Dark Matter

ASTR 333/433
Spring 2026
TR 11:30am–12:45pm
Sears 552

http://astroweb.case.edu/ssm/ASTR333/

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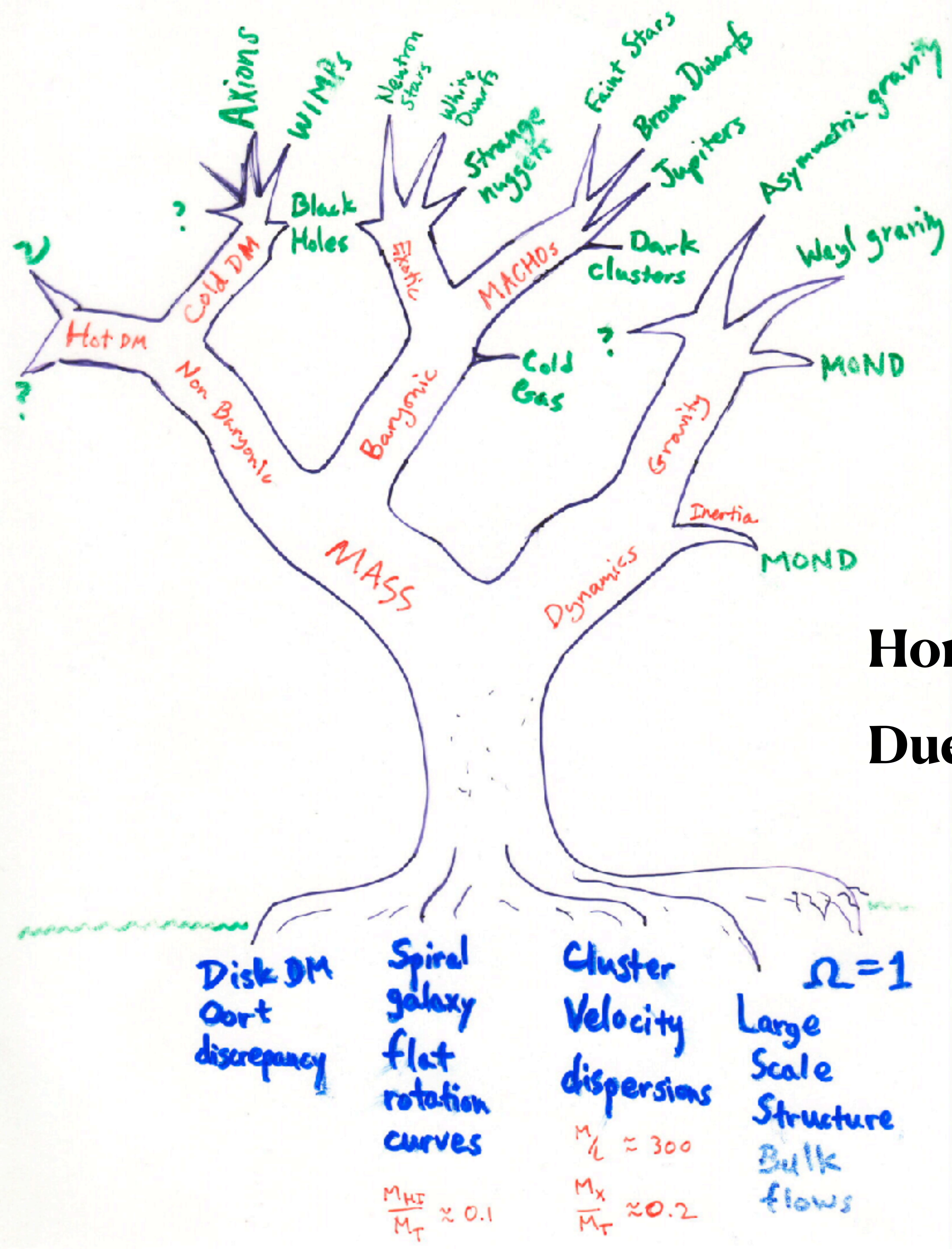
HOMEWORK DUE NEXT TIME

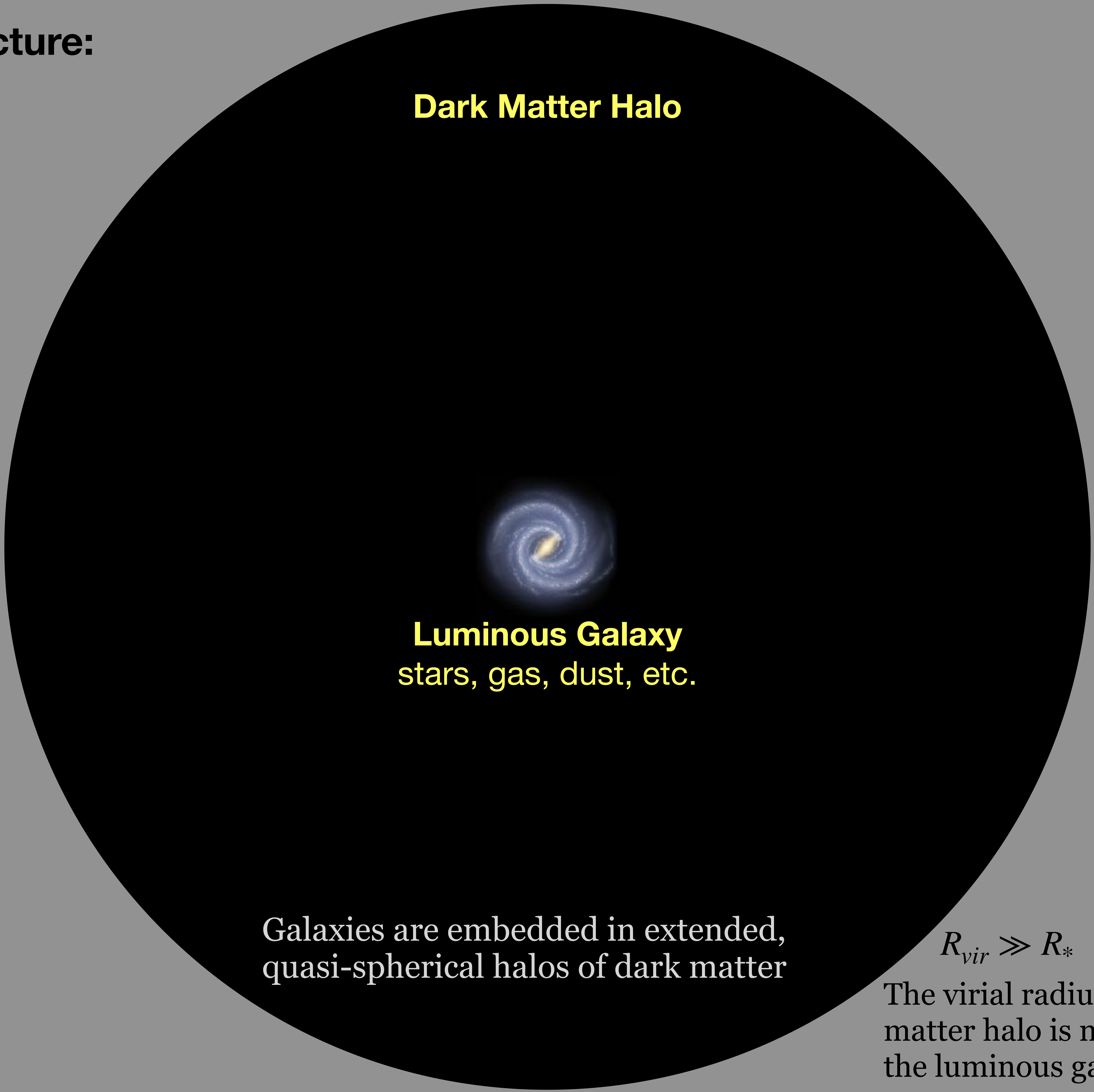
CASE WESTERN RESERVE UNIVERSITY EST. 1826



Homework 1

Due Feb. 5

Basic Picture:



Galaxies are embedded in extended, quasi-spherical halos of dark matter

$R_{vir} \gg R_*$

The virial radius of the dark matter halo is much larger than the luminous galaxy

