

DARK MATTER

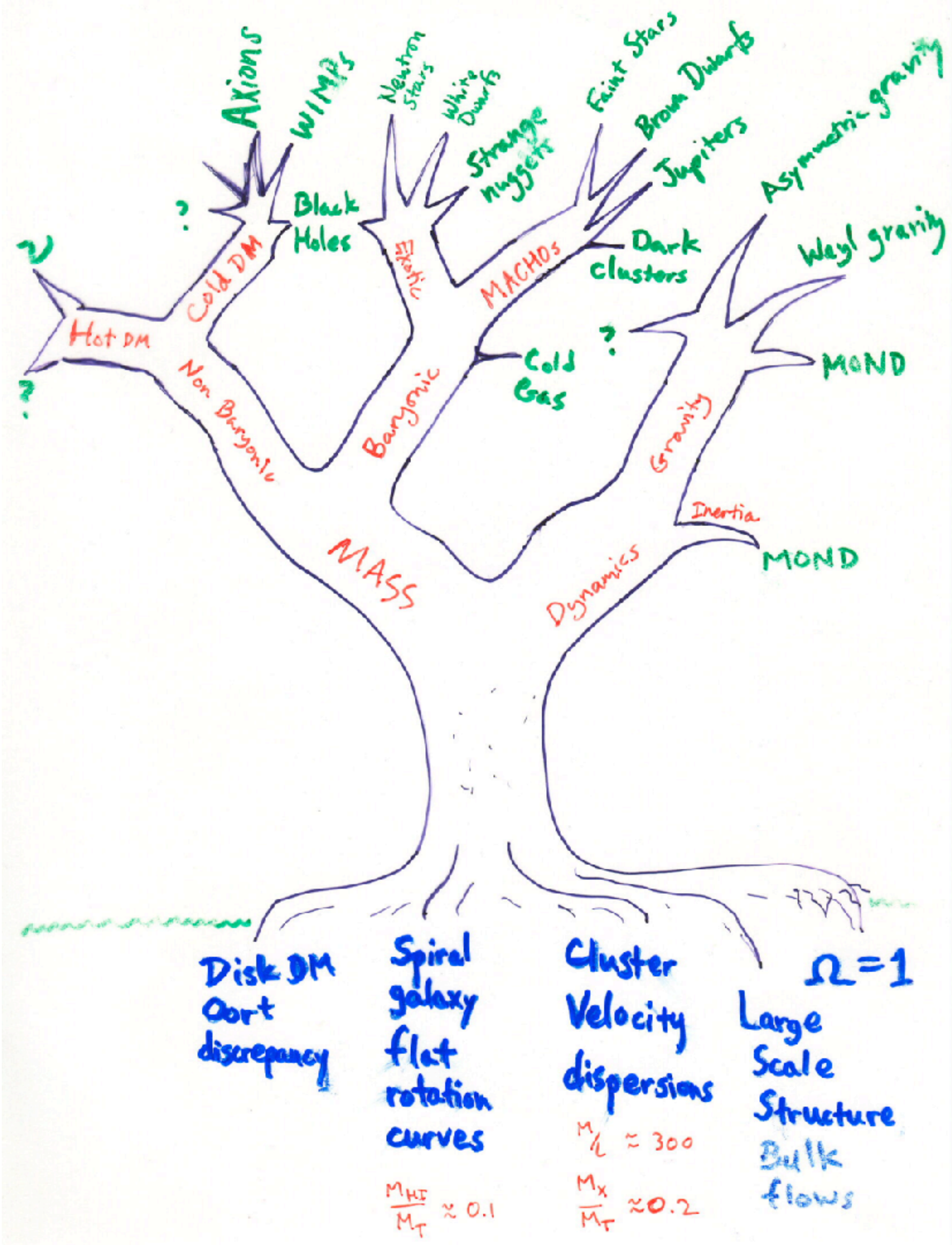
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SPRING 2026
TR 11:30AM-12:45PM
SEARS 552

<http://astroweb.case.edu/ssm/ASTR333/>

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HOMEWORK DUE



Empirical Laws of Galactic Rotation

- Flat rotation curves (Rubin-Bosma Law)

Rotation curves tend asymptotically towards a constant rotation velocity that persists to indefinitely large radii: $V(R \rightarrow \infty) \rightarrow V_f$

TF /

- Tully-Fisher relation (Luminous, Stellar Mass, and Baryonic TF relations)

BTFR

The baryonic mass of galaxies scales as the fourth power of the flat rotation velocity: $M_b = AV_f^4$

diversity

- Central density relation (lower surface brightness galaxies exhibit larger mass discrepancies)

The central dynamical surface densities of galaxies is related to their central surface brightnesses: $\Sigma_{dyn}(R \rightarrow 0) = f[\Sigma_*(R \rightarrow 0)]$

- Renzo's rule (Sancisi's Law)

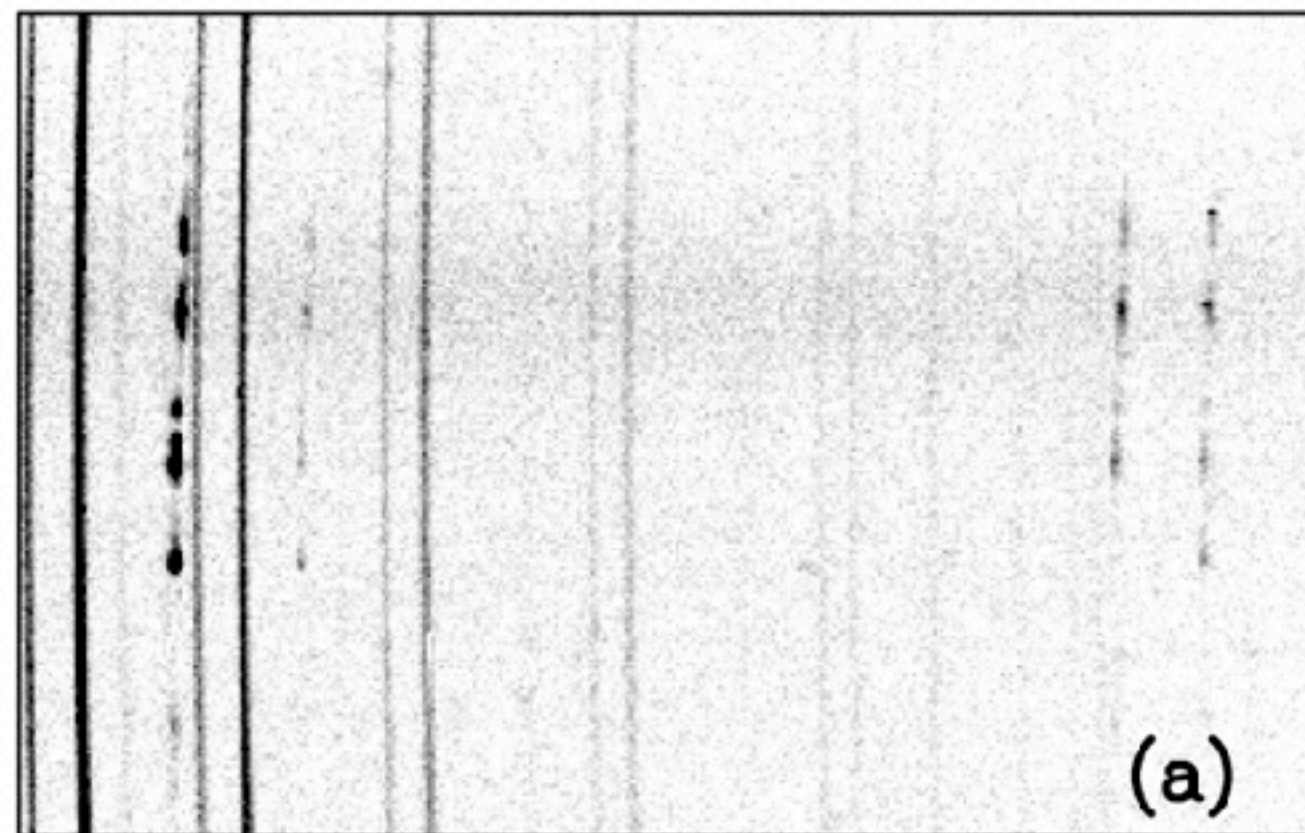
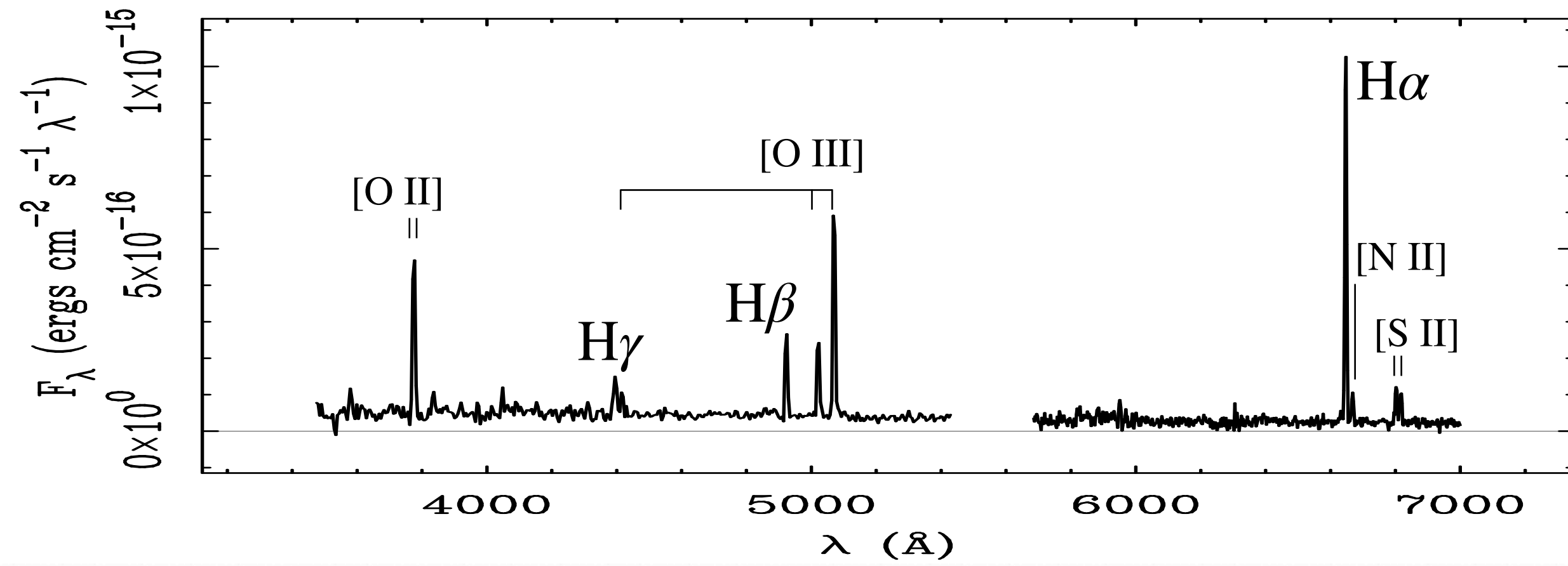
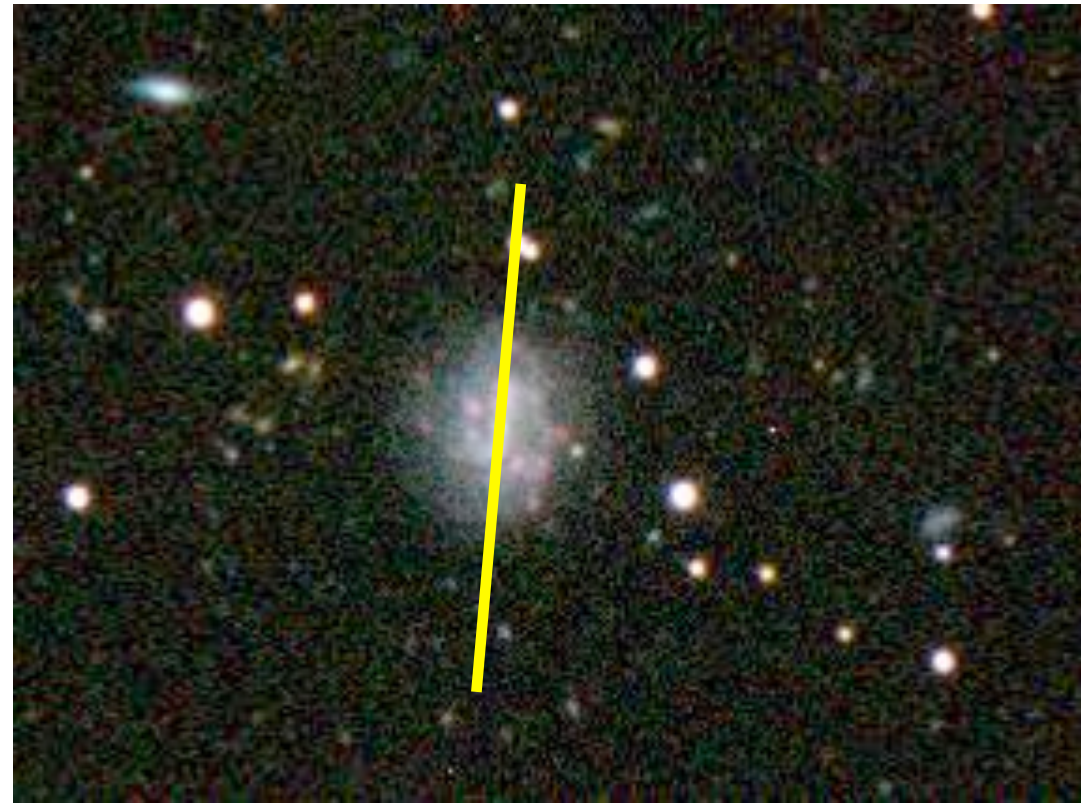
“For any feature in the luminosity profile there is a corresponding feature in the rotation curve and vice versa.” (Sancisi 2004).

RAR

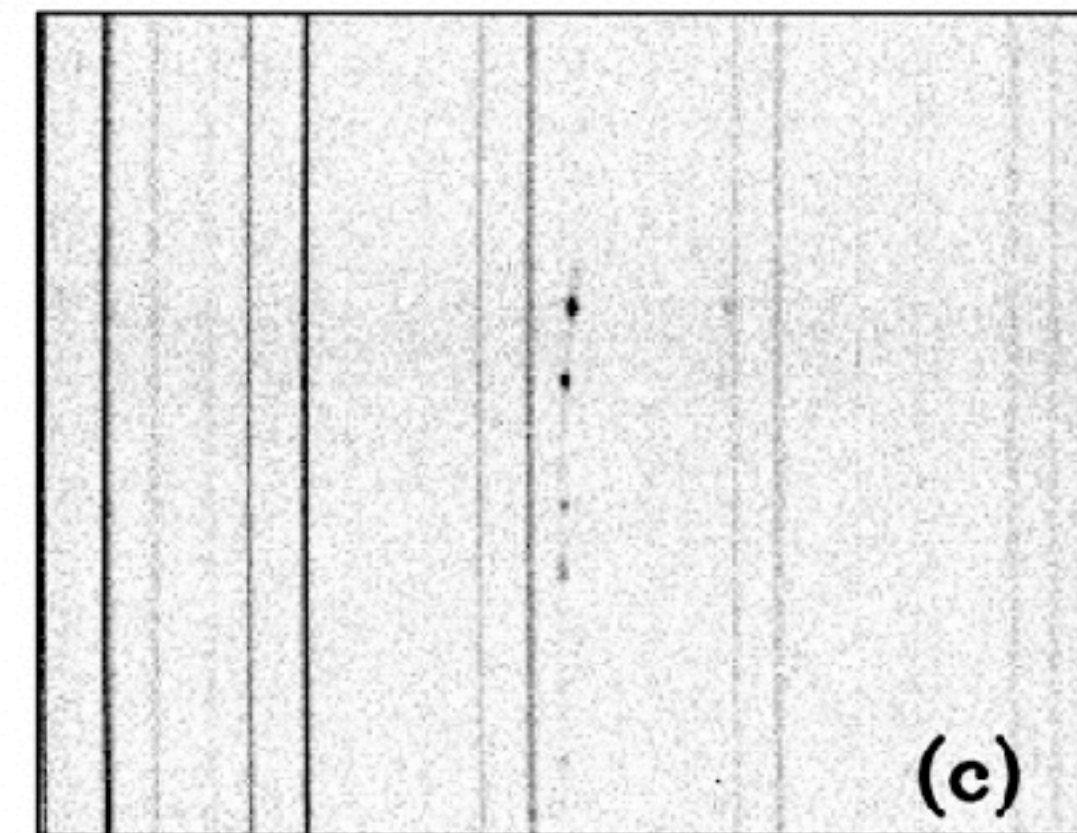
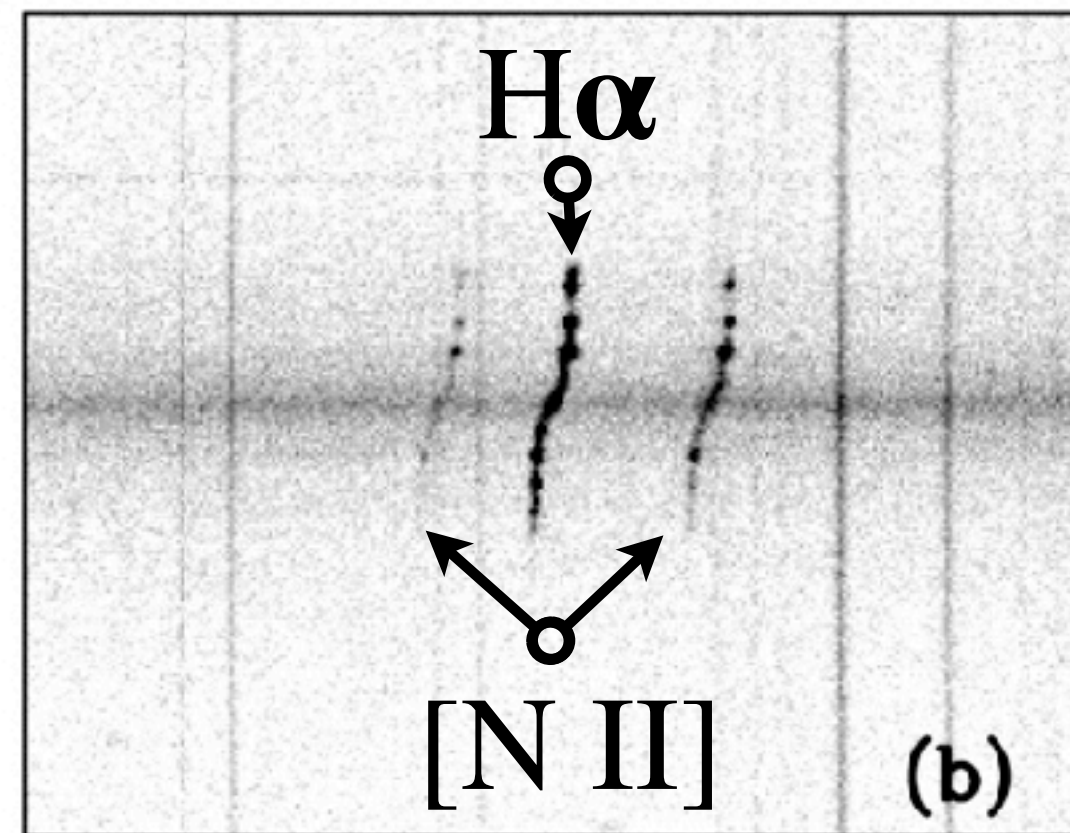
- Radial acceleration relation

