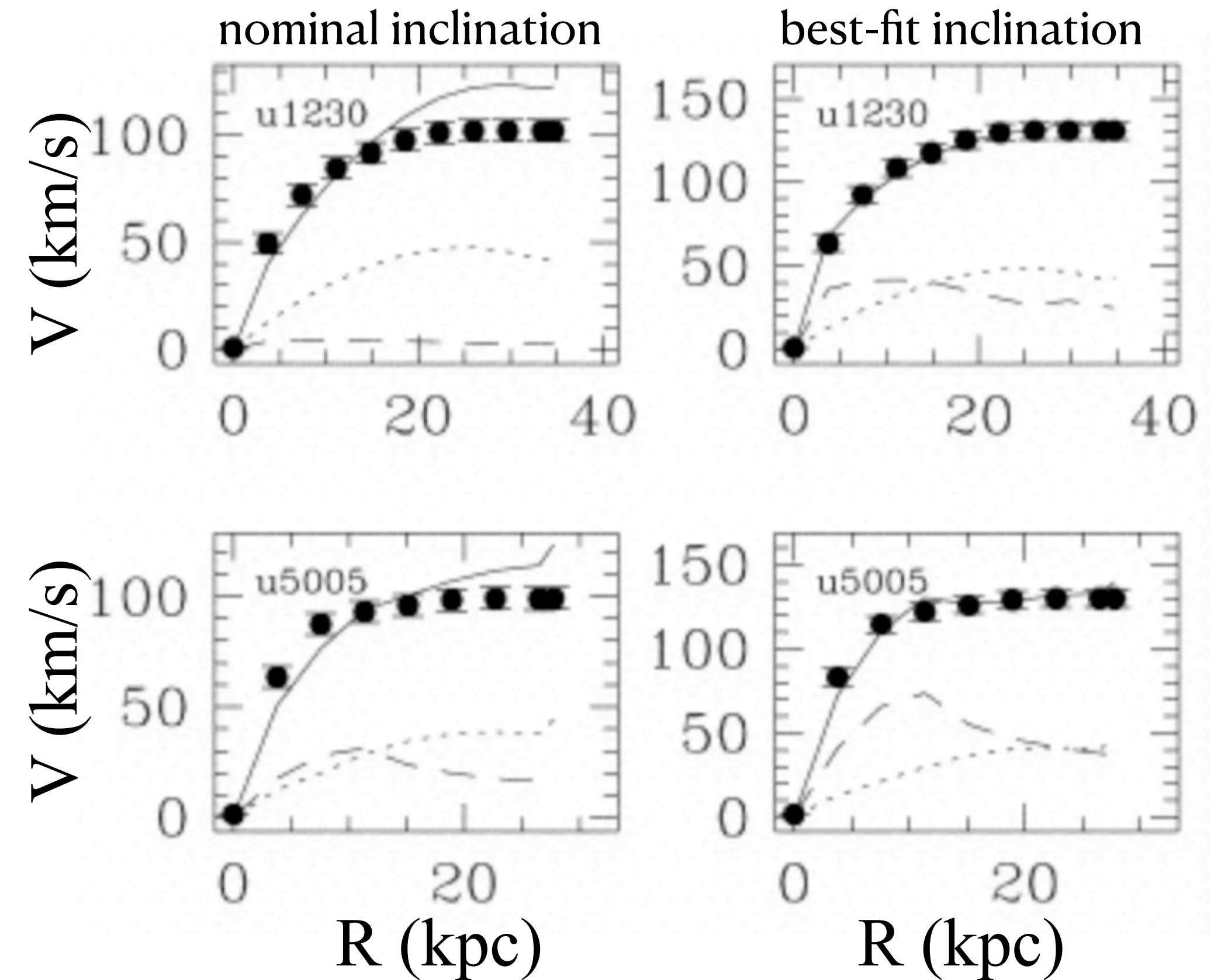
UGC 1230



UGC 5005



$$M \sim V^4 \sim \frac{v_{obs}^4}{\sin^4(i)}$$



Recall our empirical laws - these are true in the data irrespective of their interpretation

**1. Flat Rotation Curves**

The rotation curves of galaxies tend towards an approximately constant rotation speed that persists to indefinitely large radii (Rubin et al. 1978, 1980, Bosma 1981a,b).

**2. Renzo's Rule**

For any feature in the luminosity profile there is a corresponding feature in the rotation curve, and vice versa (Sancisi 2004).

**3. The Baryonic Tully-Fisher Relation (BTFR)**

The amplitude of the flat rotation speed of a galaxy correlates with its baryonic mass (the sum of stars and gas: McGaugh et al. 2000, Lelli et al. 2016b).

**4. The Central Density Relation (CDR)**

The dynamically measured central mass surface density of a galaxy correlates with its photometrically measured central surface brightness (Lelli et al. 2016c).

**5. The Radial Acceleration Relation (RAR)**

The observed centripetal acceleration correlates with that predicted by the distribution of baryonic mass (McGaugh et al. 2016, Lelli et al. 2017, Li et al. 2018).

# Predicted by MOND?

## 1. Flat Rotation Curves

*Unanticipated observation. No prediction.*

## ★ 2. Renzo's Rule

*Match between features natural and inevitable*

## ★ ★ 3. The Baryonic Tully-Fisher Relation (BTFR)

★ *TF known when MOND proposed, but not the specific BTFR*

★ *Specifically predicted lack of surface brightness residuals*

## ★ 4. The Central Density Relation (CDR)

*Specifically predicted surface brightness-acceleration connection*

## ★ 5. The Radial Acceleration Relation (RAR)

*Just MOND's interpolation function*

*A priori prediction is the gold standard of the scientific method.*

*MOND was the only theory to correctly predict many of these observations in advance.*

# Explained by dark matter?

## 1. Flat Rotation Curves

*Unanticipated observation. No prediction.  
Dark matter halos do not automatically make for flat rotation curves.*

## 2. Renzo's Rule

*Match between features should only occur where baryons dominate.*

## 3. The Baryonic Tully-Fisher Relation (BTFR)

*CDM predicted wrong slope; still struggles to match data  
There should be surface brightness residuals*

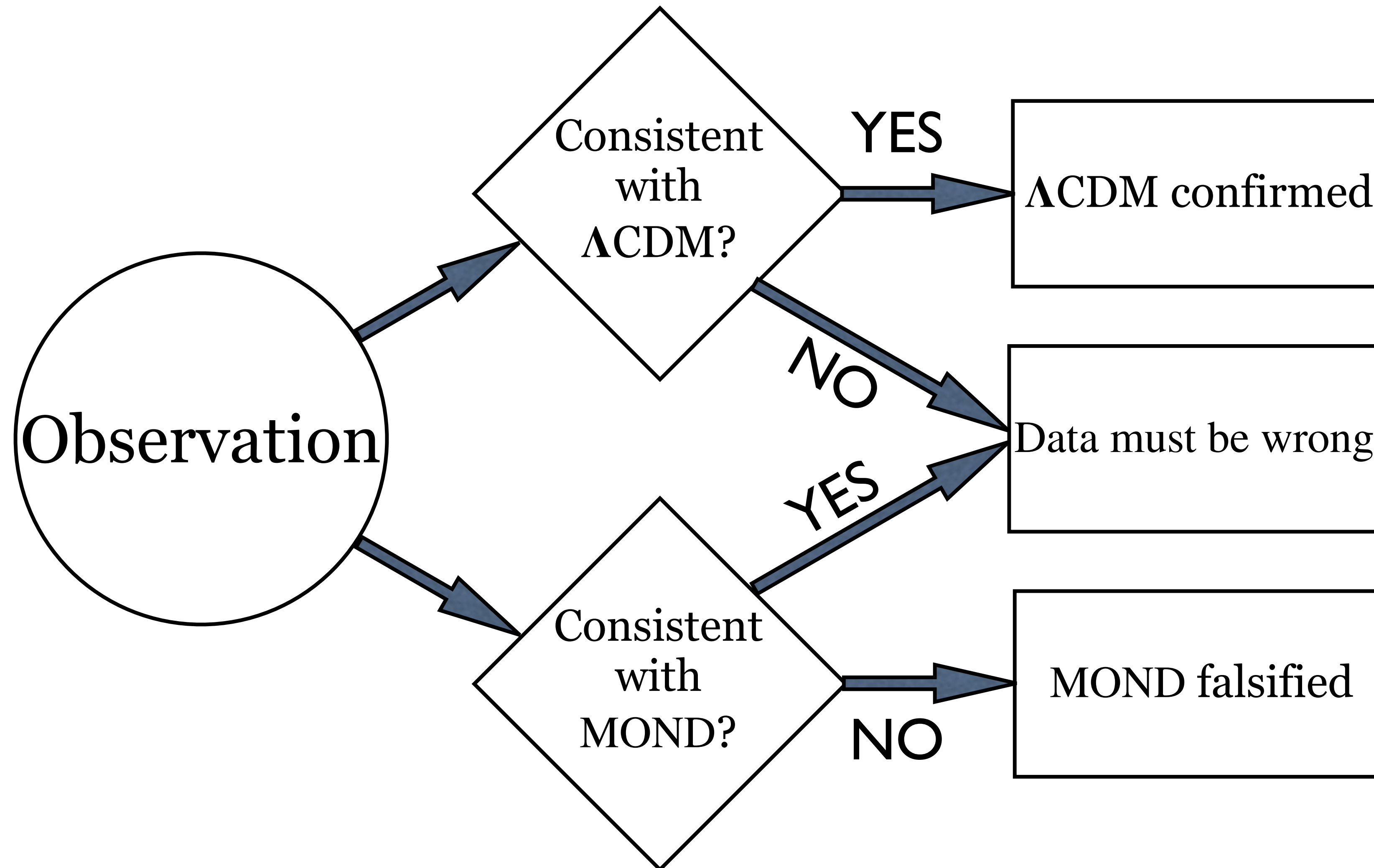
## 4. The Central Density Relation (CDR)

