DARK MATTER

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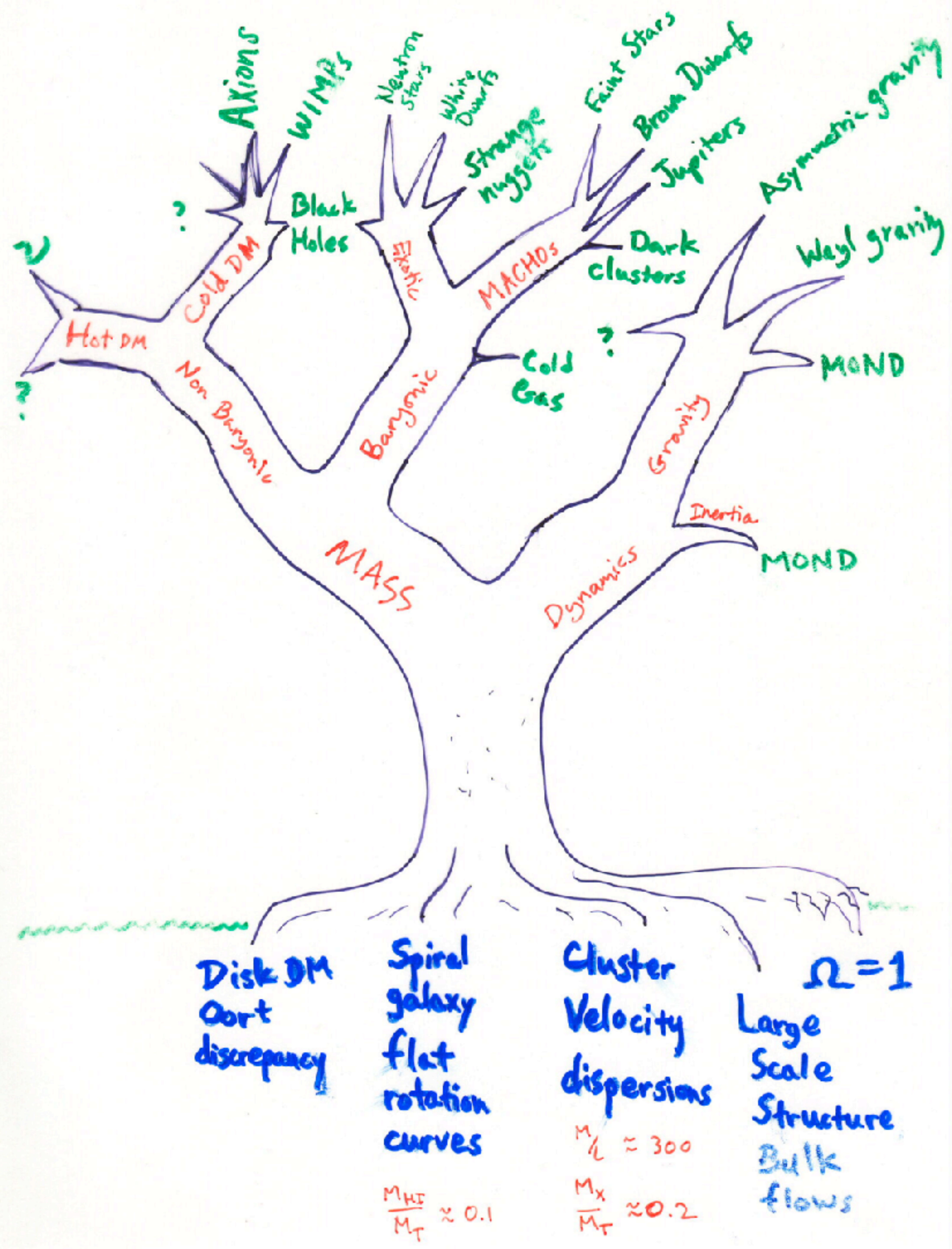


So far we have discussed

Three early observational indications of dark matter

Galactic & Extragalactic systems that evince an acceleration discrepancy

- Clusters of Galaxies
  - velocity dispersions
  - gravitational lensing
  - X-ray gas
  - SZ effect

virial theorem

$$M \approx \frac{2.5}{G}\sigma^2 R_e$$

- the Milky Way
  - Oort discrepancy
  - vertical motions exceed restoring force of stellar disk (by a modest amount)
  - Beware approximations!

$$K_z \approx -\frac{\partial \Phi}{\partial z} \approx 2\pi G\Sigma + \frac{z}{R}\frac{\partial V^2}{\partial R}$$

vertical force

- Spiral Galaxies
  - dynamically cold disks are unstable
  - need some extra something for stability