The observed centripetal acceleration is related to that predicted by the observed distribution of baryons: $g_{\text{obs}} = \mathcal{F}(g_{\text{bar}})$

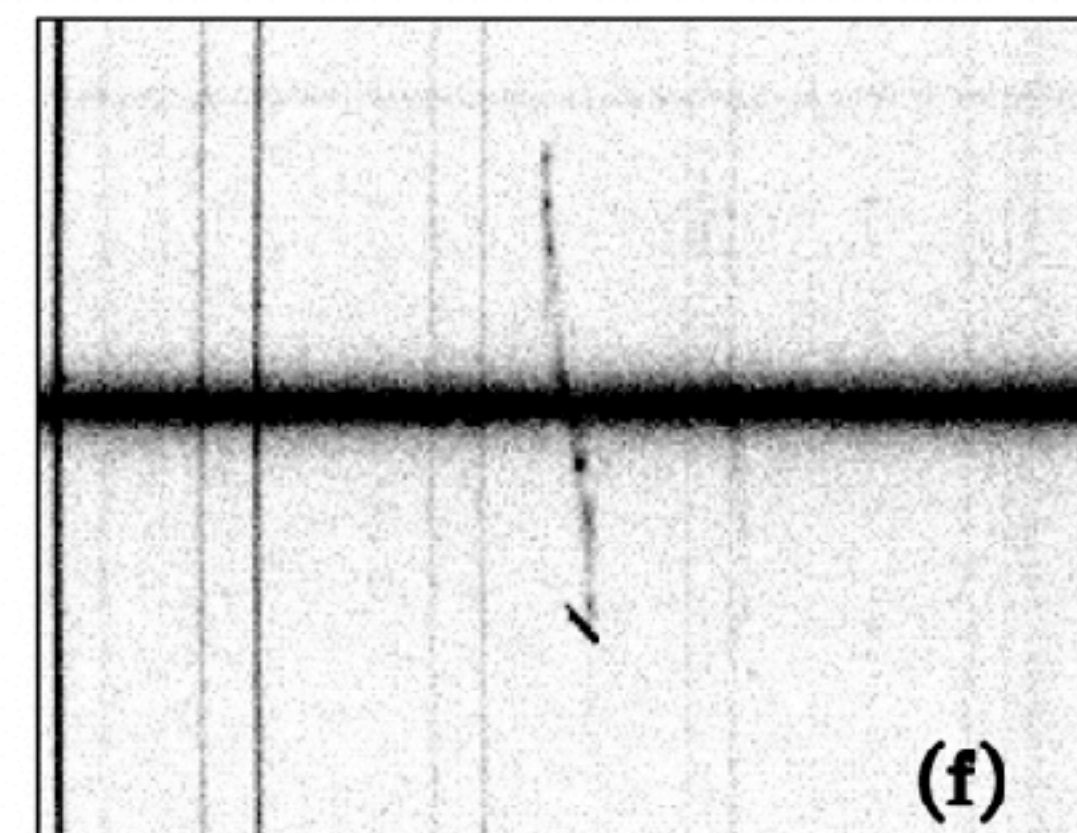
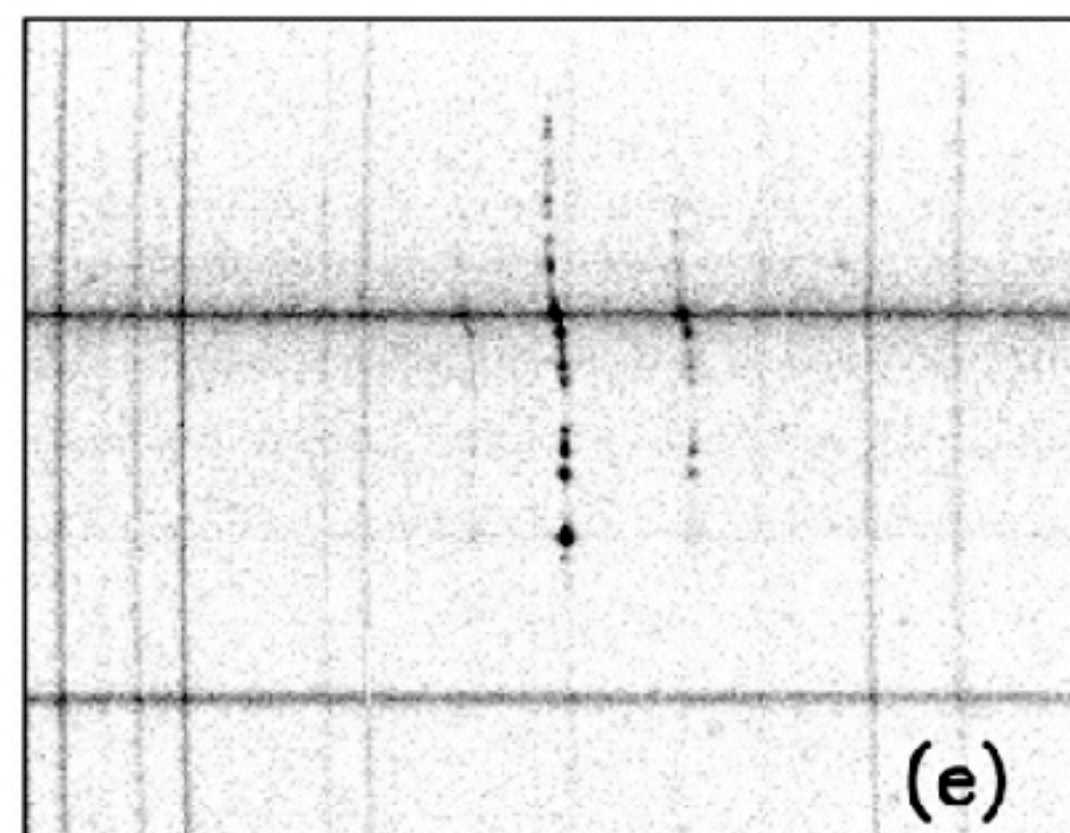
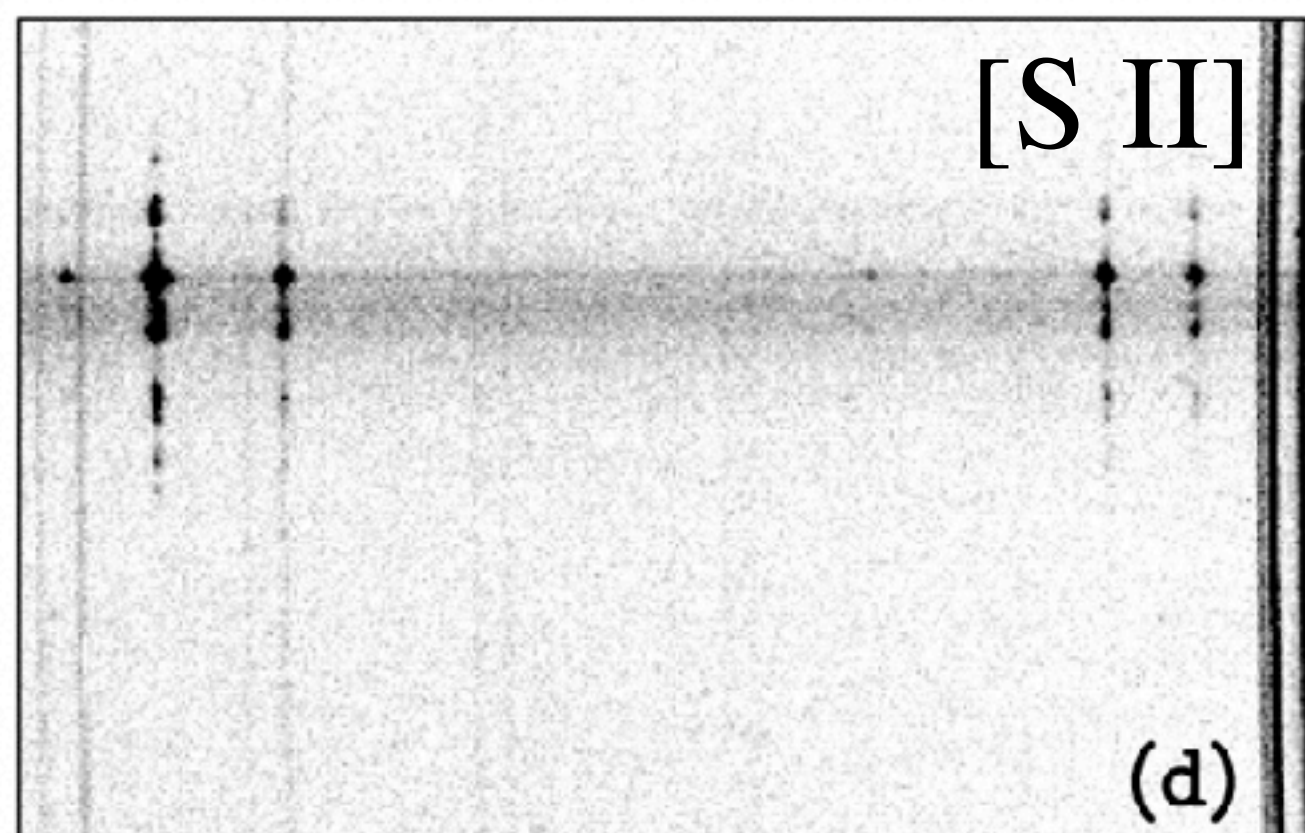
Optical data