*Not explained. Never seems to get explicitly addressed.*

## 5. The Radial Acceleration Relation (RAR)

*No satisfactory explanation.*

A simple flowchart for the interpretation of any astronomical observation



*How many predictions must a theory get right to be considered a valid possibility?*

**Table 1. MOND Predictions & Tests.**

Prediction	Test Positive?	<i>A Prior</i>
<b>MASR (Tully-Fisher)</b>		
1. Normalization	Yes	No
2. Slope	Yes	No
3. Mass & Asymptotic Speed	Yes	Yes
4. Surface Brightness Independence	Yes	Yes
<b>Rotation Curves</b>		
5. Flat Rotation Curves	Yes	No
6. Acceleration Discrepancy	Yes	Yes
7. Rotation Curve Shapes	Yes	Yes
8. Surface Brightness & Density	Yes	Yes
9. Detailed Fits	Yes	No
10. Stellar Population $Y_*$	Yes	—
11. Feature Correspondence	Yes	—
<b>Disk Stability</b>		
12. Freeman Limit	Yes	No
13. Vertical Velocity Dispersions	?	No
14. LSB Galaxy Morphology	Yes	Yes

Space of possible model galaxies  
with baryons in dark matter halos

That MOND works at all is problematic for the dark matter paradigm. To explain the data, we must pluck the MOND needle from the haystack of possible DM models.

More plausible models

MOND  
Observed Galaxies

Less plausible models

# MOND predictions and tests

in galaxies

everywhere else

System	Prediction	Result	References
<b>Rotationally Supported Galaxies</b>			
<b>Rotation Curves</b>			
	Asymptotically Flat Rotation Curves	✓	[18, 215]
	Flatness Extended by Weak Lensing	✓	[187]
	Rotation Curve Shapes	✓	[102, 108, 144, 199]
	Feature Correspondence (Renzo's rule)	✓	[95, 218]
	Detailed Rotation Curve Fits	✓	[14, 112, 225]
	Stellar Population Mass-to-Light Ratios	✓	[141, 148, 228]
<b>Mass-Asymptotic Speed Relation</b>			
	BTFR with slope 4: $M_b = AV_f^4$	✓	[139, 236, 246, 250]
	BTFR Normalization $A = (a_0 G)^{-1}$	✓	[137, 140, 250]
	BTFR from weak lensing	✓	[176, 187]
	No Second Parameter Dependence	✓	[39, 106, 277]
	Negligible Intrinsic Scatter	✓	[46, 103, 140]
<b>acceleration constant</b>			
	acceleration constant $a_0$ in the data	✓	Table 2
<b>Radial Acceleration Relation</b>			
	Newtonian limit $a \rightarrow g_{N,b}$ for $a \gg a_0$	✓	
	DML $a \rightarrow (g_{N,b} a_0)^{1/2}$ for $a \ll a_0$	✓	[233, 272, 256, 107, 141, 184]
	Variation a function $\nu(g_{N,b}/a_0)$	✓	[181]
	Negligible Intrinsic Scatter	✓	[45, 142, 47]
	Fundamental Relation	✓	[107, 249]
<b>Central Surface Density Relation</b>			
	Surface Density Scale $\Sigma_M = a_0/(2\pi G)$	✓	[108, 173]
	$\Sigma_{\text{dyn}}^0 \rightarrow \Sigma_b^0$ for $\Sigma_b^0 \gg \Sigma_M$	✓	[177, 108, 181, 161]
	$\Sigma_{\text{dyn}}^0 \rightarrow (4\Sigma_M \Sigma_b^0)^{1/2}$ for $\Sigma_b^0 \ll \Sigma_M$	✓	[40, 42, 108, 181, 143]
	Variation a function $\mathcal{S}(\Sigma_b^0/\Sigma_M)$	✓	[181]
<b>Disk Stability</b>			
	Maximum Surface Density (Freeman limit)	✓	[173, 22, 128, 144]
	Vertical Velocities & Disk Thickness	?	[128, 144]
	Radial Dependence of Oort Discrepancy	?	[16, 122]
	Galaxy Morphology	✓	[128, 257, 254]
<b>pressure-supported Galaxies</b>			
	Weak Lensing Masses	✓	[176, 187]
	Faber-Jackson Relation for ellipticals	✓	[Tian2026, 223, 226]
	Large discrepancies in hot x-ray gas in ellipticals	✓	[175]
	Small Discrepancies in Giant Ellipticals	✓	[182, 257]
	Large Discrepancies in Dwarf Spheroidals	✓	[169, 129]
	EFE in Dwarf Satellites	✓	[130]
<b>Novel Galaxy Types</b>			
	Low Surface Brightness Galaxies	✓	[41, 144]
	Gas Dominated Galaxies	✓	[137, 183, 224]
	Ultra Diffuse Galaxies	?	[61, 101, 264]
	Ultrafaint Dwarfs	?	[151, 136, 107]
	High Redshift Galaxies	✓	[110, 150]

System	Prediction	Result	References
<b>Solar System</b>			
	Cassini Gravitational Quadrupole	✗	[49, 84, 85]
	Planet 9	✓	[24, 154]
	Long Period Comets	✗	[266]
<b>Binary Stars</b>			
	Close Binary Orbits	✓	(Newtonian)
	Wide Binary Orbits	?	[31, 86, 203, 33, 89, 87, 32, 88, 204, 30, 38]
	Wide Binary Extent	✓	[239]
<b>Tidal tails of star clusters</b>			
	Asymmetry of tails	✓	[252, 96, 97]
<b>Survival of dwarfs in galaxy clusters</b>			
	Disrupting dwarfs in the Fornax cluster	✓	[7]
<b>Binary &amp; Interacting Galaxies</b>			
	Pairwise Velocities	✓	[157, 234, 13, 242]
	Dynamical Friction	?	[194, 217, 253, 255]
	Tidal Tail Morphology	✓	[255]
	Tidal Dwarf Galaxies	?	[75, 104]
	Polar Rings	✓	[117]
	Planes of Satellites	?	[11, 12]
<b>Groups of Galaxies</b>			
	Dynamical Masses	✓	[157, 163, 168, 170, 171]
	Weak Lensing Masses	✓	[118]
<b>Clusters of Galaxies</b>			
	Dynamical Masses	✗	[156, 115]
	Hot-gas hydrostatic Masses	✗	[2, 227, 219, 251]
	Weak Lensing Masses	✗	[186]
	Mass-Temperature Relation Slope	✓	[126, 132]
	Collision Velocities	✓	[3, 5, 6]
	Bulk Velocities	✓	[92, 189]
<b>Large Scale Structure</b>			
	EFE from LSS	✓	[150, 220]
	Galaxy Power Spectrum	✓	[34, 36, 35, 62]
	Baryon Acoustic Oscillation	?	[231, 94, 116, 195]
	Large, Mostly Empty Voids	?	[10]
	Massive Galaxies at $z \gtrsim 10$	✓	[132, 196]
	Massive Clusters of Galaxies at $z \gtrsim 2$	✓	[220, 150]
	Early Emergence of Cosmic Web	✓	[5, 72, 71, 70]
	Early Reionization	✓	[59, 90, 240]
	Dark Ages & Cosmic Dawn	?	[134]
		?	[138]
<b>Cosmology</b>			
	Geometry	?	[17, 44, 241]
	Expansion History	?	[17, 44, 241]
	Background Radiation	?	[17, 44, 241]
	Big Bang Nucleosynthesis	✓	[220]

# Disk Stability

- MOND stabilizes disk in the low acceleration regime
- High acceleration objects suffer usual Newtonian instabilities
- Predicts upper limit to disk surface brightness
- Freeman's surface brightness marks transition between stable and unstable regimes

$$\Sigma \lesssim \Sigma_{\dagger} = a_0/G$$

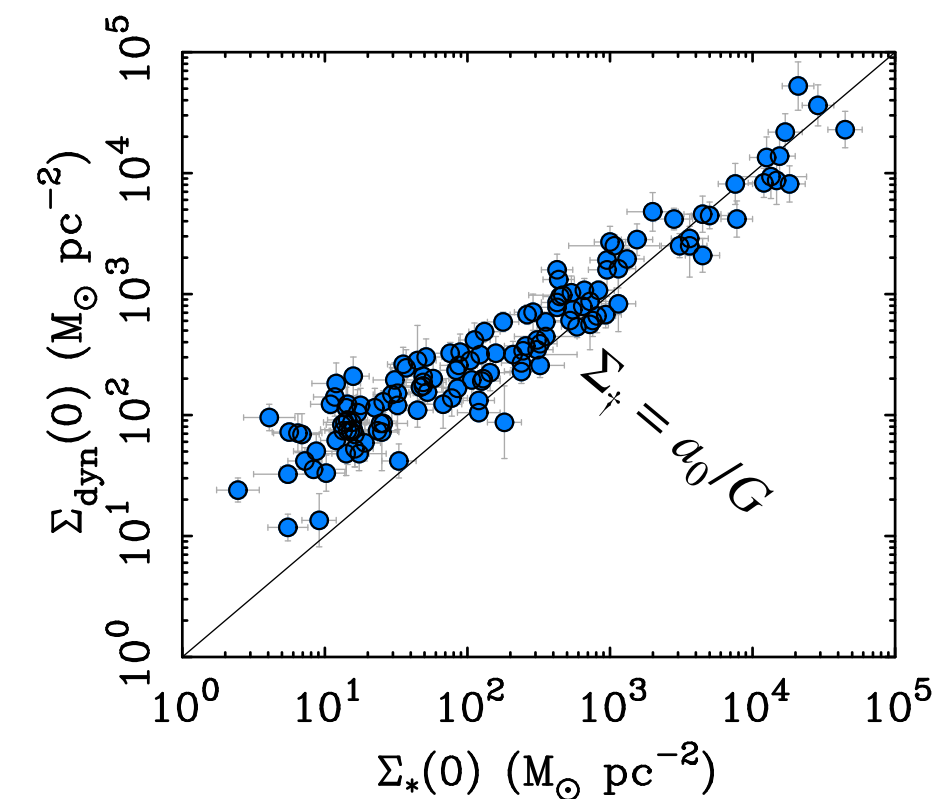
- Stability properties differ from DM case

- similar at high surface brightness

$$t = \frac{T}{|W|} \approx 0.14$$

- less added stability at low surface brightness (low acceleration)

$$t = \frac{T}{|W|} \ll 0.14$$



## Disk Stability in MOND

Brada (1998)

Brada & Milgrom (1999, 2000)

MOND adds stability roughly comparable to that added by a dark matter halo of  $\sim 3$  times the disk mass, enclosed by the disk radius.