Ostriker & Peebles $t = \frac{T}{|W|} \lesssim 0.14$

Toomre $Q = \frac{\sigma_r \kappa}{3.36 G\Sigma}$

Goldreich & Tremaine $X_m = \frac{\kappa^2 R}{2\pi m G\Sigma}$

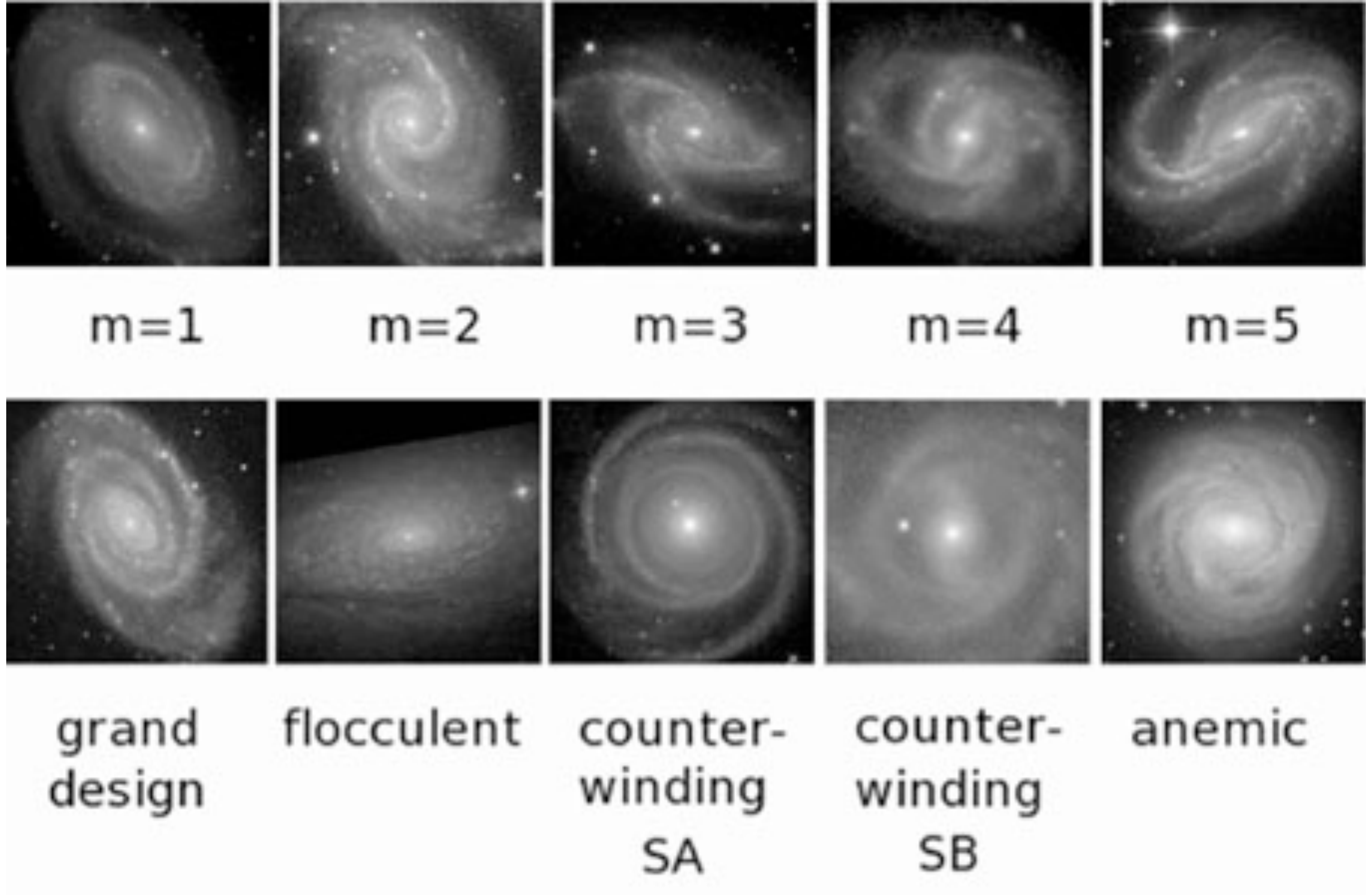
Spiral arm
type & multiplicity

A bar is an m=2 mode

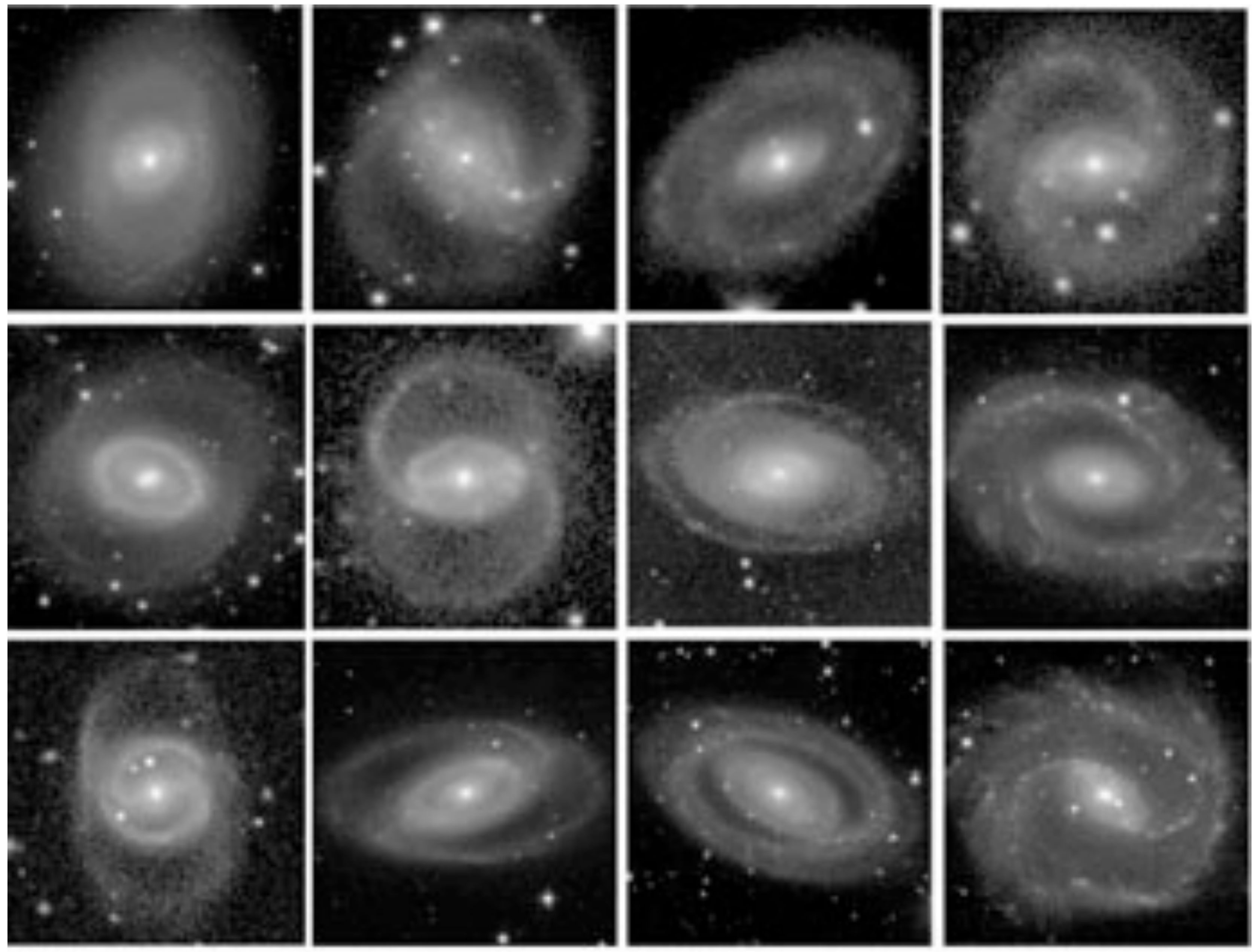
A two-armed spiral is an m=2 mode with a pitch angle p > 0.

A four armed-spiral has m=4, etc.

$$A_m = \frac{1}{N}\sum_{j=1}^{N} e^{i[m\theta_j + p\ln(R_j)]}$$



rings



Disk self-gravity drives bars and also spiral structure. Need a dark matter halo to suppress the rate of growth of these modes (but see Sellwood 2016 on live halos). But need some disk self-gravity to drive the observed features -

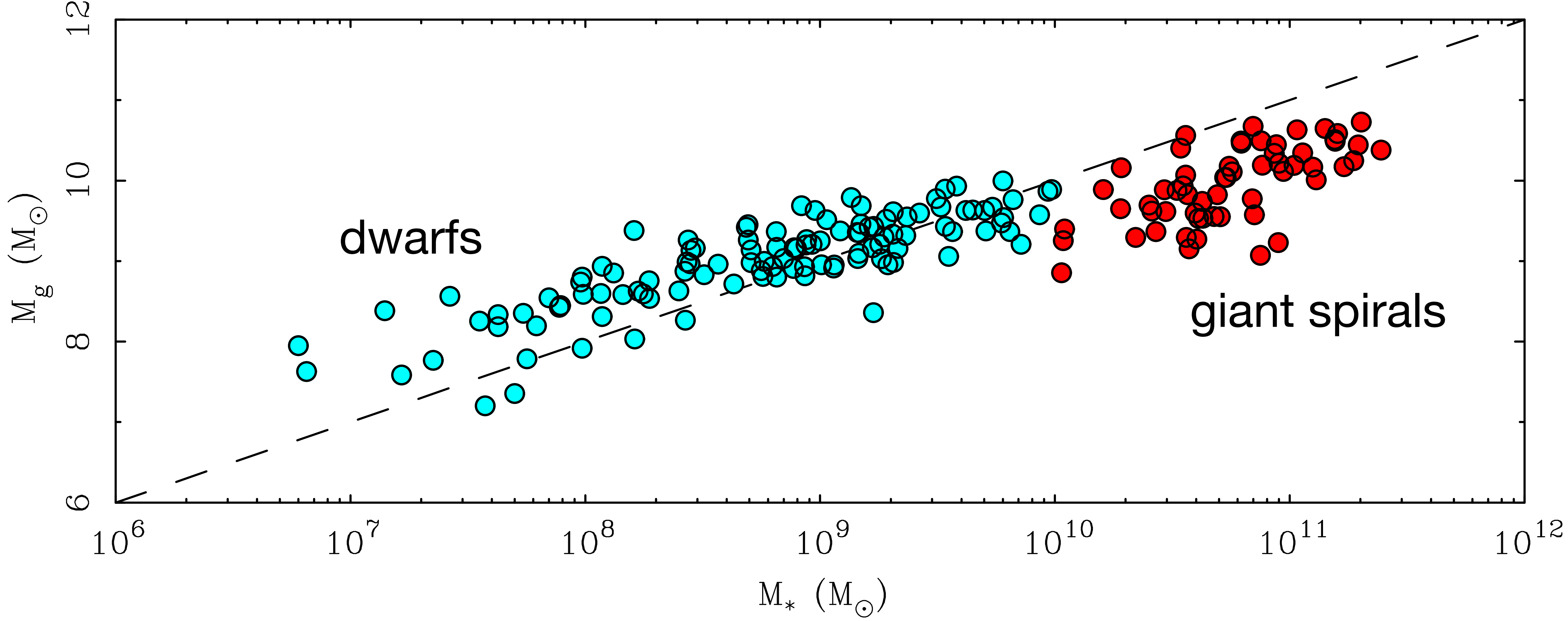
Athanassoula et al. (1987, A&A, 179, 23) find the disk mass has to be within a factor of 2 of maximum disk.

Fuchs (2003, Ap&SS, 284, 719) finds LSB disks need to be heavier than expected by stellar population models in order to drive the observed structure.

see also Tiret & Combes
(2007, A&A 464, 517; 2008, A&A, 483, 719)

The number of galaxies...

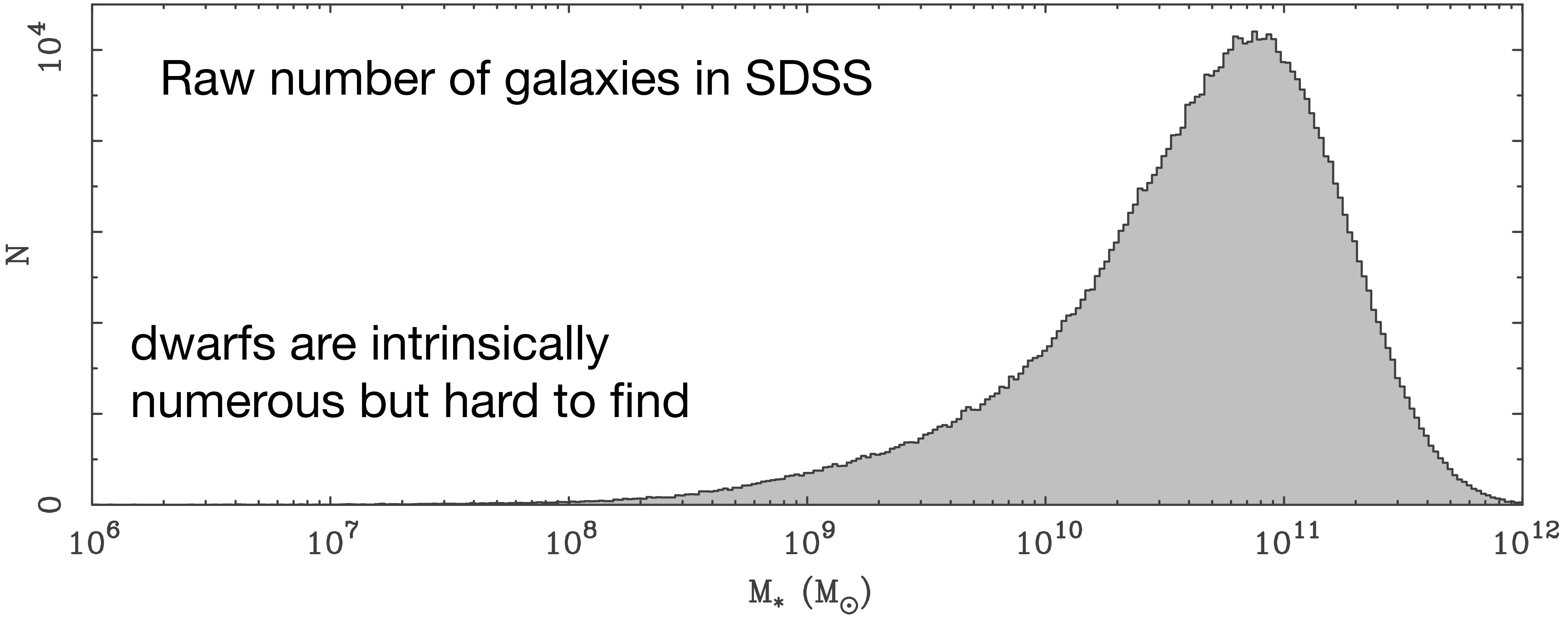
Beware selection effects! Catalogs are always dominated by brightest objects



gas mass

dwarfs

giant spirals



Number

Raw number of galaxies in SDSS

dwarfs are intrinsically
numerous but hard to find

stellar mass

The apparent numbers of galaxies in magnitude-limited samples decreases with decreasing mass, while their intrinsic numbers increase.

