Dark Matter

ASTR 333/433
Spring 2026
TR 11:30am–12:45pm
Sears 552

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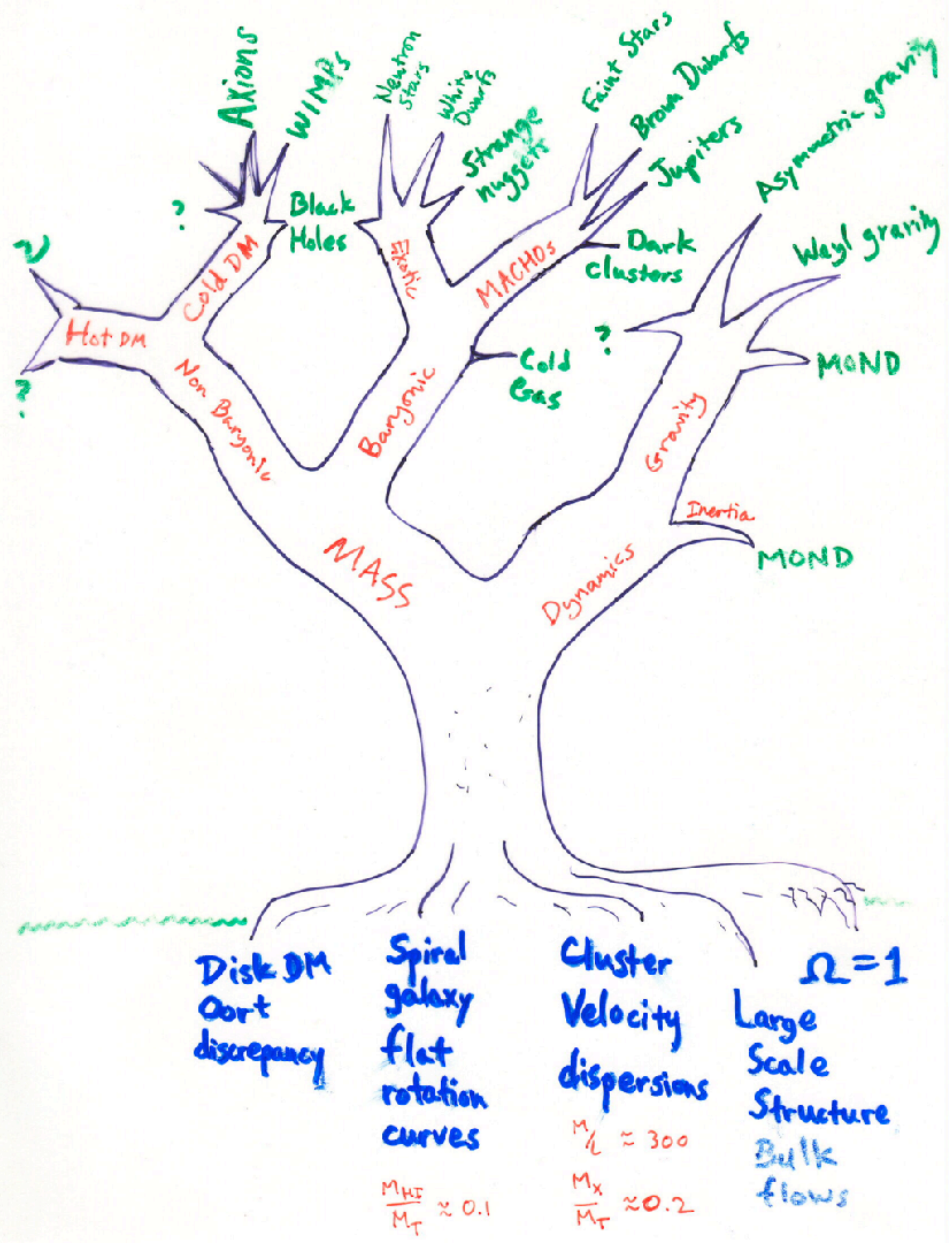


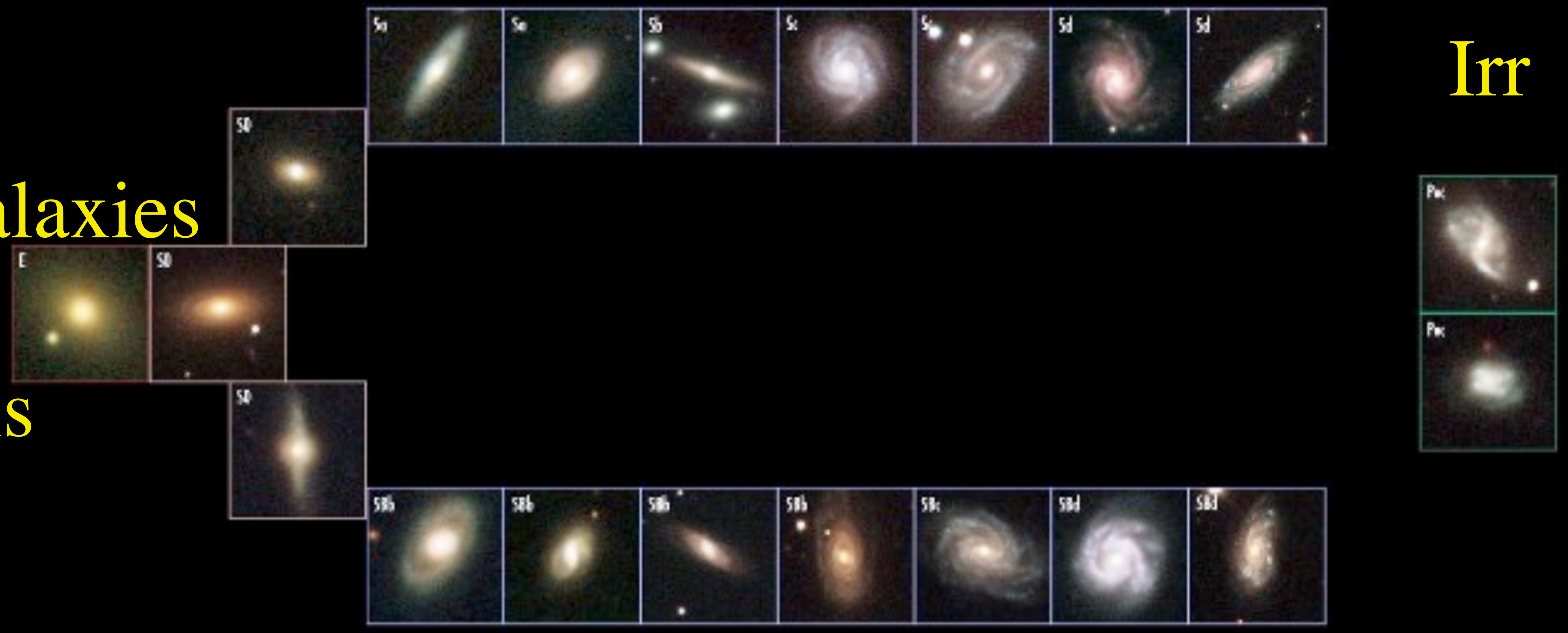
The original Hubble sequence stopped at Sc; subsequently extended to later types