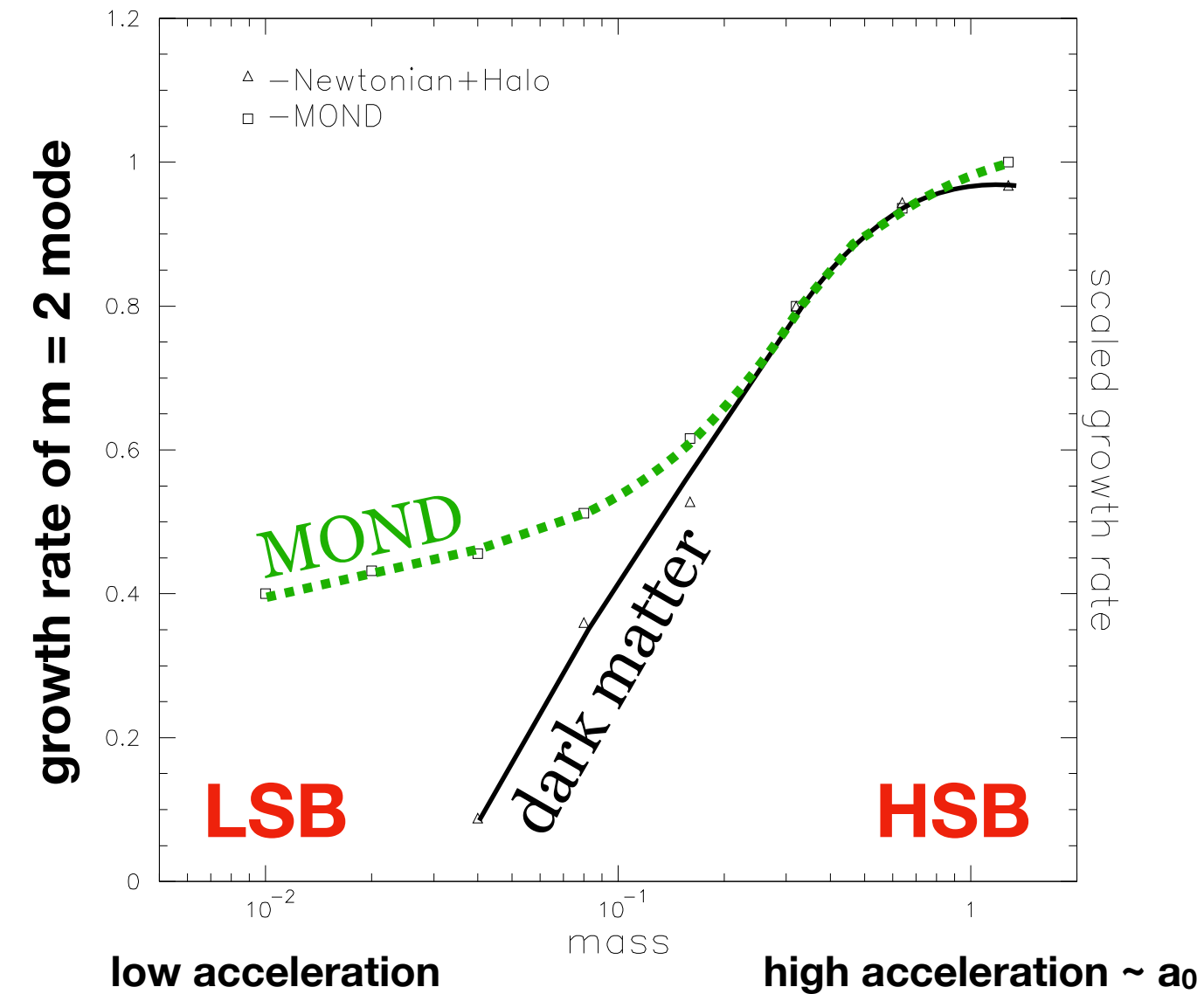


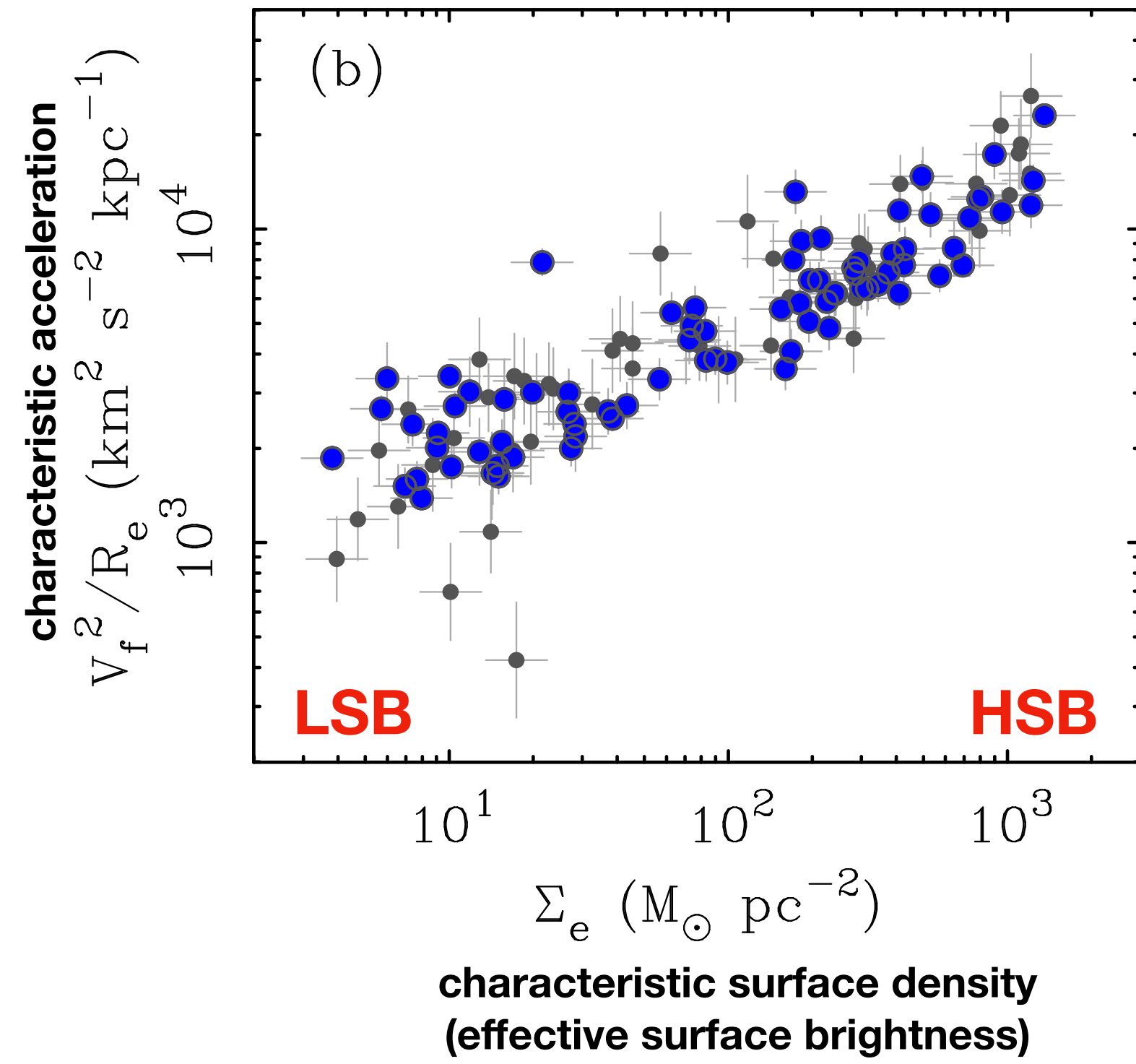
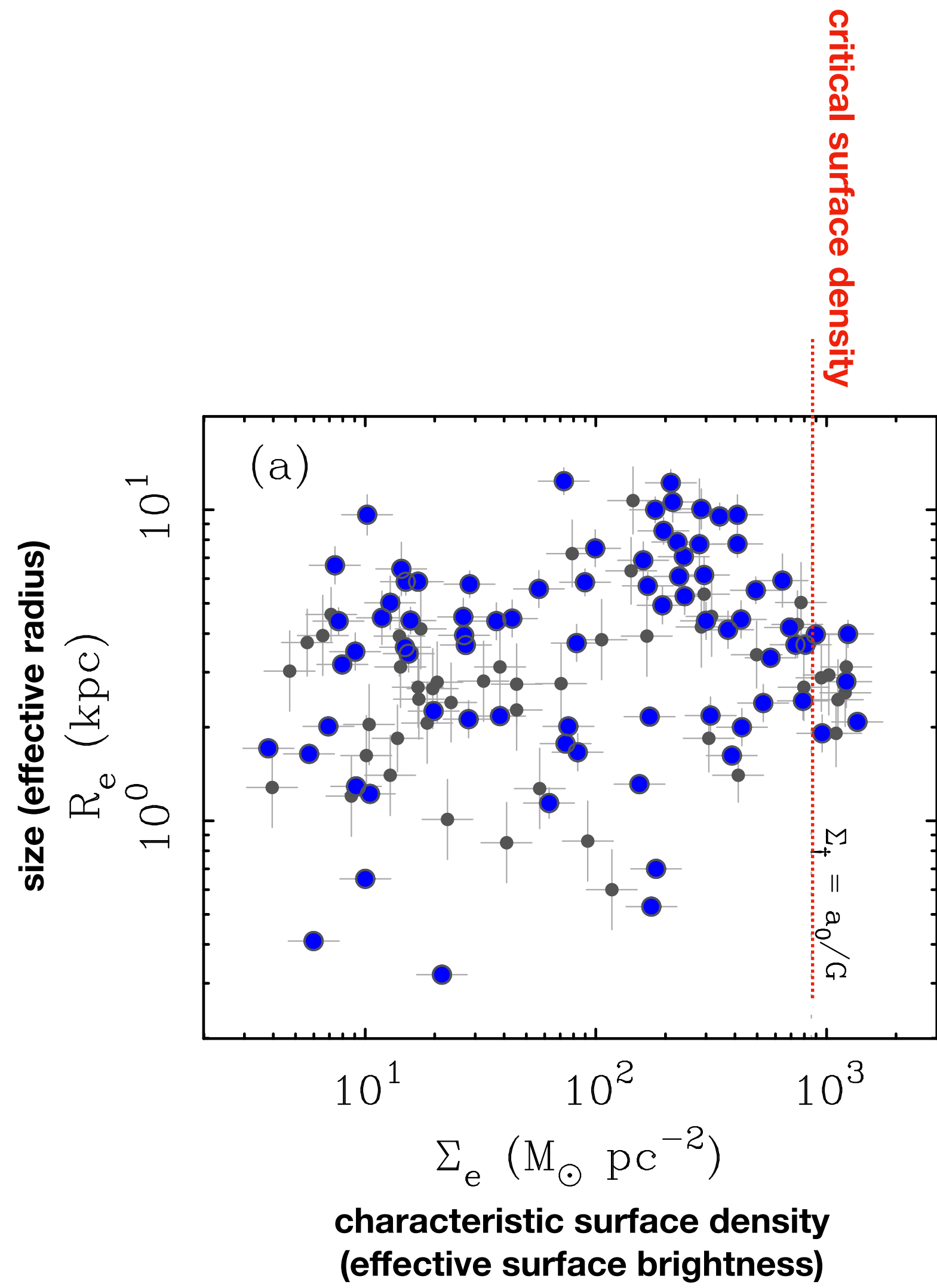
Figure 11: The growth rate, in units of the dynamical time, for the  $m=2$  mode as a function of the total mass of the disk.  $\square$  MOND,  $\triangle$  Newtonian + Halo.