Approximately fit by a **Schechter function**:

$$\phi(L) = \left(\frac{\phi^*}{L^*}\right)\left(\frac{L}{L^*}\right)^{\alpha} e^{-\left(\frac{L}{L^*}\right)}$$

in terms of absolute magnitude

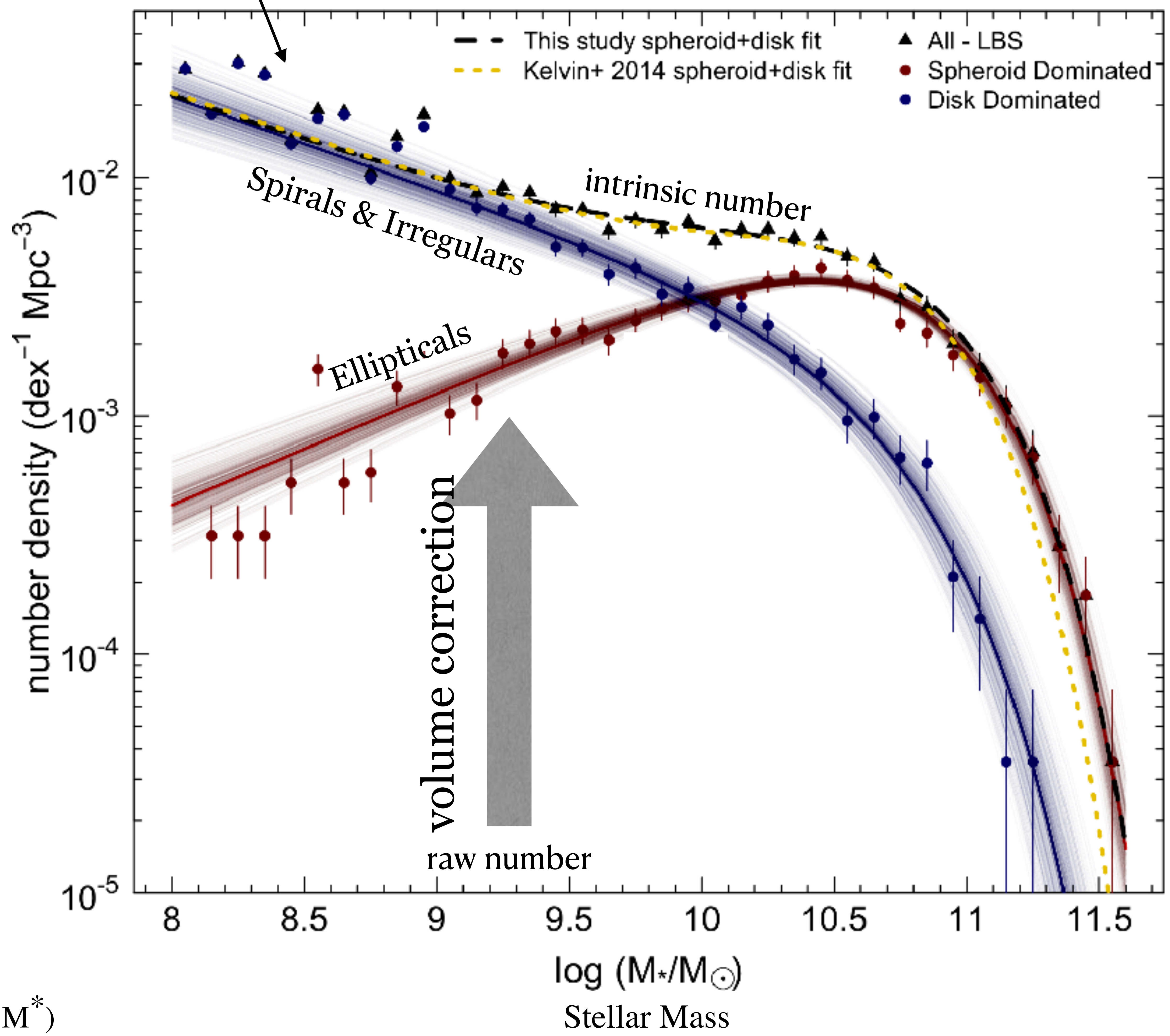
$$\mathrm{M} - \mathrm{M}_{\odot} = -\,2.5\log\left(\frac{L}{L_{\odot}}\right)$$

$$\phi(\mathrm{M}) = \frac{\ln(10)}{2.5}\phi^* 10^{0.4(\alpha+1)(\mathrm{M}-\mathrm{M}^*)} e^{-10^{0.4(\mathrm{M}-\mathrm{M}^*)}}$$