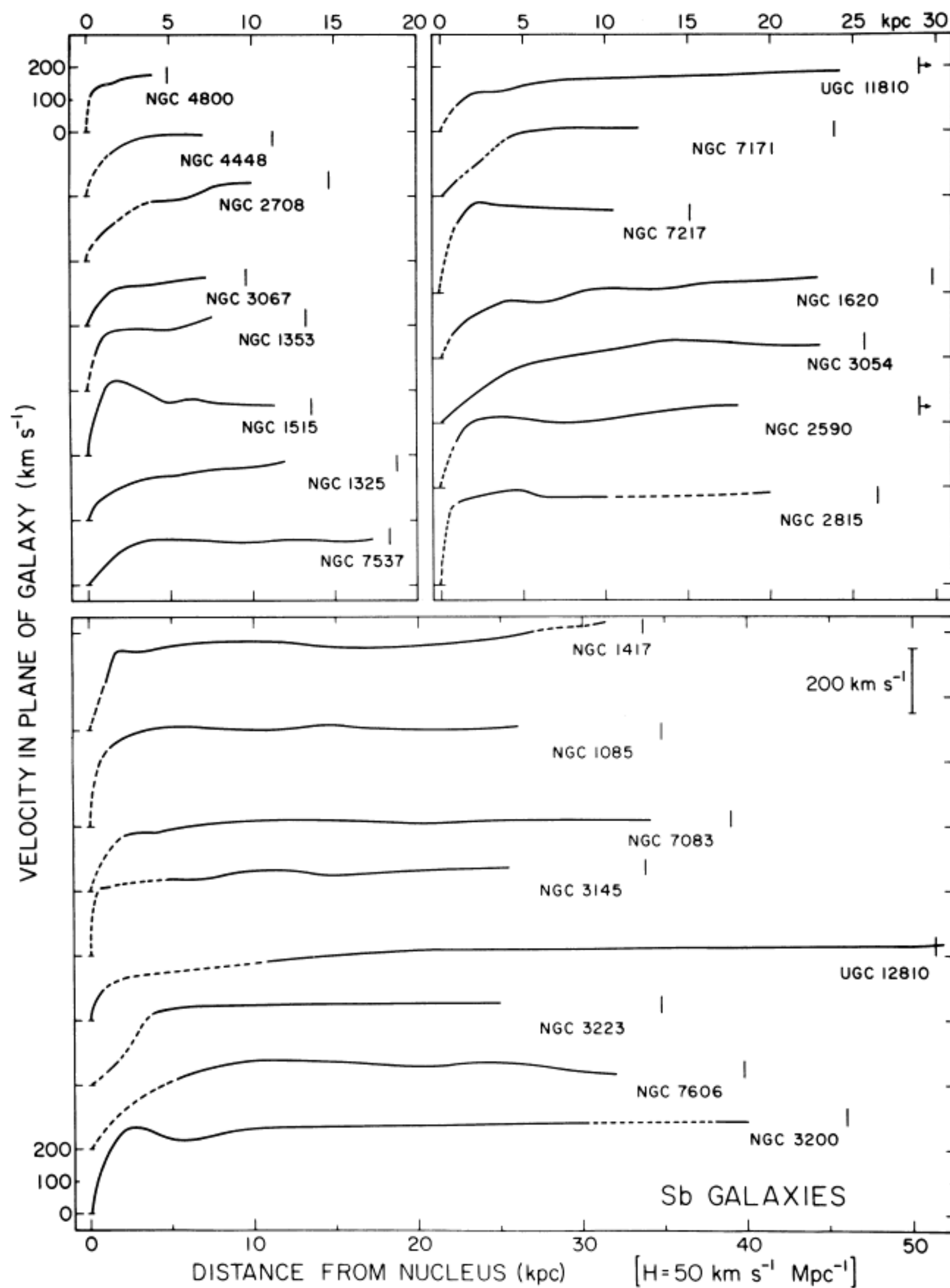
position along slit



wavelength



Optical data



Rotation curves tend to become flat at large radii

$$V \propto \text{const}$$

$$M \propto R$$

$$\rho \propto R^{-2}$$

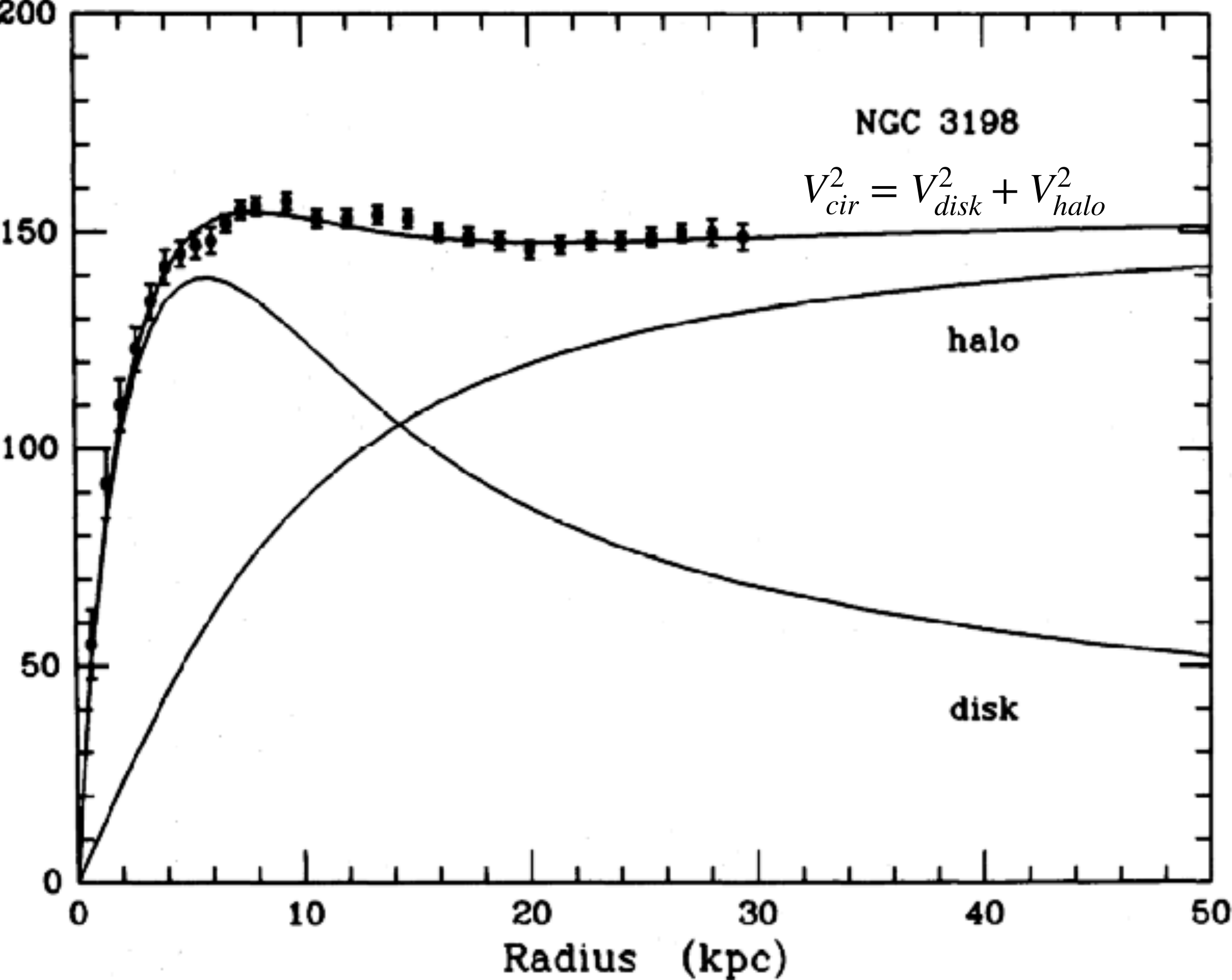
Optical data from Rubin, Thonnard, & Ford 1978, *ApJ*, **225**, L107

FIG. 3.—Mean velocities in the plane of the galaxy, as a function of linear radius for 23 Sb galaxies, arranged approximately according to increasing luminosity. Adopted curve is rotation curve formed from the mean of velocities on both sides of the major axis. Vertical bar marks the location of R_{25} , the isophote of $25 \text{ mag arcsec}^{-2}$, corrected for effects of internal extinction and inclination. Regions with no measured velocities are indicated by dashed lines.

Radio data

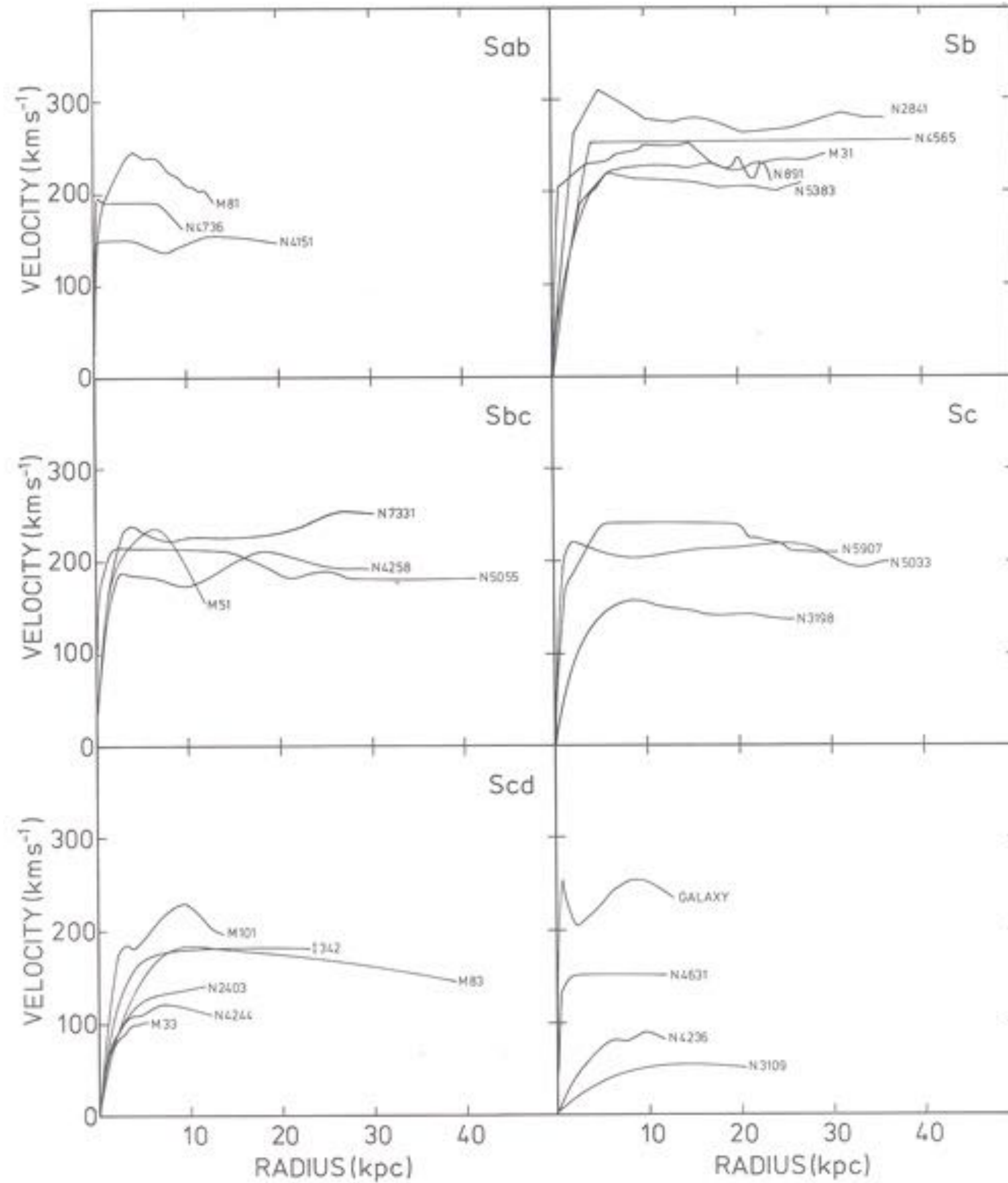
van Albada et al. (1985)

DISTRIBUTION OF DARK MATTER IN NGC 3198



Radio data

Radio data from
Bosma 1981, *AJ*, **86**, 1825



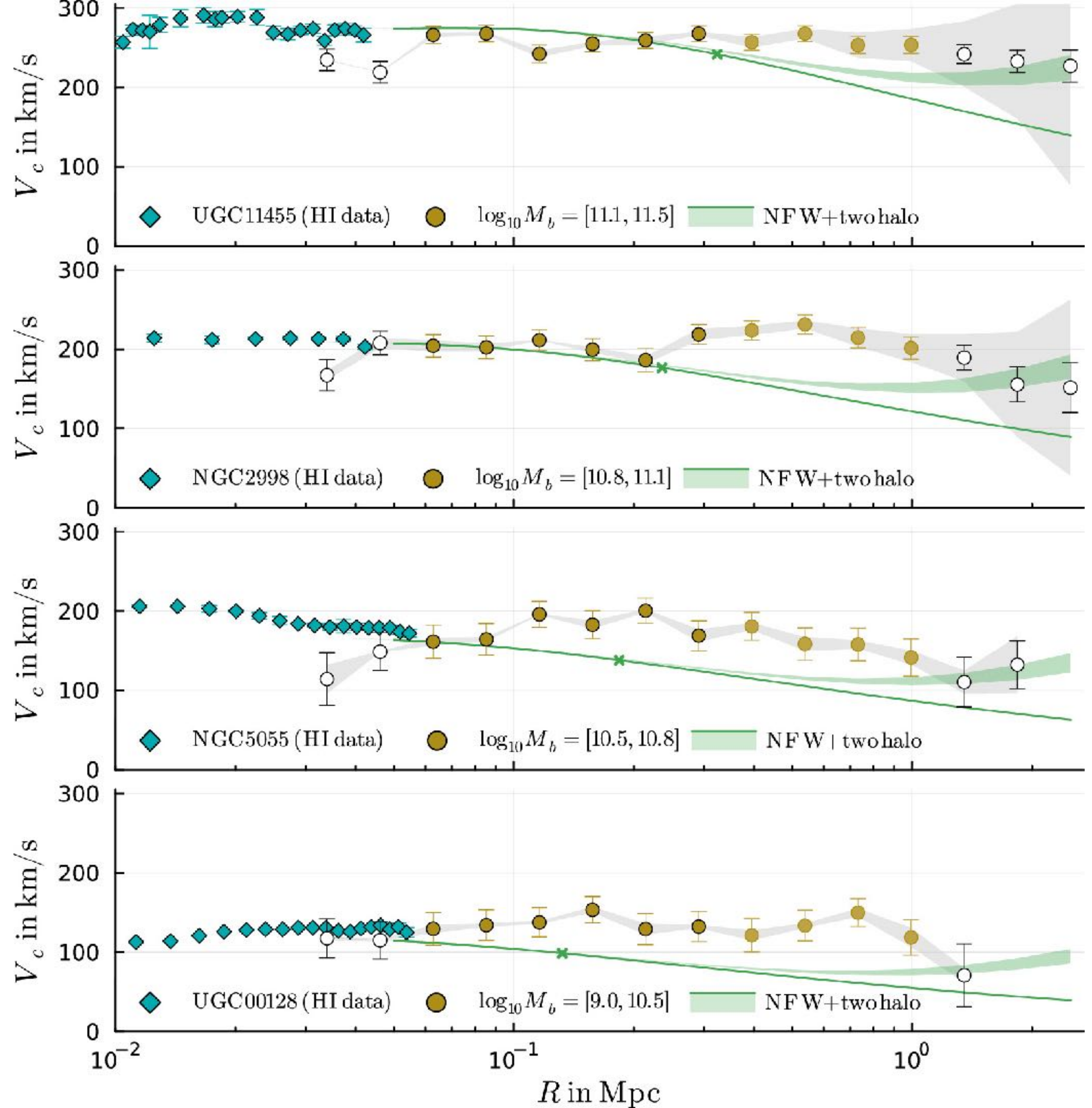
Rotation curves stay flat
to the largest radii probed

Historically, 21cm data were an
important independent validation
that flat rotation curves persisted
to much larger radii than could be
explained by the observed
luminous mass.

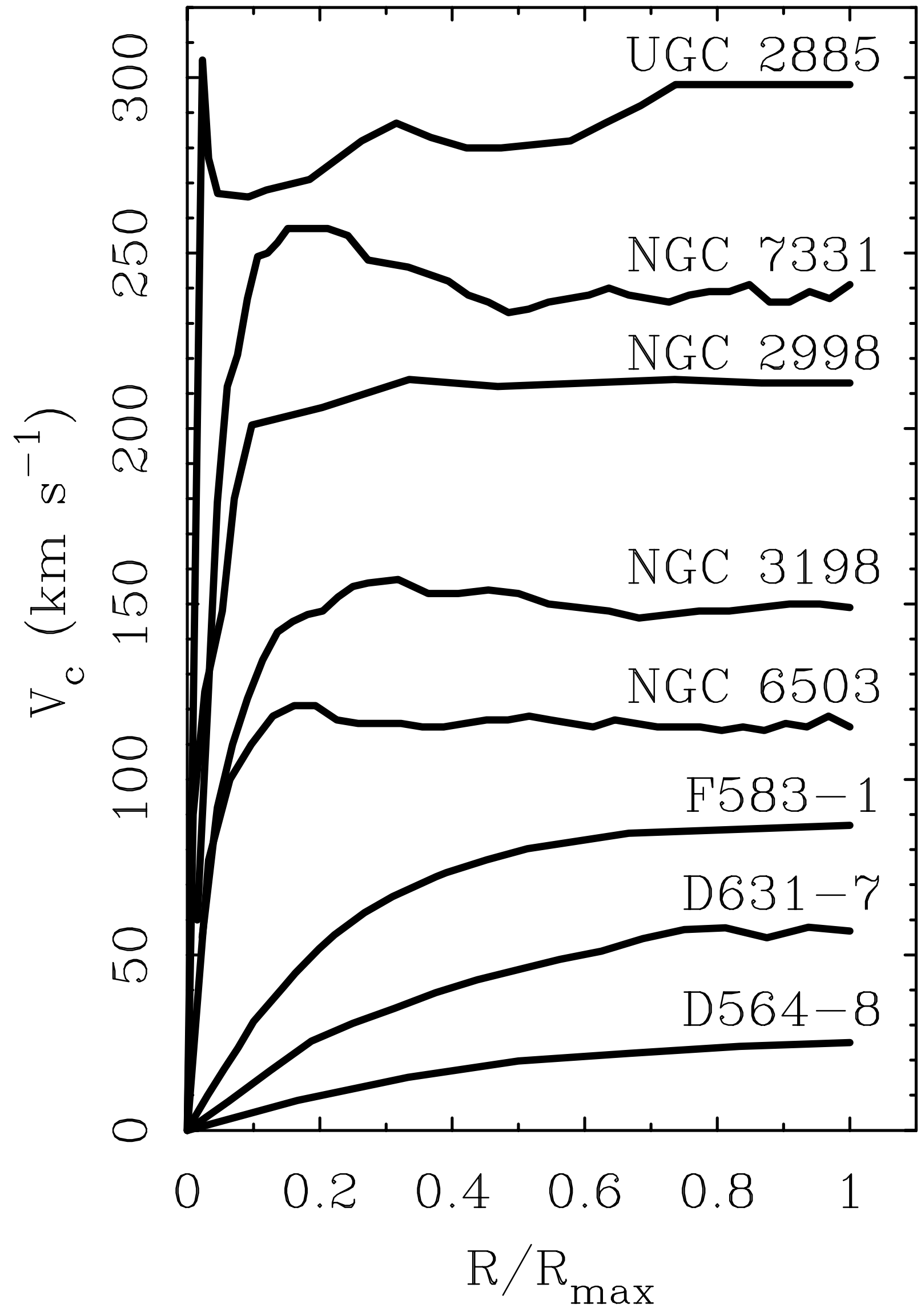
See IAU Symposium 100 pp. 87-88
(Kalnajs on mass models)

Gravitational Lensing data

flatness extends...
even to Mpc scales!?