- **Stability properties**

- similar to DM at high surface brightness
- less added stability at low surface brightness (low acceleration)

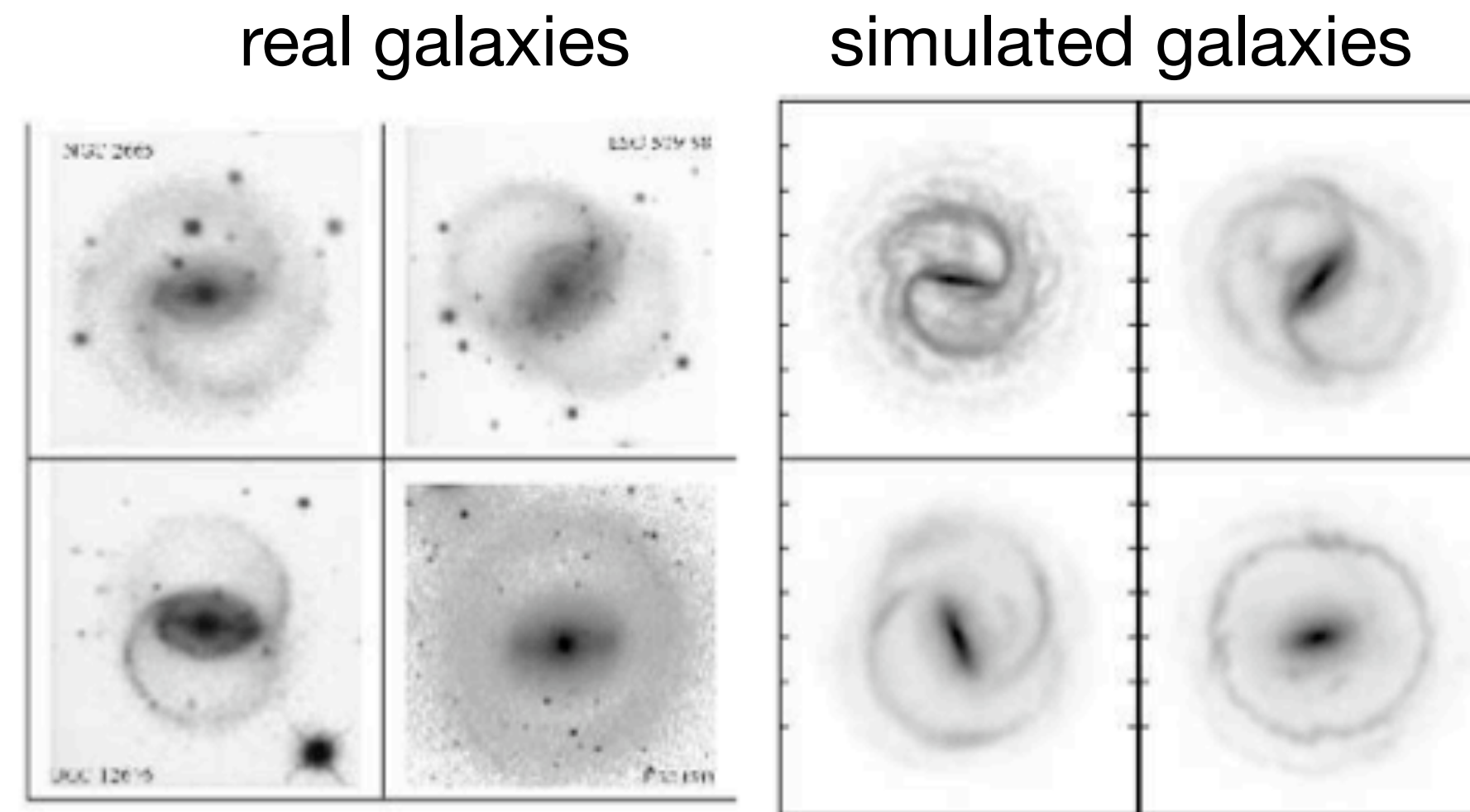
m	Q	time step scaling	Growth rate		halo mass at R=1
			MOND	Newt+DM	
0.005	2.55	1			
0.01	2.5	0.84	0.4		
0.02	2.4	0.7	0.43		
0.04	2.25	0.58	0.46	0.09	0.18
0.08	2.0	0.48	0.51	0.36	0.23
0.16	1.79	0.39	0.62	0.53	0.28
0.32	1.62	0.3	0.8	0.8	0.31
0.64	1.53	0.22	0.94	0.94	0.31
1.28	1.5	0.16	1.0	0.97	0.27

Table 1: The growth rate, in units of dynamical time, for the  $m = 2$  mode, and model parameters for the different mass models.