faint end slope α

L^* characteristic luminosity

Galaxy mass function from the GAMA survey

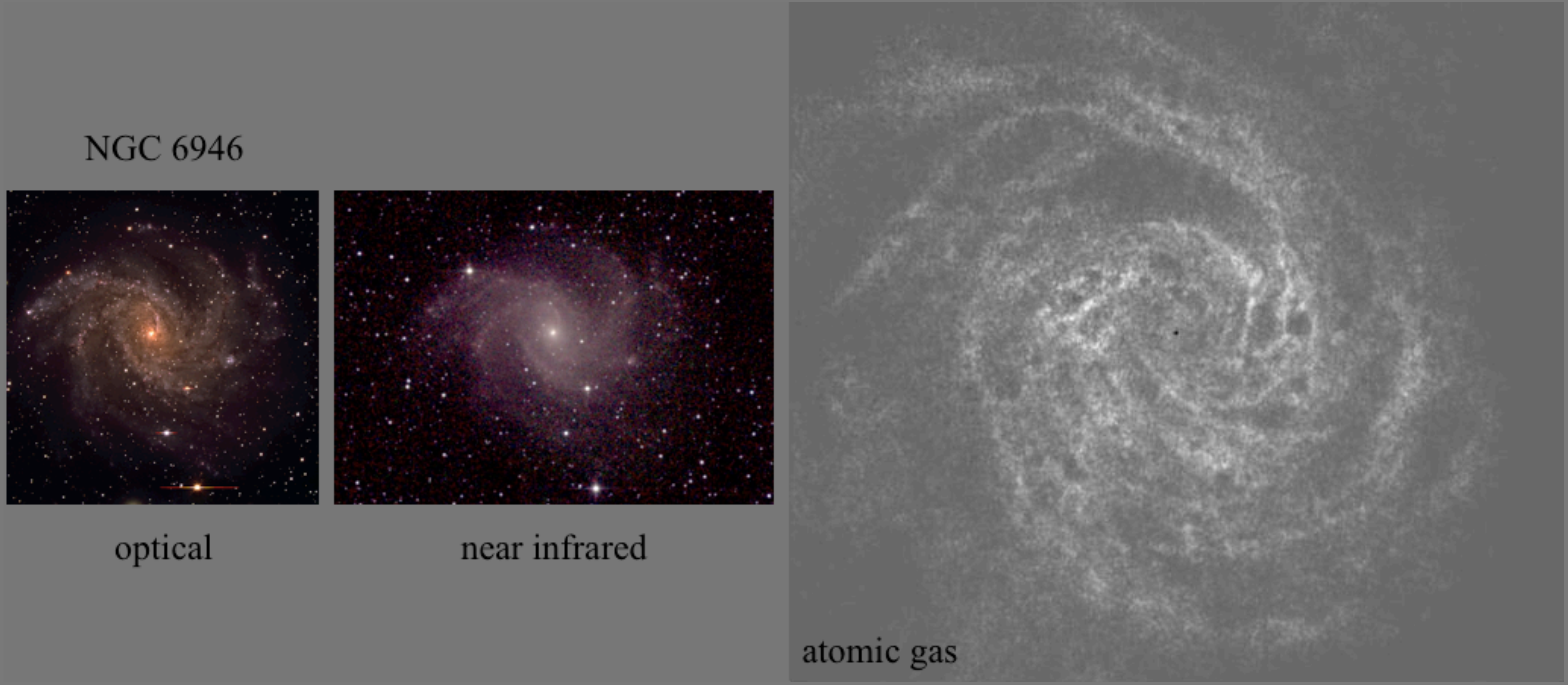


ϕ^* characteristic density

Moffett et al. 2016, MNRAS, 457, 1308

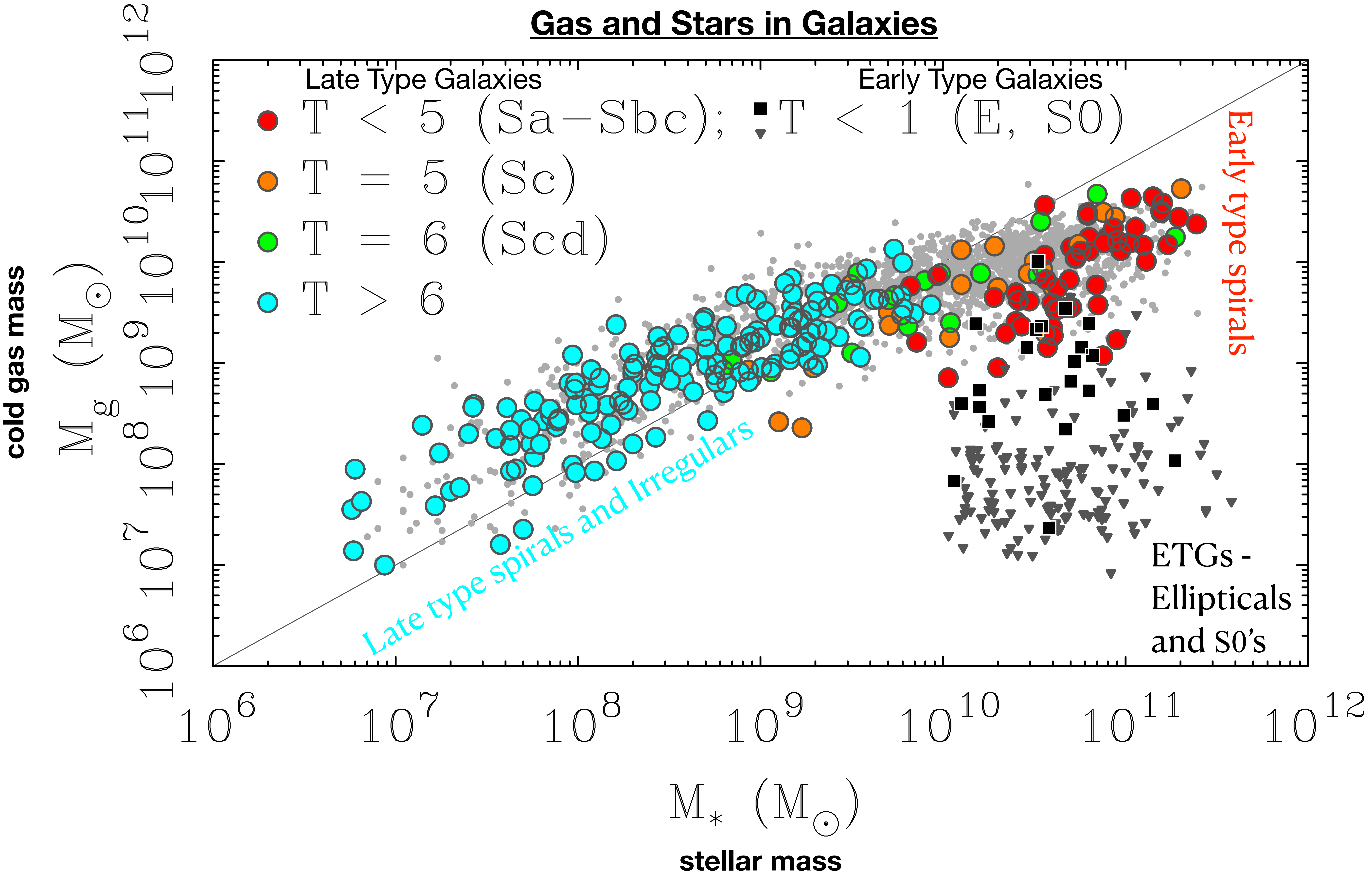
Be careful not to confuse the Schechter absolute magnitude M* with the stellar mass M_*!

Galaxies are made of gas as well as stars



NGC 6946 stars & gas

Gas and Stars in Galaxies



Baryonic Mass Content of Galaxies

$$M_b = M_* + M_g = \Upsilon_* L + \frac{1}{X}\left(M_{HI} + M_{H_2}\right)$$

$X \approx 0.73$ (hydrogen fraction)

- **Stars**
  - Υ_*^i is the stellar mass-to-light ratio in photometric band i

$$M_* = \Upsilon_*^i L_i \qquad L_i = 4\pi D^2 F_i$$

- **Gas**
  - *Atomic gas* - H I
    - $M_{HI} = 2.36 \times 10^5\, D^2\, F_{HI}$
  - *Molecular gas*
    - $M_{H_2} = 1.1 \times 10^4\, D^2\, F_{CO}$
  - *Ionized gas* - H II
    - negligible at small radii
    - may be considerable at very large radii in the circumgalactic medium (CGM)

also scales with stellar mass

$$M_{H_2} \approx 0.07\, M_*$$

ISM

The stuff between the stars

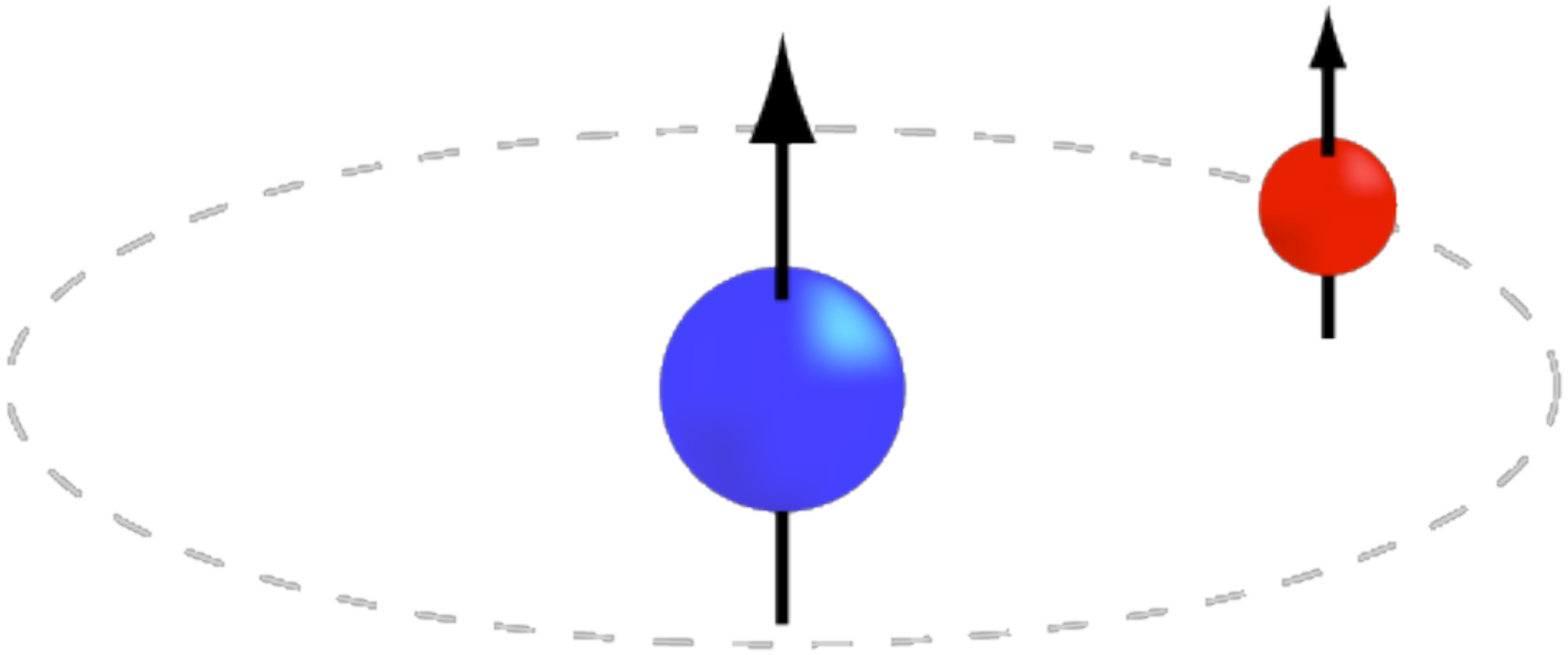
Atomic gas (H I)
Molecular gas (H_2)
Ionized gas (H II)
Dust

Explanatory links at NRAO

H I: http://www.cv.nrao.edu/course/astr534/HILine.html

H_2: http://www.cv.nrao.edu/course/astr534/MolecularSpectra.html

HI: atomic hydrogen in the interstellar medium



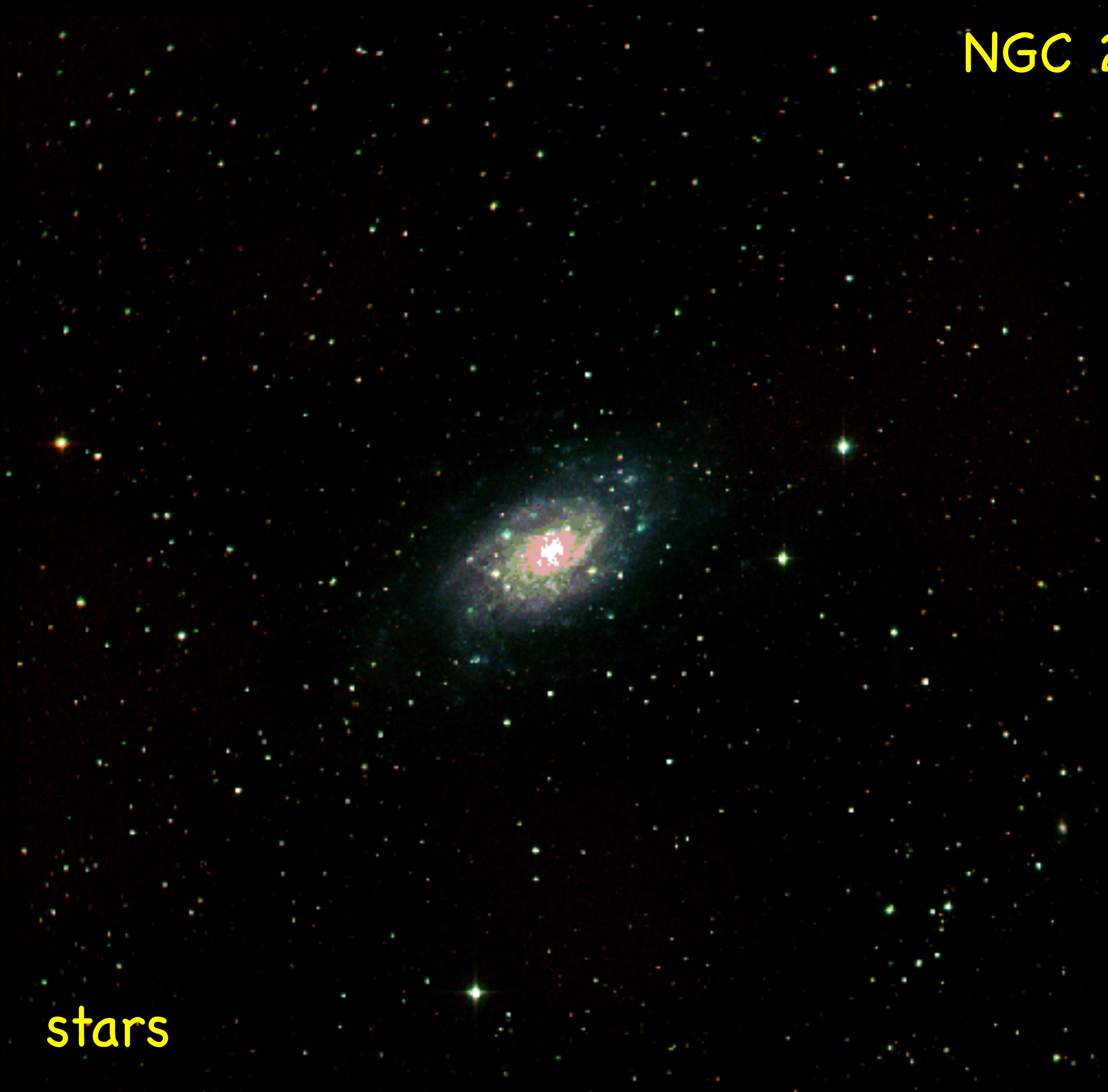
21 cm emission from hyperfine transition: parallel to anti-parallel spins