Dark Matter Halo models

pseudo-isothermal

older
empirically motivated

$\rho(r) \sim r^{-2}$ gives a flat rotation curve

3D density profiles

$$\rho(r) = \frac{\rho_0}{1 + (r/R_c)^2}$$

theoretically reminiscent of an isothermal distribution

Both models have 2 parameters - a characteristic density and scale radius

NFW

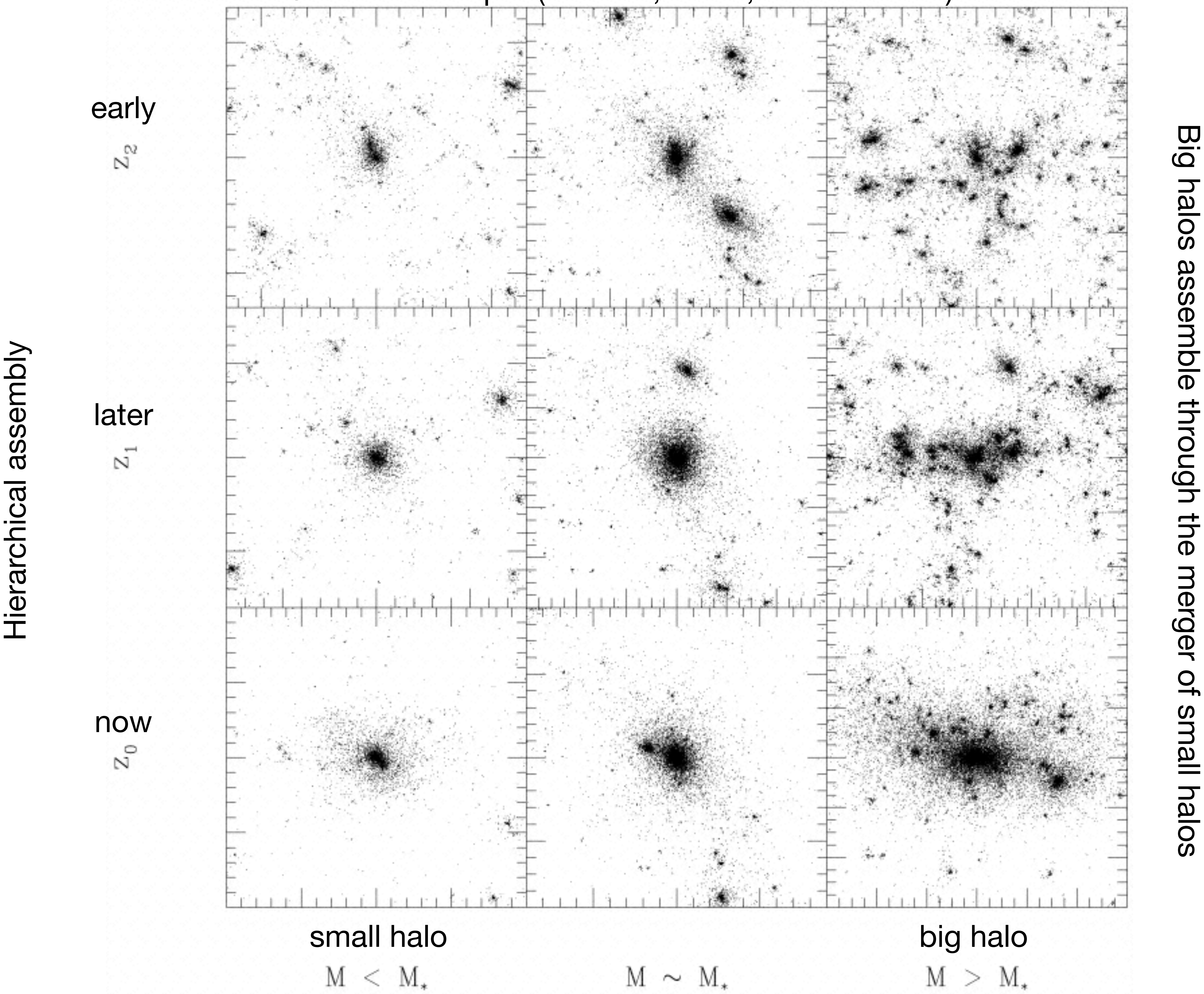
now old new normal
theoretically motivated

NFW arises in pure, dark matter only (DMO) simulations

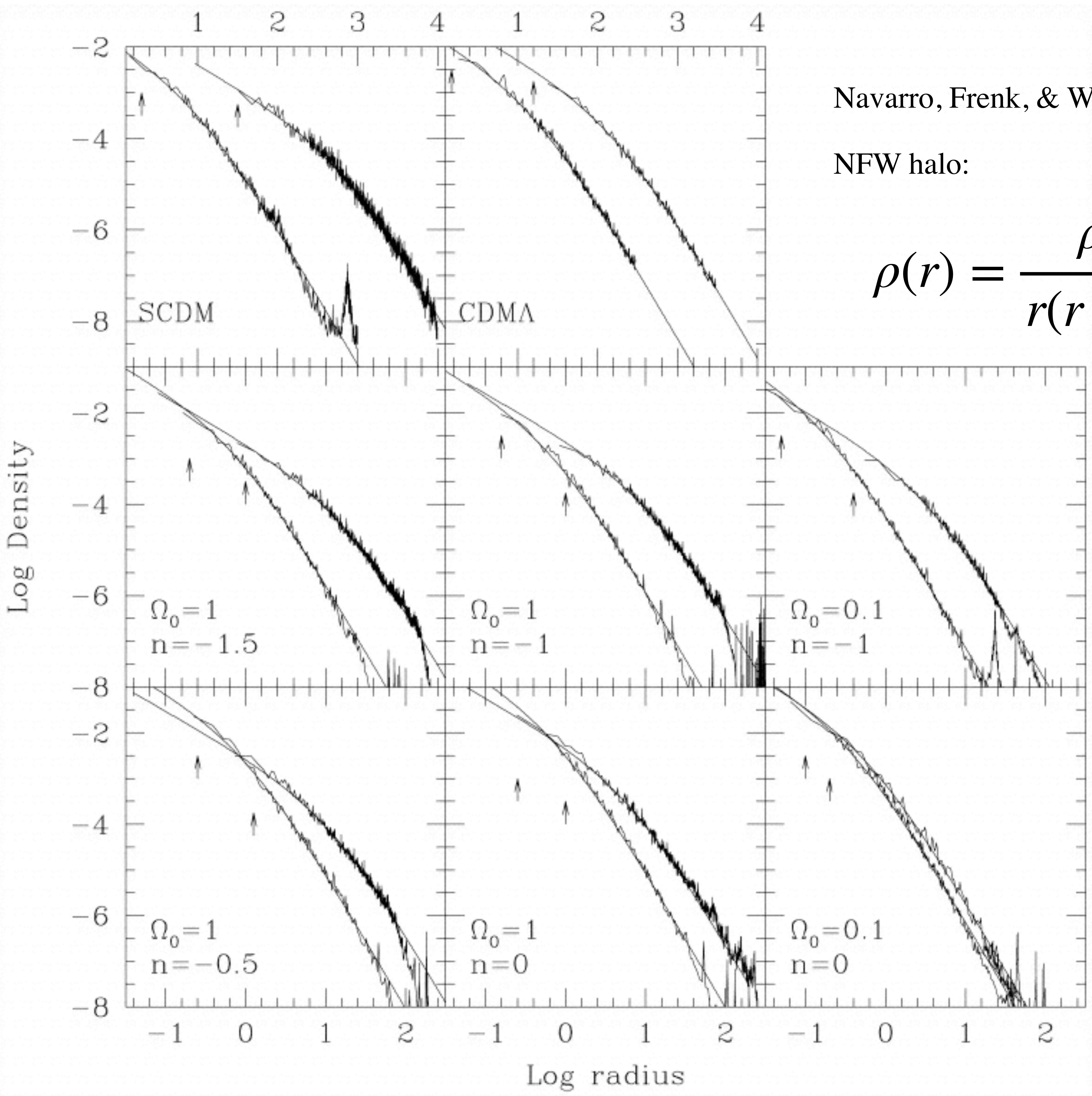
$$\rho(r) = \frac{\rho_s r_s^3}{r(r + r_s)^2}$$

an analytic approximation to the results of numerical simulations

Simulation output (Navarro, Frenk, & White 1997)



Density profiles of simulated dark matter halos



Navarro, Frenk, & White (1997)

NFW halo:

$$\rho(r) = \frac{\rho_s r_s^3}{r(r + r_s)^2}$$

See “NFW math” link on the lecture notes page.

Mass models

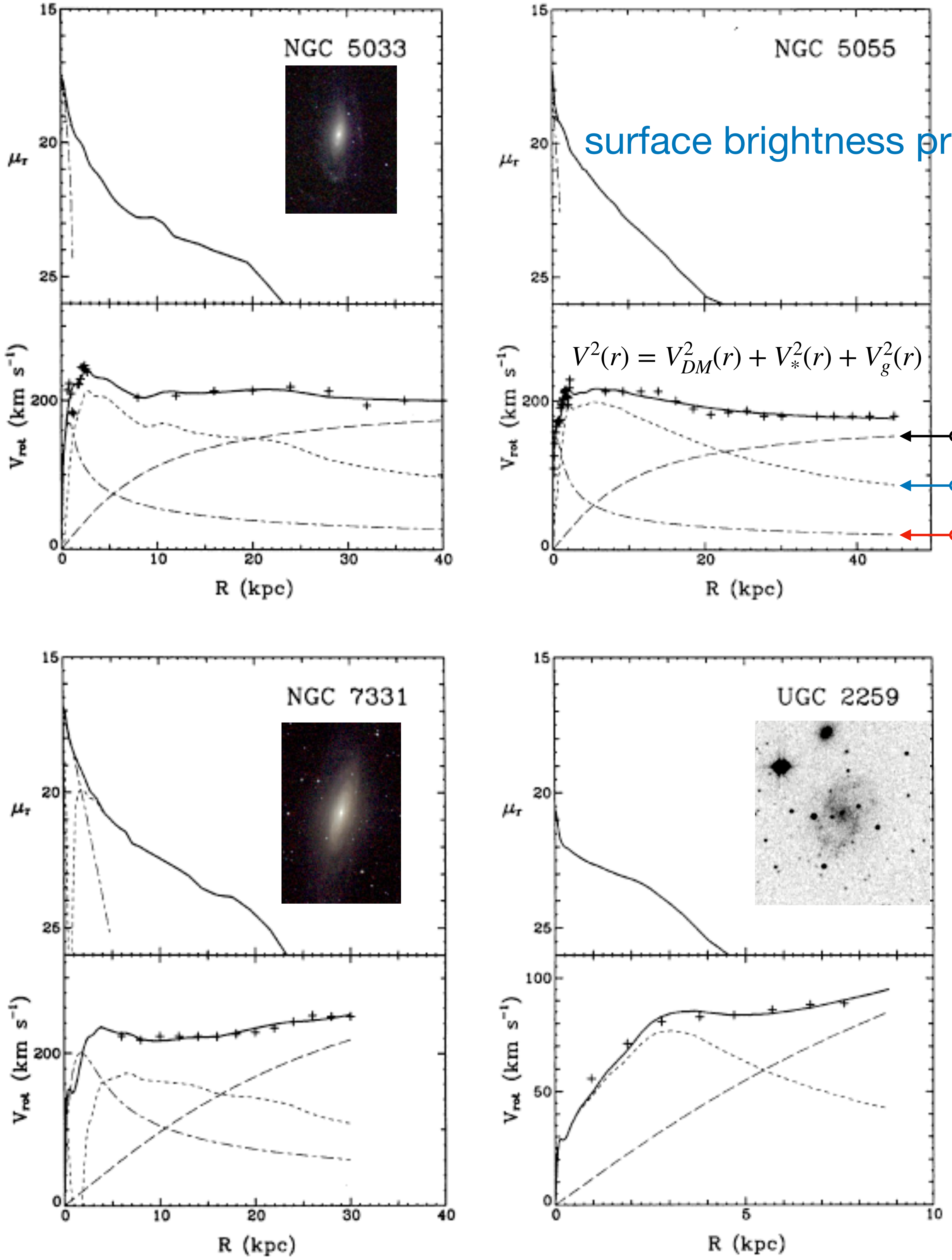
Kent (1987)