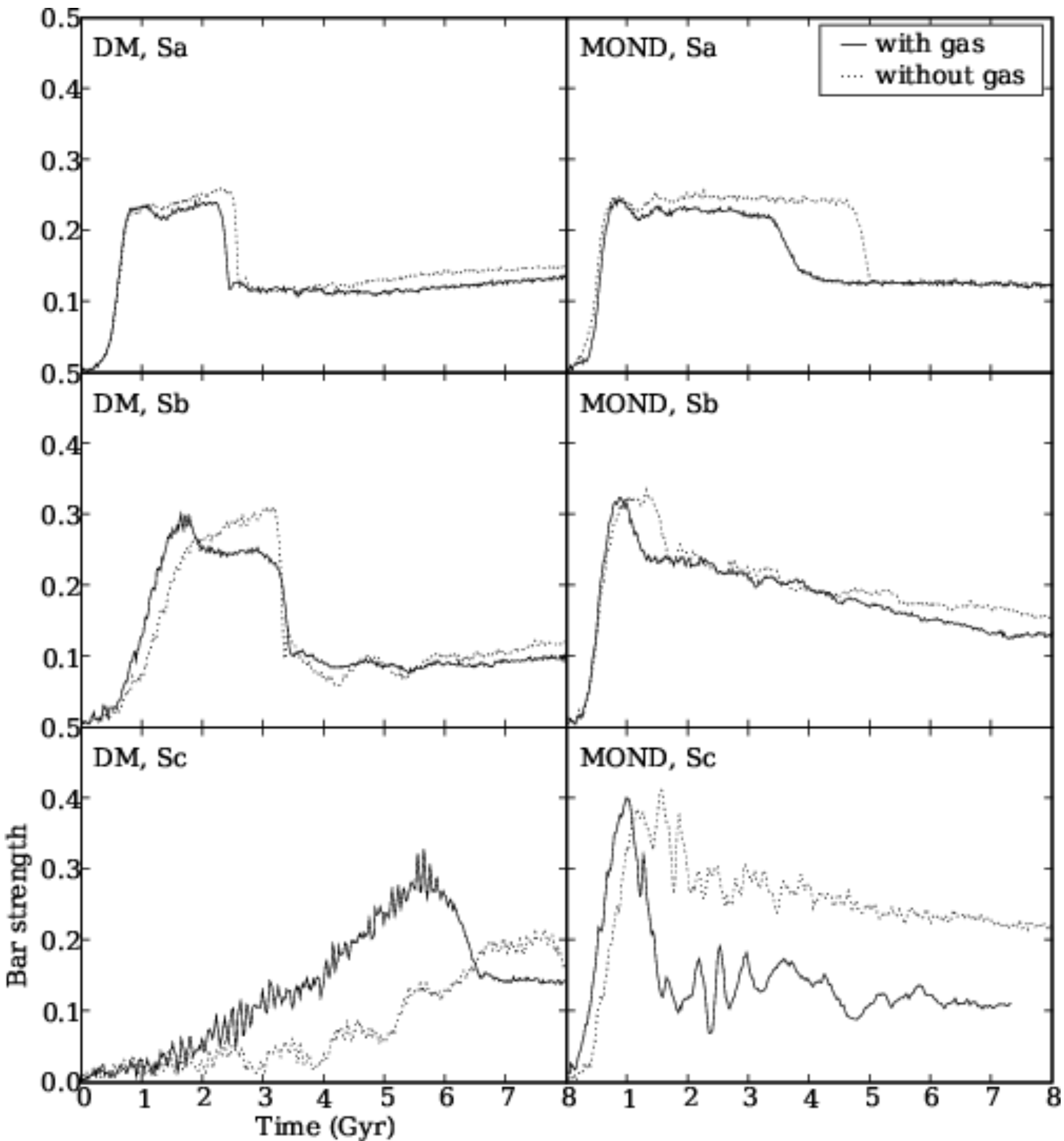
**Tiret & Combes (2007, 2008)**

MOND numerical simulations of galaxy morphology

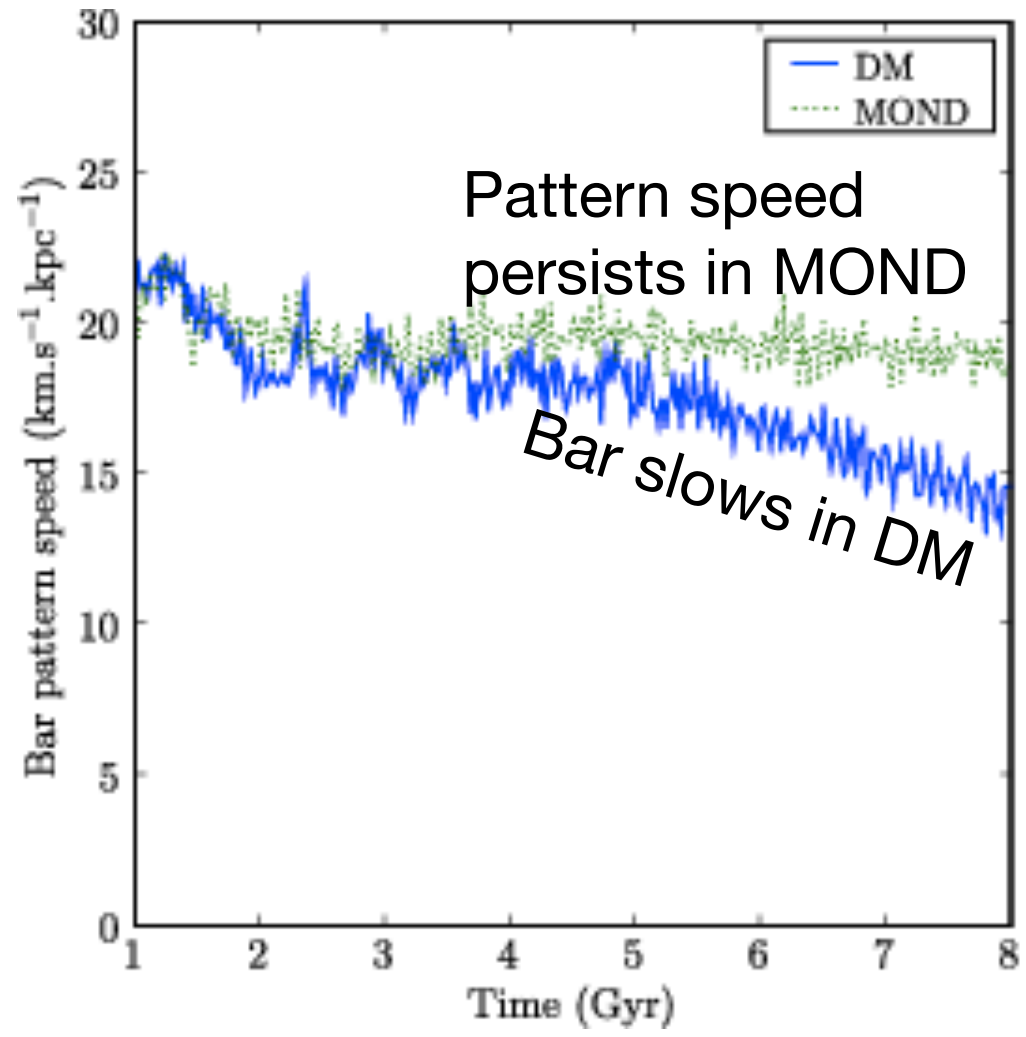


**Fig. 3.** Several examples showing the morphological structures of NGC 2665, ESO 509-98, UGC 12646 and NGC 1543 (top panel) compared to simulated galaxies in MOND (bottom panel). Rings and pseudo-rings structures are well reproduced with modified gravity.