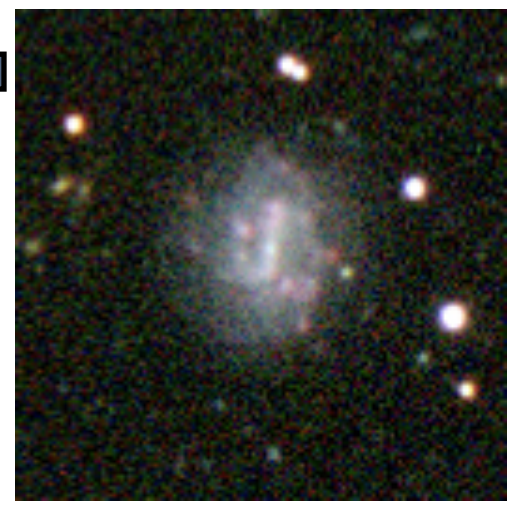
Tully-Fisher: Rotation curve amplitude correlates with observed mass:



star dominated HSB



gas dominated LSBs



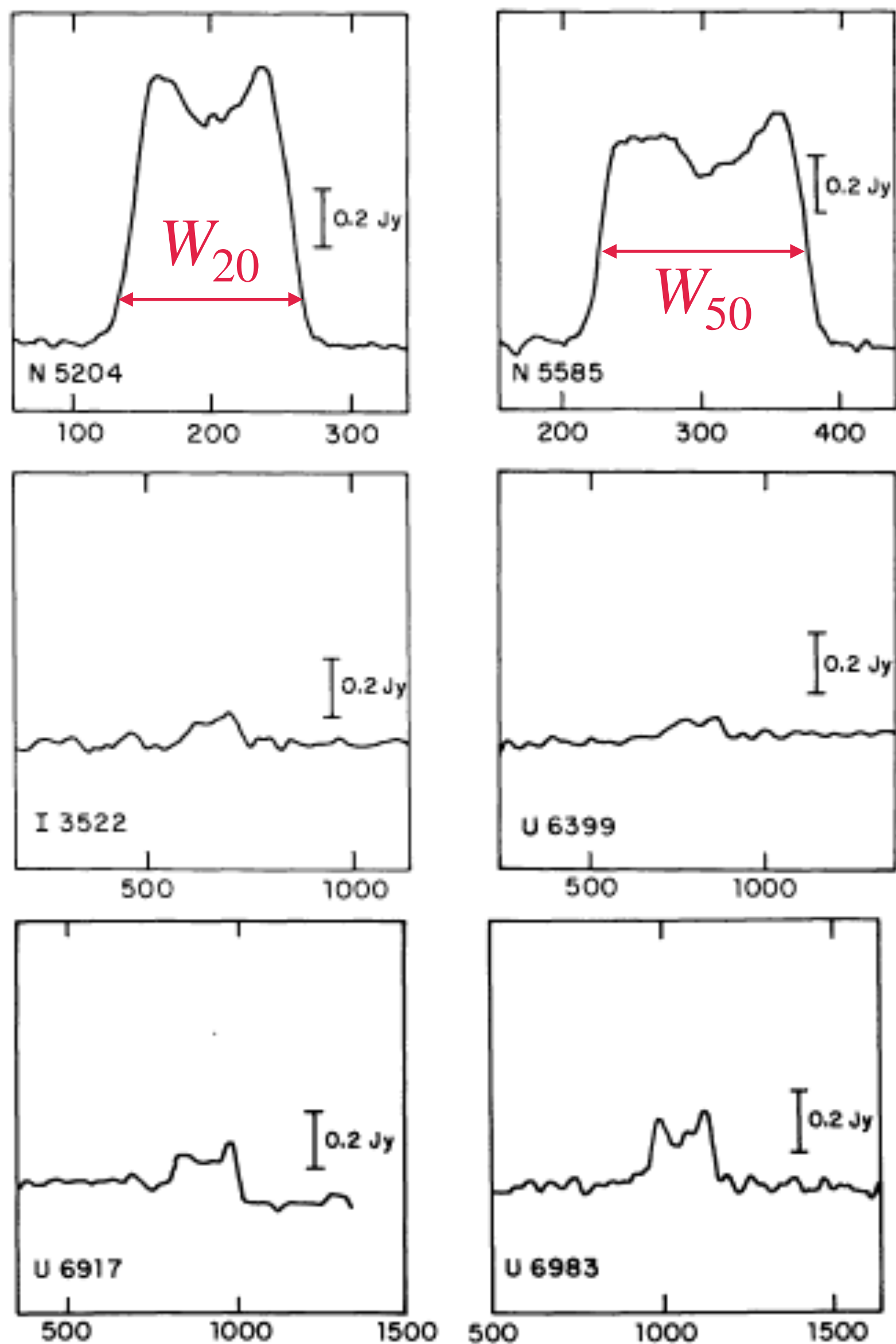
Flat rotation curves continue to occur in quite small systems (e.g., Leo P with $V_f \sim 15$ km/s)

Tully & Fisher (1977)

Great for distance scale work.

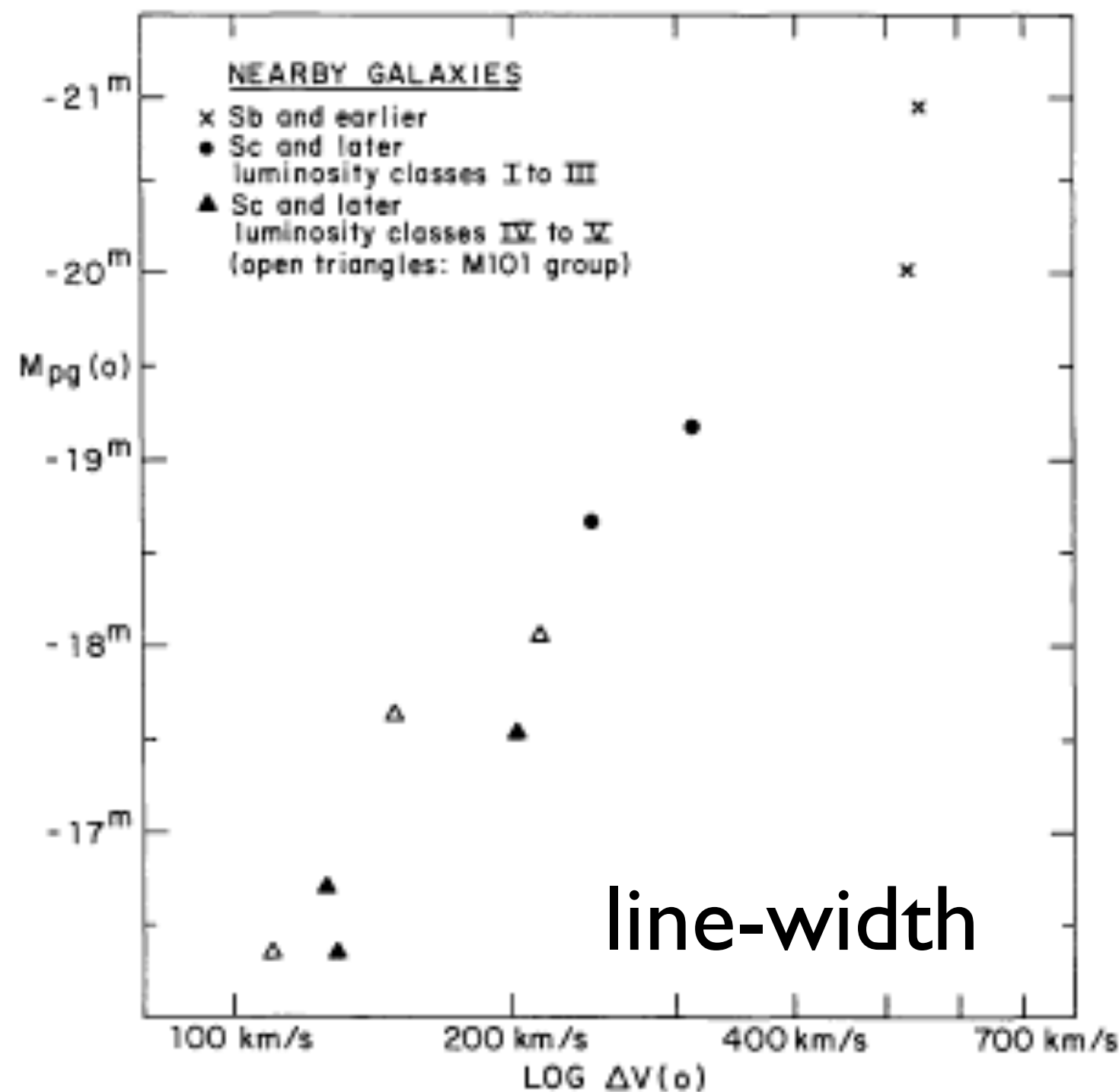
But why does it happen?

line-width



R. B. Tully and J. R. Fisher: Distances to Galaxies

Absolute Magnitude



line-width

Fig. 1. Absolute magnitude – global profile width relation for nearby galaxies with previously well-determined distances. Crosses are M31 and M81, dots are M33 and NGC 2403, filled triangles are smaller systems in the M81 group and open triangles are smaller systems in the M101 group

others from ST I and ST III]; (4) photographic magnitudes (Holmberg, 1958); (5) magnitude corrections due to galactic extinction according to the precepts in ST I [based on Sandage (1973), except that the source for M31 and M33 is McClure and Racine (1969), and for NGC 2403 is Tammann and Sandage (1968)]; (6) magnitude corrections due to galactic absorption as a function of inclination according to the precepts used by Sandage and Tammann (1974d, hereafter ST IV)

we take these uncertainties to be dominated by those

We have precisely the relations presented in ST I, not in total agreement with the complete lack of correlation to V (e.g., Fisher and Tully with them that, ultimately, the corrections to make is not of the results we are getting from the Virgo cluster, we are clearly comparable as that the two Sb systems correction (following modestly affect (steeply not the measurement because both the nearby Sb systems, at similar with similar inclination

In explicitly following given in ST IV we determine between the inclination and the value of ξ used to discuss we want to scale, but for the proper inclination is needed.

In Figure 1, correlations plotted against corrected for the local sample to distinguish Sb's, Sc I to as the three members to give a realistic estimate points, but if the vertical

Observables

- Luminosity (must calibrate with known D)
 - Band pass (*BVRIJHK*) [slope varies with band]
 - Mass - stars, gas, stars+gas
- Rotation Velocity
 - line-widths; rotation curves
 - W_{20} , W_{50} ; V_{flat} , $V_{2.2}$, V_{max}
 - inclination corrections $1/\sin(i)$
 - turbulence/non-circular motions

Luminosity measures

- Band pass
 - slope becomes steeper from bluer to redder bands (B / H)
 - internal extinction is a concern, especially for blue bands and highly inclined galaxies
- Mass
 - Can convert luminosity to stellar mass by estimating the stellar M/L via population modeling.
 - IMF biggest systematic uncertainty

What we measure

- Luminosity
- Stellar Mass
- Gas: HI, H₂
- Rotation speed
 - line-width
 - rotation curve

Uncertainties

- Distance
- Stellar M_{*}/L
- HI flux, X-factor
- velocity dispersion
- inclination
- asymmetric drift

Rotation curve data from

Boomsma et al (2008) [HI]

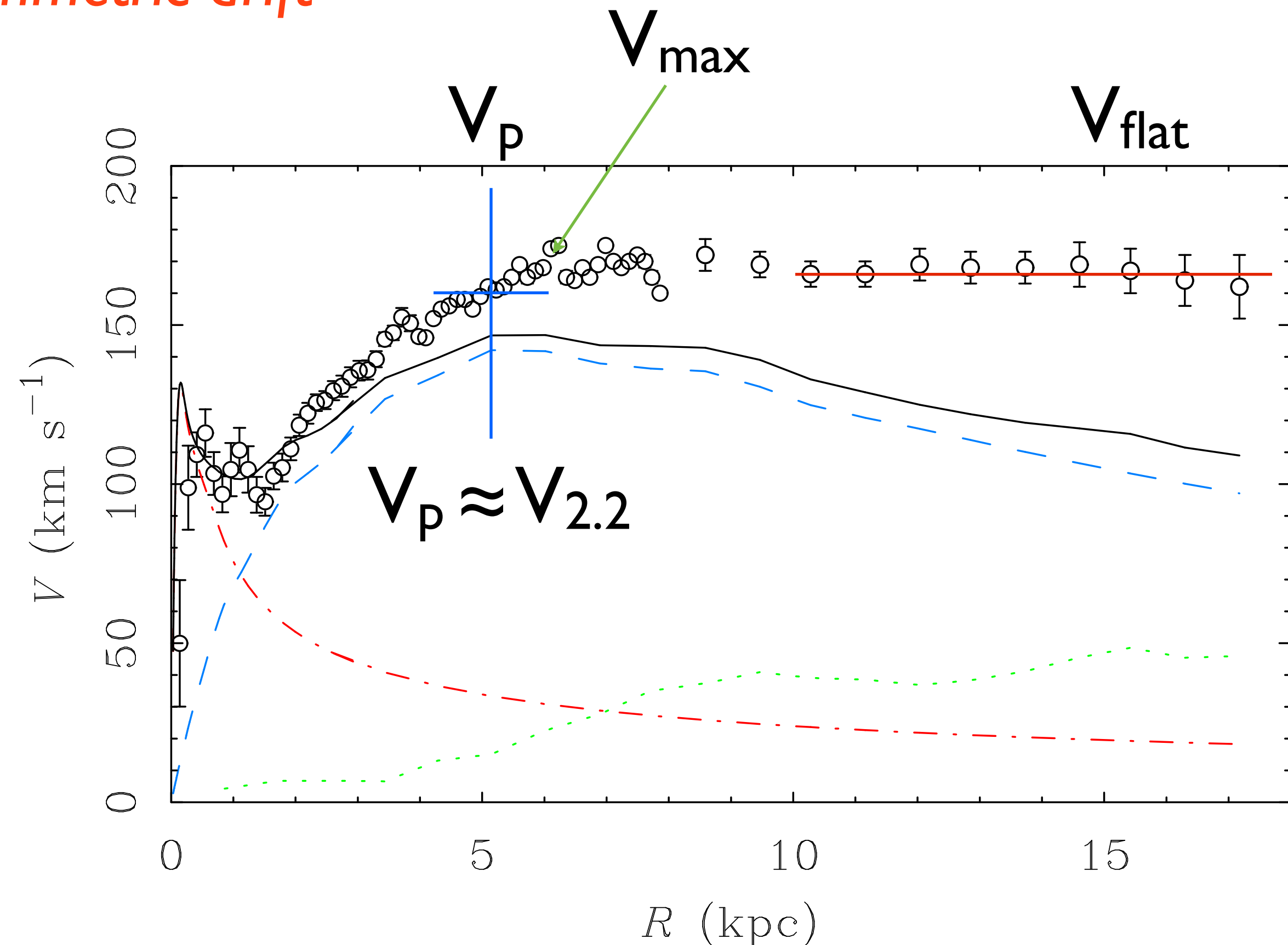
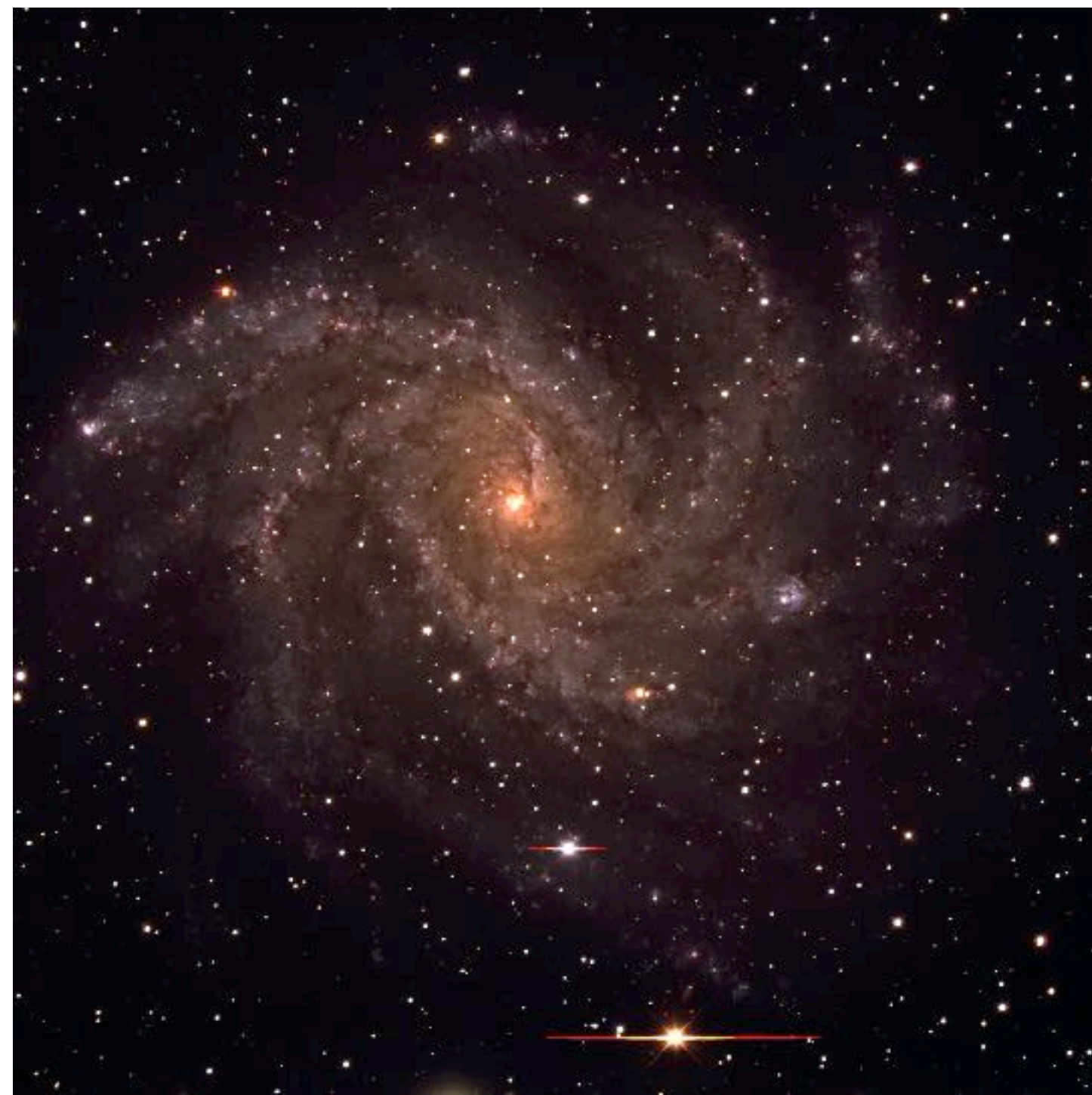
Daigle et al (2006) [Ha]