Late Type Galaxies
LTGs

Sa Sb Sc Sd Sm

Irr

Early Type Galaxies
ETGs
Ellipticals



early type spirals

late type spirals & irregulars

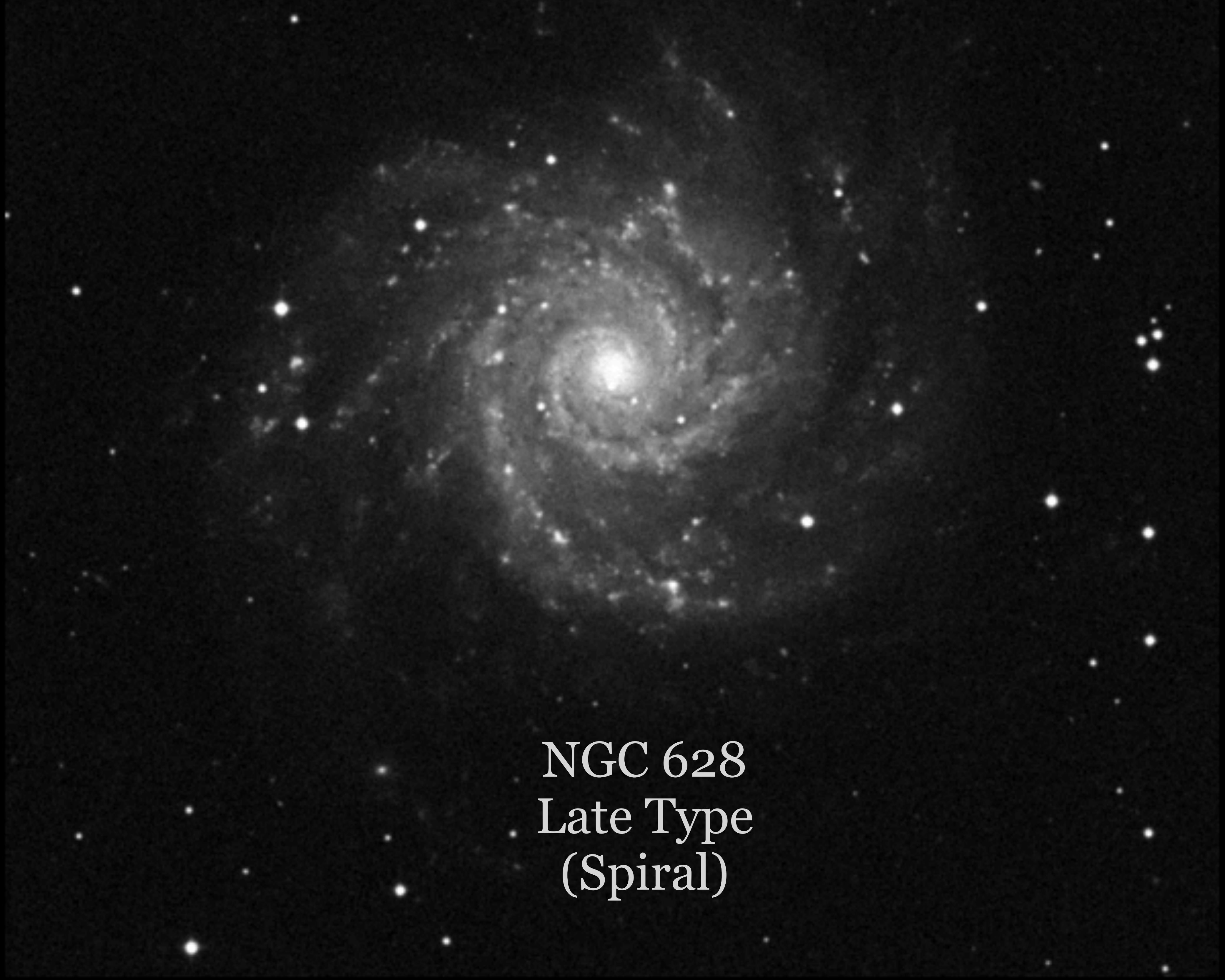
dynamically hot: $V/\sigma \lesssim 1$

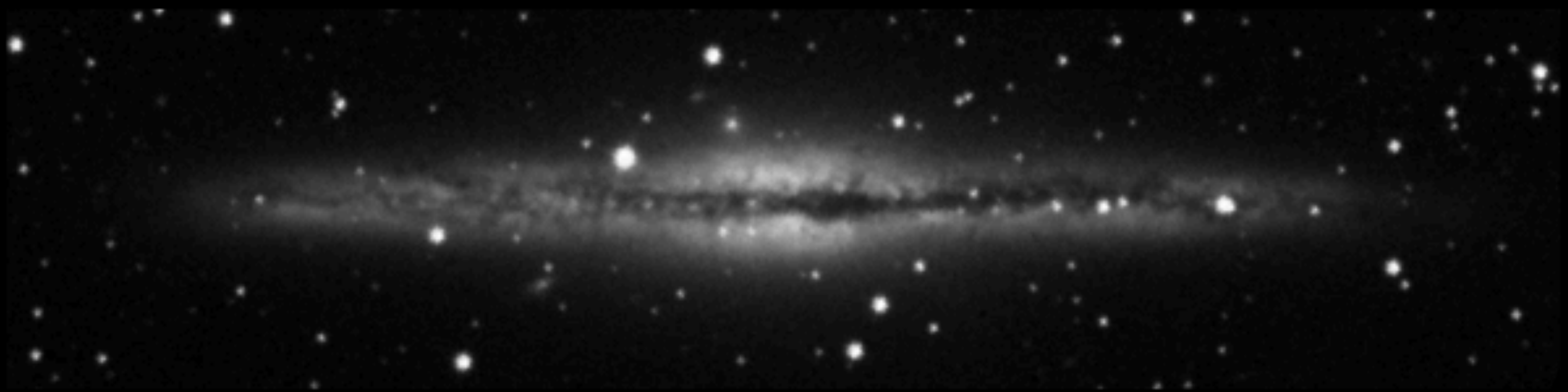
dynamically cold: $V/\sigma \gg 1$

can have a mix of dynamically hot (bulge) and cold (disk) components

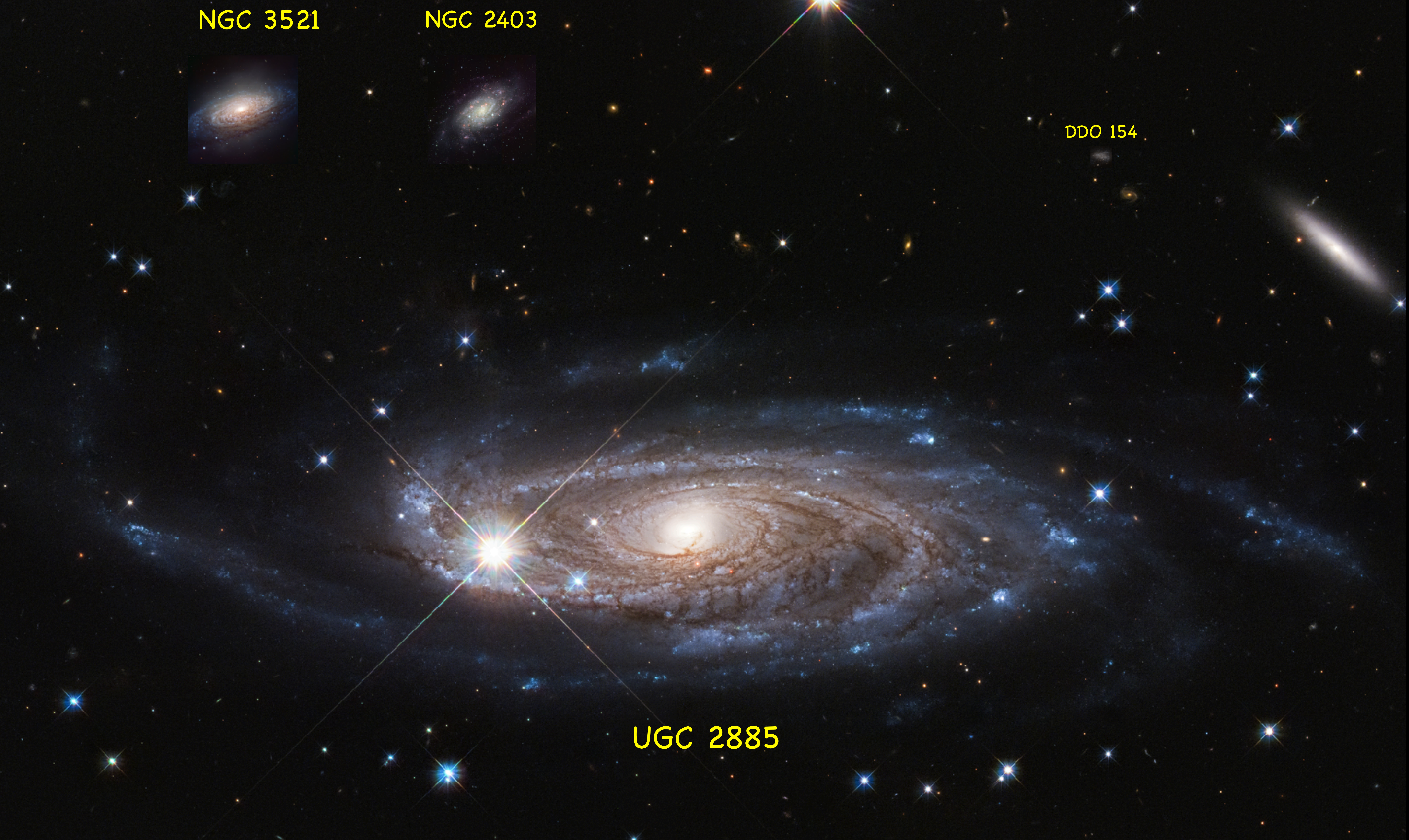
Early Type Galaxies
pressure supported
dynamically hot, eccentric, randomly oriented orbits

Late Type Galaxies
rotationally supported
dynamically cold, quasi-circular orbits in the same plane





NGC 891
(Edge-on Disk)


NGC 3521
NGC 2403
DDO 154
UGC 2885

Galaxies exist over a huge dynamic range in

Luminosity
$1x10^{7} < L_{[3.6]} < 5x10^{11}\ L_{\odot}$

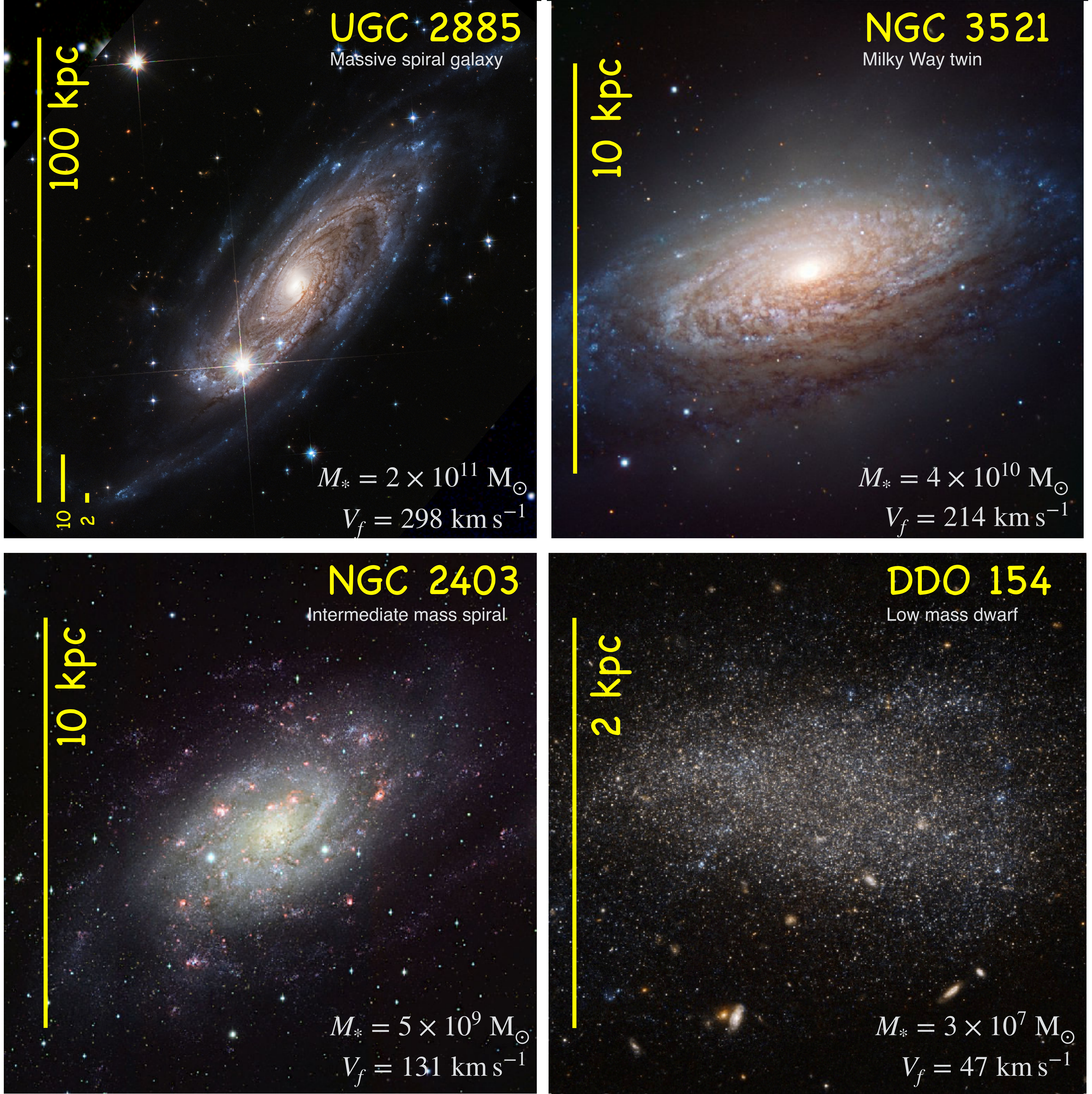
Gas mass
$1x10^{7} < M^{*} < 5x10^{10}\ M_{\odot}$