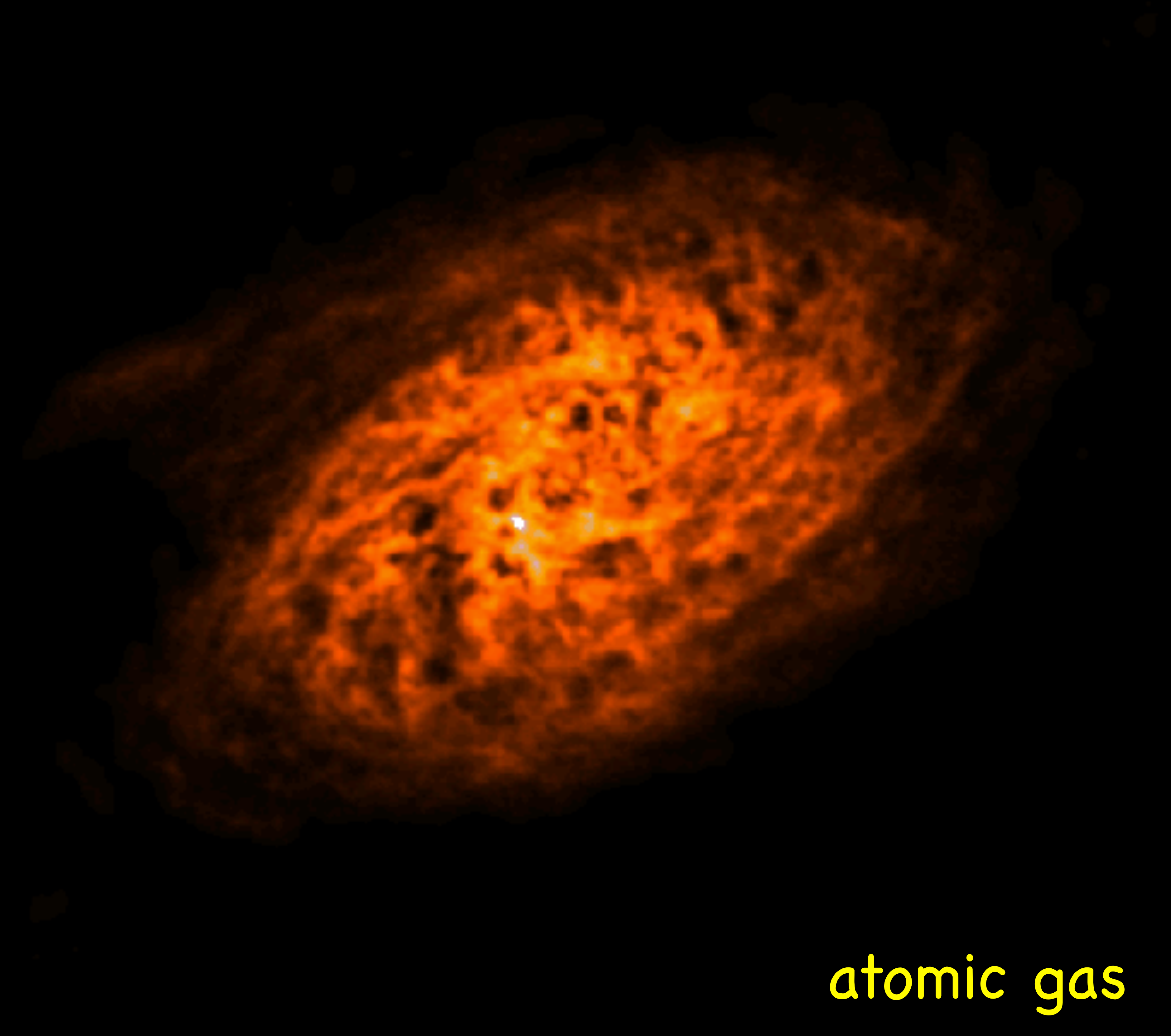
$$\nu = \frac{8}{3} g_I \frac{m_e}{m_p} \alpha^2 R_m c = 1420.405751 \text{ MHz}$$

The 21 cm line is in the radio at 1420 MHz

The atomic gas of the ISM is often more extended than the stars

NGC 2403

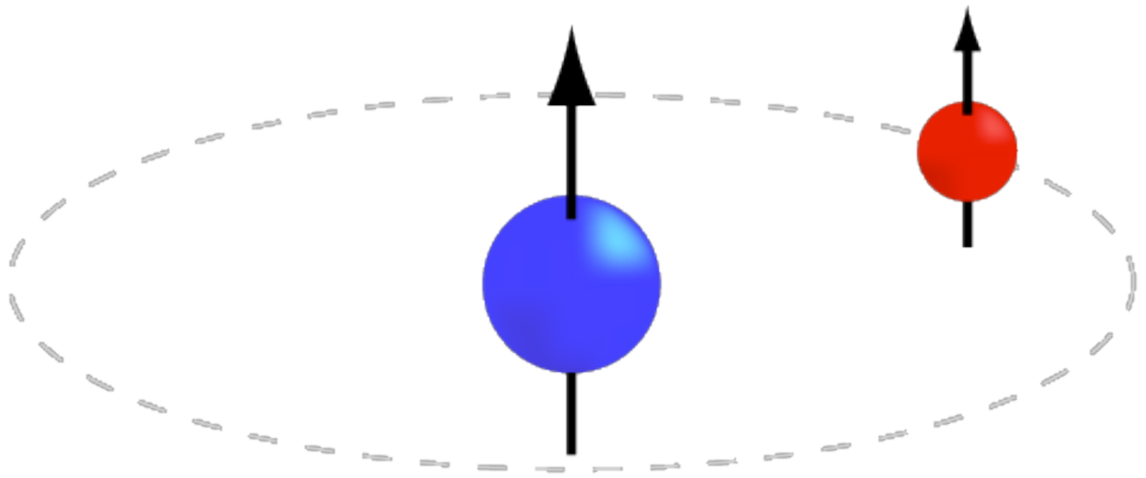




Fraternali, F., Oosterloo, T., Sancisi, R., van Moorsel, G.A. 2001, ApJ, 562, L47 emission coefficient

$$A_{UL} = \frac{64\pi^4}{3hc^3}\nu^3|\mu^*|^2$$

Bohr magneton



The radiative half-life of this transition is 11 Myr. This is readily maintained in equilibrium even in a cool (~100 K), diffuse ISM (< 1 atom/cc)