Blais-Ouellette et al (2004) [Ha]

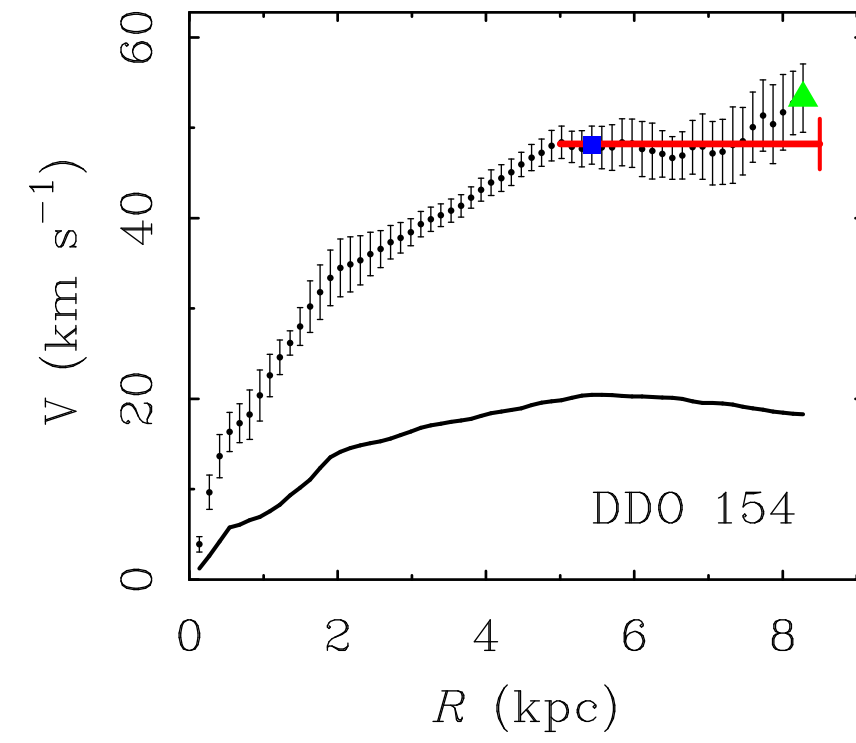
Mass model built from

2MASS K-band data (SSM)

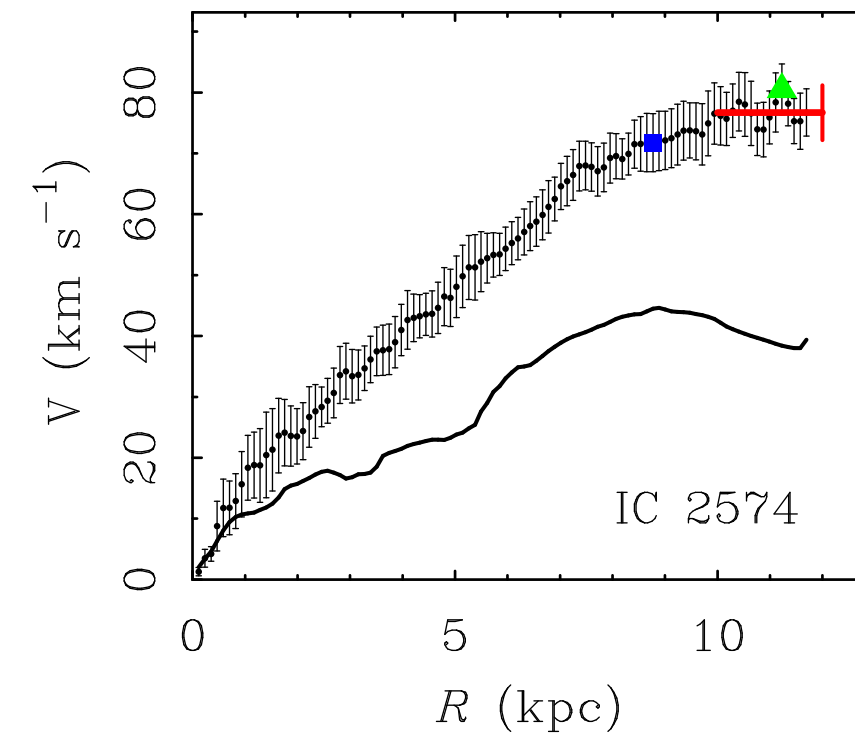
NGC 6946



outer (flat) velocity



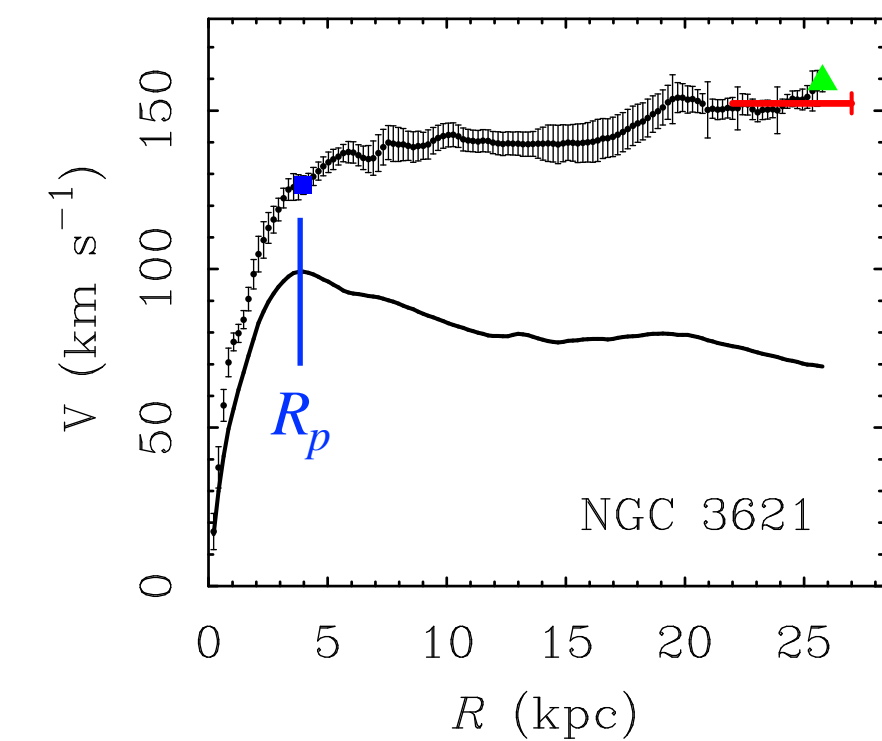
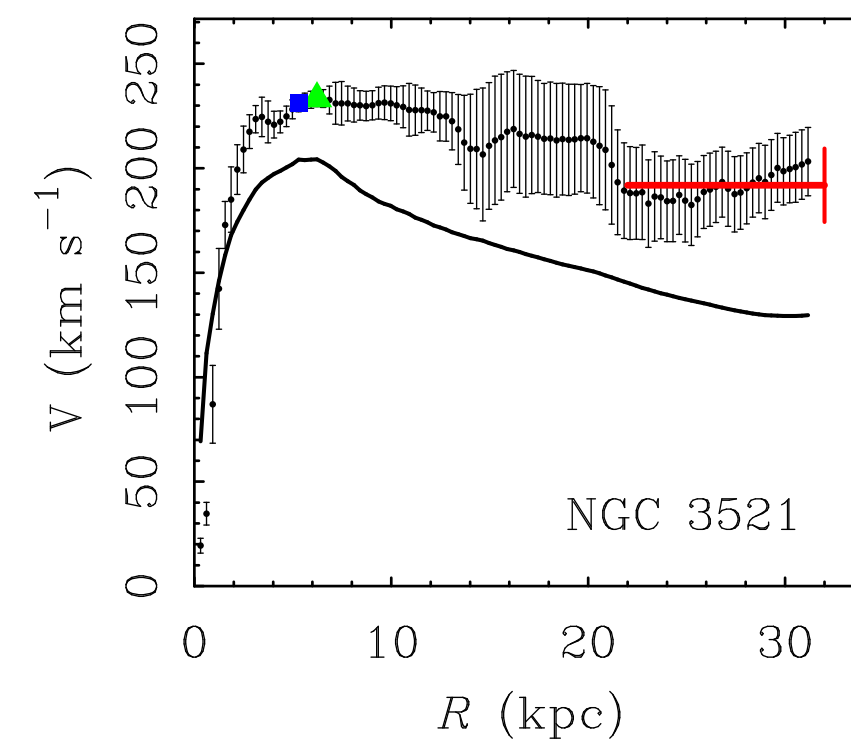
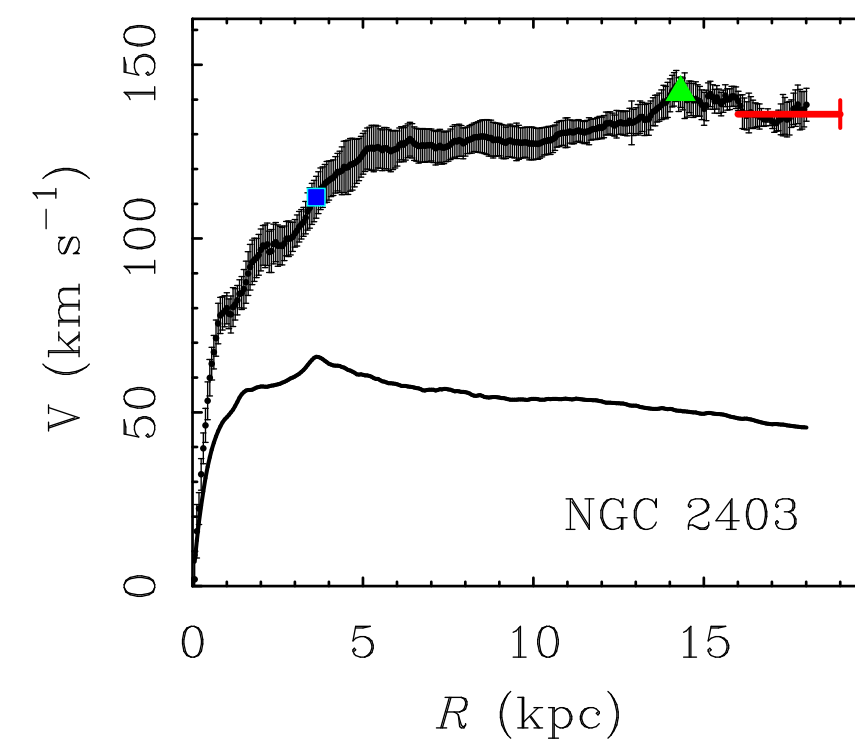
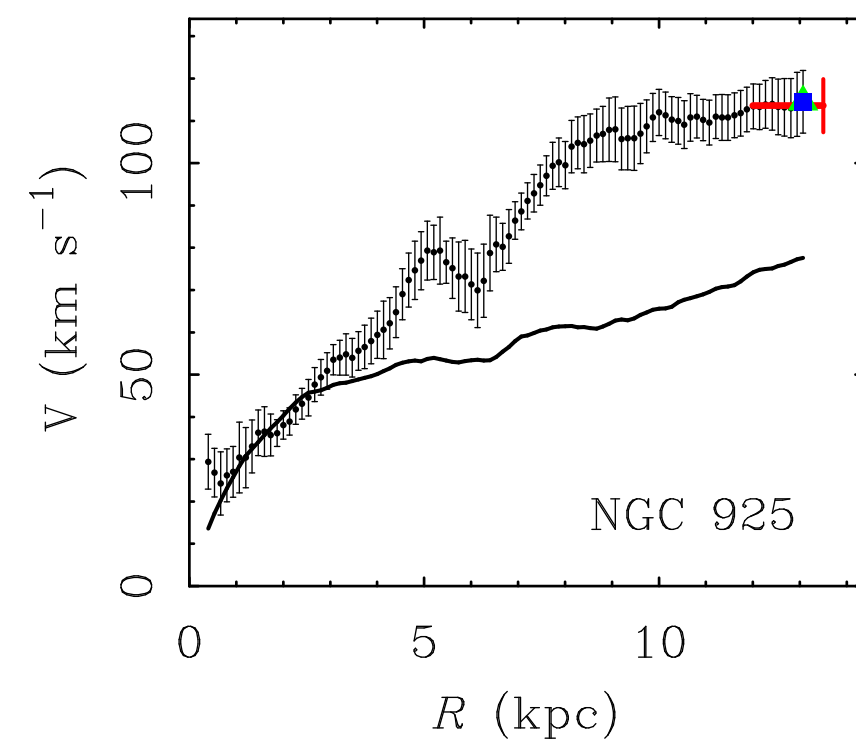
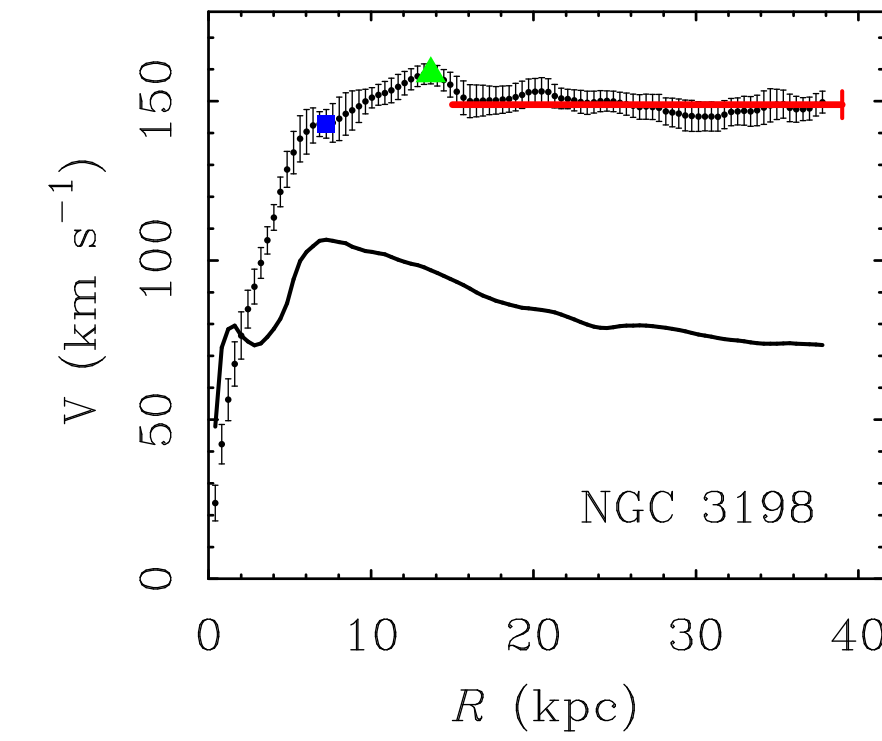
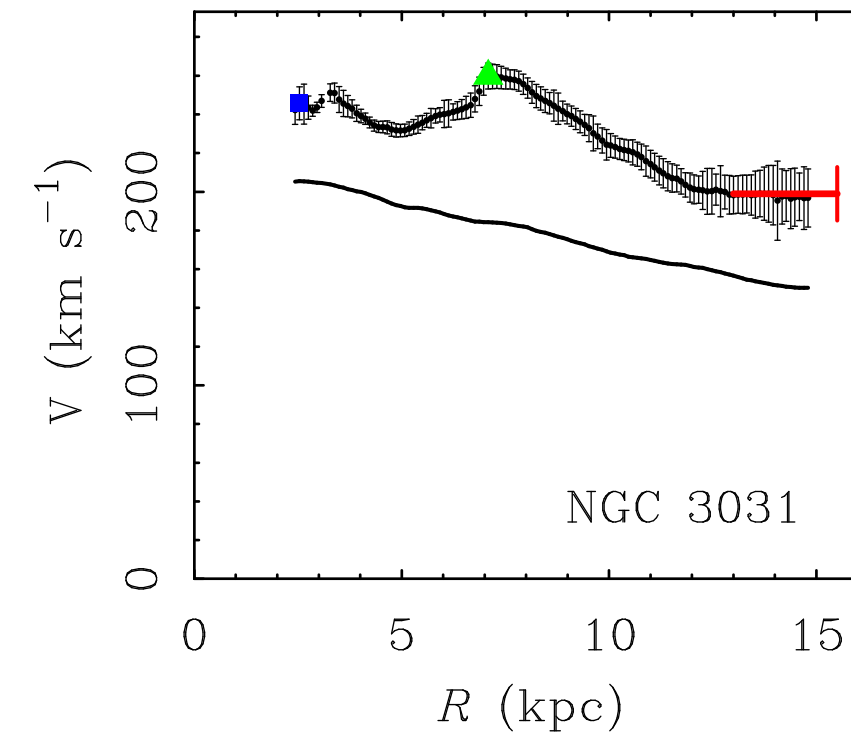
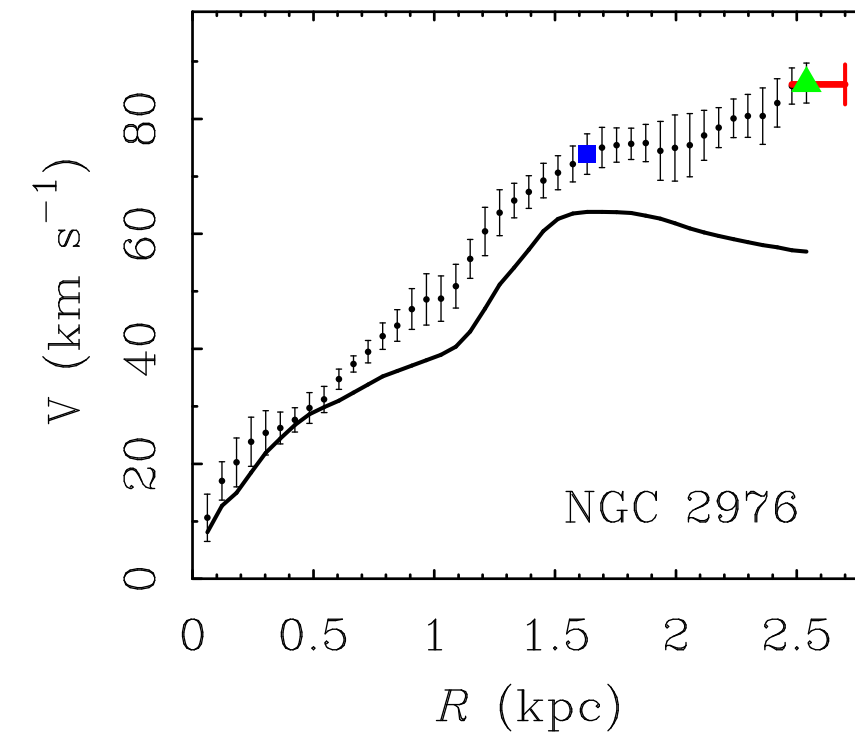
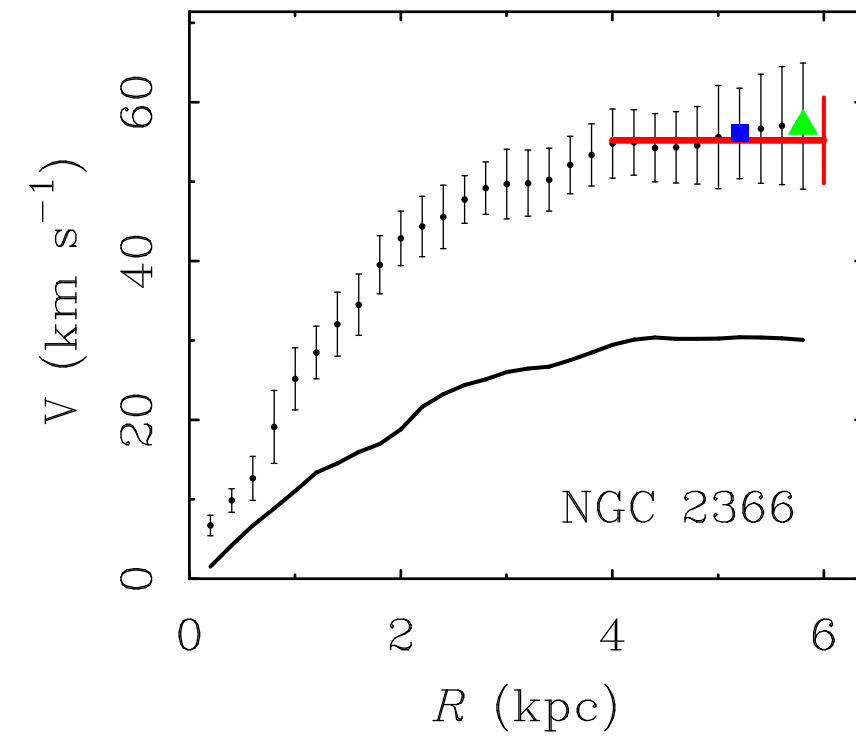
maximum velocity