Rotation curves fit with pseudo-isothermal halos

$$\rho(r) = \frac{\rho_0}{1 + (r/R_c)^2}$$

$$V_{\rm iso}(r) = V_\infty \sqrt{1 - \frac{R_c}{r} \tan^{-1}\left(\frac{r}{R_c}\right)}$$

$$V_\infty = \sqrt{4\pi G \rho_0 R_c^2}$$



surface brightness profile

dark matter halo

stellar disk

stellar bulge

$$V^2(r) = V_{DM}^2(r) + V_*^2(r) + V_g^2(r)$$

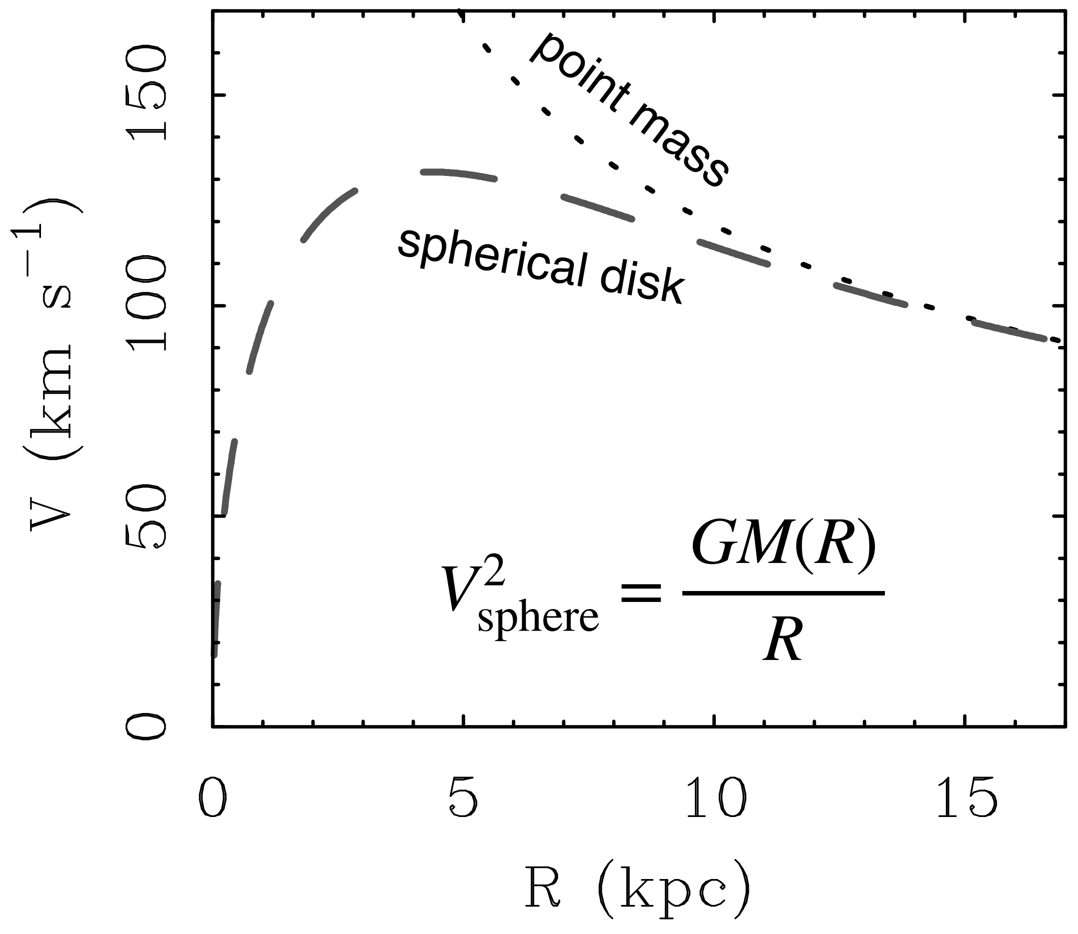
velocities sum in quadrature since $V^2 \sim \dfrac{M}{r}$

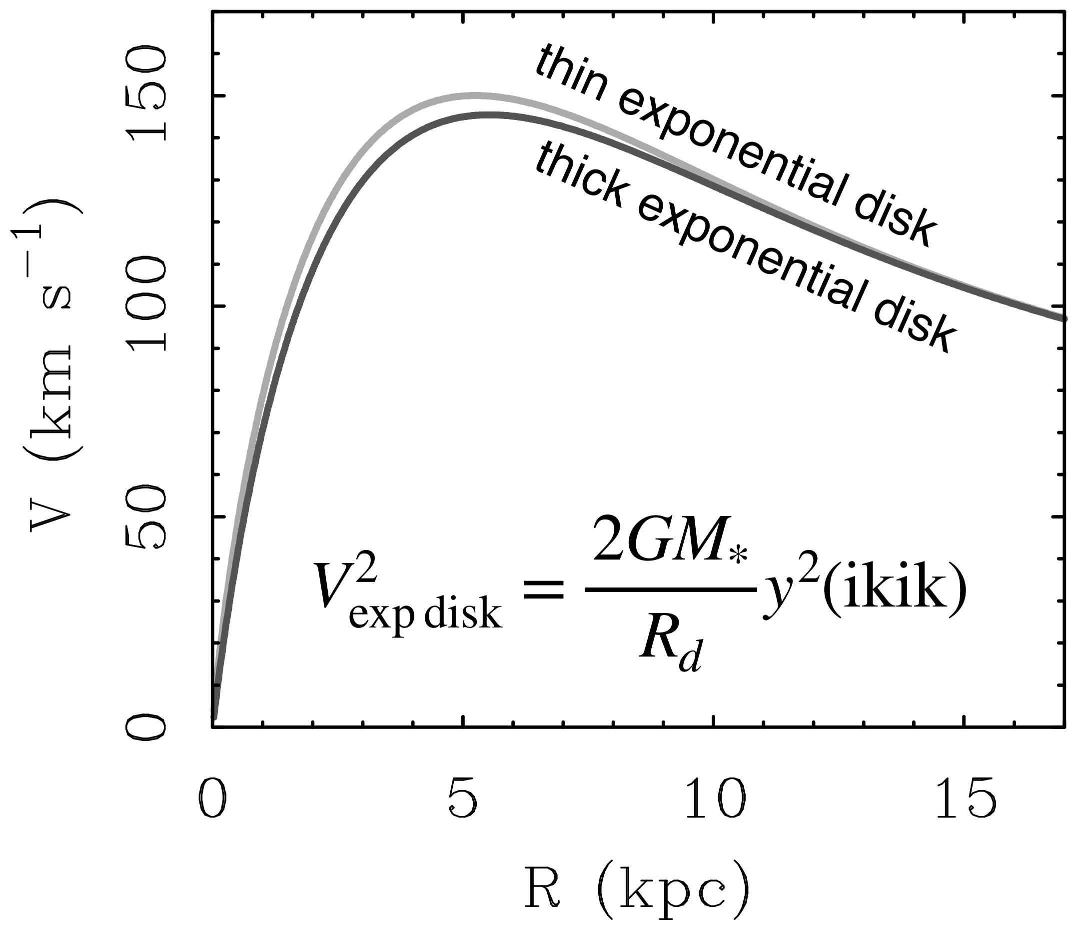
Progressive approximations in mass modeling

Appear in homework:

- Point Mass
- “spherical” disk
- thin exponential disk
- thick exponential disk
- surface density $\Sigma(R)$
- 2D $\Sigma(R, \phi)$ [e.g., bars]
- 3D $\rho(R, \phi, z)$
- 3D + non-equilibrium

We numerically solve the Poisson equation to obtain the gravitational potential Φ_* from the observed surface density $\Sigma_*(R)$