Surface brightness
$5 < \mu_{e} < 3x10^{3}\ L_{\odot}\,pc^{-2}$

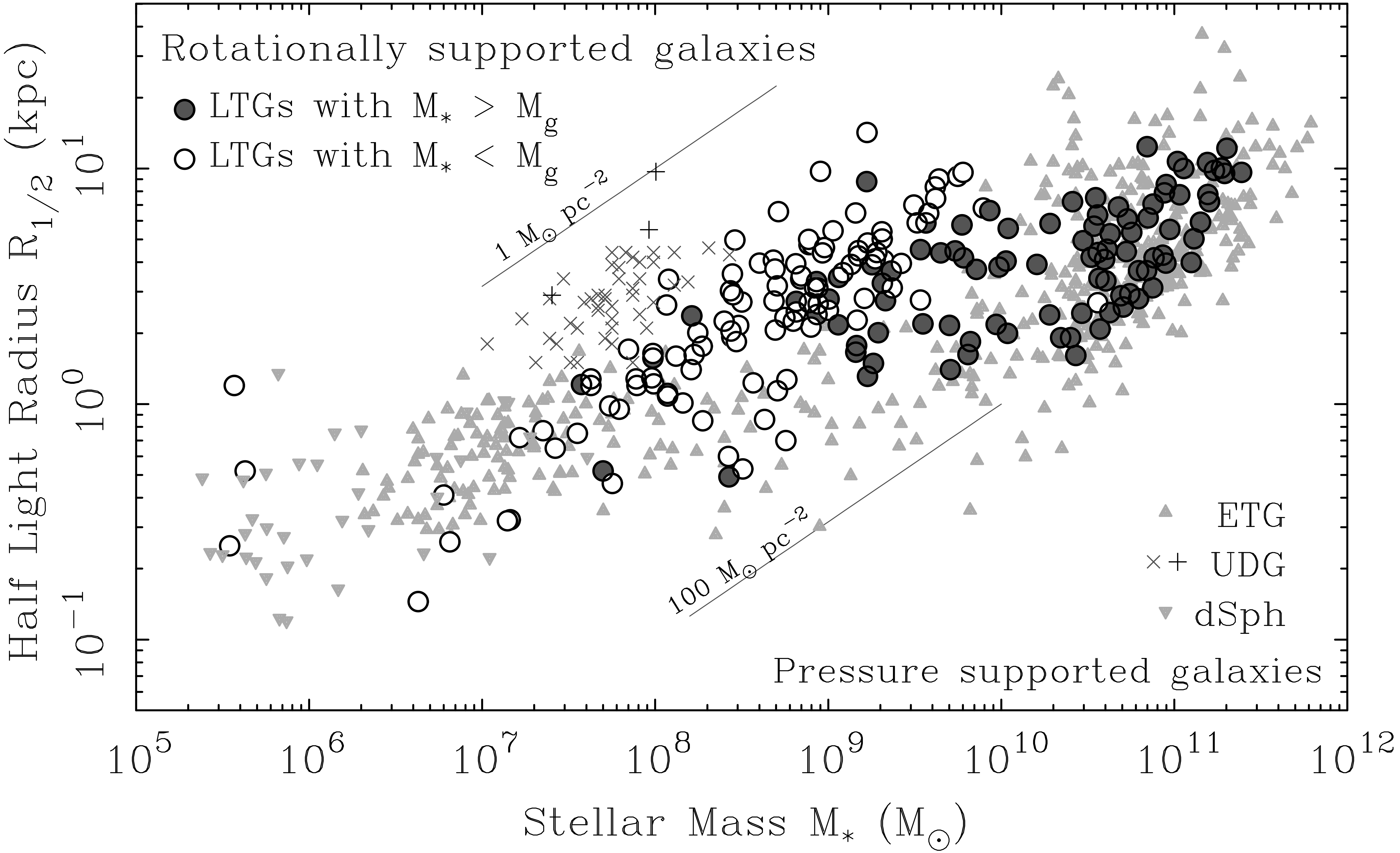
Gas fraction
$0.03 < f_{g} < 0.97$

Rotation velocity
$15 < V_{f} < 300$ km/s

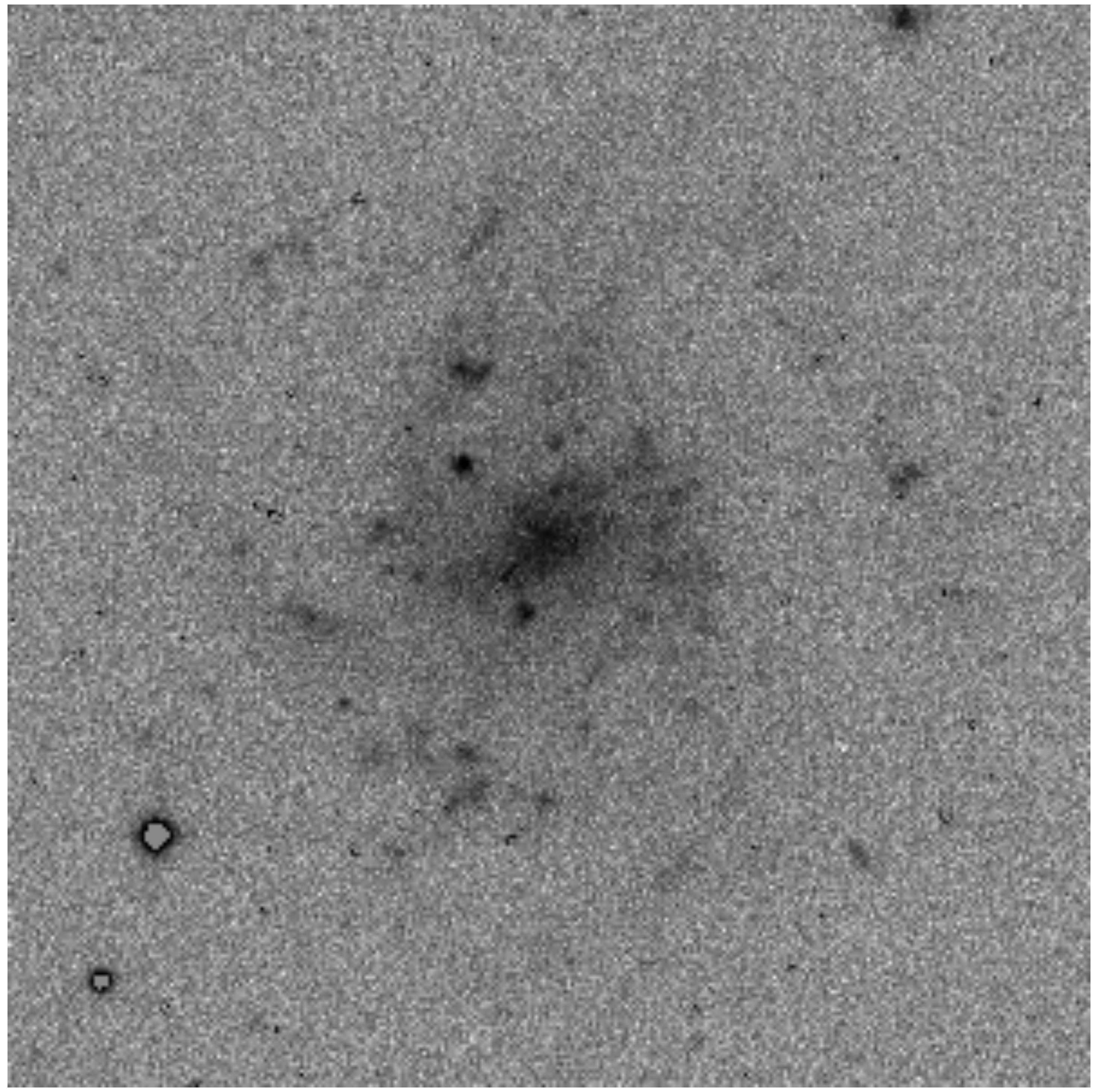
and probably more -
the faint/dim end is always limited by selection effects.

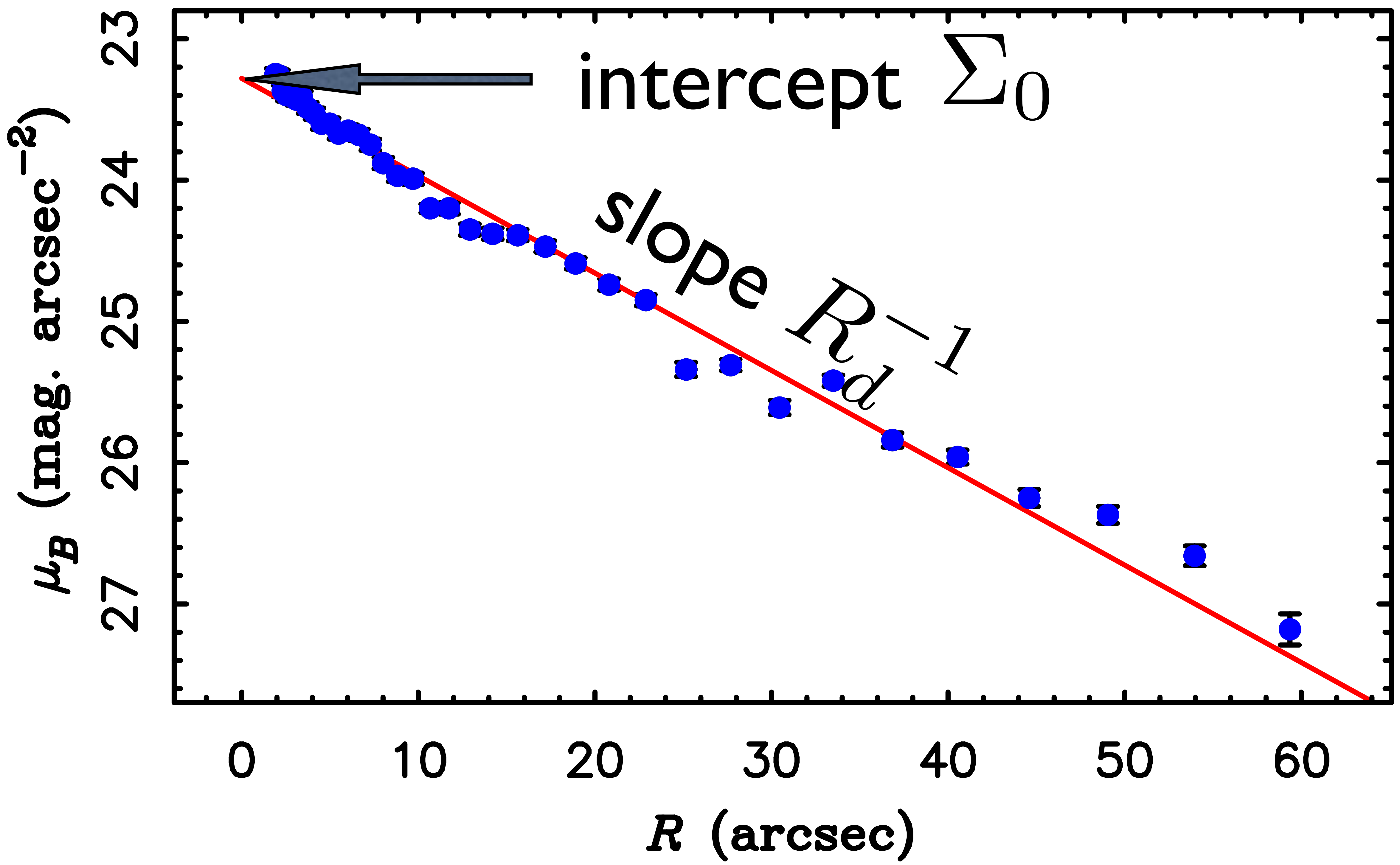


Sizes and masses of galaxies



Late Type Galaxies are typically Exponential disks





$$\Sigma(R) = \Sigma_0 e^{-R/R_d}$$

Azimuthally averaged light distribution approximately exponential for spiral disks.

Surface brightness: conversion between linear Σ in $L_\odot$ pc^{-2} and logarithmic μ in magnitudes arcsecond^{-2}: $\mu = 21.57 + M_\odot - 2.5\log\Sigma$ where the absolute magnitude of the sun $M_\odot$ is bandpass-specific. See useful numbers page and astronomical magnitude systems.

The surface brightness profile is obtained by fitting ellipses to galaxy images, as in this example from Schombert (2007) using ARCHANGEL.