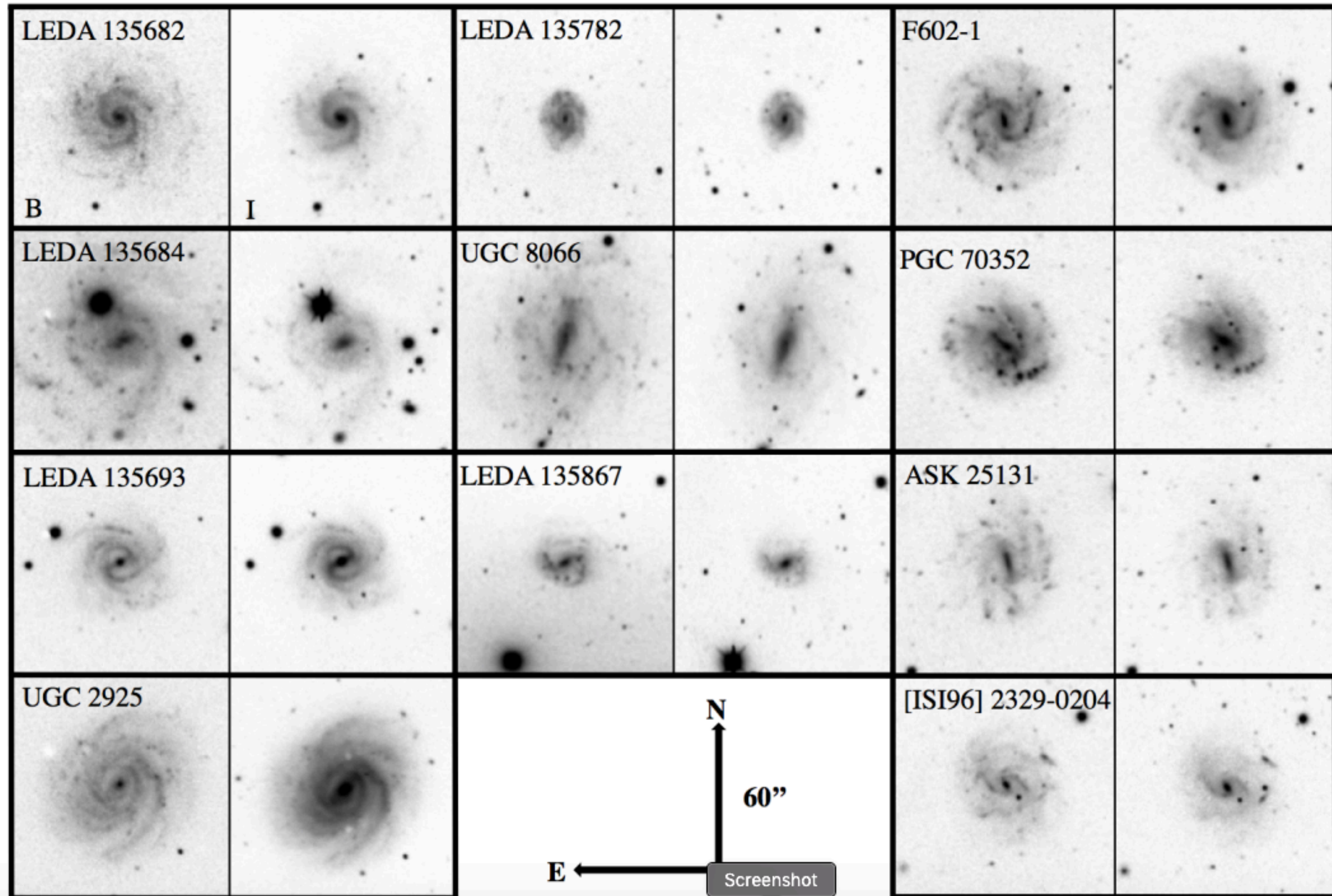
### Bar strength



### Bar pattern speed

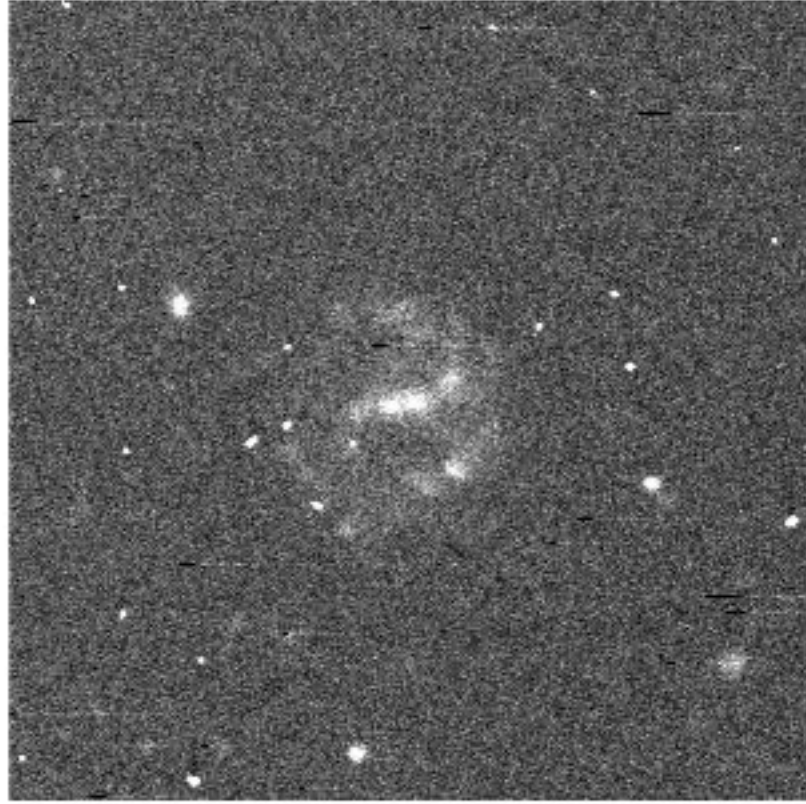


# Bars in LSB galaxies