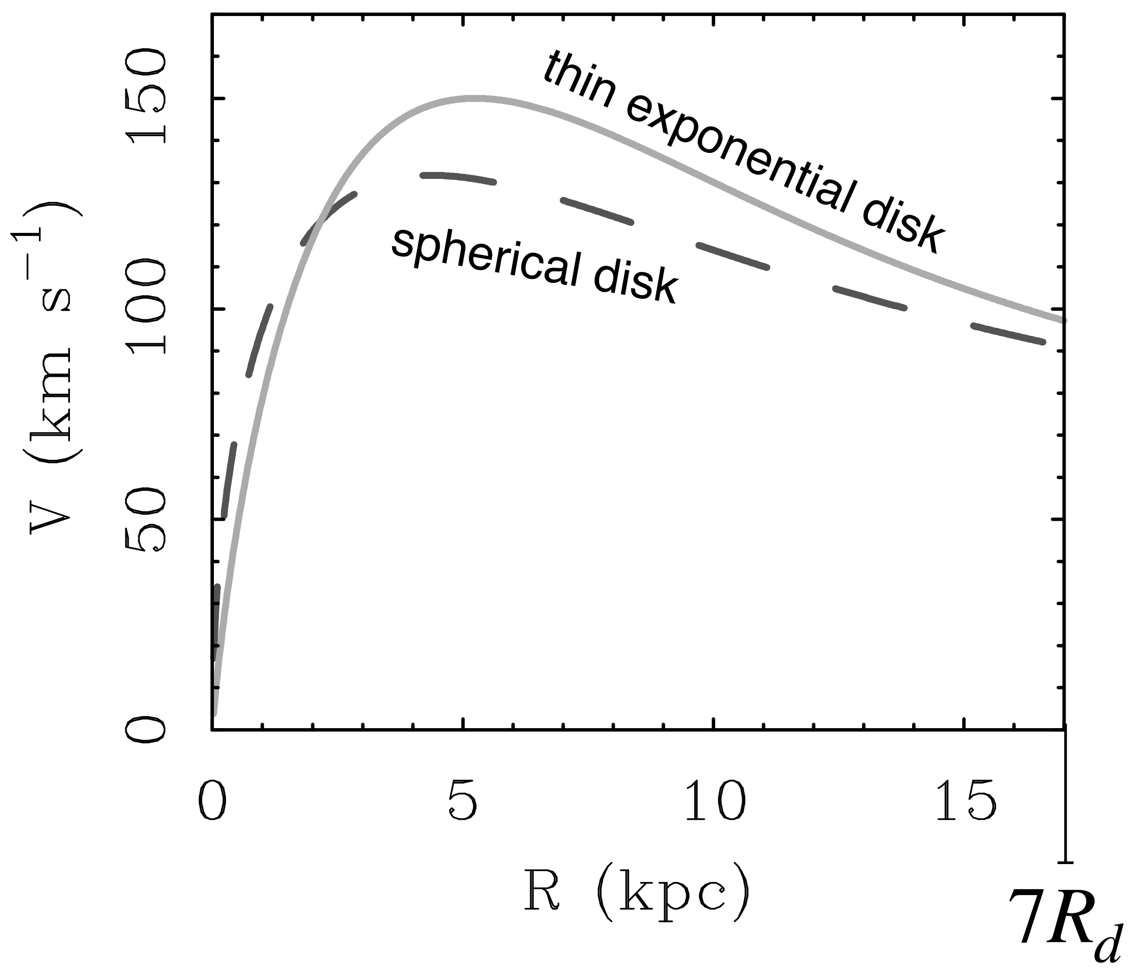


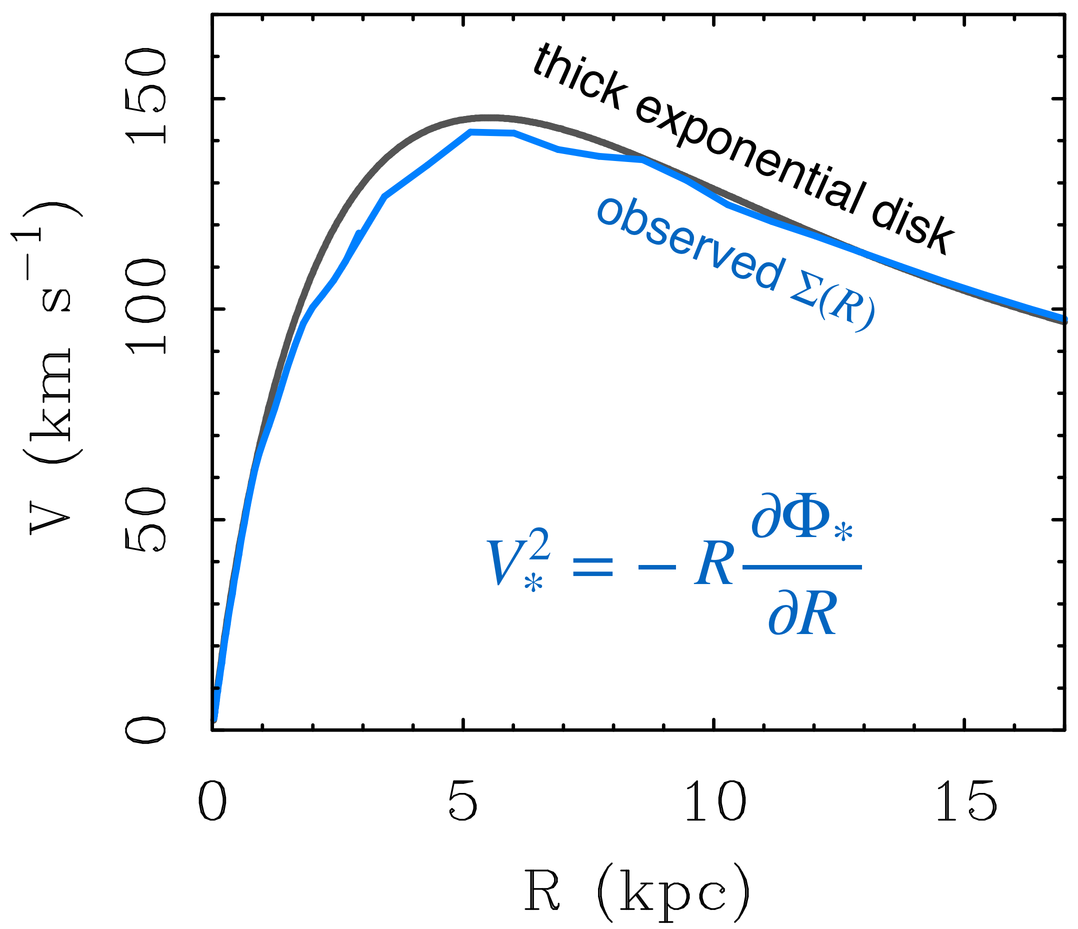
$${\rm ikik} = [I_0(y)K_0(y) - I_1(y)K_1(y)]$$

$$y = \frac{R}{2R_d}$$

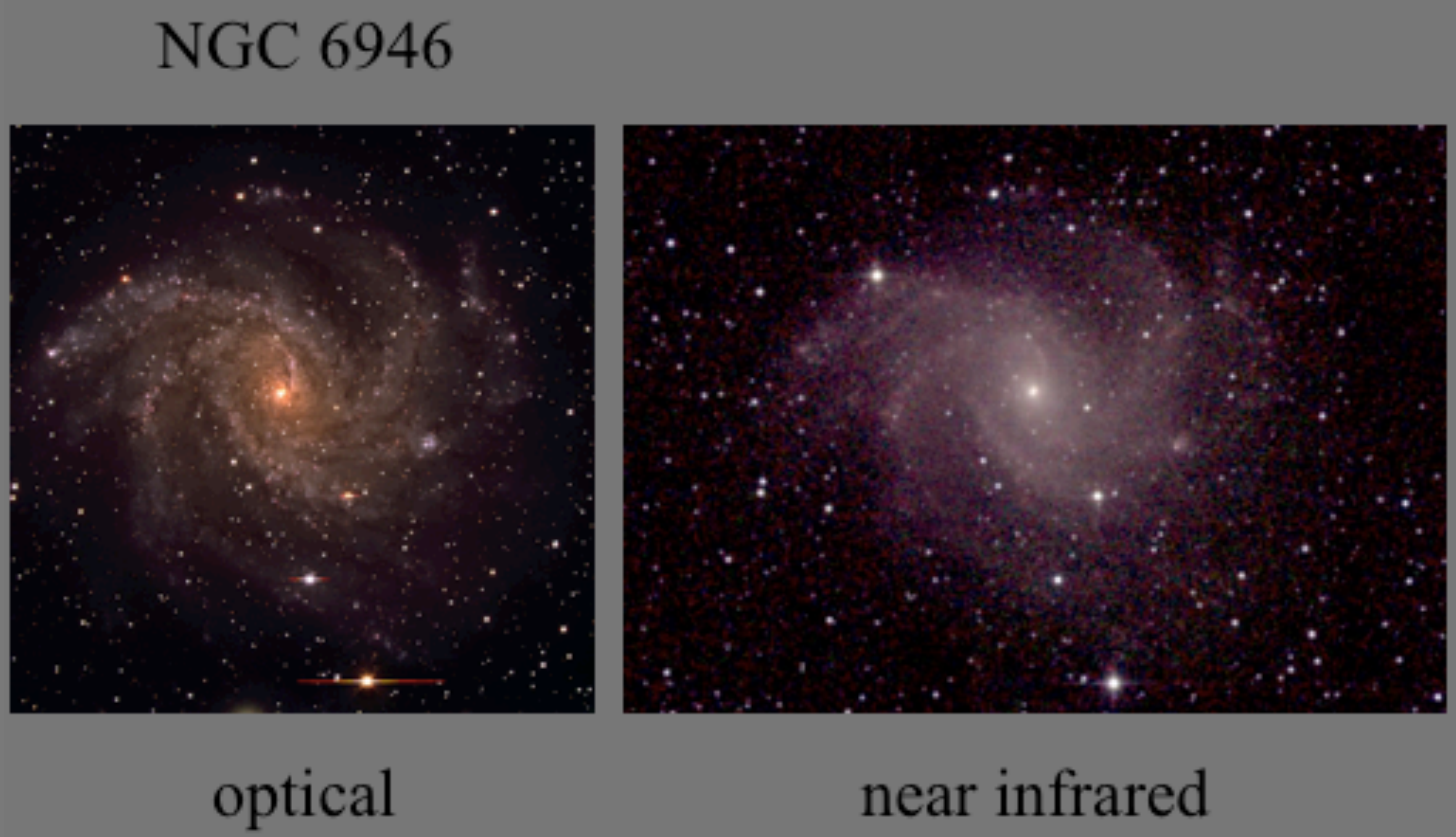
Examples for the size and mass of NGC 6946