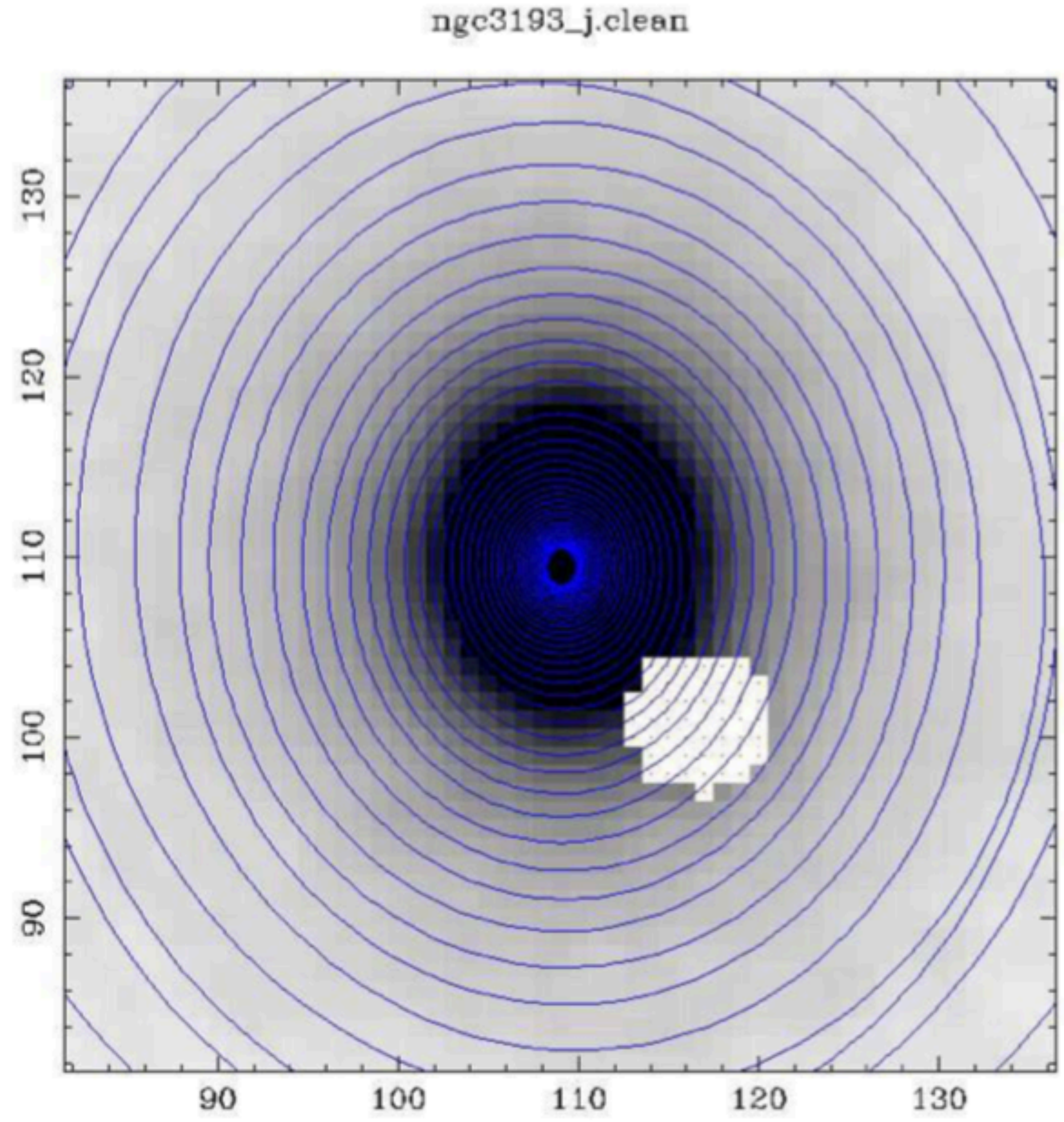
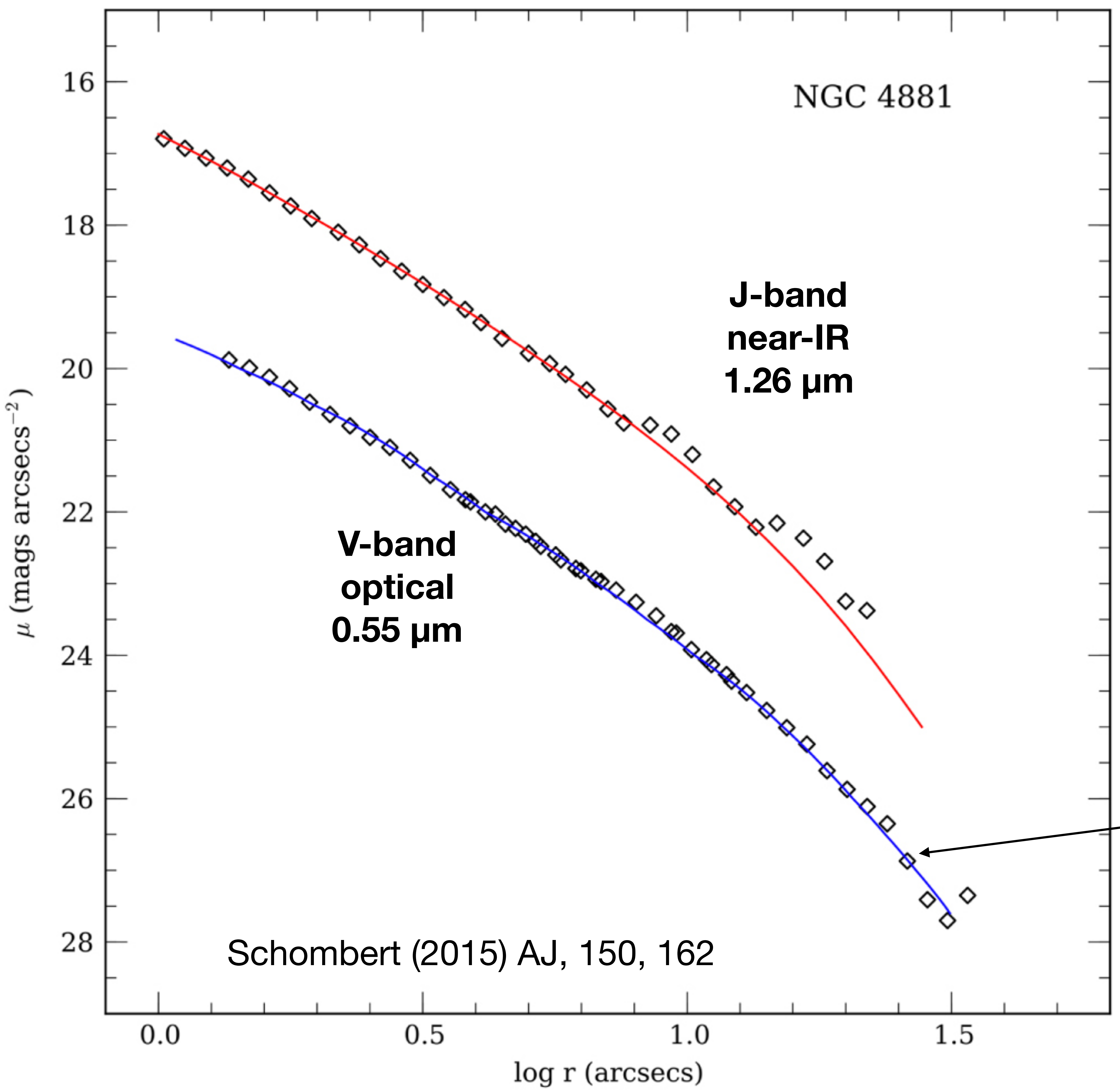
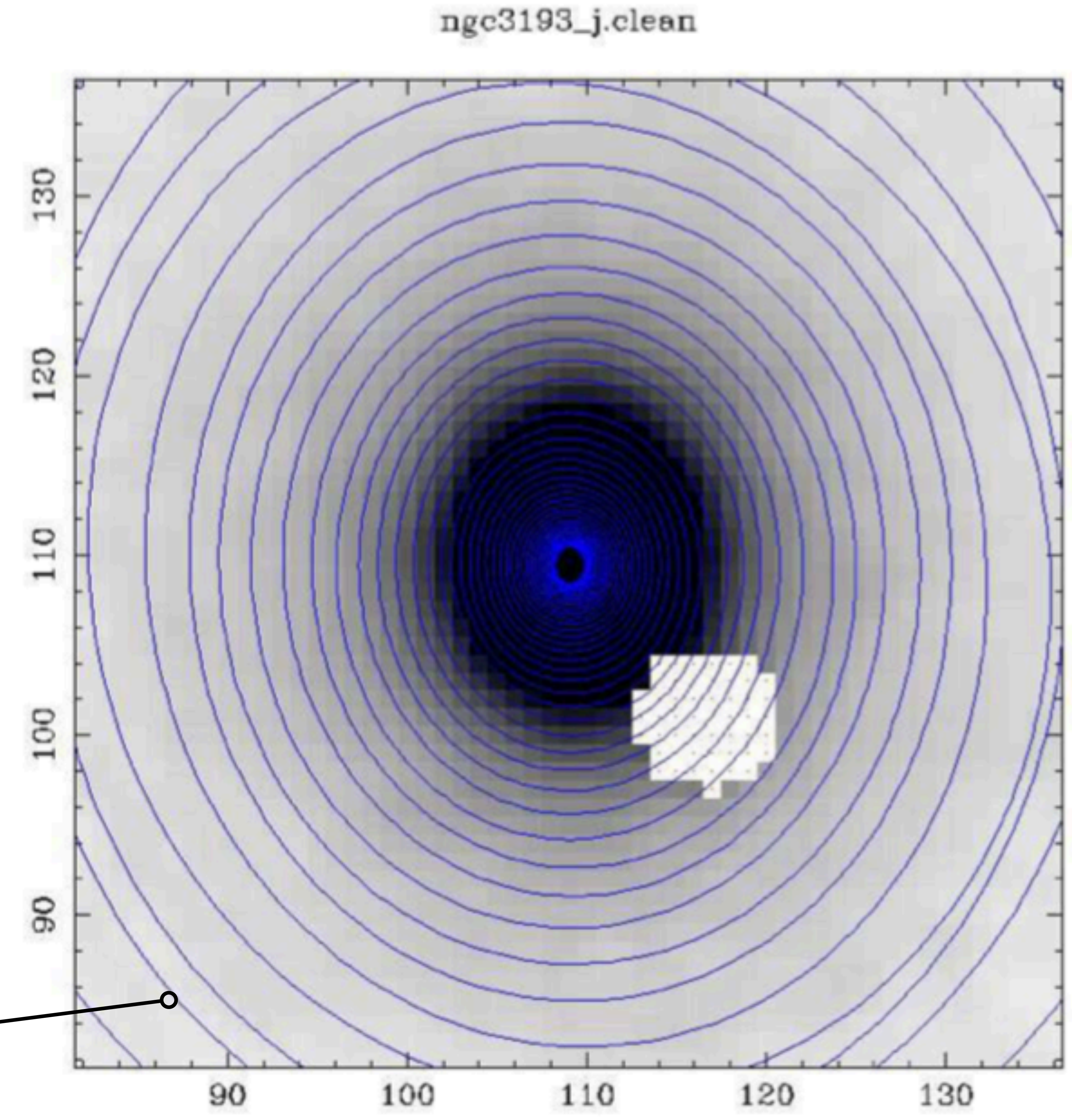


Fig. 2.— The resulting ellipse fits to NGC 3193's core region. While the automatic masking of the contaminating star is not perfect, it is sufficient to maintain a high quality fit.