peak velocity

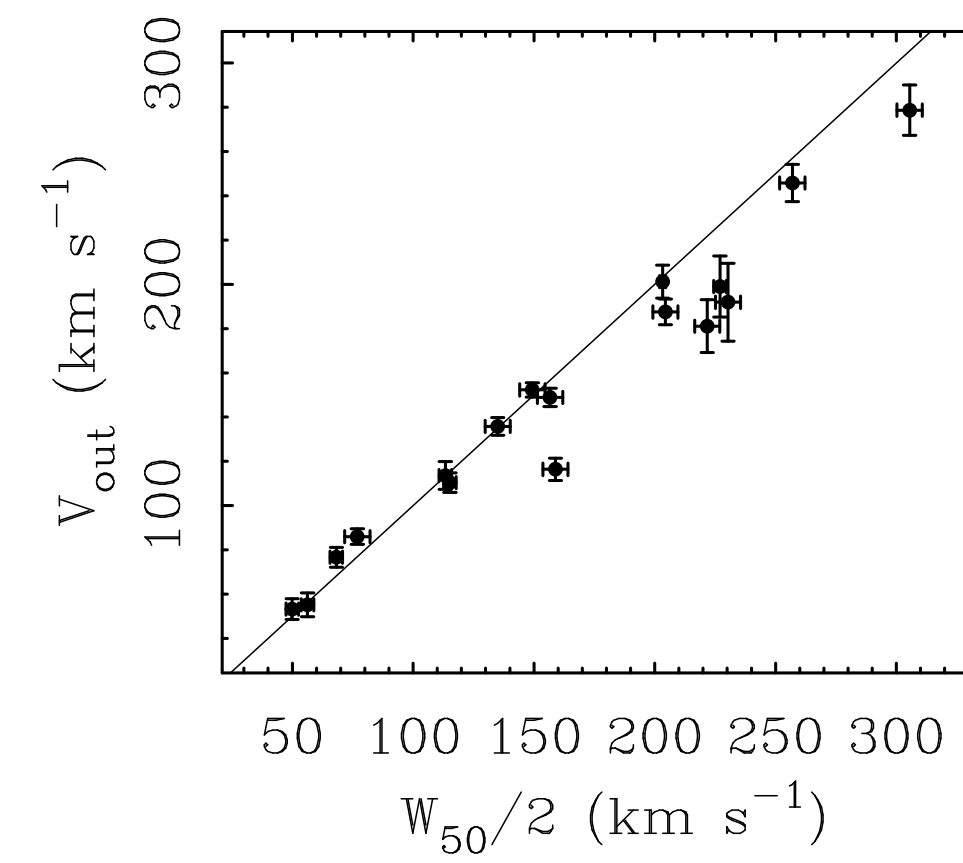
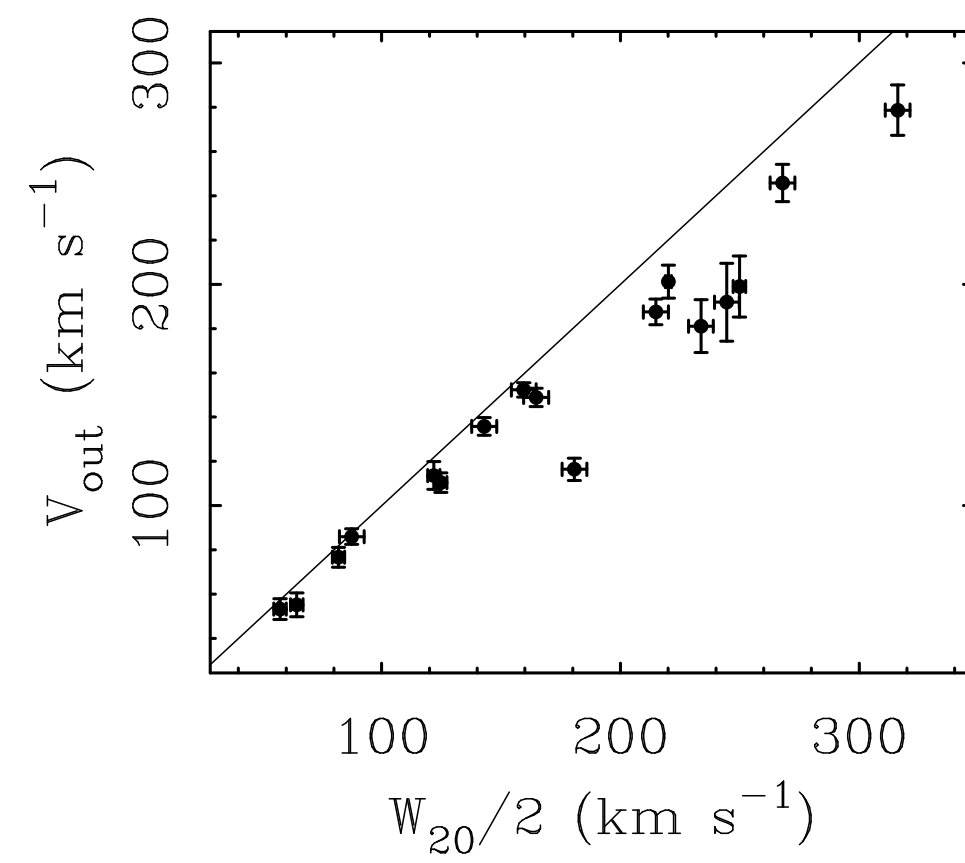


THINGS data (Walter et al 2008)



Velocity estimators:

V_{flat}

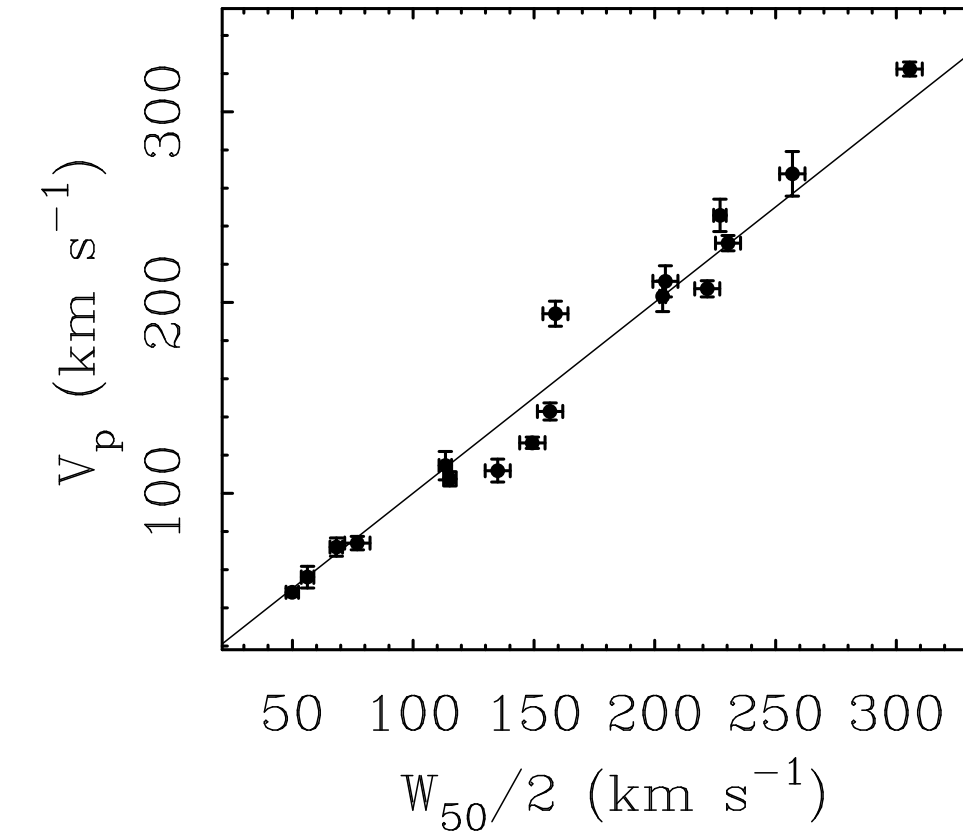
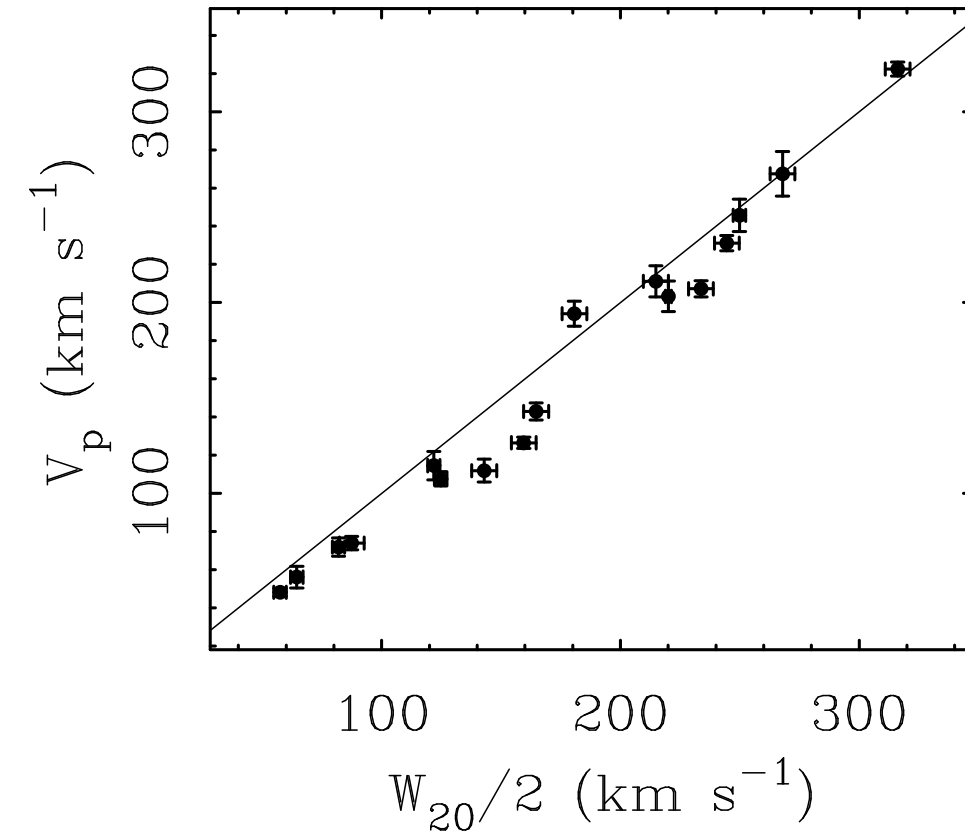


THINGS data
(Walter et al 2008)

W_{20}

W_{50}

V_p

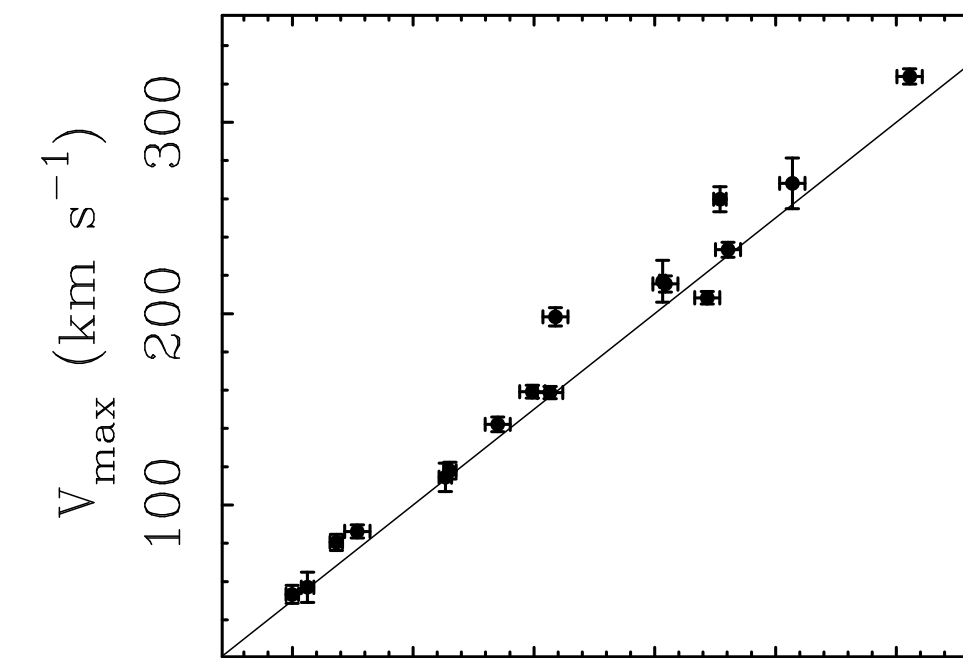
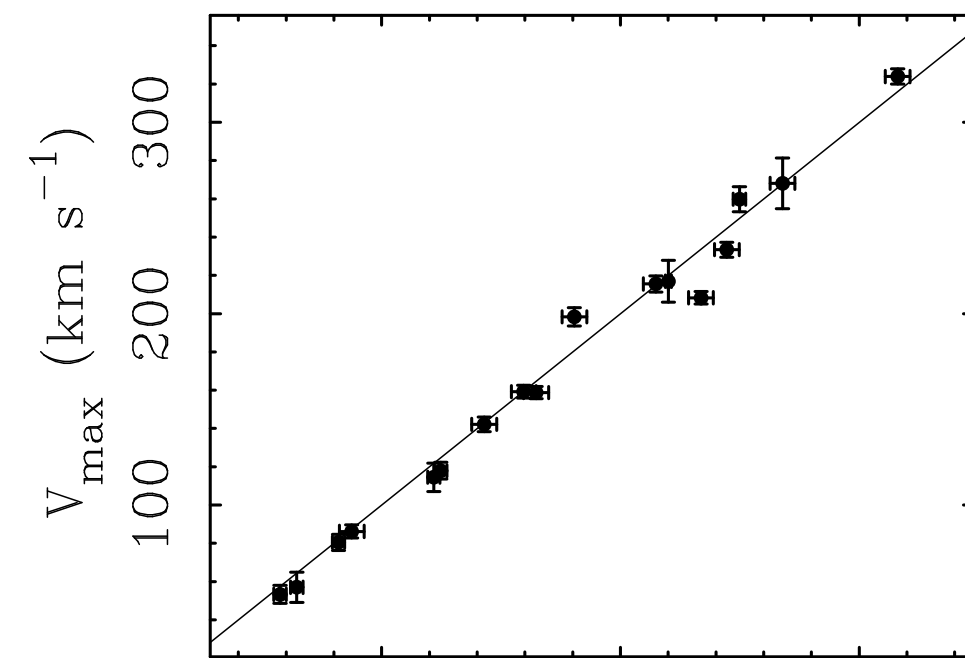


Different velocity measurements correlate but are not identical. TF relations fit using linewidths will differ from those fit using resolved rotation curves.