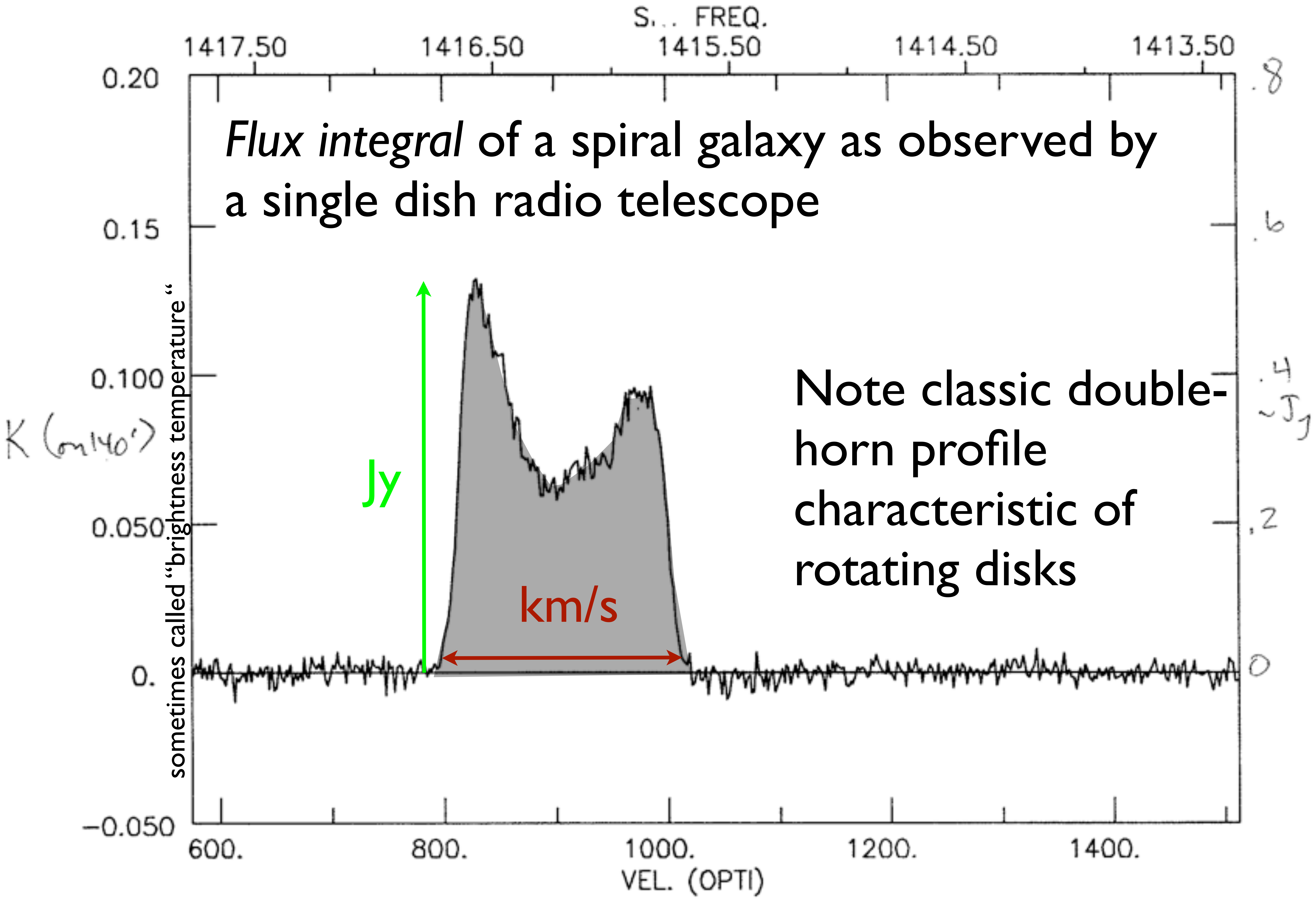
Counting 21 cm photons is equivalent to counting hydrogen atoms - a direct relation to mass!

$$M_{HI} = 2.36 \times 10^5 D^2 F_{HI}$$

Gives mass in solar masses for
D in Mpc and measured flux
F_{HI}, the flux integral in Jy-km/s

$1\ \mathrm{Jy} = 10^{-26}\ \mathrm{W\,m^{-2}\,Hz^{-1}}$

Flux integral of a spiral galaxy as observed by a single dish radio telescope



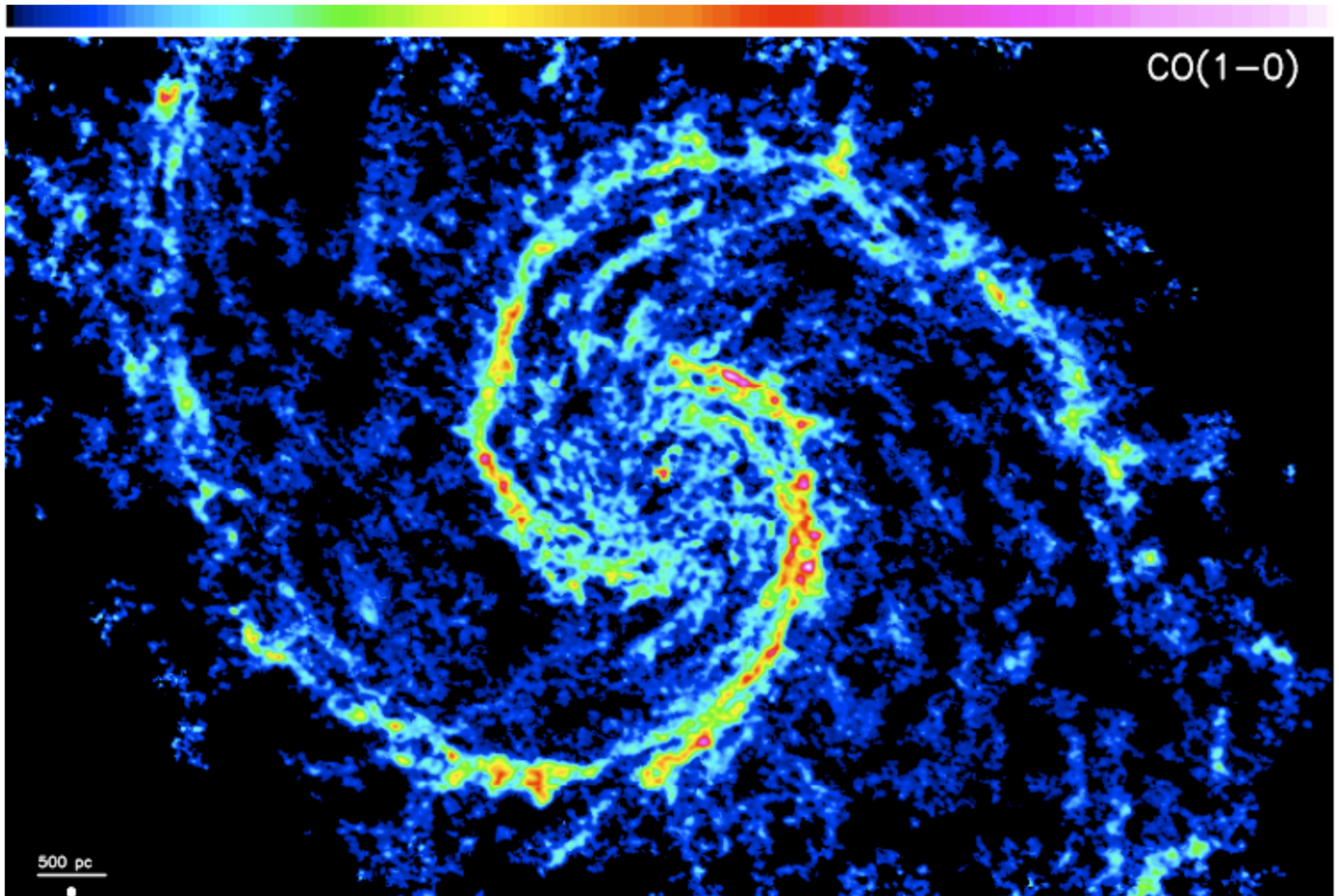
Note classic double-horn profile characteristic of rotating disks

Molecular ISM

Cold (~ 30 K), "dense" (> 100 molecules/cc) phase of the ISM

Very clumpy, with low filling factor - much of the
H_2 mass is in Giant Molecular Clouds ($\sim 10^6 M_\odot$). This is where stars form.

M51 seen in CO



Diatomic molecules (H_2, N_2, O_2) boring - or at least hard to excite, as they have no dipole moment.

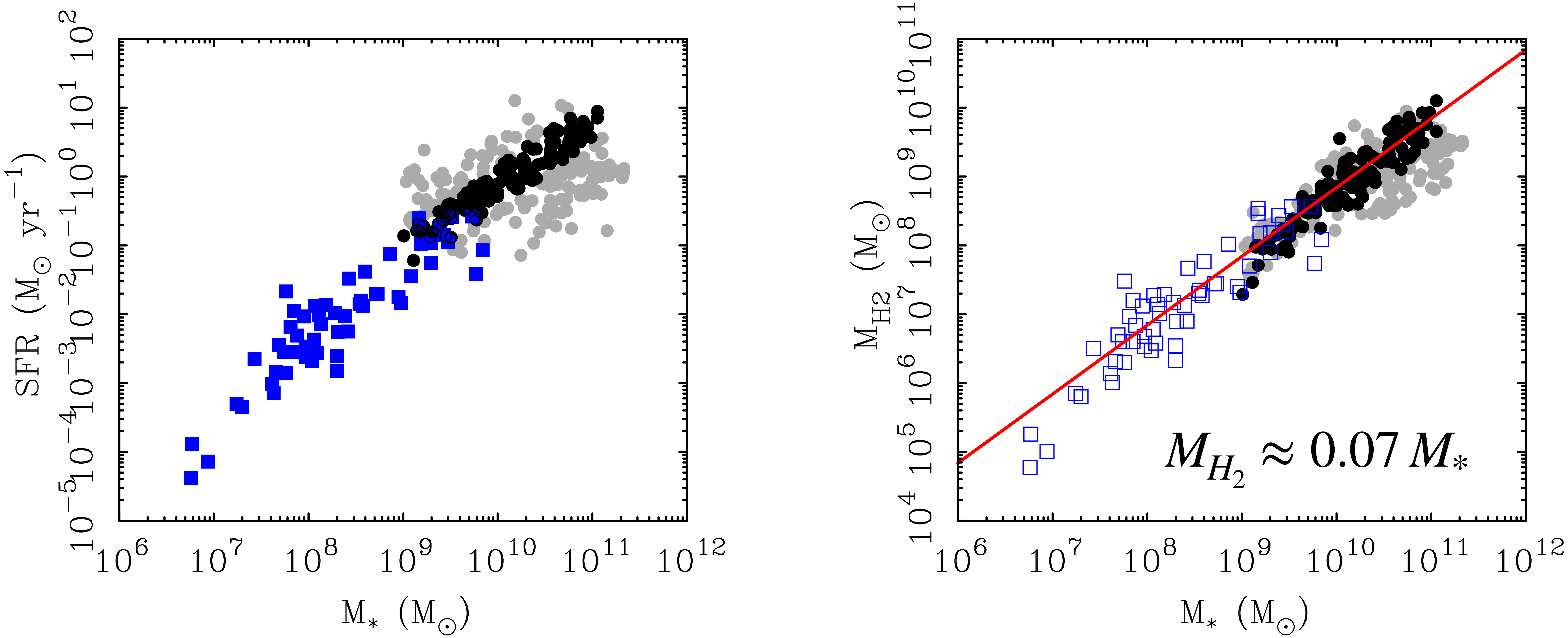
Polar molecules (esp. CO) have a permanent dipole moment thanks to asymmetry so have a rich rotational spectrum (typically in the mm or cm wavelengths).

$$M_{H_2} = 1.1 \times 10^4 D^2 F_{CO}$$

assuming the conversion factor $X_{CO} = 2.8 \times 10^{20} \mathrm{cm}^{-2} (\mathrm{K\ km/s})^{-1}$

which is calibrated by estimating the virial mass of nearby molecular clouds

Often CO observations are not available, in which case one approach is to use scaling relations: the amount of molecular gas is proportional to the star formation rate and the stellar mass.