Early Type Galaxies typically modeled as de Vaucouleurs $r^{1/4}$ profiles

see Schombert (2015) AJ, 150, 162

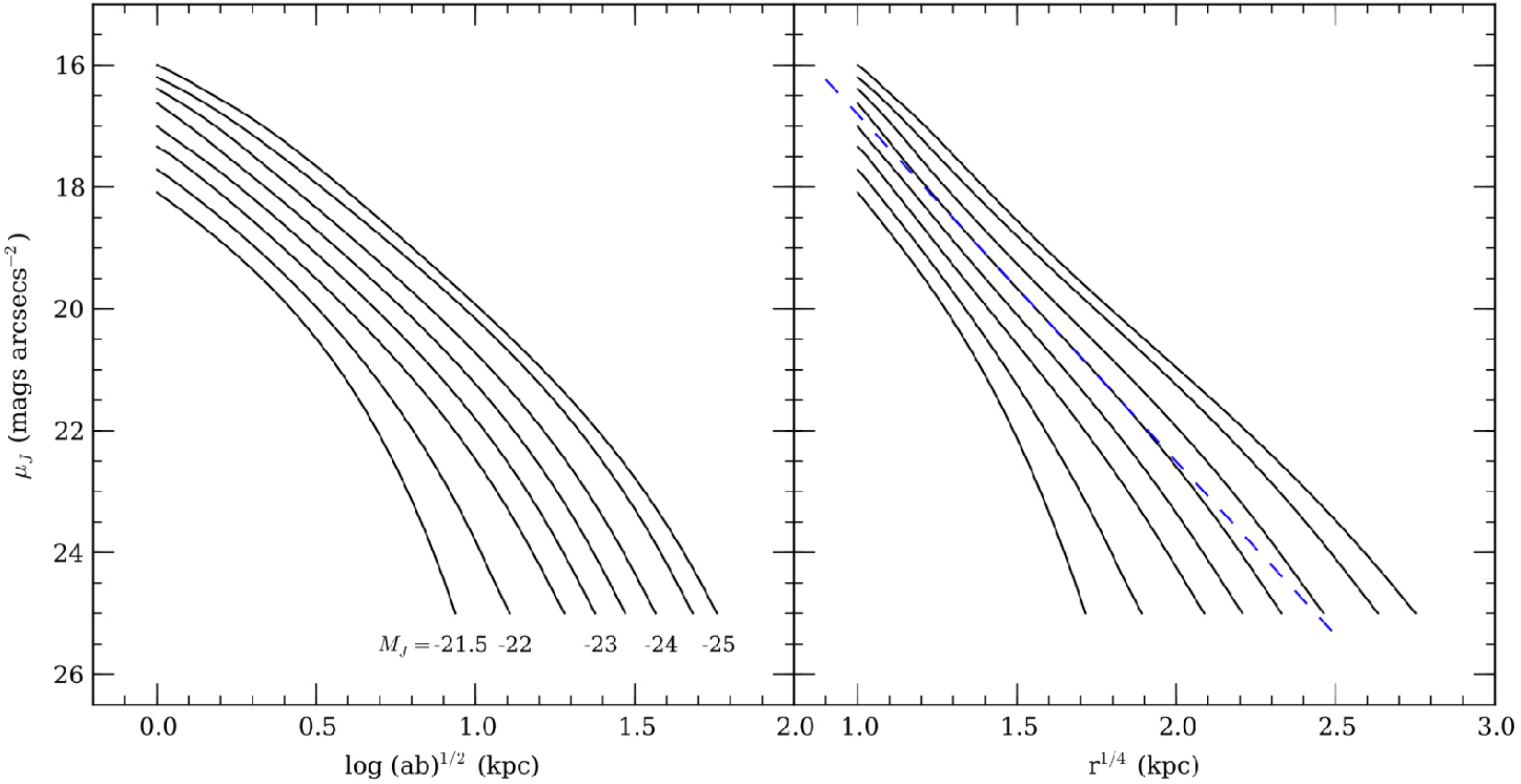




each point corresponds to one ellipse on the image

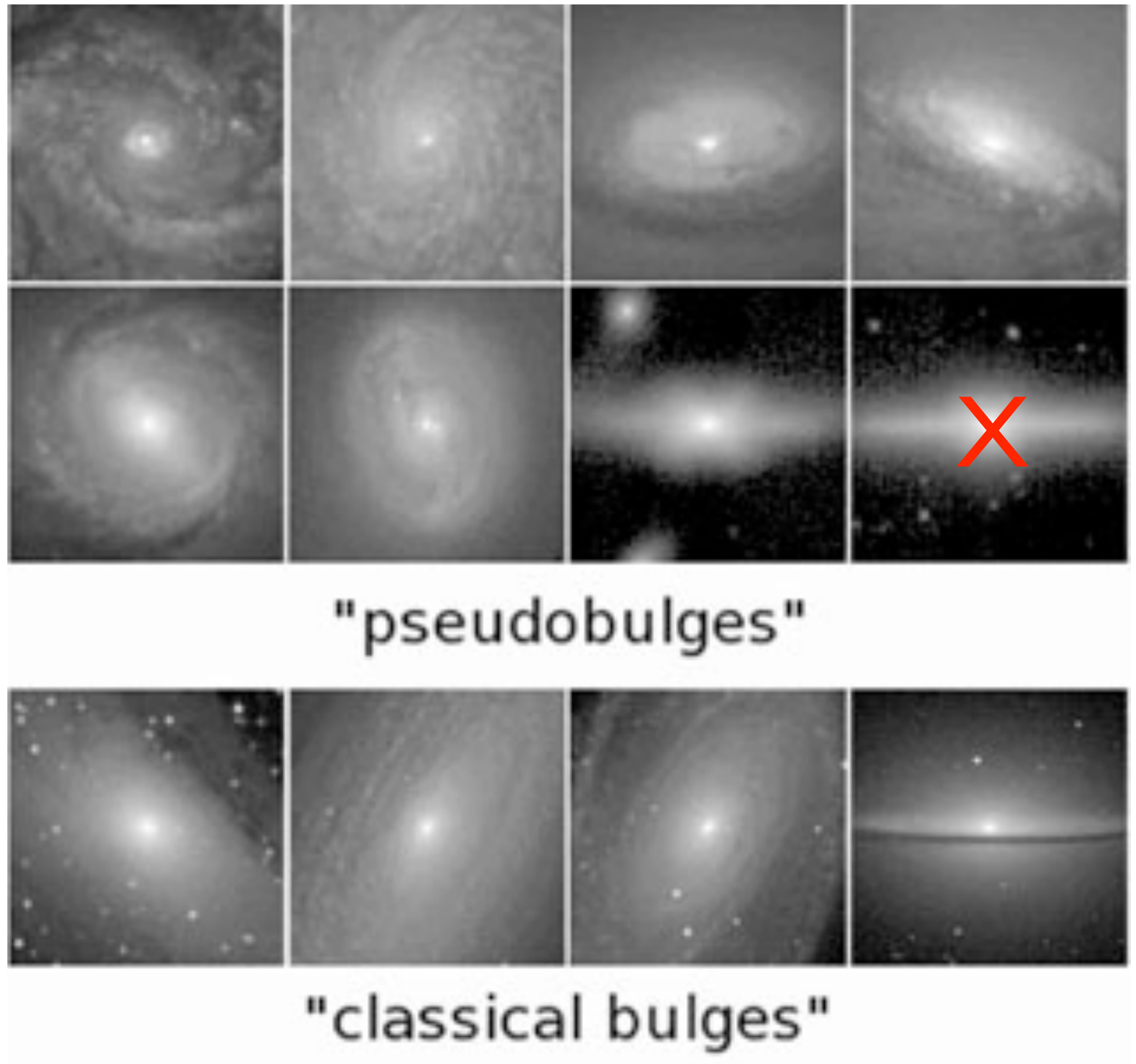
ETG profiles vary systematically with luminosity

Schombert (2015) AJ, 150, 162



Profiles would be straight lines here if the de Vaucouleurs profile were a perfect representation of ETGs

Classical bulges tend to have Sersic indices close to n=4 (de Voucoulers profile)



X/peanut shape characteristic of bars seen edge-on

Pseudo-bulges have various Sersic indices, often closer to n=1 (exponential) than to n=4 (de Voucoulers profile)

Oort limit - imagine the disk as a plane parallel slab

First, think of balancing KE with PE for a small mass m orbiting a big mass M: $\frac{1}{2}mv^2 \sim \frac{GMm}{r}$

So we can solve for the big mass M: $v^2 \sim \frac{2GM}{r}$

Now, instead of a big mass M, think of a circular patch of radius r and surface density Sigma (in M_{sun}/pc^2). It has a total mass: $M \sim \Sigma_0 \pi r^2$

So plug that in and get $v^2 \sim 2\pi G \Sigma_0 r$

Or, now thinking about a group of stars: $\sigma_z^2 \sim 2\pi G \Sigma_0 z_0$

So if we measure velocity dispersions and scale heights for groups of stars, we can measure the mass density of the Galaxy's disk. This was first done in the early 1960s by Jan Oort and is called the Oort limit. A recent (and more sophisticated) analysis gives ~ 70 M_{sun}/pc^2.

Now let's just add up all the mass we see:

Stars	25 M_{sun}/pc^2
Stellar remnants (mostly WDs)	20 M_{sun}/pc^2
Gas (HI+H2)	5 M_{sun}/pc^2
Total	50 M_{sun}/pc^2

From **Sparke & Gallagher**

Cylindrical coordinates

Poisson equation: $\nabla^2\Phi = 4\pi G\rho$

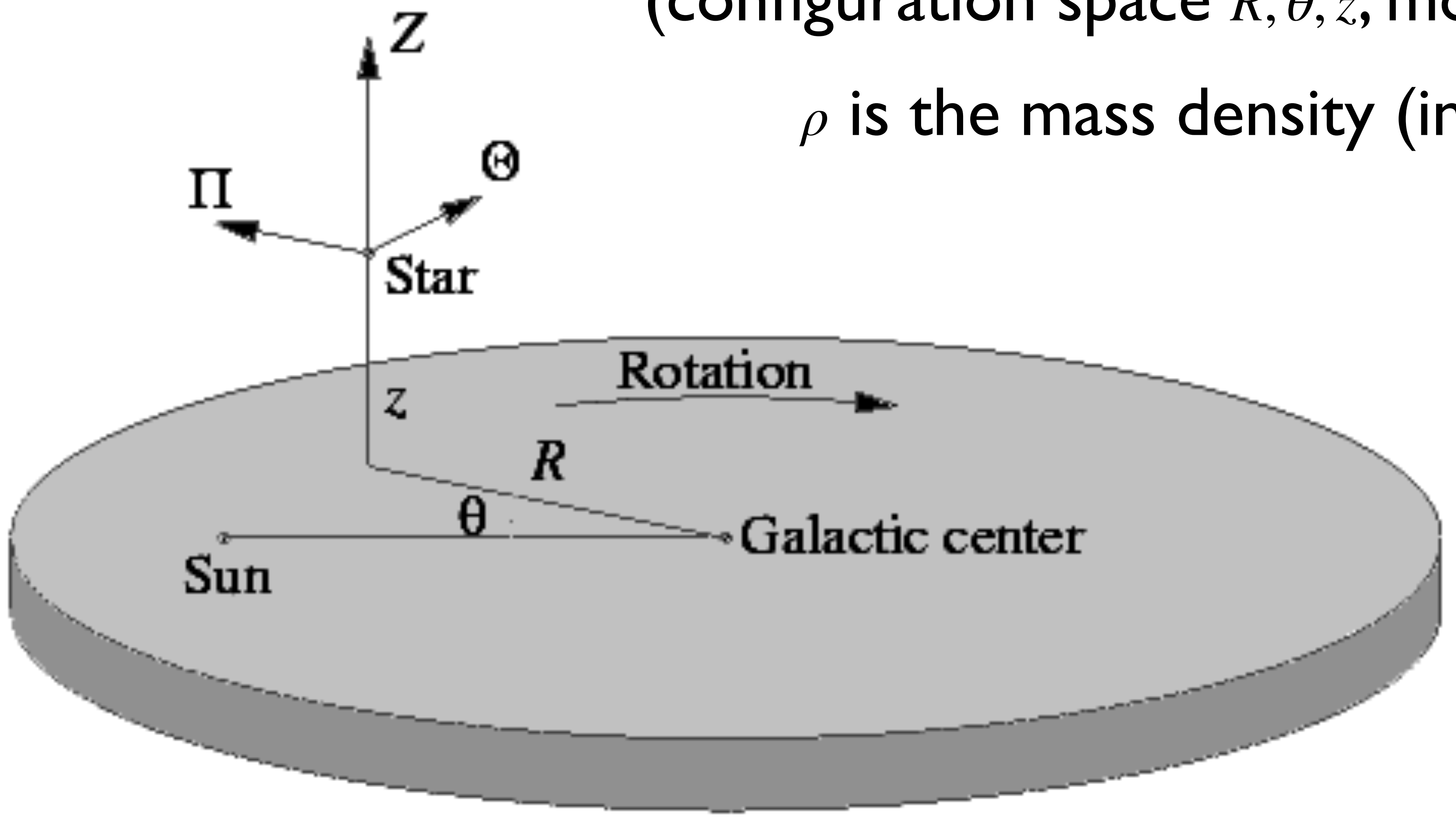
in cylindrical coordinates $$\nabla^2\Phi = \frac{1}{R}\frac{\partial}{\partial R}\left(R\frac{\partial\Phi}{\partial R}\right) + \frac{1}{R^2}\frac{\partial^2\Phi}{\partial\theta^2} + \frac{\partial^2\Phi}{\partial z^2}$$