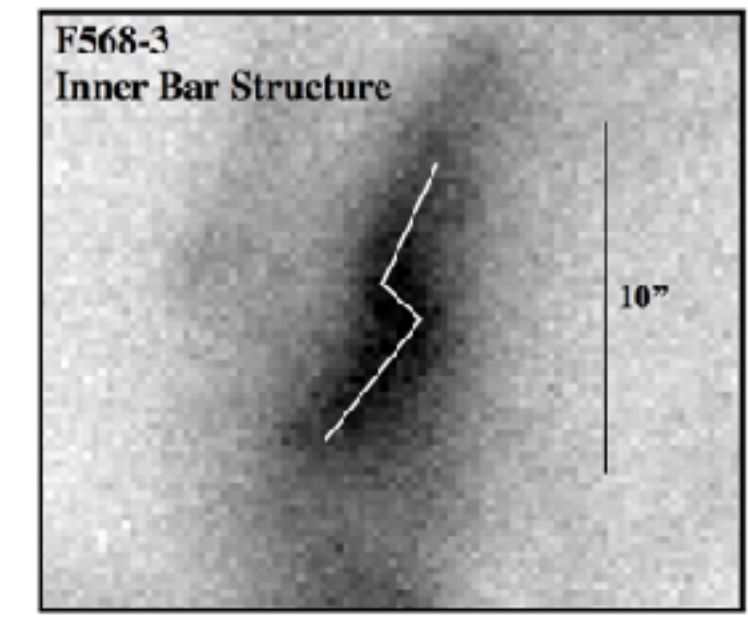
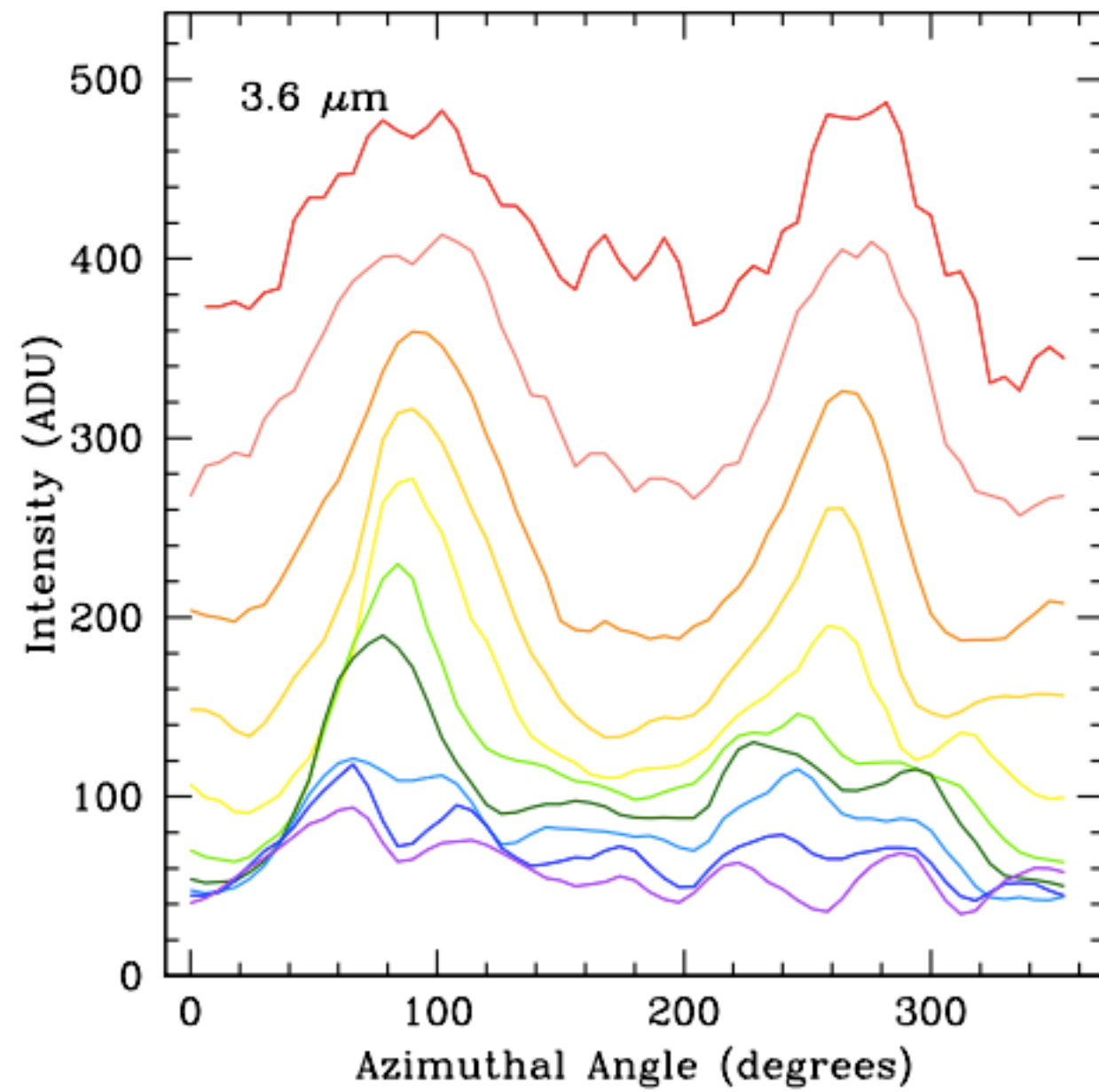
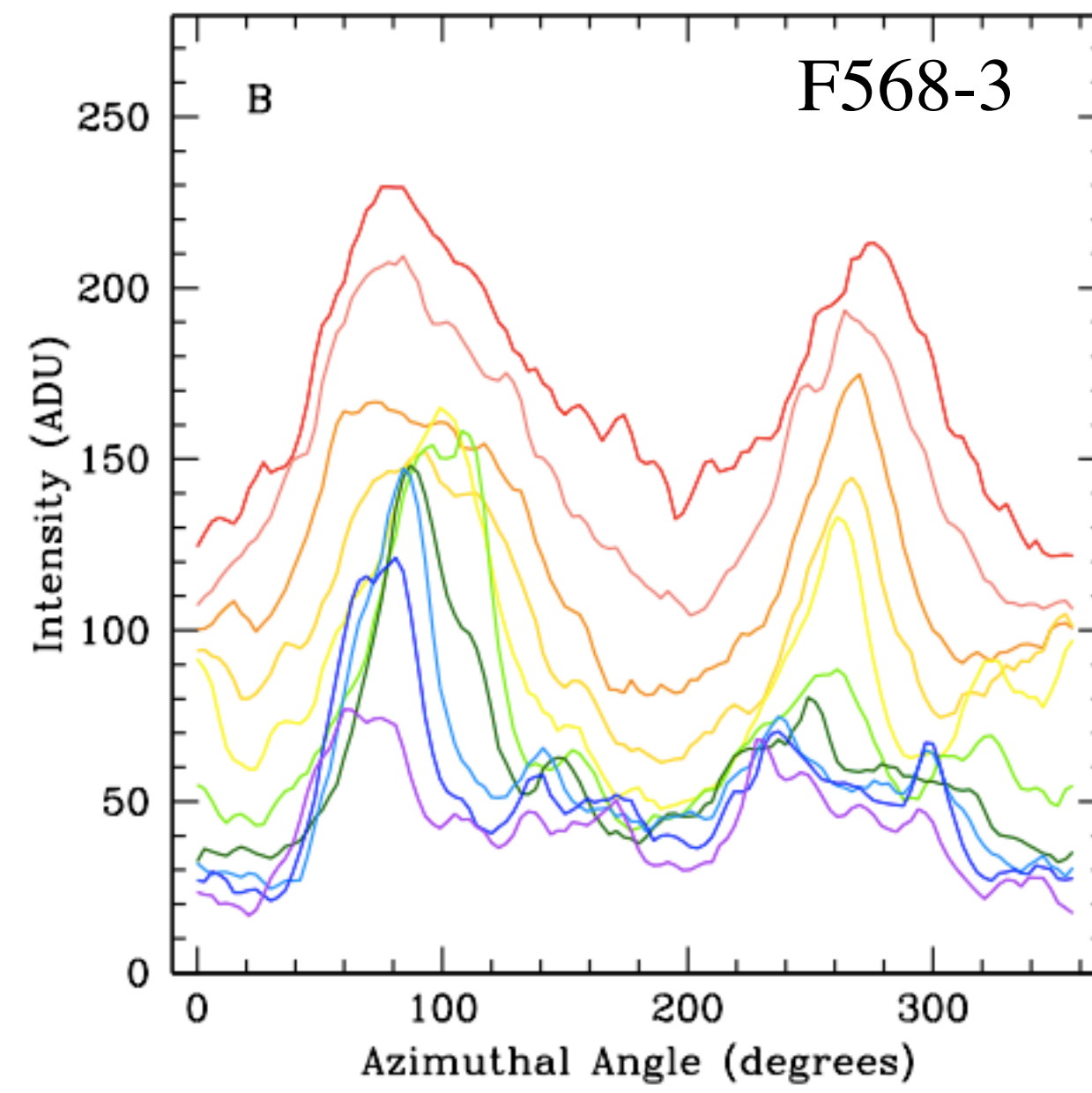
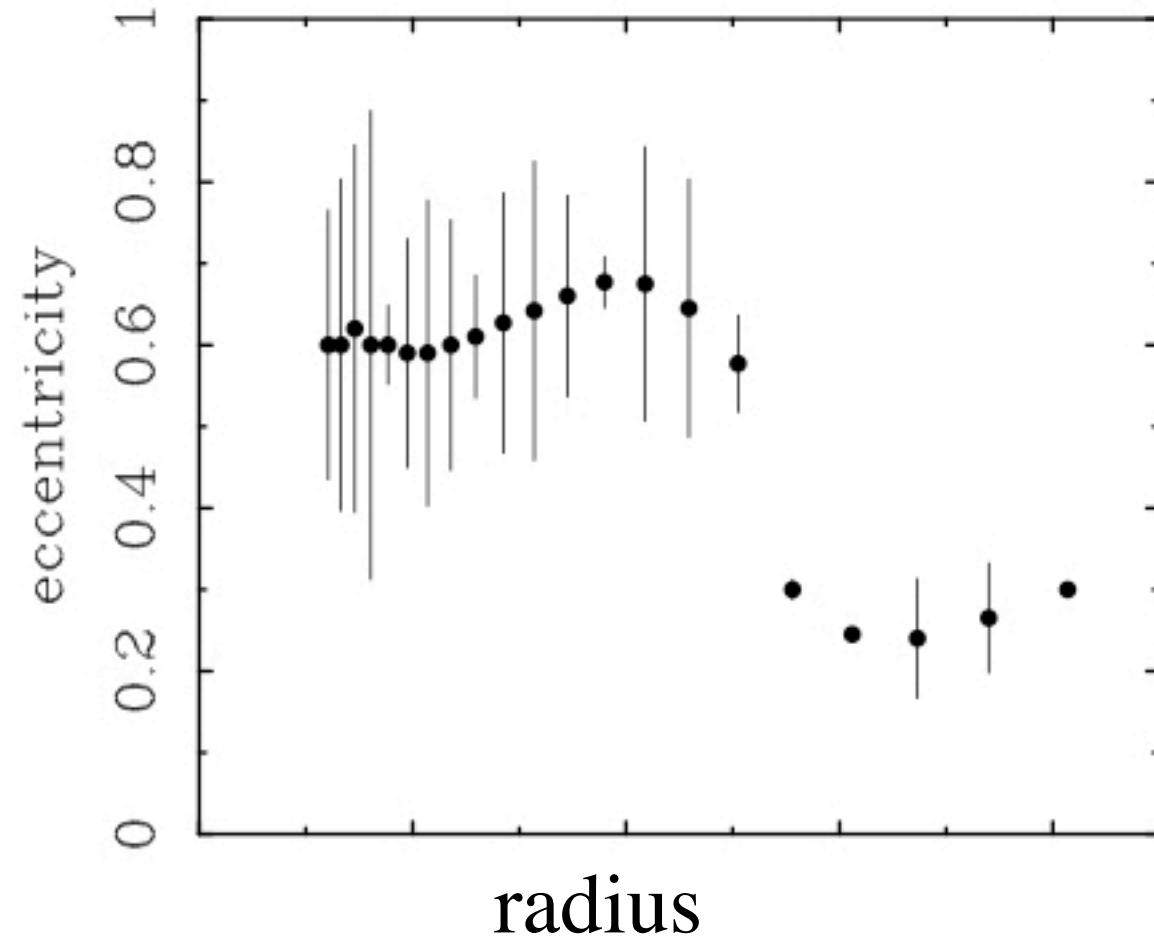