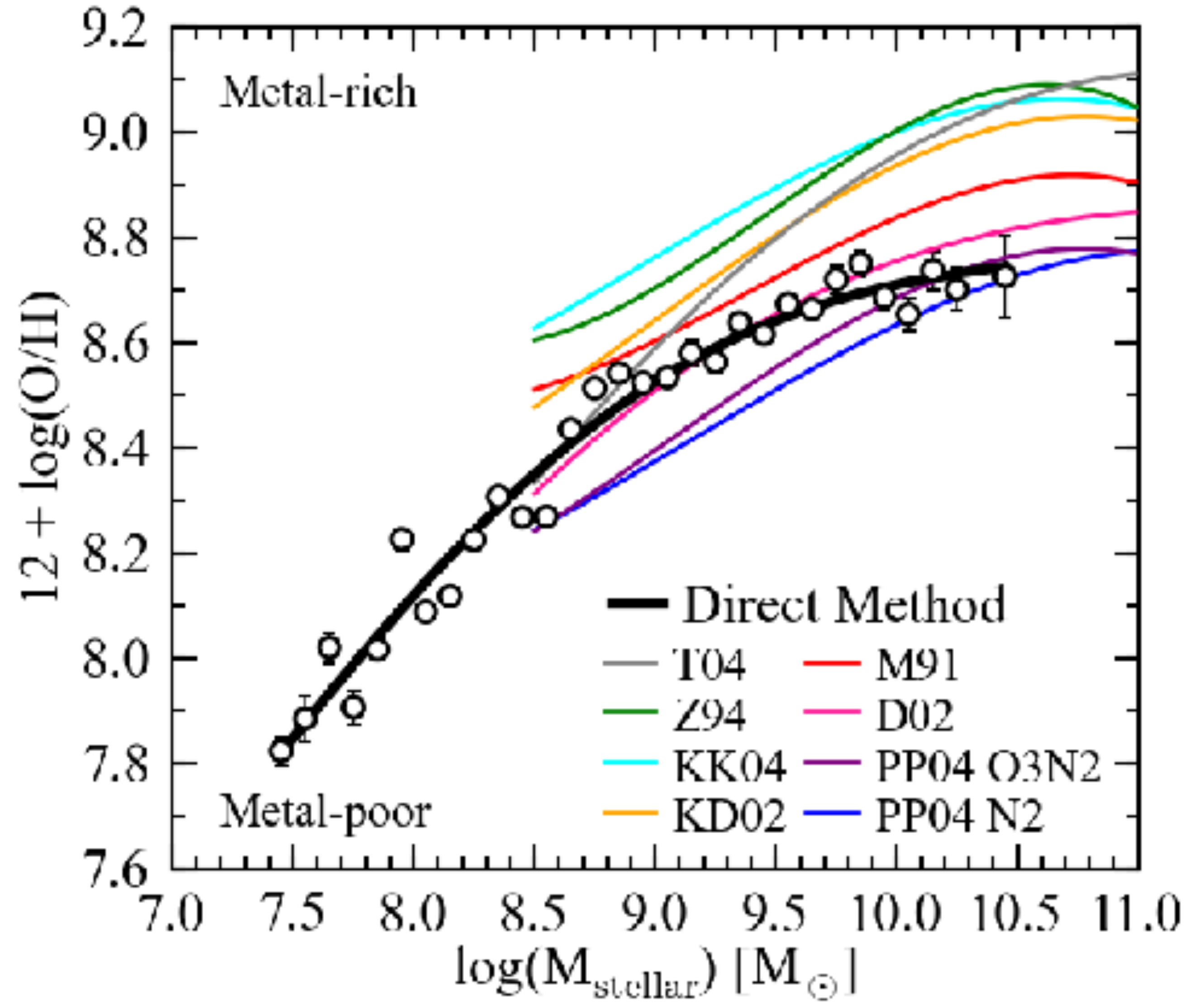
McGaugh et al. (2020, RNAAS, 4, 45)

Metallicity Dependence of the Hydrogen Fraction

Typically we measure the mass of hydrogen gas (e.g., M_{HI}). This needs to be corrected to account for the presence of helium and metals.

X = hydrogen fraction (primordial fraction 3/4)
Y = helium fraction (primordial fraction 1/4)
Z = everything else

Andrews & Martini (2013, ApJ, 765, 140)



As galaxies evolve, they form stars which make metals. Consequently, the metallicity correlates with stellar mass

$$X = 0.75 - 38.2\left(\frac{M_*}{M_0}\right)^{\alpha}$$

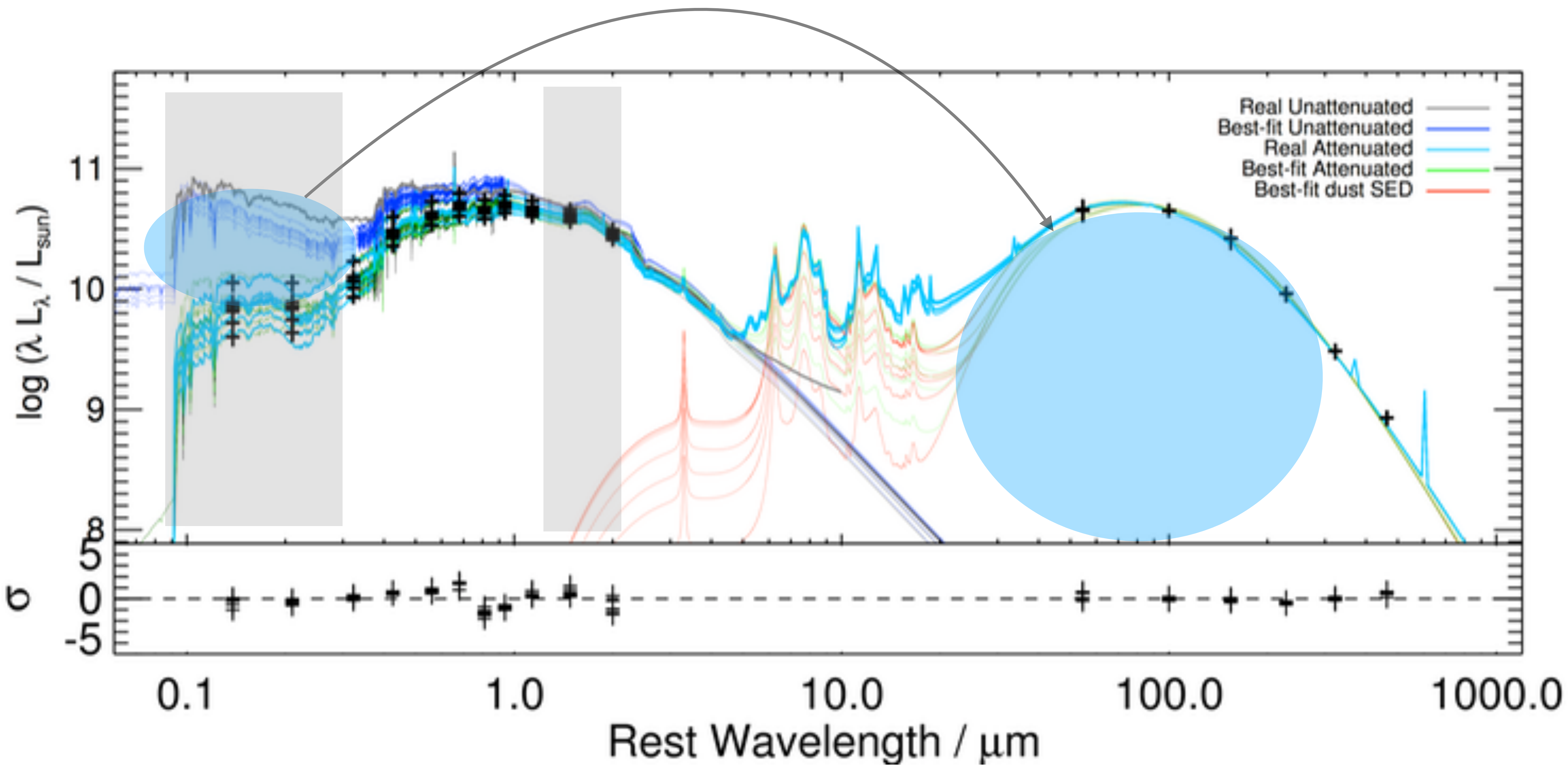
with $\alpha = 0.22$ and $M_0 = 1.5 \times 10^{24}\ \mathrm{M}_\odot$

For a low mass dwarf galaxy, X^{-1} = 1.34, while for a Milky Way mass galaxy, X^{-1} = 1.41.