Let's define a coordinate system:

Galactocentric cylindrical coordinates

We wish to extract the gravitational potential Φ from observations of the 6D phase space of stars (configuration space R, θ, z; momentum Π, Θ, Z)

ρ is the mass density (including dark matter)



Newtonian gravity is linear, so you can treat each source of the gravitational potential separately and sum them up:

$\rho = \rho_* + \rho_g + \rho_{DM}$;

$\Phi = \Phi_* + \Phi_g + \Phi_{DM}$.

Position : (R, θ, z)

- R = galactocentric distance
- theta = azimuthal coordinate
- z = height above/below the plane

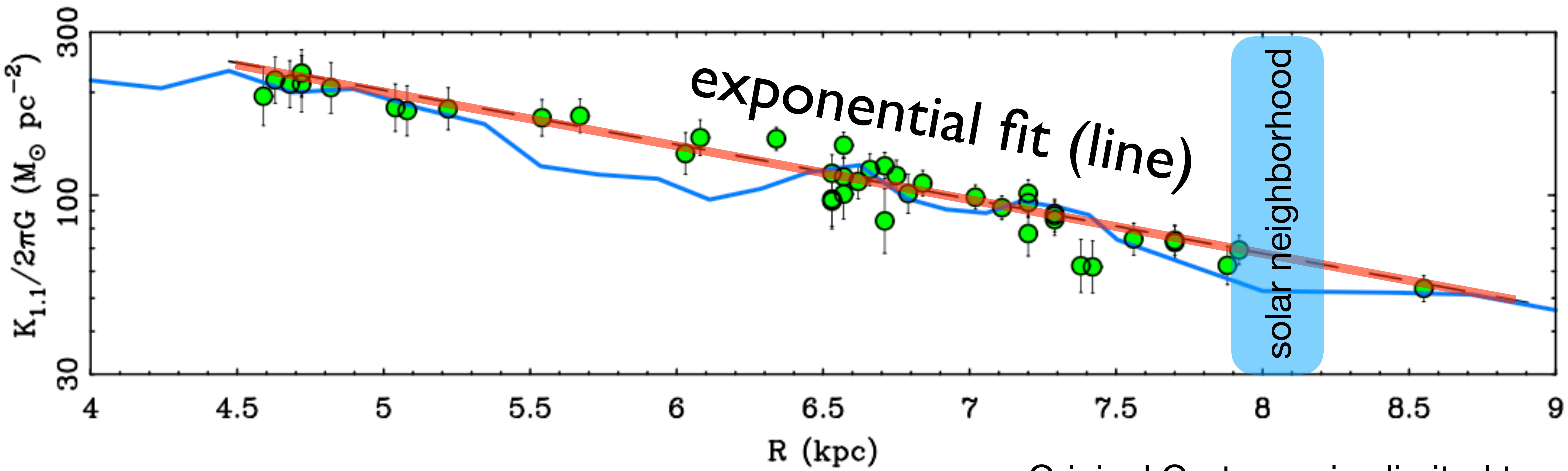
Velocity : (Π, Θ, Z)

- Pi = velocity in/out from center
- Theta = tangential velocity
- Z = velocity up and down

OR Cartesian (X,Y,Z; U,V,W) centered on either the sun or the Galactic Center.

Assuming axial symmetry and that $\Phi(R,z)$ is separable, the vertical force is

$$K_Z = 2\pi G\Sigma + \frac{Z}{R}\frac{\partial V^2}{\partial R}$$



Original Oort exercise limited to the solar neighborhood. Can now expand to other radii.

$$\Sigma(R) = \Sigma_\odot e^{-\frac{(R-R_\odot)}{R_d}}$$

$$\Sigma_\odot = 38\ \mathrm{M_\odot\ pc^{-2}} \quad \text{(stars only)}$$

$$R_d = 2.15\ \mathrm{kpc} \quad \text{Bovy \& Rix (2013)}$$

Continues to improve with surveys like Gaia and APOGEE, e.g.,

Price-Whelan et al. (2021, ApJ, 910, 17)
Eilers et al. (2019, ApJ, 871, 120)
McGaugh (2019, ApJ, 885, 87)

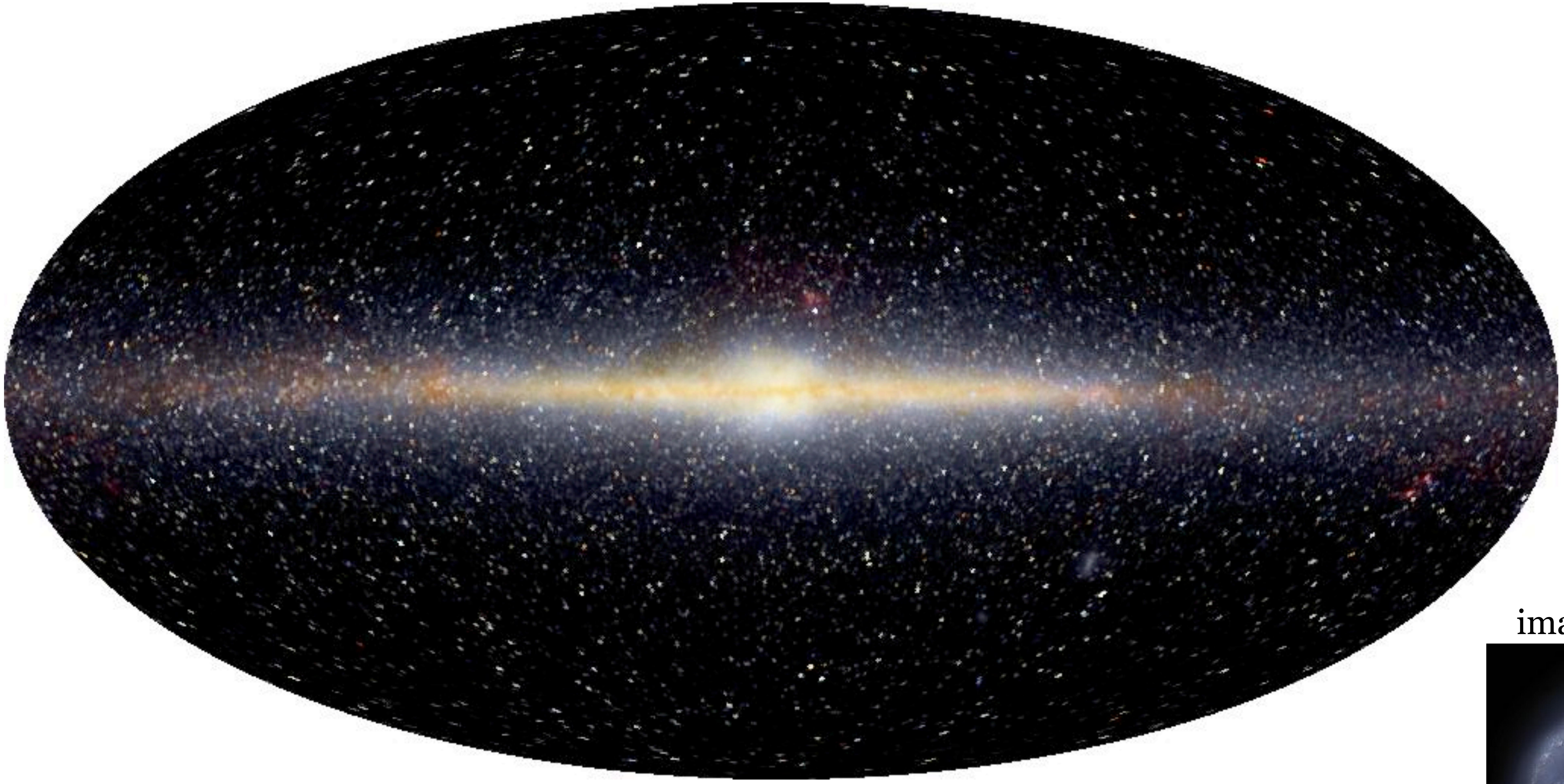
Disk Stability

NGC 628: a spiral galaxy

NGC 1300: a barred spiral galaxy

The Milky Way is a barred spiral

observed from within



Peanut-shaped bulge is the signature of a bar seen edge-on.
Our viewing angle is 20 - 30^{o} from the major axis of the bar.