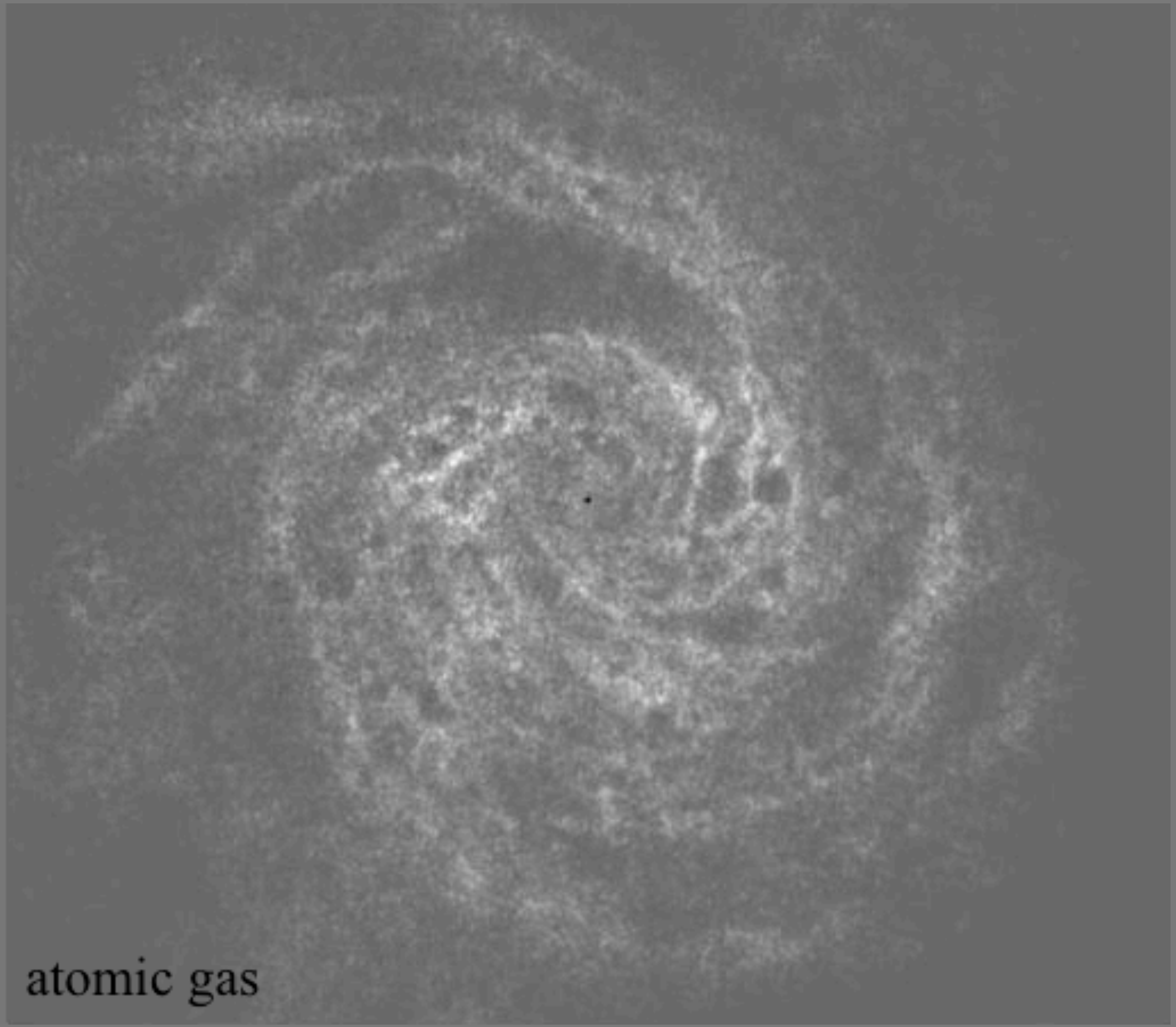
$$M_* = 3.3 \times 10^{10}\ {\rm M}_\odot$$
$$R_d = 2.44\ {\rm kpc}$$





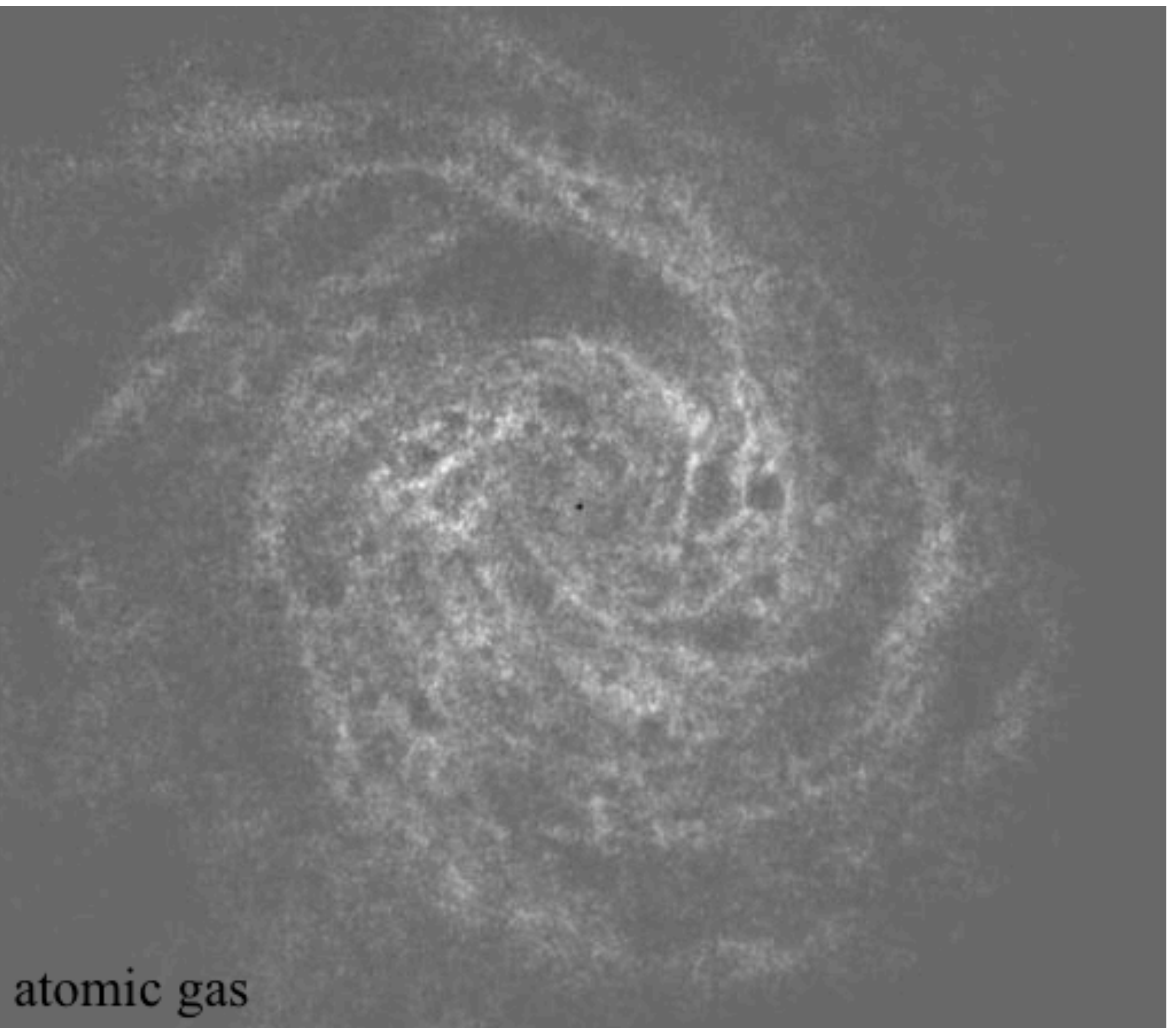
In practice, solve the Poisson equation numerically for each component separately