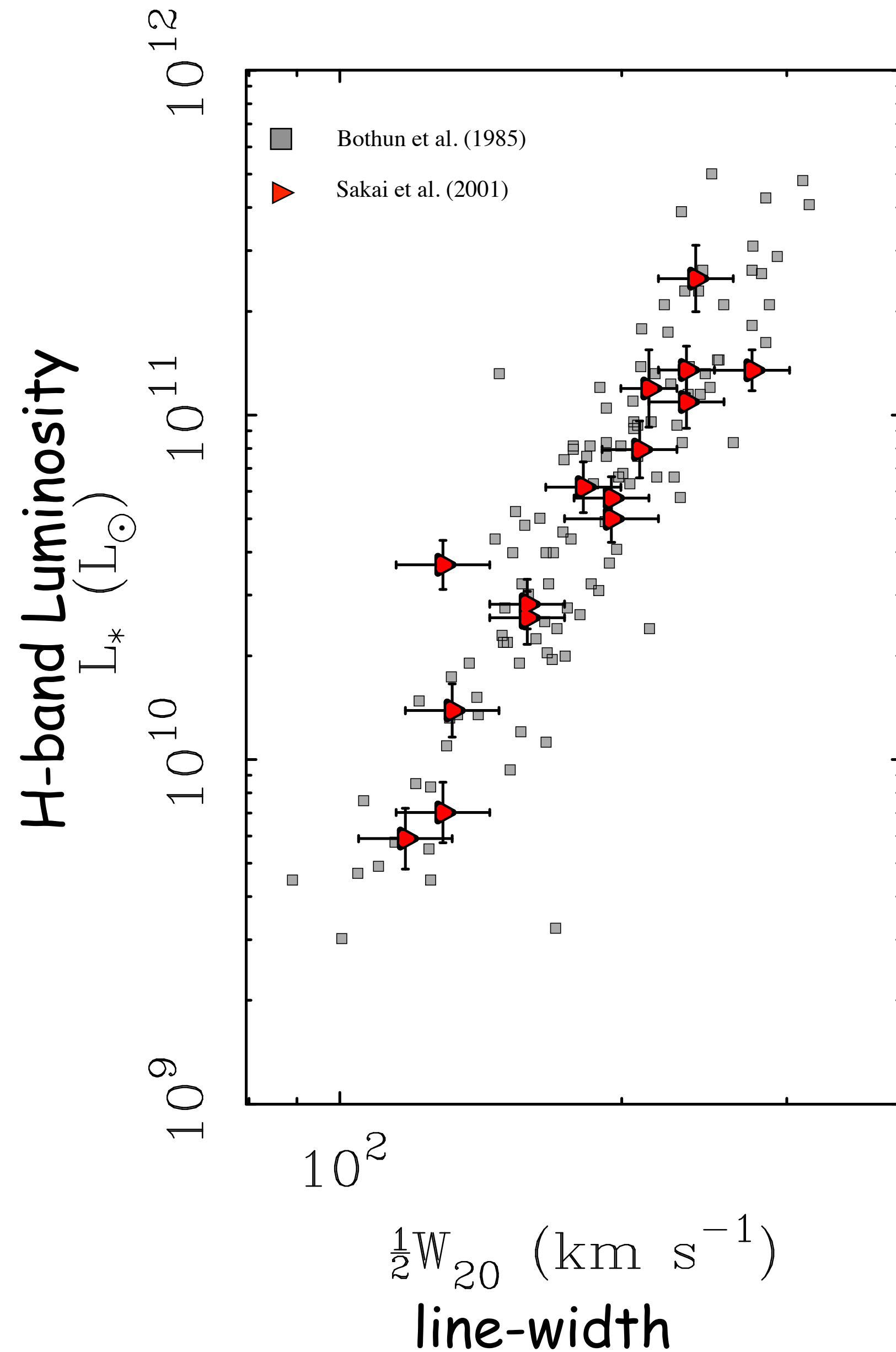
W_{20}

W_{50}

V_{max}



Tully-Fisher relation



Luminosity and line-width are presumably proxies for stellar mass and rotation velocity.

$$L = \frac{M_*}{(M_*/L)}$$

mass-to-light ratio of stellar population
(bandpass dependent)

$$M_* = f_d (f_b M_{200})$$

detected baryon fraction

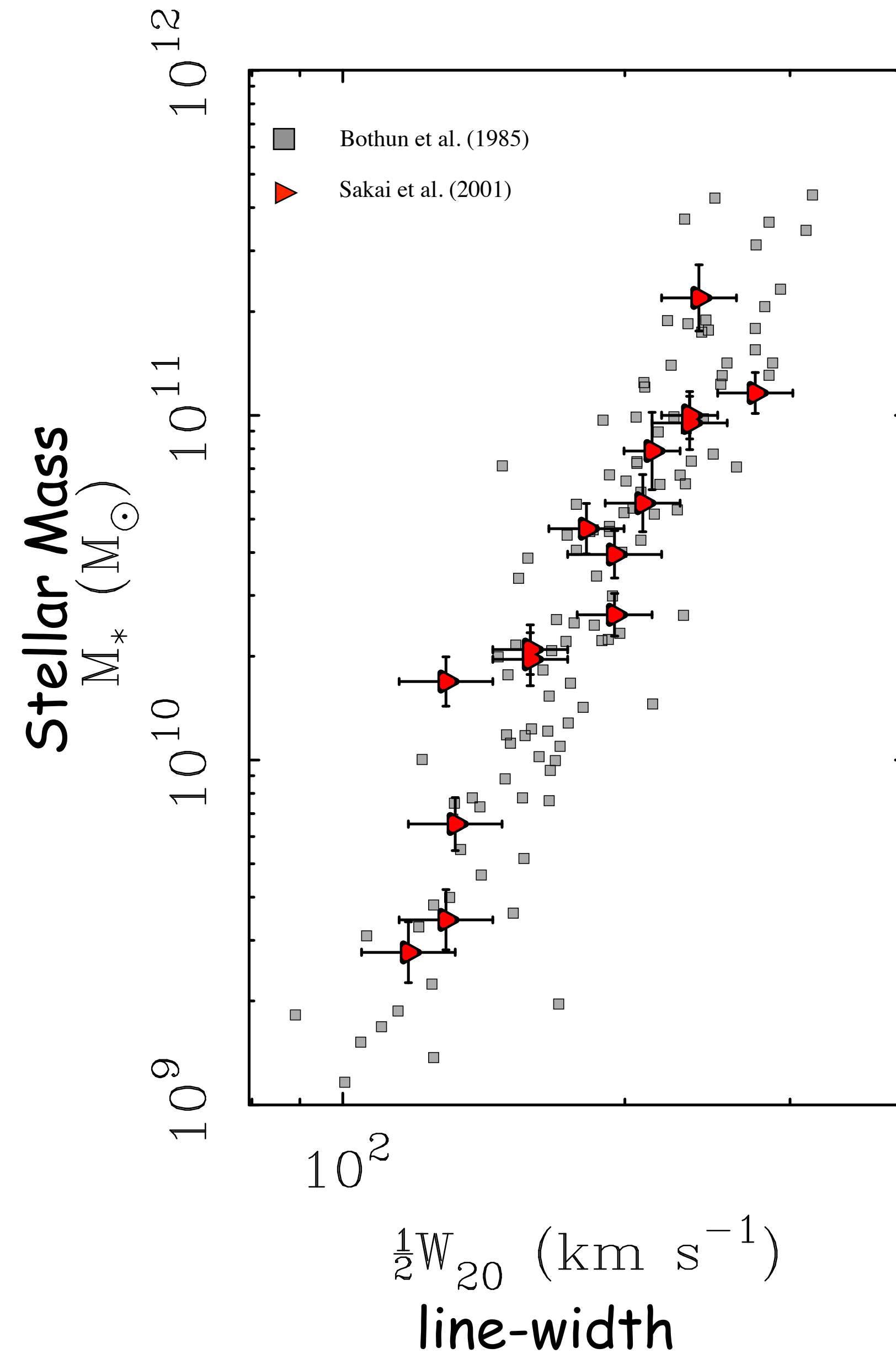
cosmic baryon fraction

$$W_{20} \propto V_{max} \sim V_f$$

$$V_f = f_V V_{200}$$

$$V_{max} \stackrel{?}{=} f_{V_m} V_{max,halo}$$

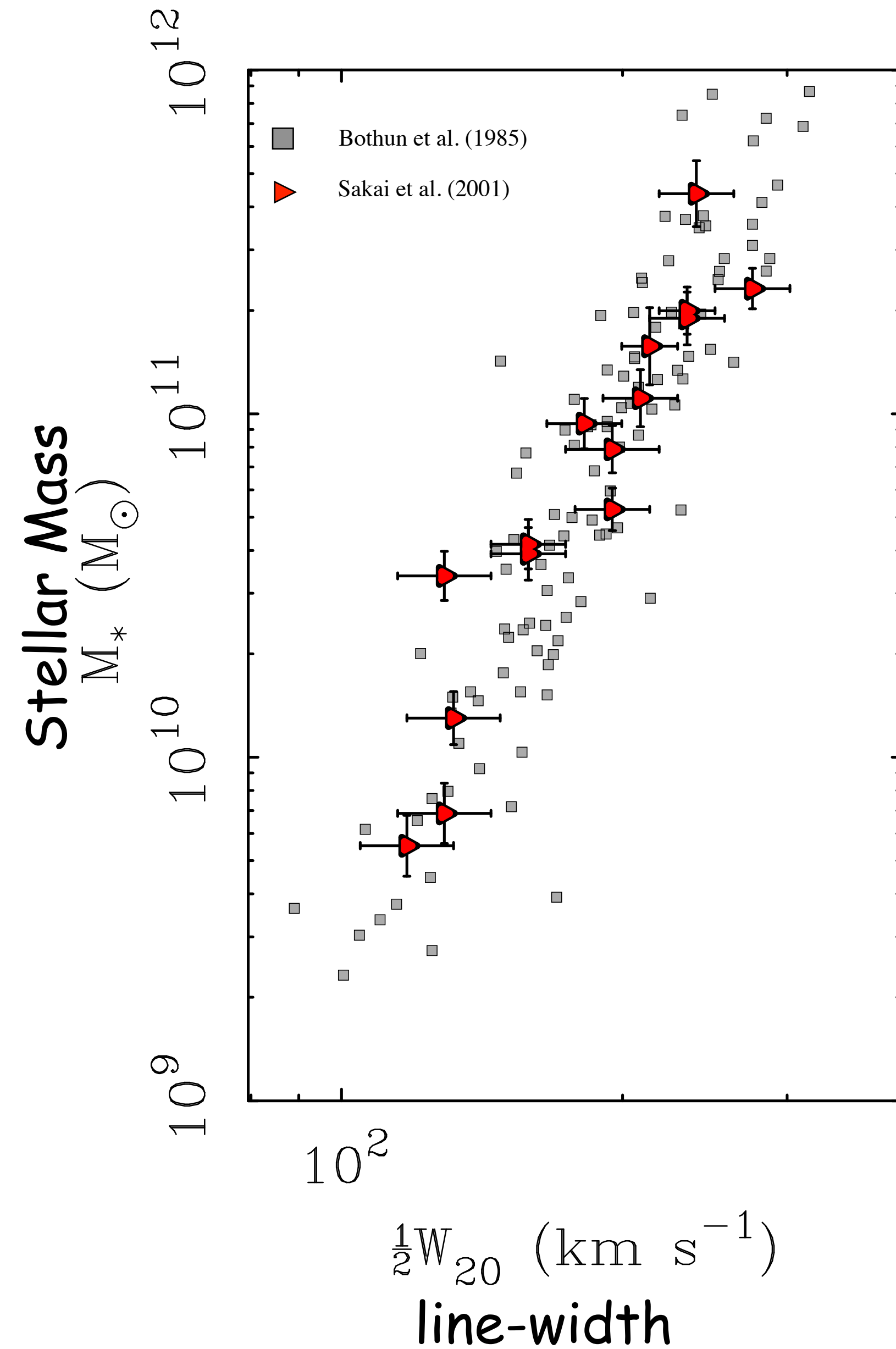
Stellar Mass Tully-Fisher relation



nominal M^*/L (Kroupa IMF)

$$M_* = \left(\frac{M_*}{L} \right) L$$

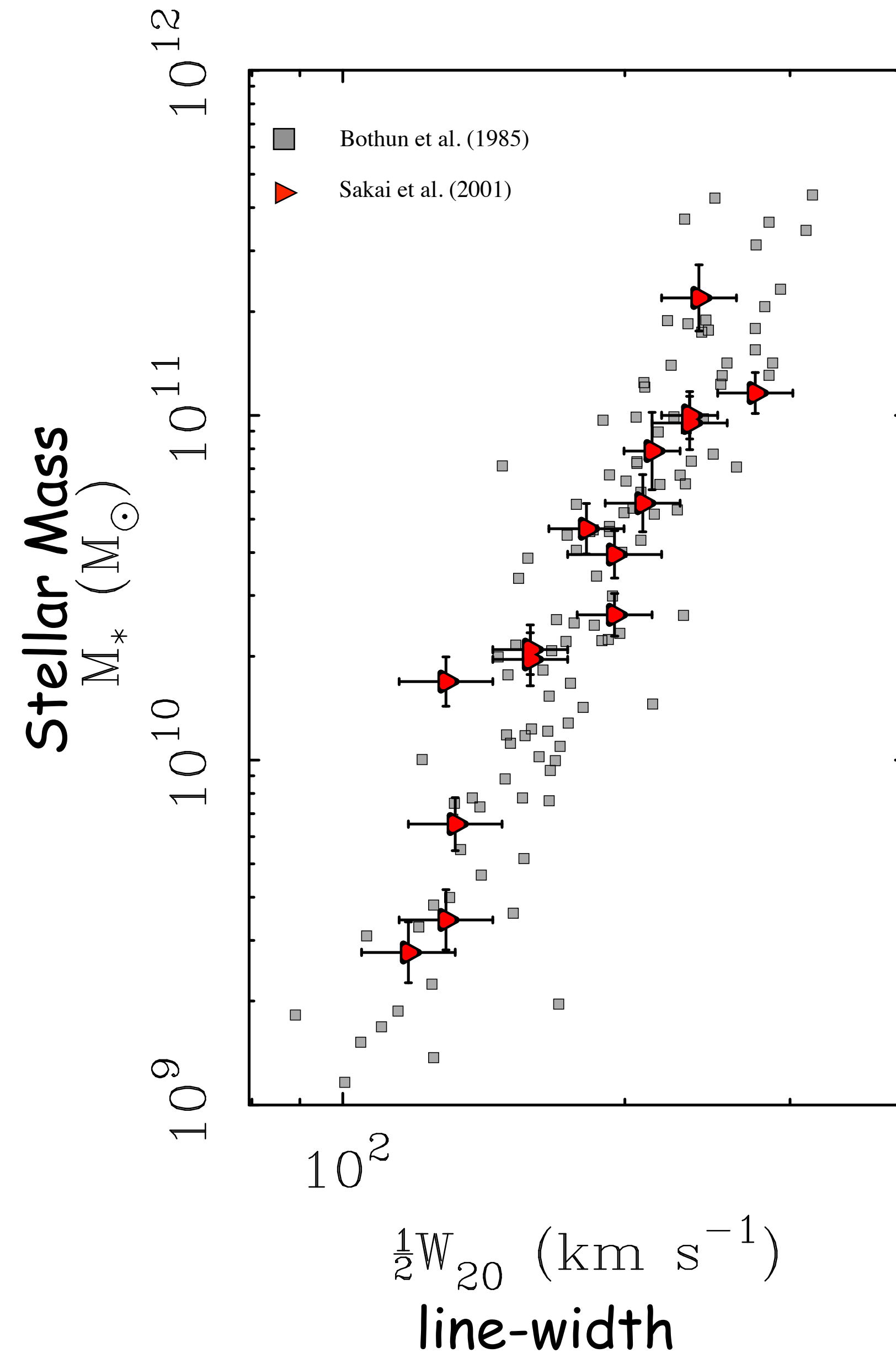
Stellar Mass Tully-Fisher relation



double M^*/L

...but stellar mass is completely dependent on choice of mass-to-light ratio (and degenerate with distance)