imagined face-on

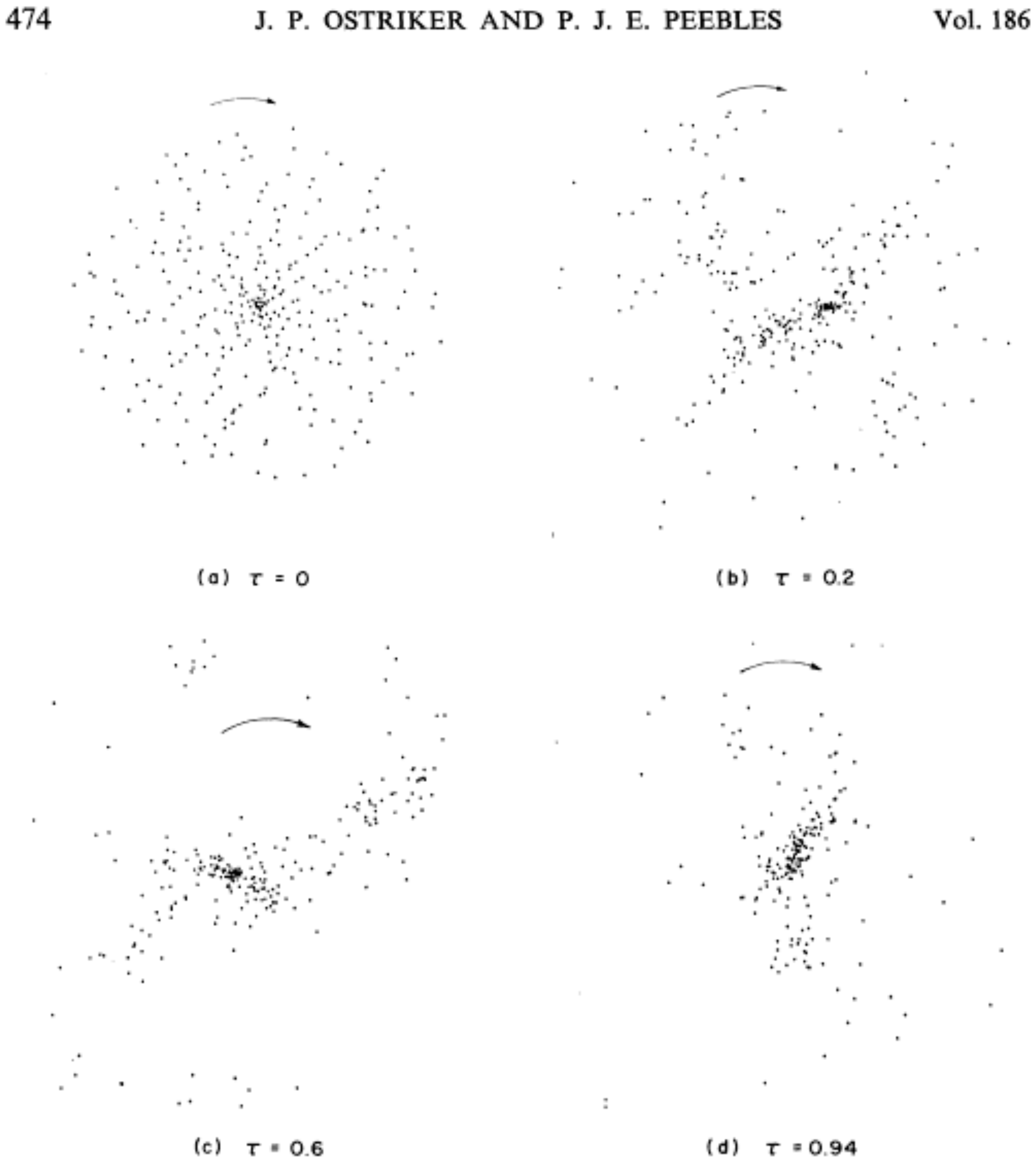
The Bar Instability

Spiral disks unstable to the development of m=2 bar modes.

Left to themselves, spiral disks fall apart in just a few dynamical times (< 1 Gyr for the Milky Way).

Cold disks are unstable if left to themselves, So Ostriker & Peebles suggested embedding them in dark matter halos.

By “cold” we mean that ordered rotation exceeds random motions: $V \gg \sigma$.



Ratio of kinetic energy in rotation to potential energy

$$t = \frac{T}{|W|}$$

O&P found we need

$$t < 0.14$$

for stability rather than the observed

$$t \approx 0.5$$

- need a big dark matter halo to contribute to the potential energy

Ostriker & Peebles (1973)

Sellwood (2016)

http://burro.astr.cwru.edu/Academics/Astr222/Galaxies/Spiral/nohalo.mpg

http://burro.astr.cwru.edu/Academics/Astr222/Galaxies/Spiral/halo.mpg

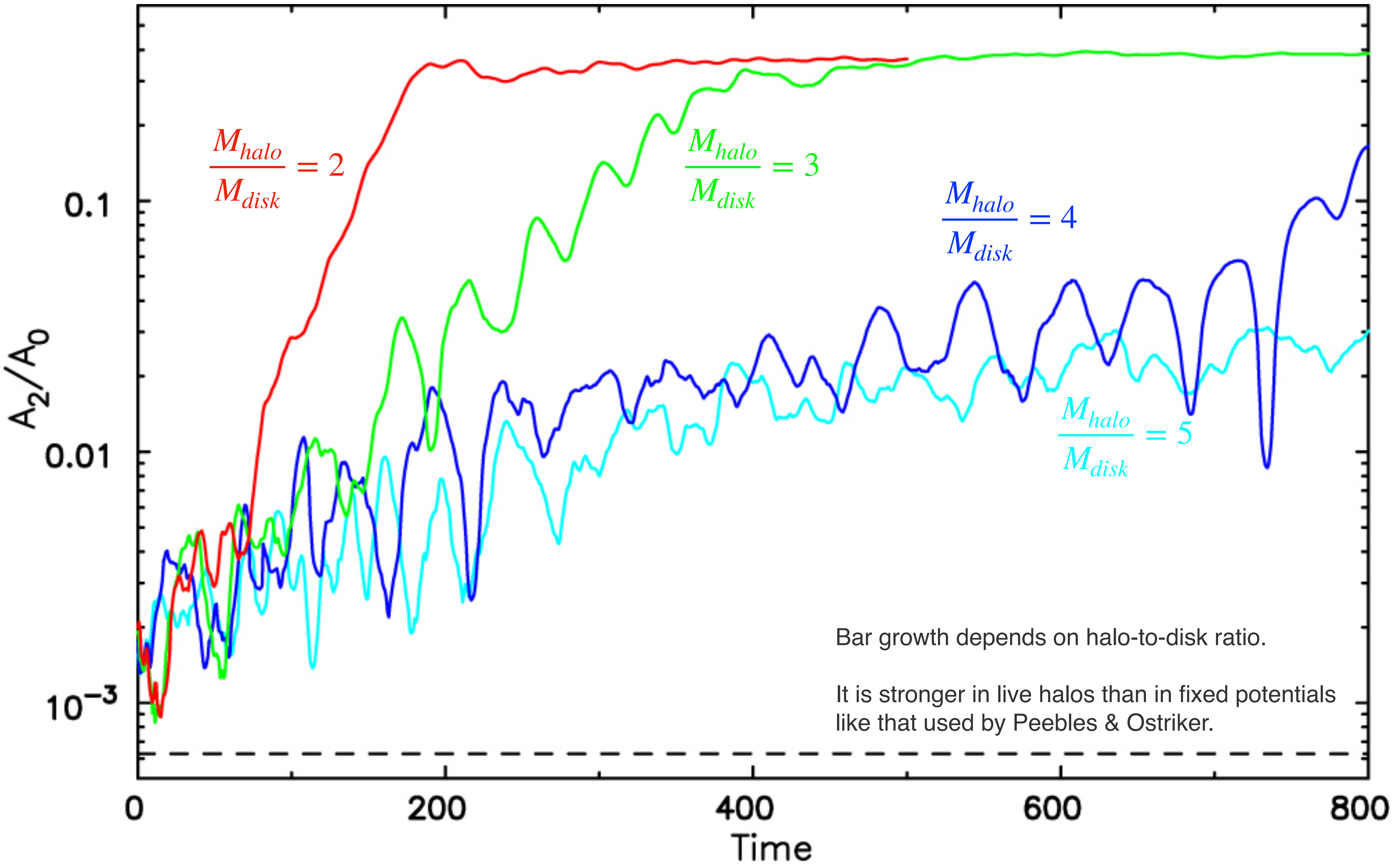
Sellwood (2016)

Time evolution of the bar amplitude in models with differing $\frac{M_{halo}}{M_{disk}}$

bar amplitude depends on halo-to-disk mass ratio

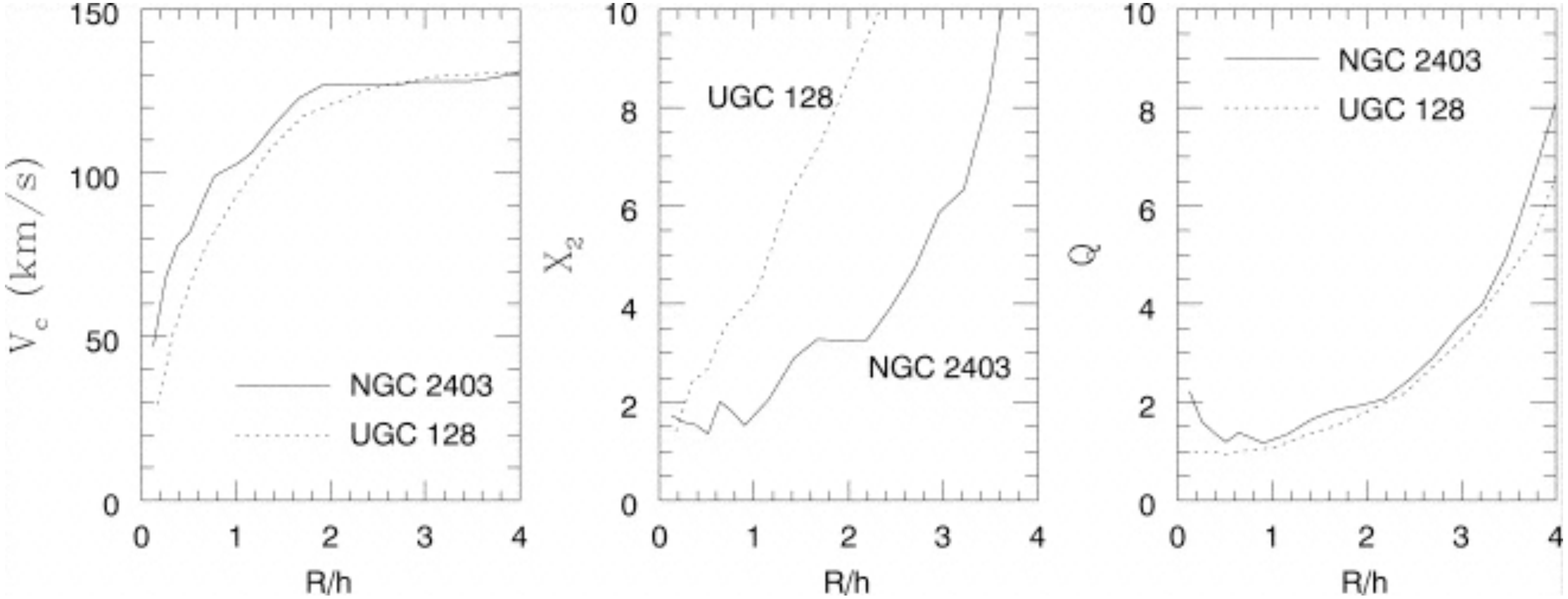
$$A_m = \frac{1}{N}\sum_{j=1}^{N} e^{i[m\theta_j + p\ln(R_j)]}$$