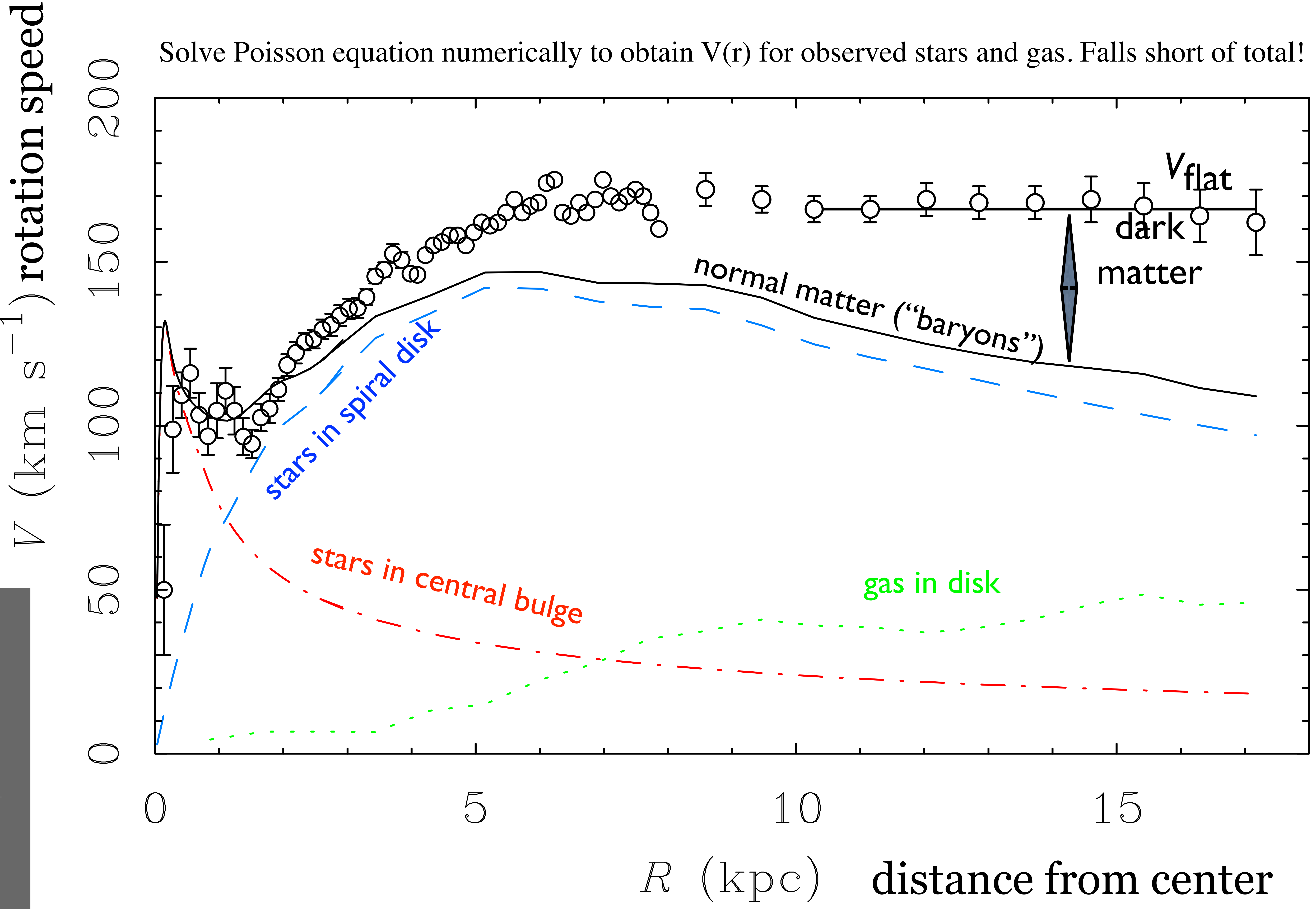


NGC 6946 stars & gas

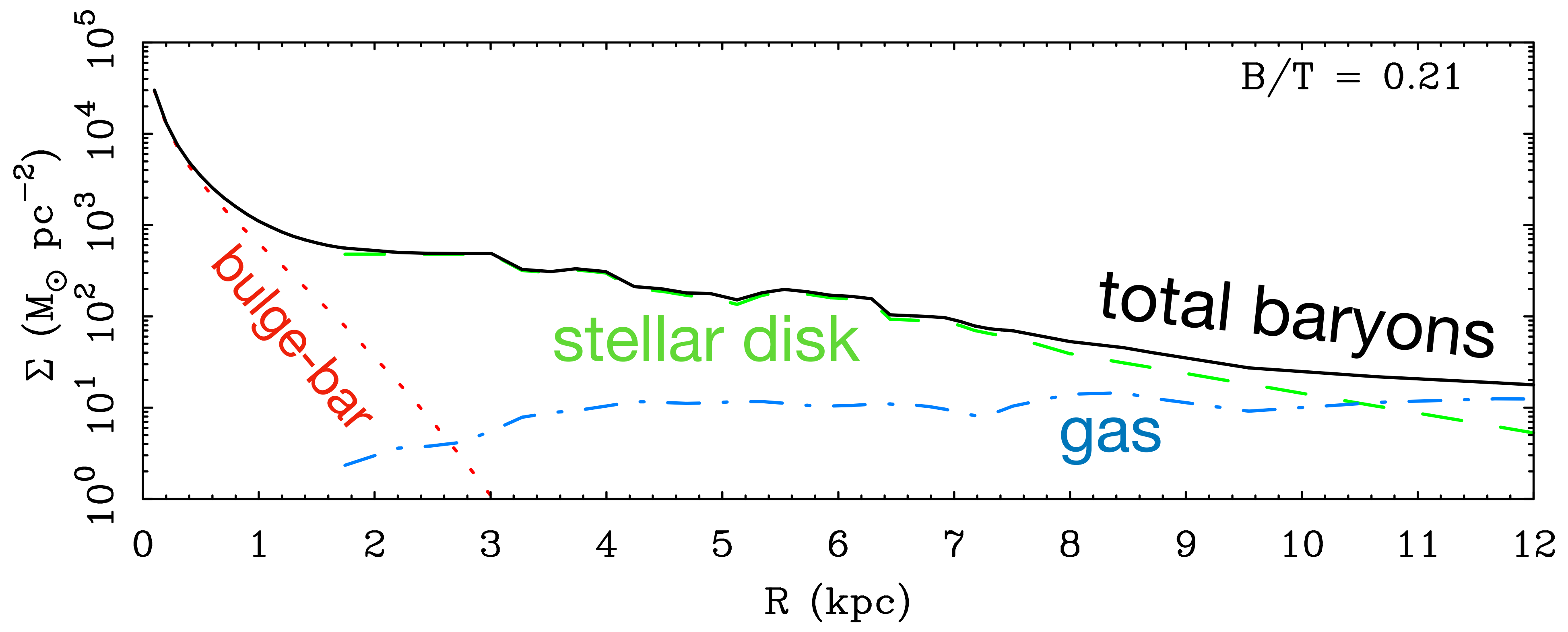
NGC 6946



Solve Poisson equation numerically to obtain V(r) for observed stars and gas. Falls short of total!



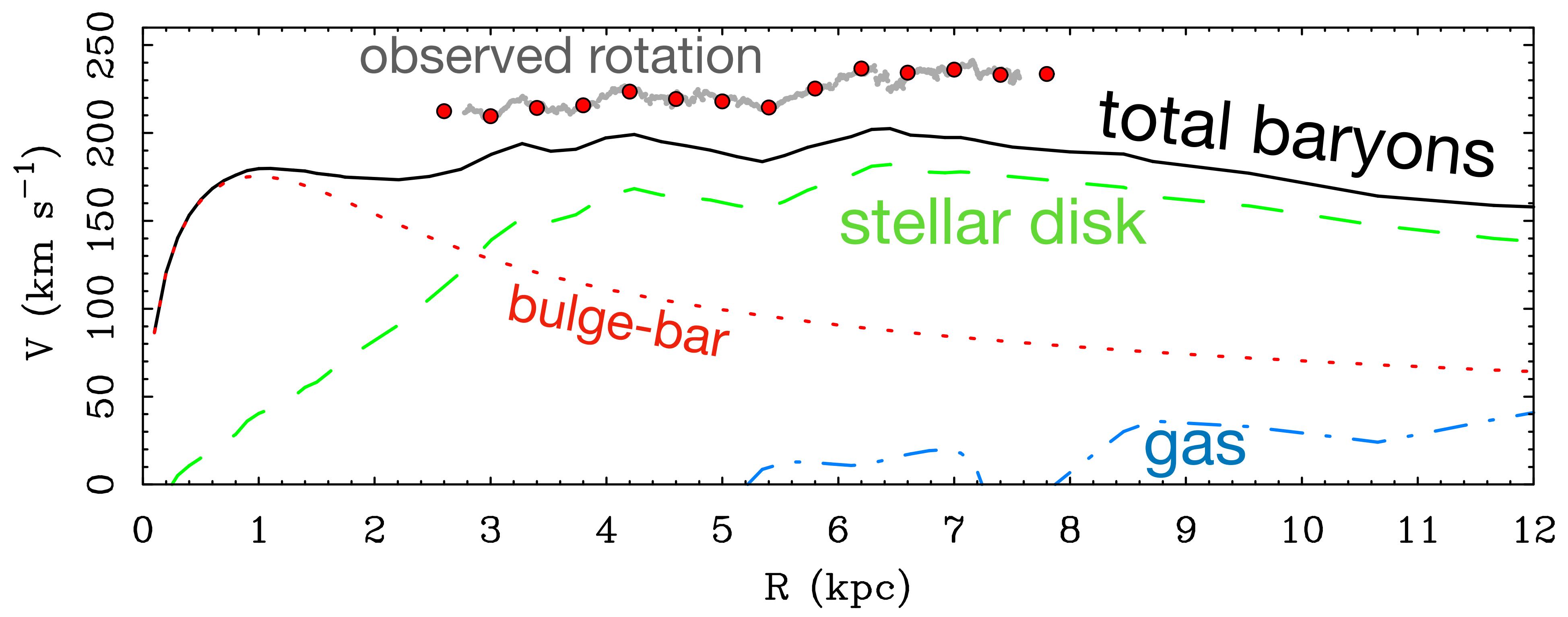
Mass modeling - Milky Way example



Surface density profile

$\Sigma(R)$ *from observed surface brightness profile*

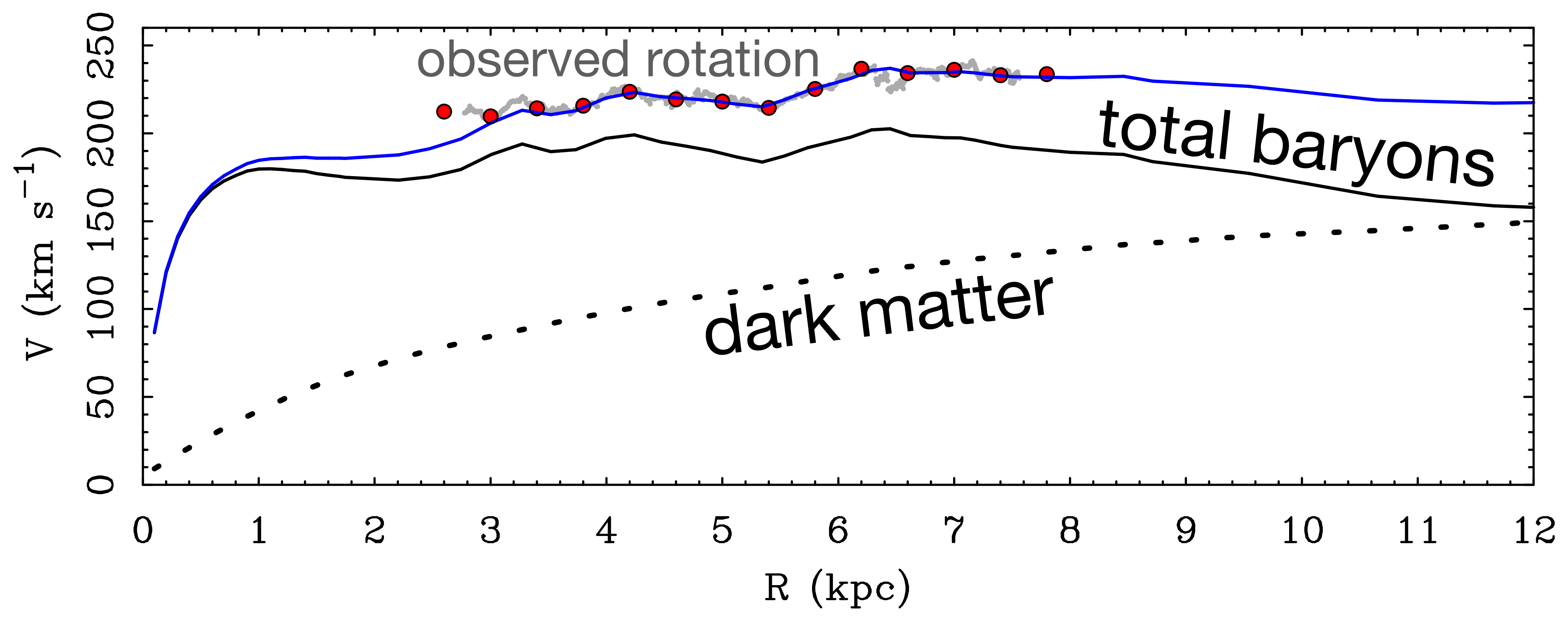
The main uncertainty is the mass-to-light ratio of the stars.