# Bars in LSB galaxies

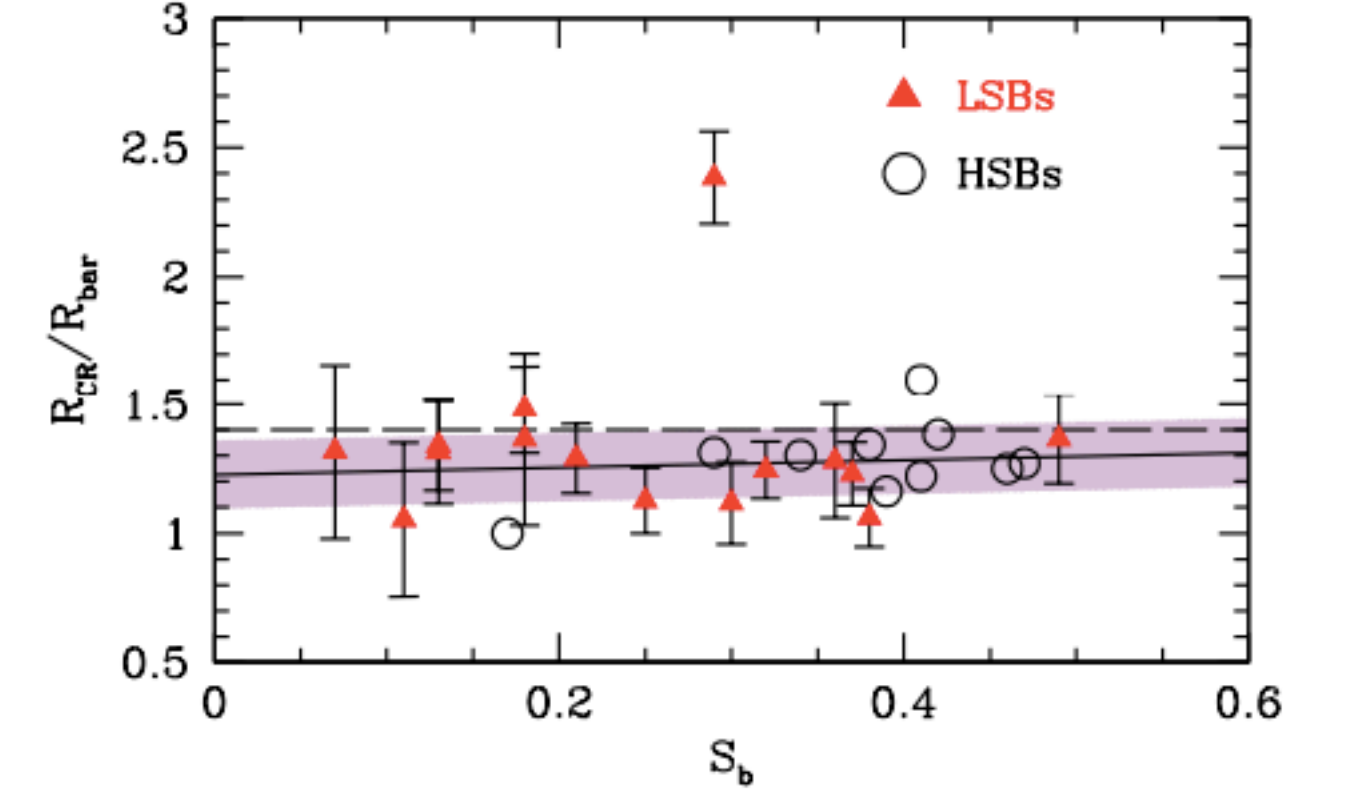
F577-V1



Eccentricity changes suddenly at the end of the bar



Bar lengths/pattern speeds  
Bars in LSBs fast; no sign of dynamical friction

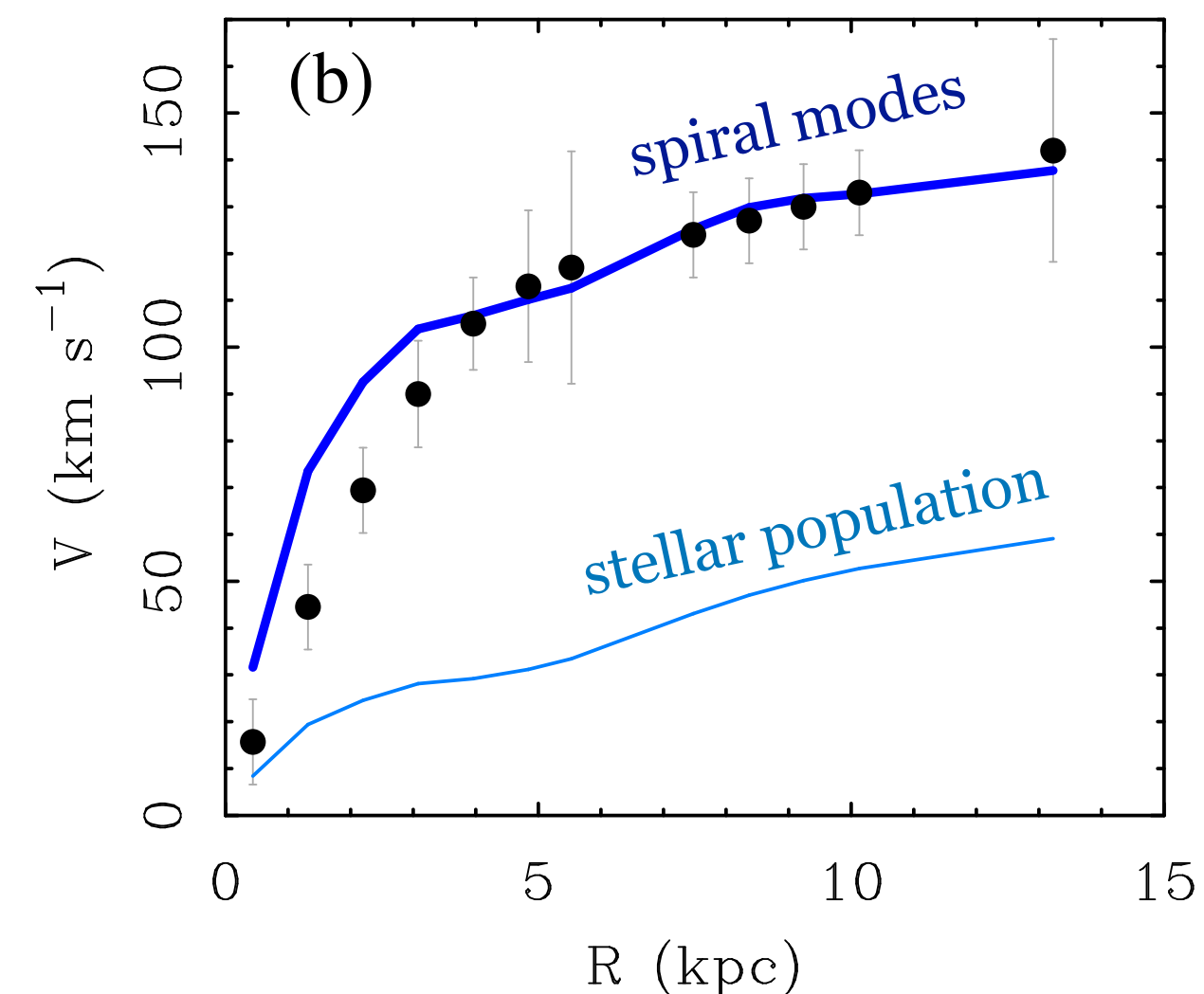
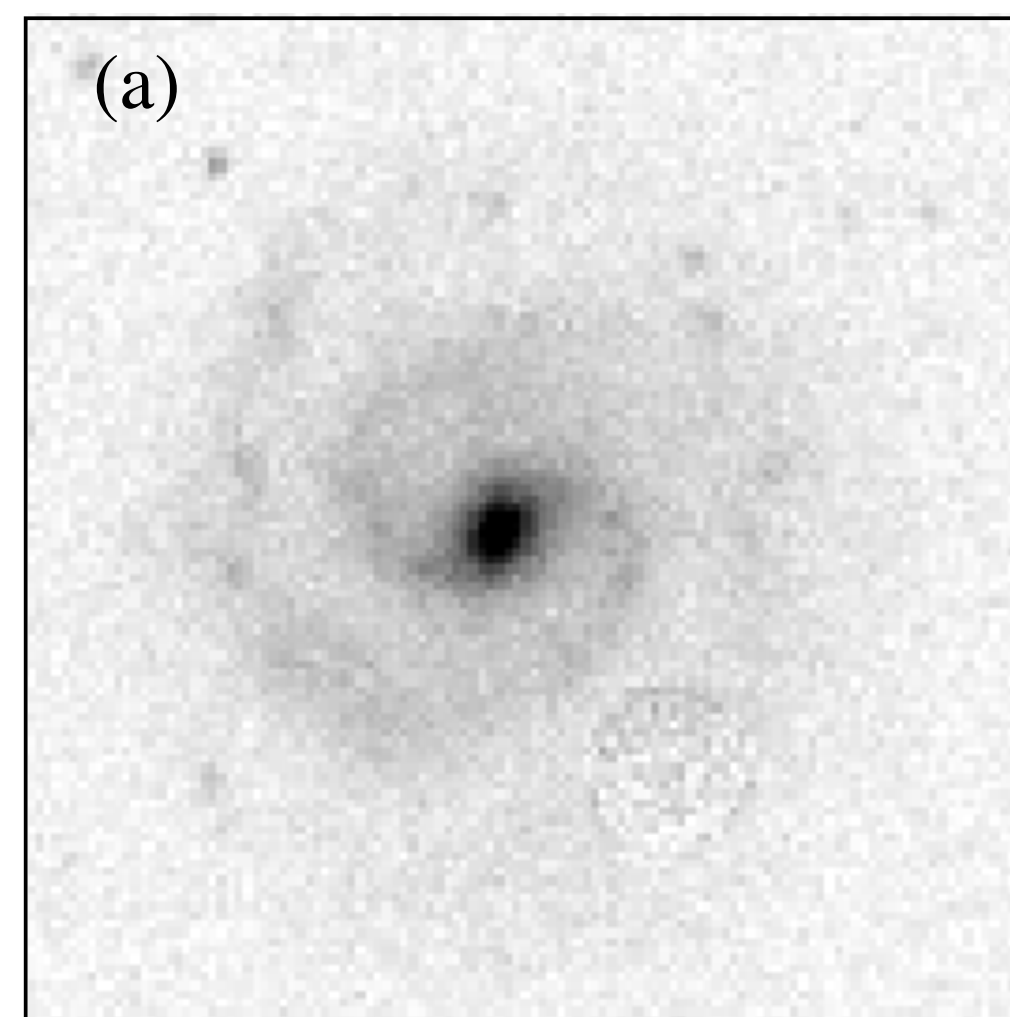


**Figure 10.** Relative bar pattern speed ( $\mathcal{R} = R_{CR}/R_{bar}$ ) as a function of bar strength ( $S_b$ ) for our sample (red triangles) and HSBs from [Aguerri et al. \(1998\)](#) (open circles). The solid line indicates the fit to the HSBs and LSBs, excluding the outlier discussed in the text:  $\mathcal{R} = 1.23 + 0.14S_b$ . The shaded region shows the scatter in the relation:  $\sigma = 0.13$ . The horizontal dashed line is the separator between fast and slow bars (i.e.  $\mathcal{R} = 1.4$ ).

The different stability properties at high and low surface brightness predict different morphologies. In DM, bars and spiral modes should be strongly suppressed. To generate them will require increasing the disk mass over that expected for ordinary stellar populations.

“In LSB disks, it is conceivable that the minimum disk mass required to generate spiral arms might exceed the maximum disk mass allowed by the rotation curve.” (McGaugh & de Blok 1998)

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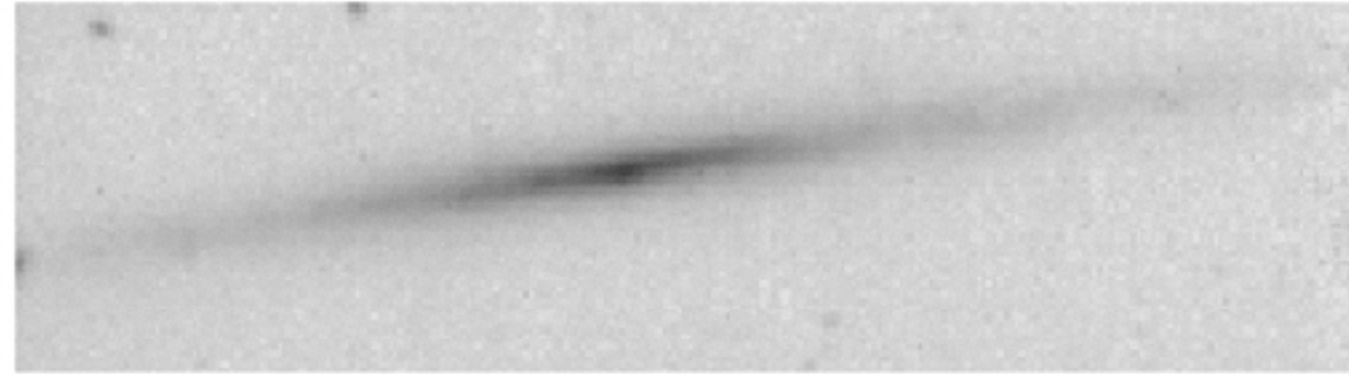
The **disk mass** required to drive the observed spiral arms is much larger than that expected for the **stellar population**.

*In this case, more disk mass is required than expected for stars (Fuchs 2003). Taken at face value, this is a contradiction to the existences of dark matter.*

Galaxy disks should flare less in MOND than in Newtonian dynamics.

Equivalently, they can sustain higher velocity dispersions without become unduly thick.

Super thin galaxy UGC 7321



Matthews, Gallagher, & van Driel (1999)

The outer, LSB regions of disks should have velocity dispersions of  $\sim 2$  km/s conventionally;  $\sim 7$  km/s is typically observed.

Conventionally, non-gravitational forces are invoked to explain the difference. These are not necessary in MOND.