A bar is an m=2 mode with p=0



bar amplitude depends on disk surface mass density

NGC 2403: high surface brightness
UGC 128: low surface brightness



$$X_m = \frac{\kappa^2 R}{2\pi m G \Sigma}$$

stable for $X > 3$ for a flat RC;
$X > 1$ suffices for rising RC

global *stability criterion*

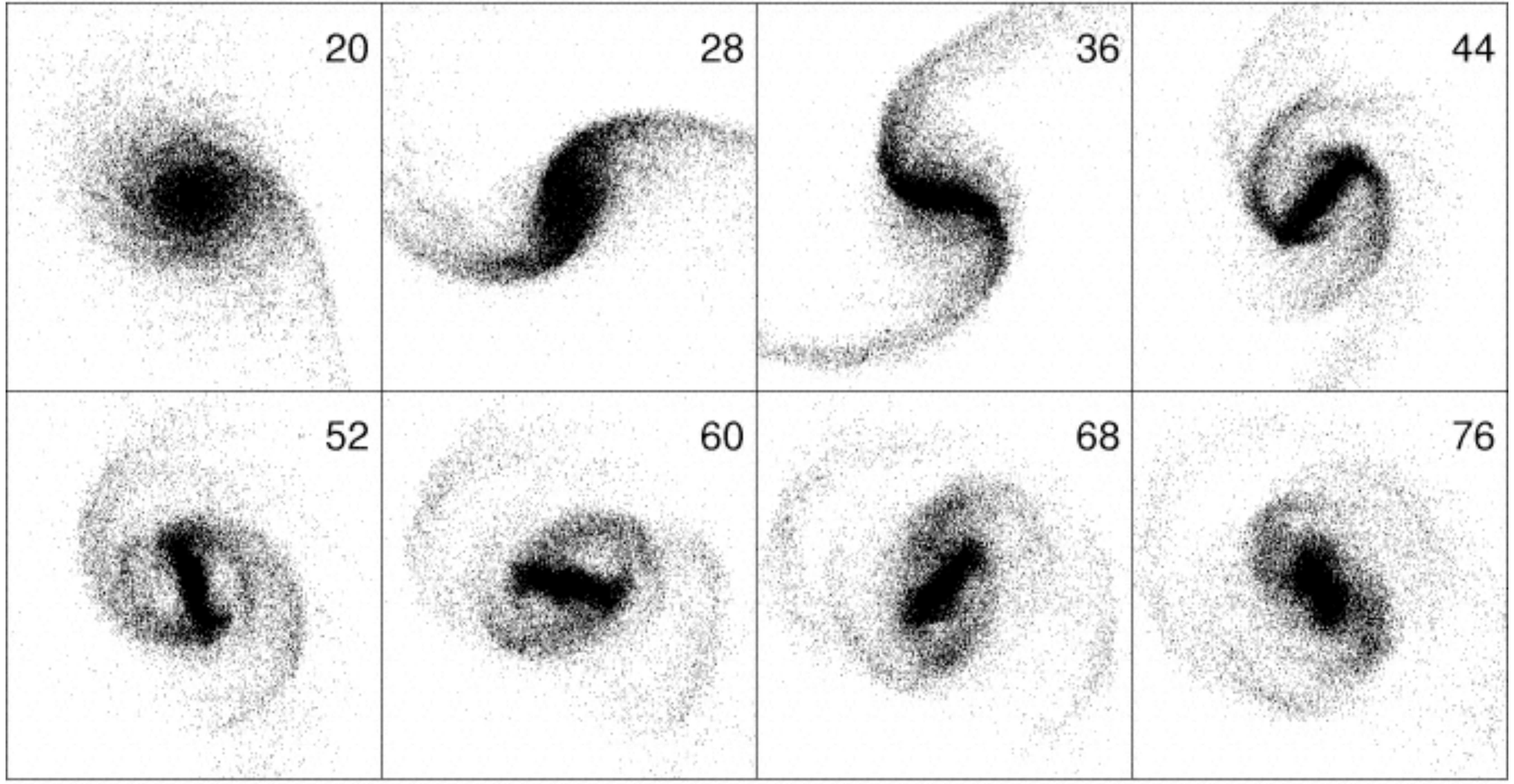
$$Q = \frac{\sigma_r \kappa}{3.36 G \Sigma}$$

stable for $Q \gtrsim 1$

local *stability criterion*

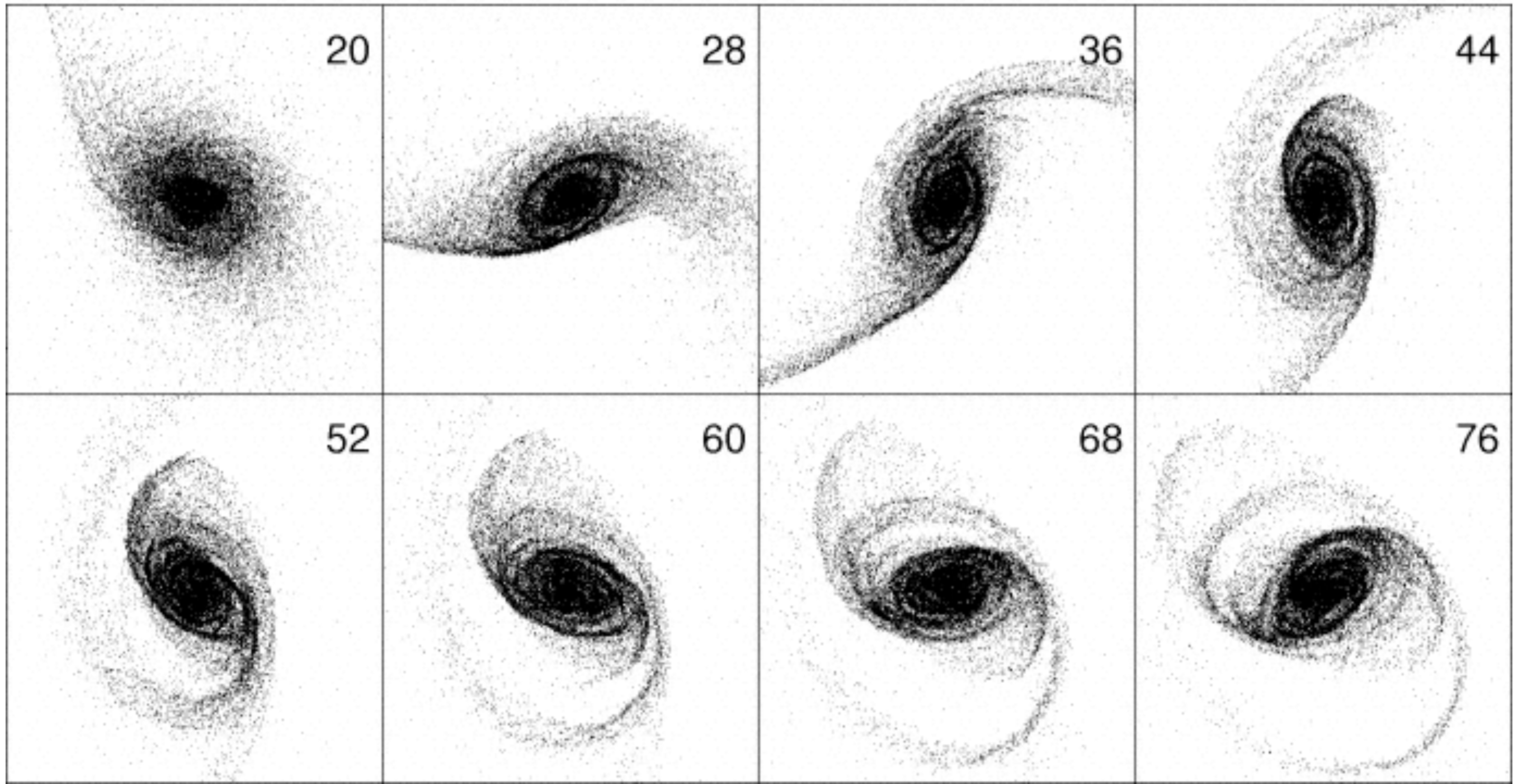
High surface density

Low X_2, marginal stability



Low surface density

High X_2, high stability



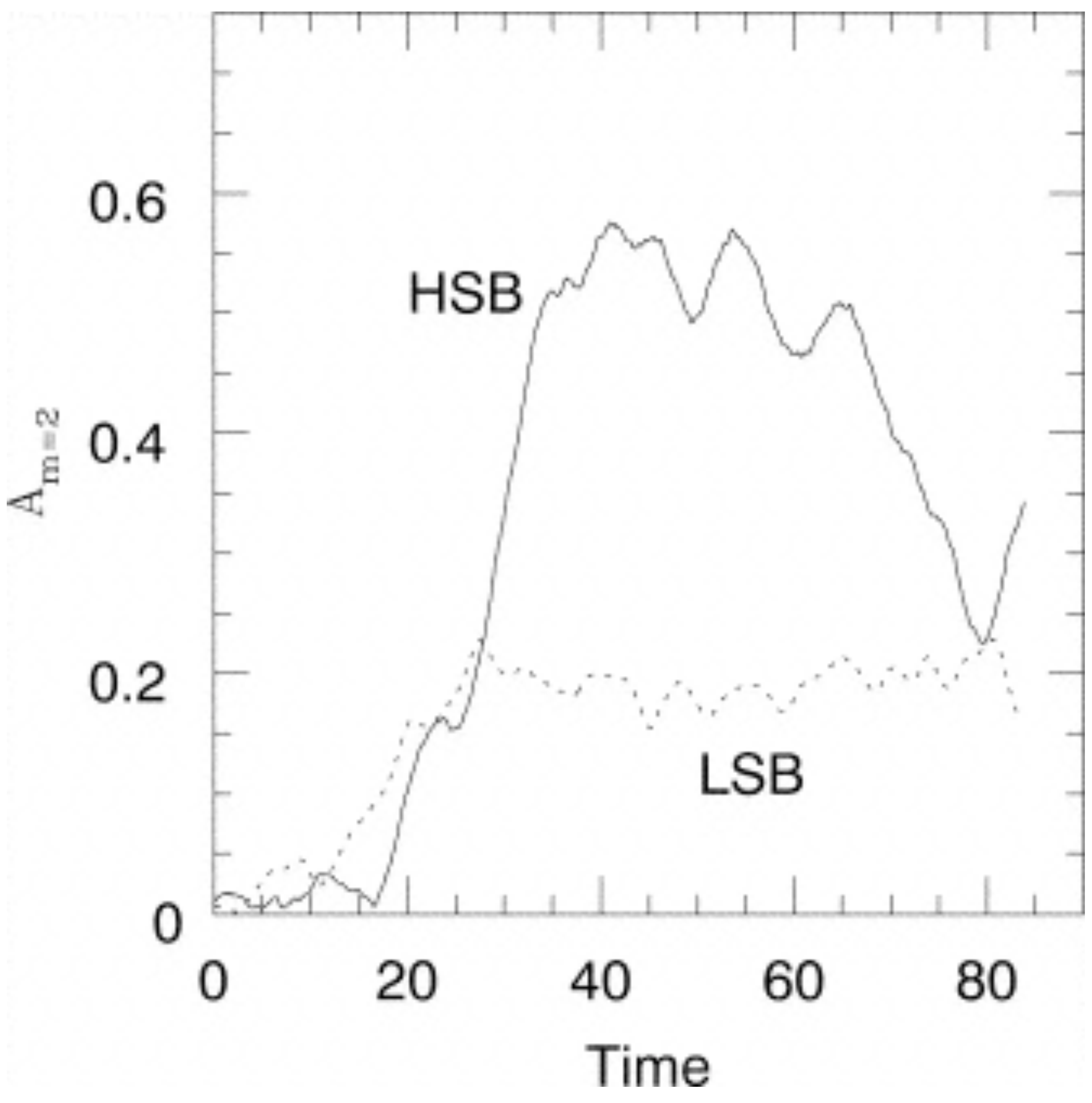
A bar is an m=2 mode

$$A_m = \frac{1}{N}\sum_{j=1}^{N} e^{i[m\theta_j + p\ln(R_j)]}$$

A two-armed spiral is an m=2 mode with a pitch angle p > 0.

A four armed-spiral has m=4, etc.

Amplitude of Bar



Low X_2, marginal stability, high A_2

High X_2, high stability, low A_2

https://www.youtube.com/watch?v=gSwiXwP56js

http://www.youtube.com/watch?v=byI9yhITDsM