Mass model

$$\frac{V^2}{R} = -\frac{\partial \Phi}{\partial R} = 2\pi G \Sigma(R)$$

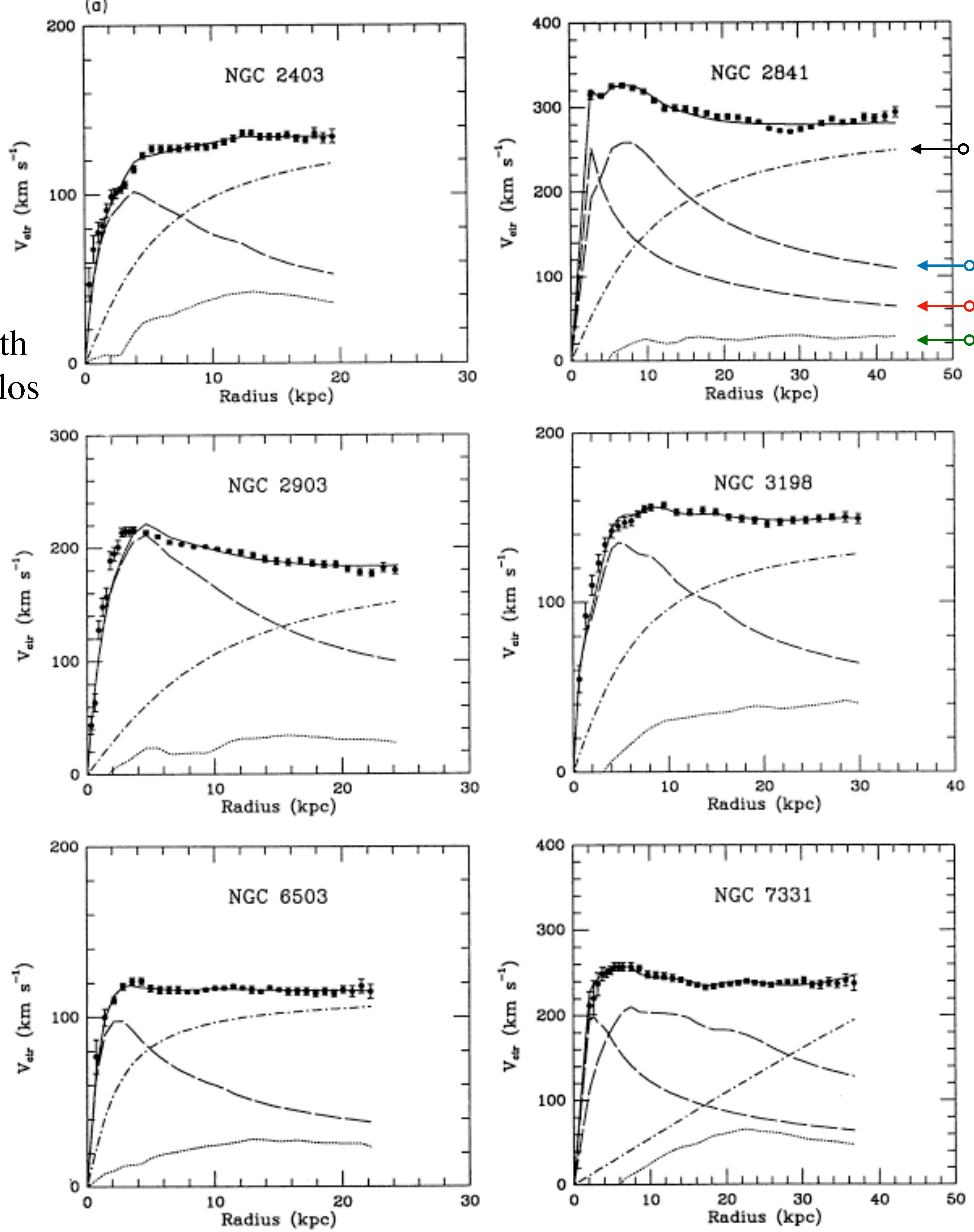
$$V_{DM}^2 = V_{obs}^2 - V_{bar}^2$$



Mass model with DM halo

Total rotation decomposed into baryonic and dark components

Rotation curves fit with pseudo-isothermal halos

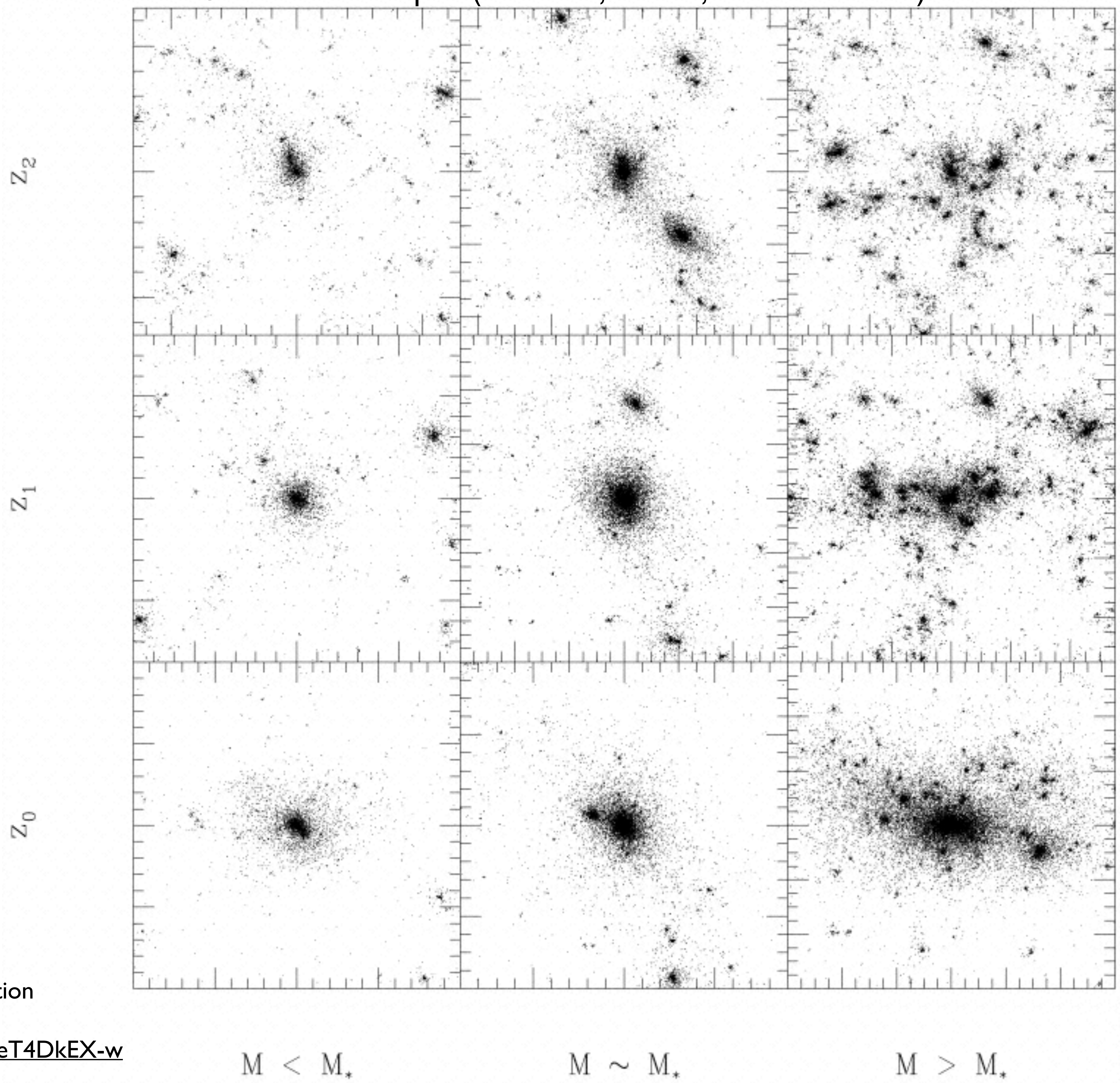


Begeman, Broeils, & Sanders (1991)

It is often assumed that $V_\infty = V_f$ but typically we only observe the rising part of the DM rotation curve

Simulation output (Navarro, Frenk, & White 1997)