(McGaugh et al. 2020, RNAAS, 4, 45)

Dust: the dust itself has negligible mass, but it can affect mass-to-light ratio estimates for stars

Dust-absorbs UV & optical radiation; re-emits in the IR



■ Lousy spot for measuring stellar mass - blue & UV wavelengths

■ Sweet spot for measuring stellar mass near-IR: 2-4 microns open cluster

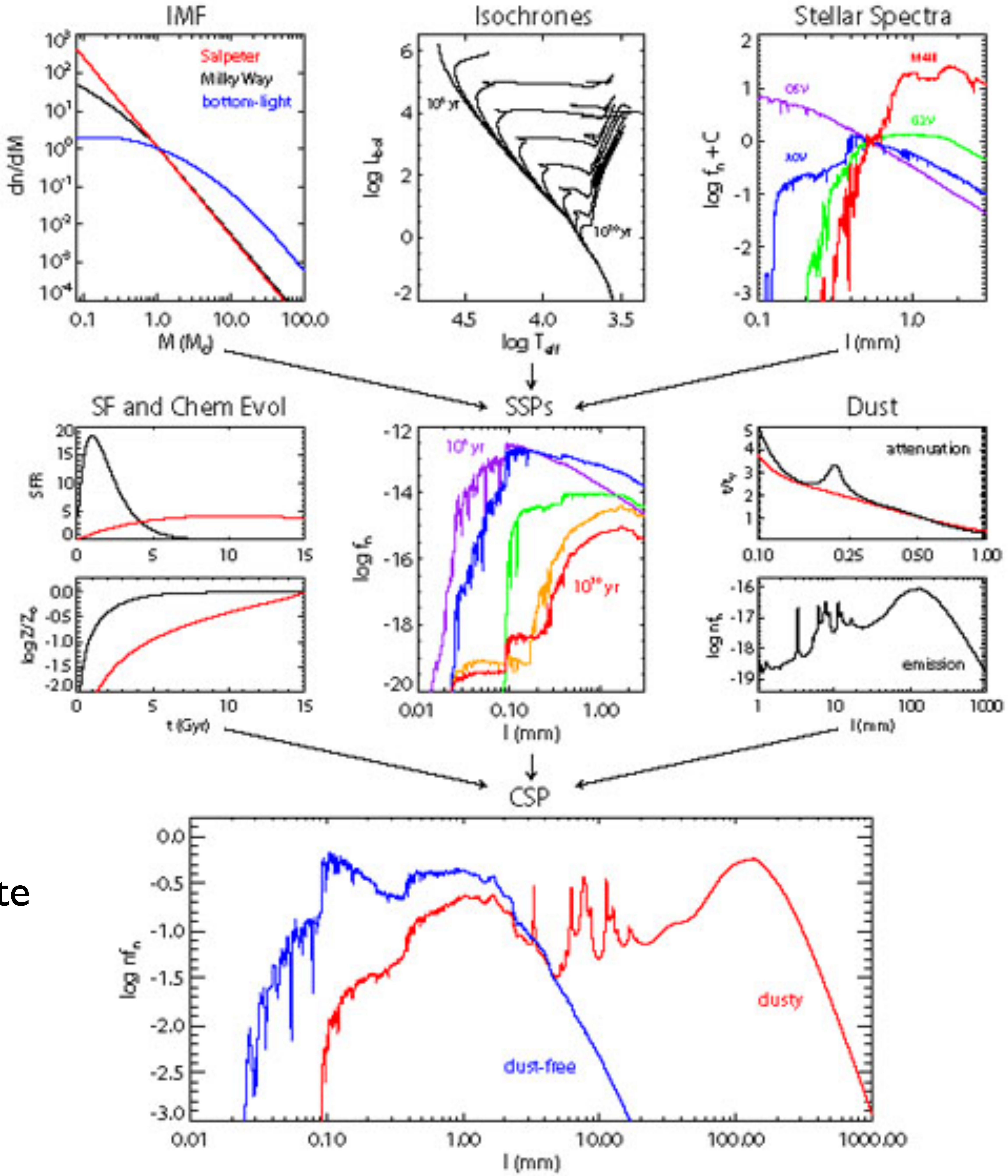
Stellar populations

- Simple Single Population (SSP)
  - – stars of all masses born at the same time
  - – e.g., a star cluster

- Complex stellar population
  - – Convolution of many star forming events
  - – need to know
    - IMF (initial mass function)
    - Birthrate (star formation rate history)

globular cluster

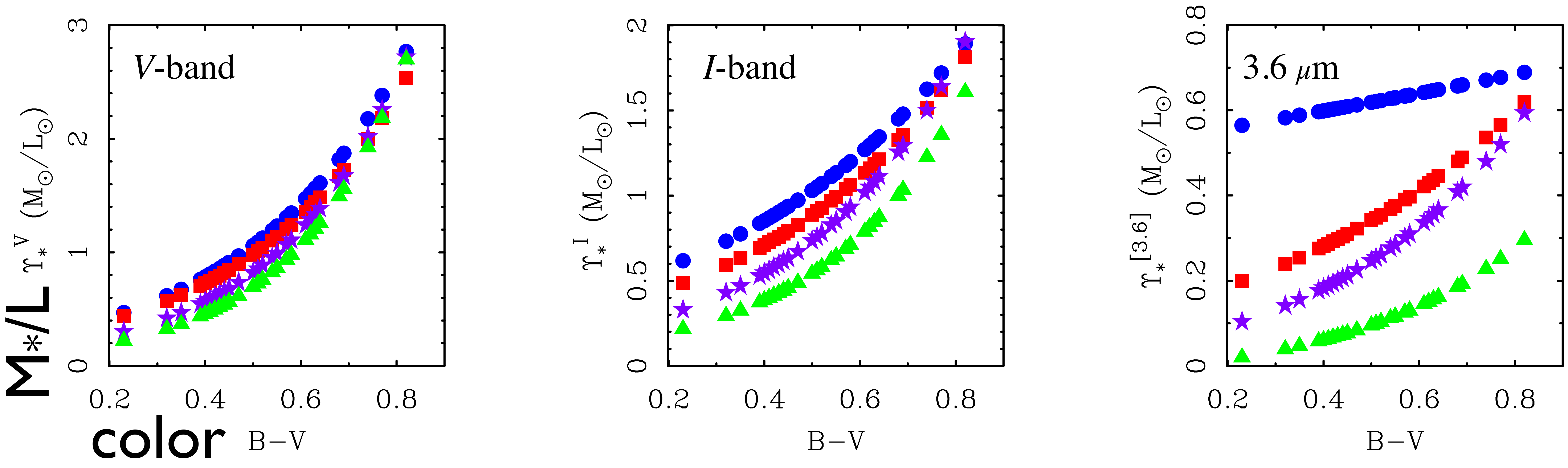
Stellar population synthesis modeling:



Stellar population synthesis modeling is on way to estimate the stellar mass-to-light ratio.

Stellar population models

Typically, redder colors mean higher mass-to-light ratios



Color-M/L relation: $\log \Upsilon_*^i = a_i + b_i\,(B - V)$

Can use multiple colors, but most of the information is in the first one.

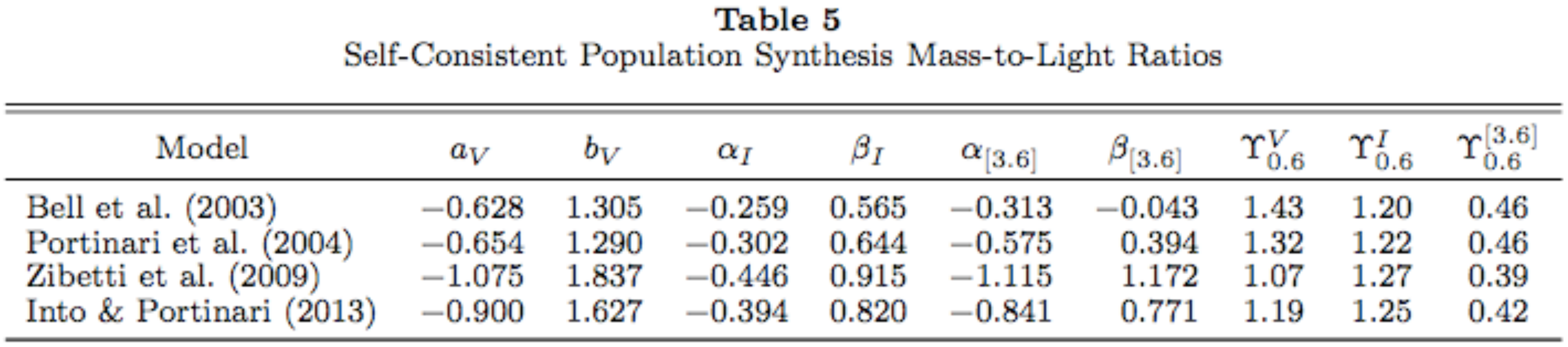


Table 5
Self-Consistent Population Synthesis Mass-to-Light Ratios

Model	a_V	b_V	α_I	β_I	$\alpha_{[3.6]}$	$\beta_{[3.6]}$	$\Upsilon_{0.6}^V$	$\Upsilon_{0.6}^I$	$\Upsilon_{0.6}^{[3.6]}$
Bell et al. (2003)	−0.628	1.305	−0.259	0.565	−0.313	−0.043	1.43	1.20	0.46
Portinari et al. (2004)	−0.654	1.290	−0.302	0.644	−0.575	0.394	1.32	1.22	0.46
Zibetti et al. (2009)	−1.075	1.837	−0.446	0.915	−1.115	1.172	1.07	1.27	0.39
Into & Portinari (2013)	−0.900	1.627	−0.394	0.820	−0.841	0.771	1.19	1.25	0.42

Baryonic Mass of Galaxies

$$M_b = M_* + M_g = \Upsilon_* L + X^{-1}\left(M_{HI} + M_{H_2}\right)$$

$$X^{-1} \approx 1.33 \; - \; 1.42$$

- **Stars** $M_* = \Upsilon_*^i L_i$ $L_i = 4\pi D^2 F_i$
  - Υ_*^i is the stellar mass-to-light ratio in photometric band i
- **Gas**
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also scales with stellar mass $M_{H_2} \approx 0.07 \, M_*$