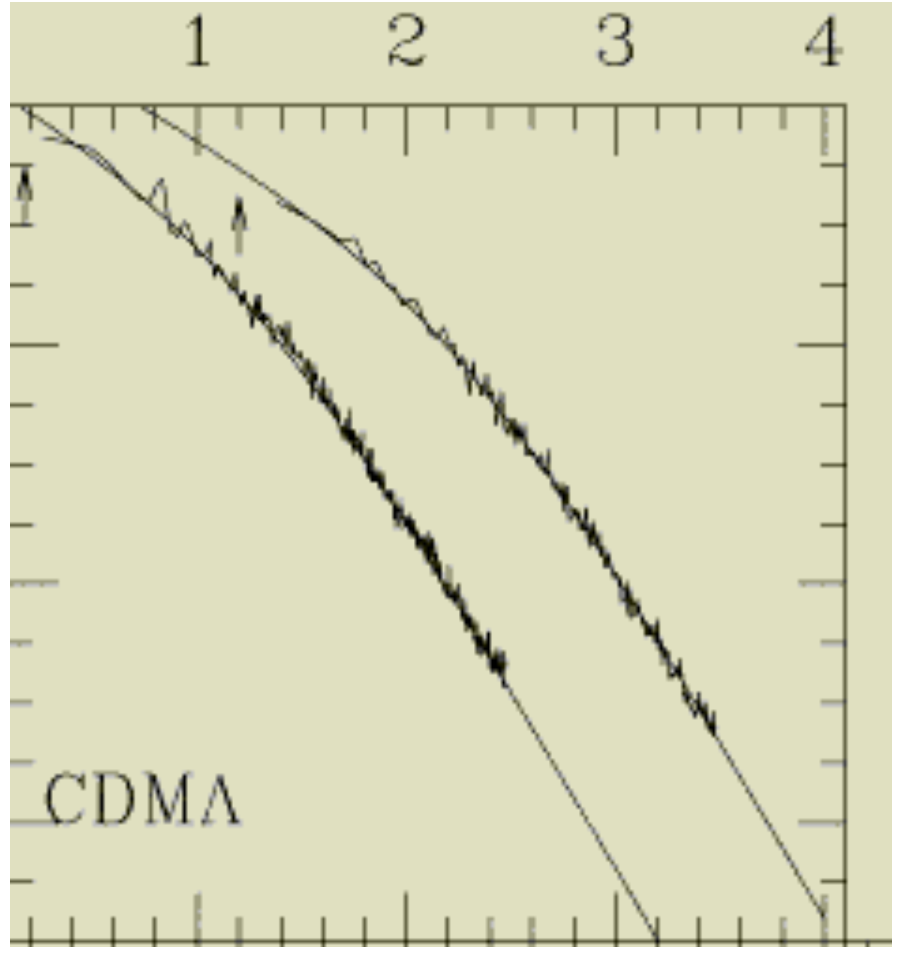
NFW



Aquarius large scale structure simulation

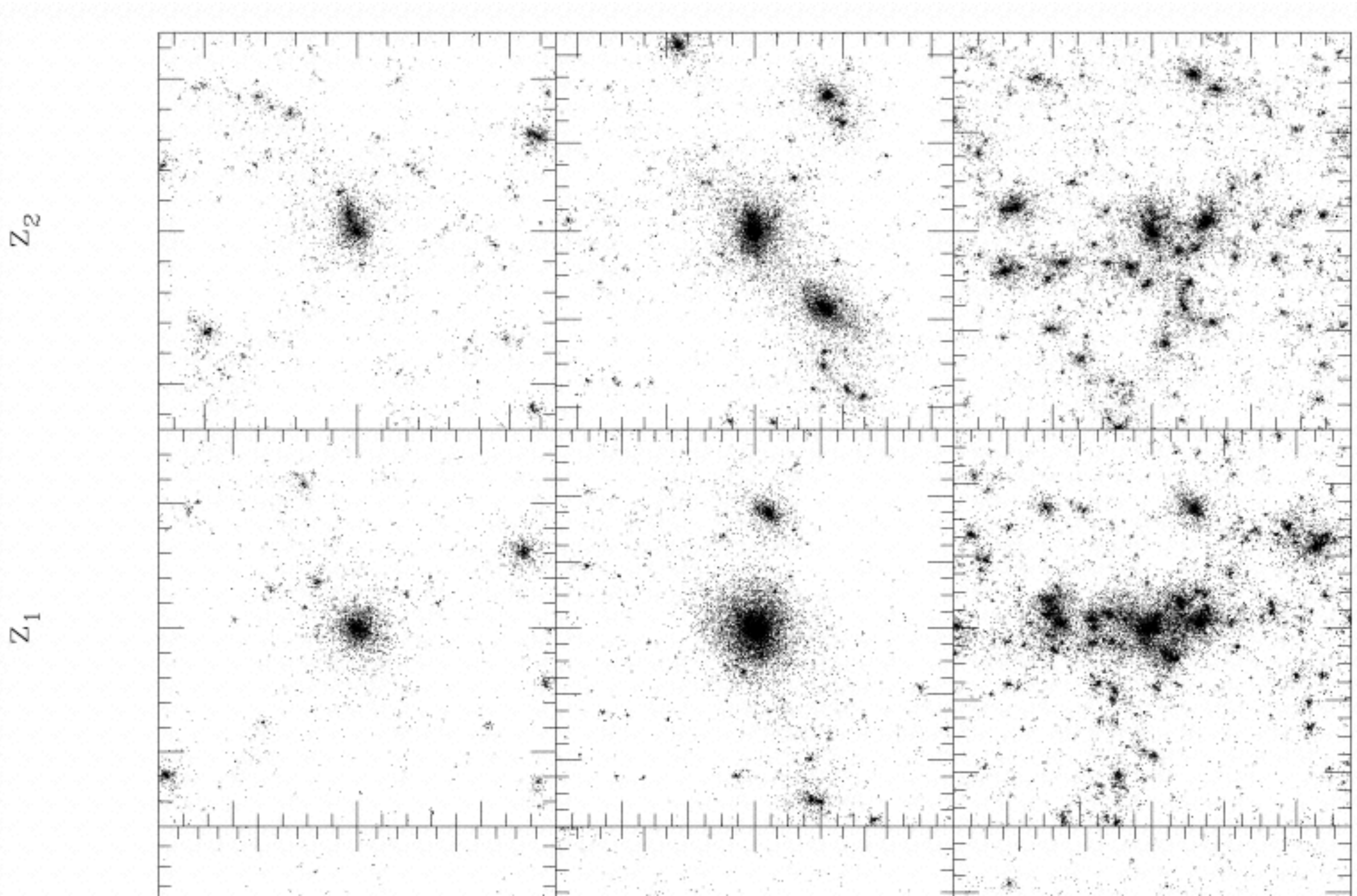
http://www.youtube.com/watch?v=2qeT4DkEX-w

NFW



NFW halo

$$\rho(r) = \frac{\rho_s r_s^3}{r(r+r_s)^2}$$



Can also define an overdensity Δ

$$M_\Delta = \frac{4\pi\Delta}{3}\rho_{crit}\, r_\Delta^3$$

Conventionally take `virial' $\Delta = 200$

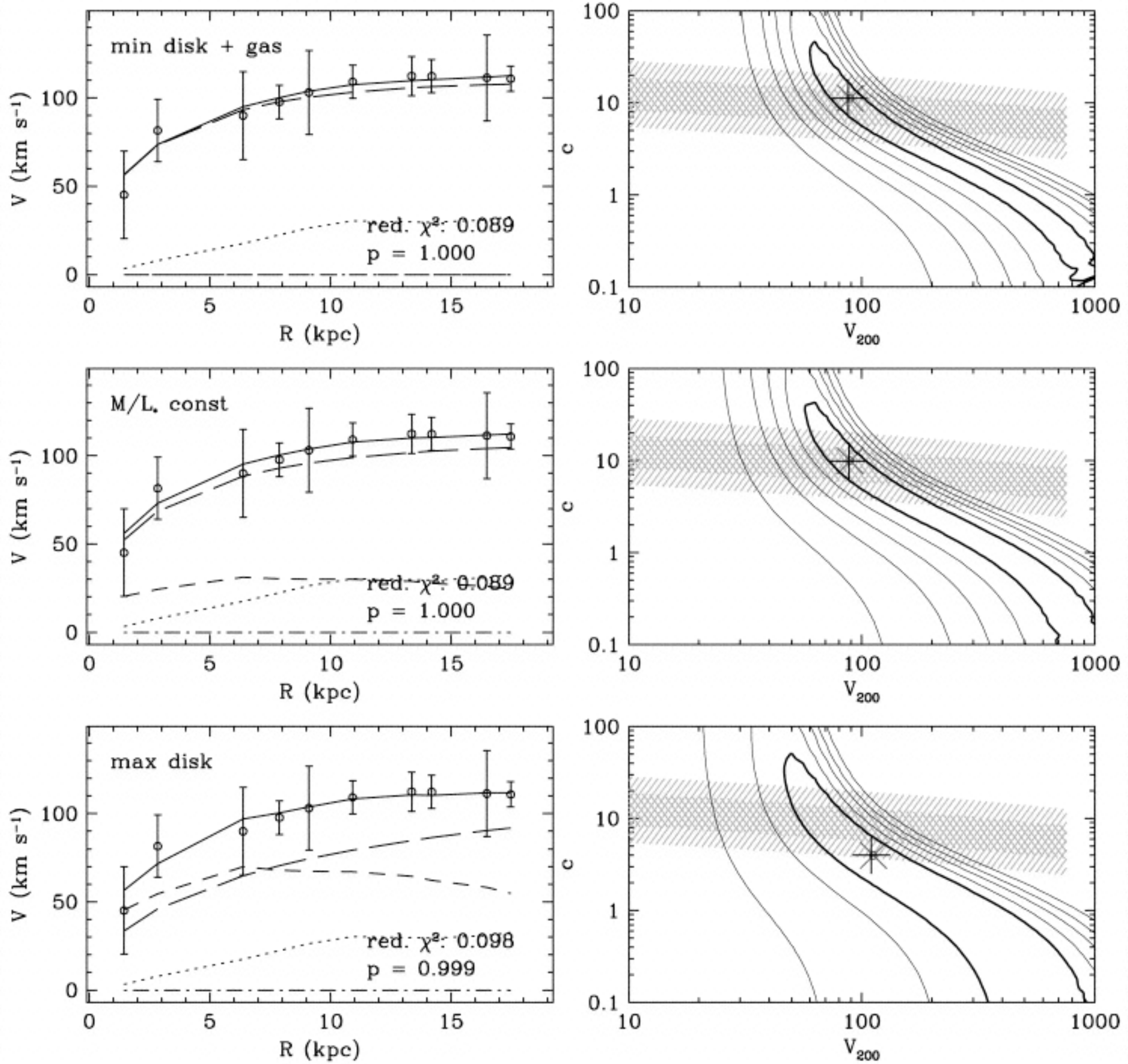
$$V_{200}^2 = \frac{G\,M_{200}}{r_{200}}$$

halo mass $M_{200} = (3.3\times10^5\ \mathrm{M_\odot\,km^{-3}\,s^3})\,V_{200}^3$

concentration $c = \frac{r_{200}}{r_s}$

NFW
fits

LSB
F563-1



considerable parameter degeneracy!