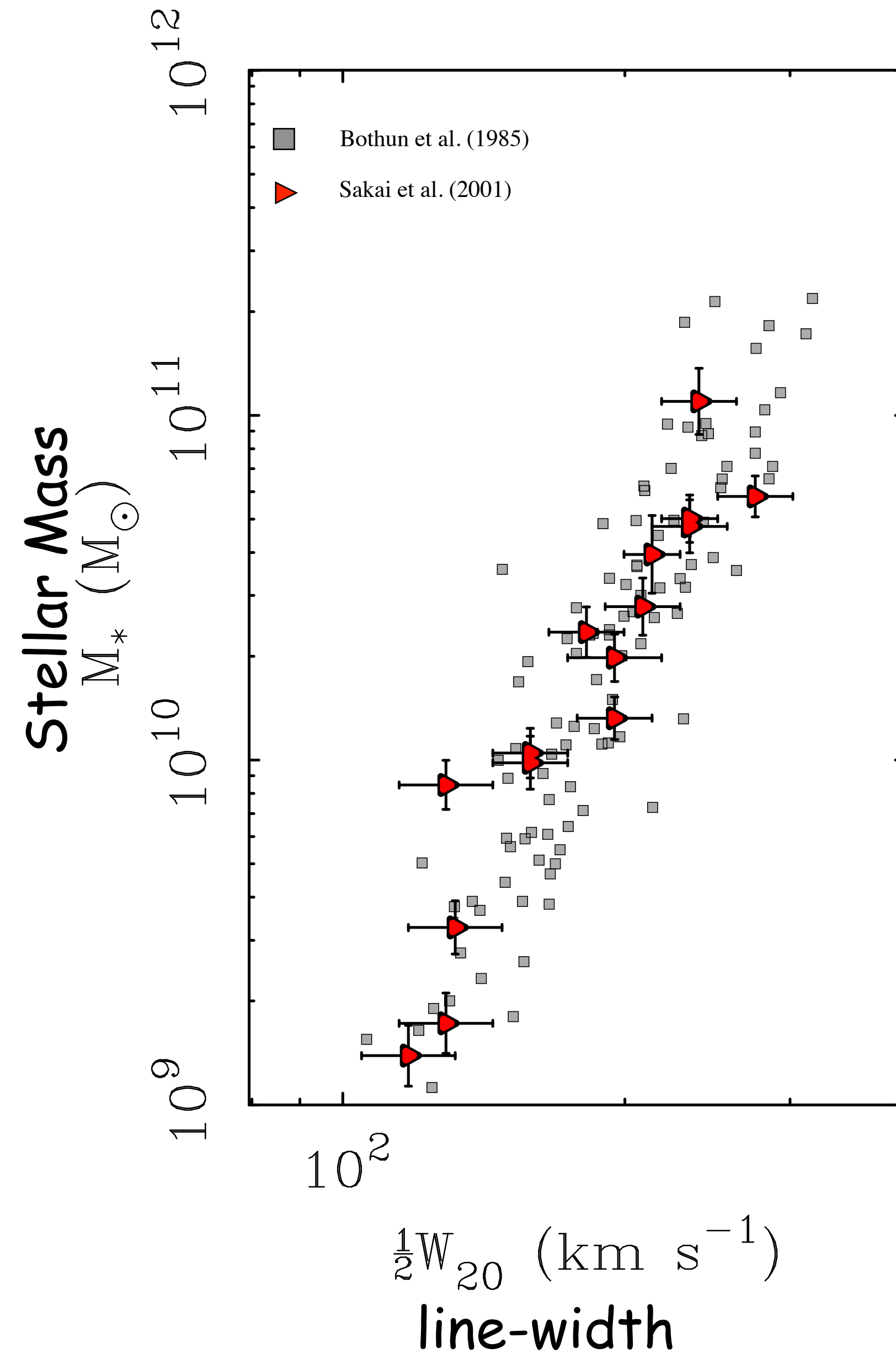
Stellar Mass Tully-Fisher relation



nominal M^*/L

...but stellar mass is completely dependent on choice of mass-to-light ratio (and degenerate with distance)

Stellar Mass Tully-Fisher relation



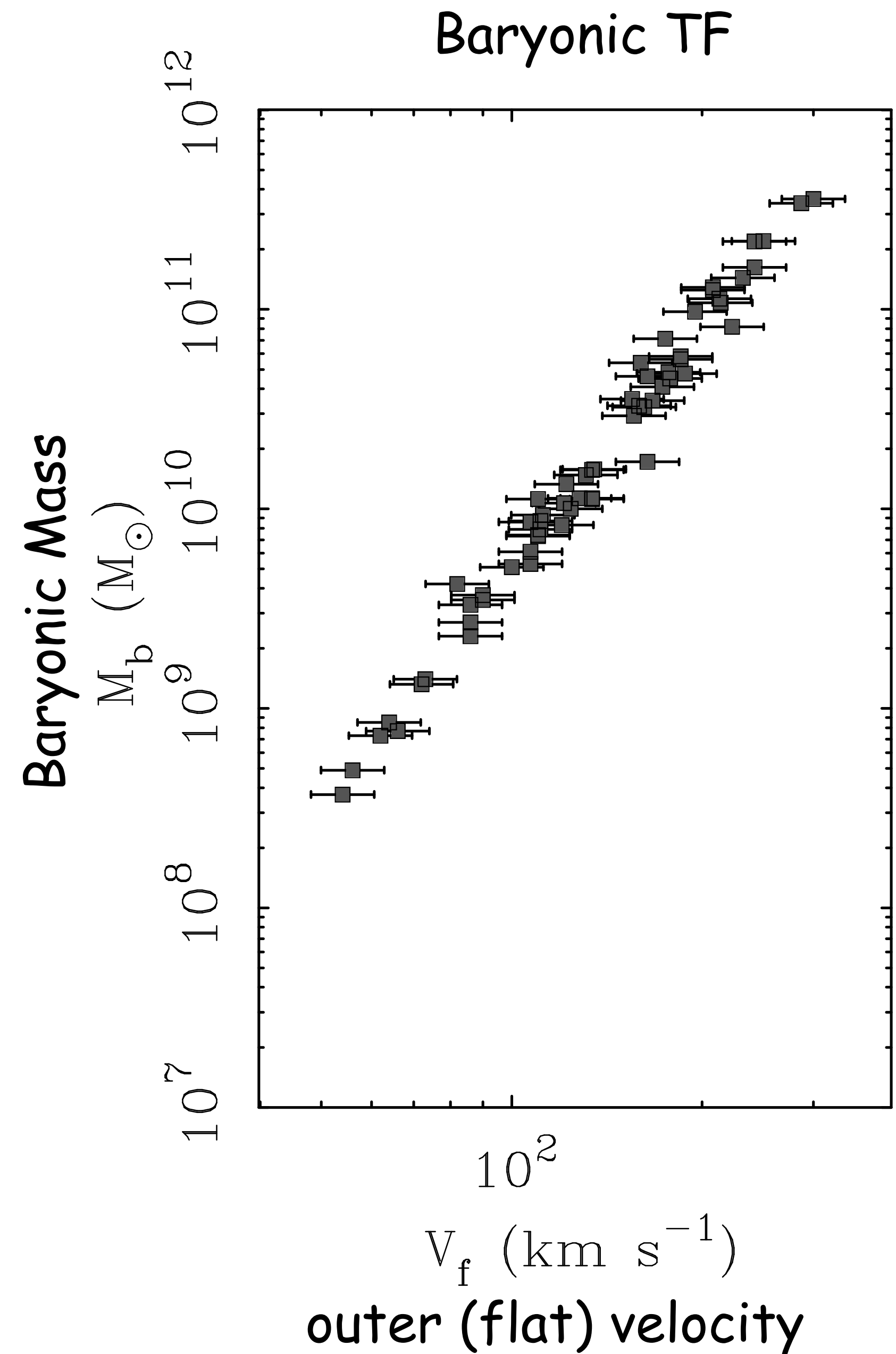
half M^*/L

...but stellar mass is completely dependent on choice of mass-to-light ratio (and degenerate with distance)

Adding gas to stellar mass restores a single continuous relation for all rotators.

$$M_b = M_* + M_g$$

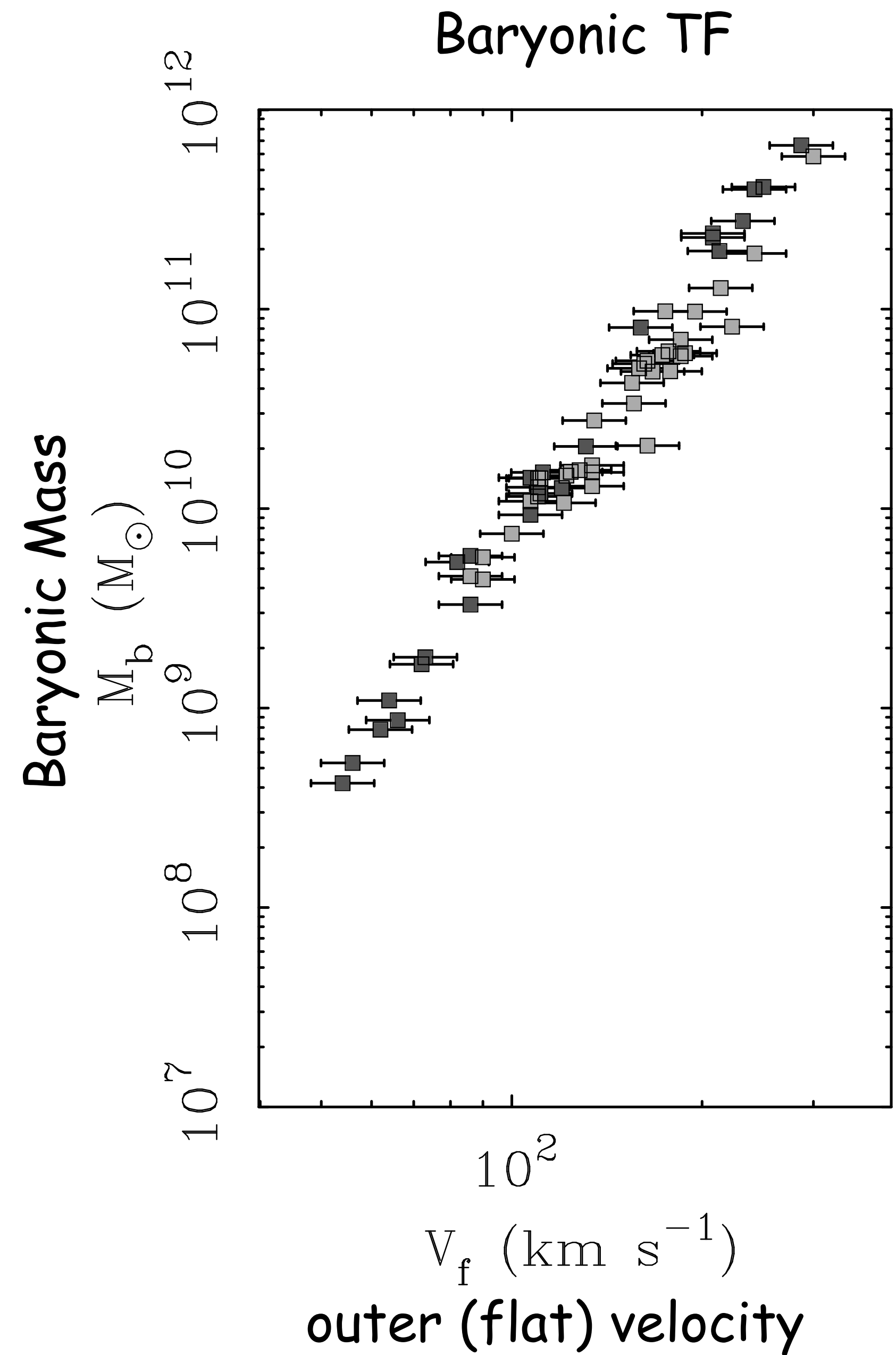
Baryonic mass is the important physical quantity. It doesn't matter whether the mass is in stars or in gas.



Twice Nominal M^*/L

Now instead of a translation, the slope pivots as we vary M^*/L .

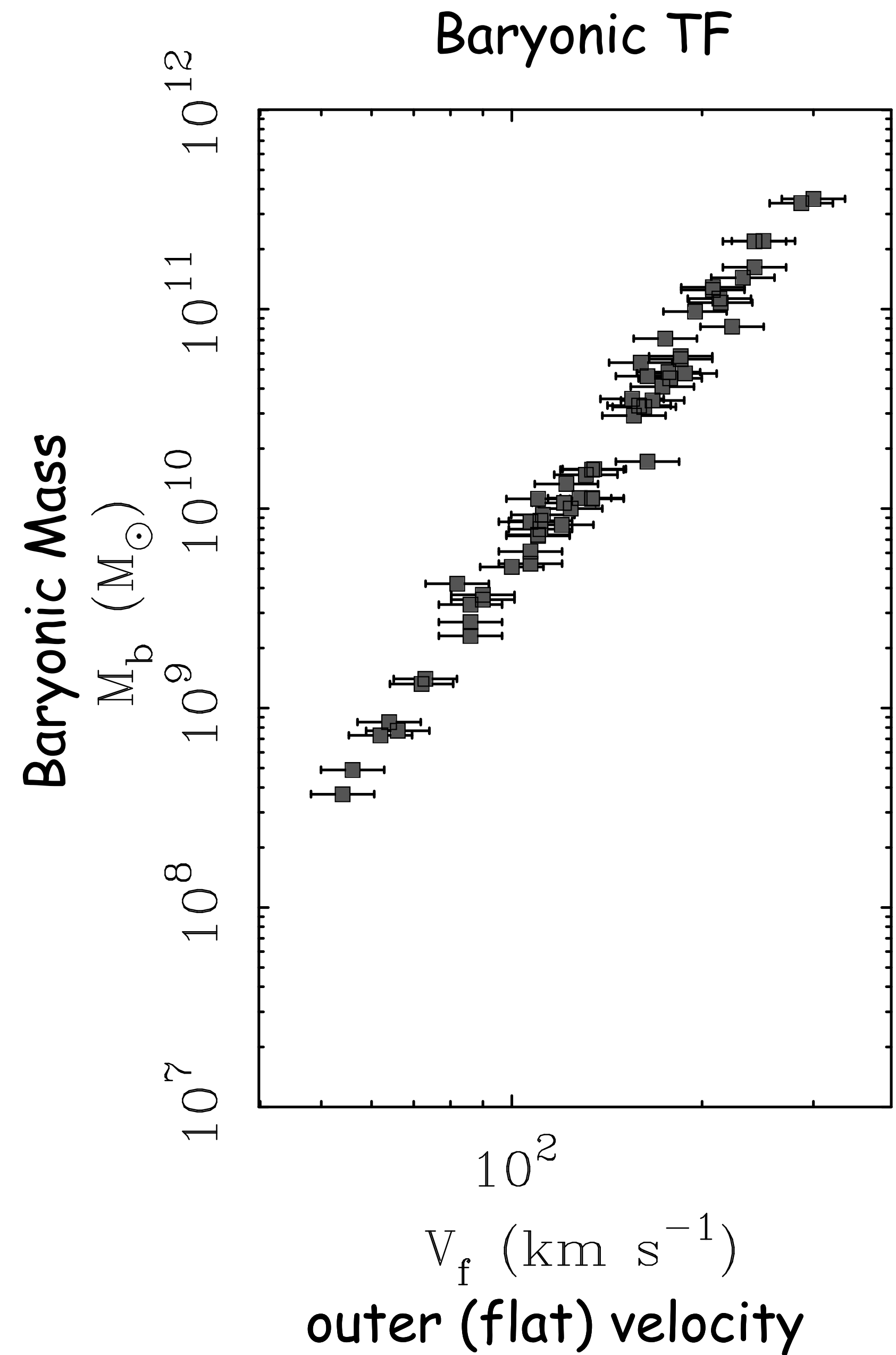
Scatter increases as we diverge from the nominal M^*/L .



Nominal M^*/L

Now instead of a translation, the slope pivots as we vary M^*/L .

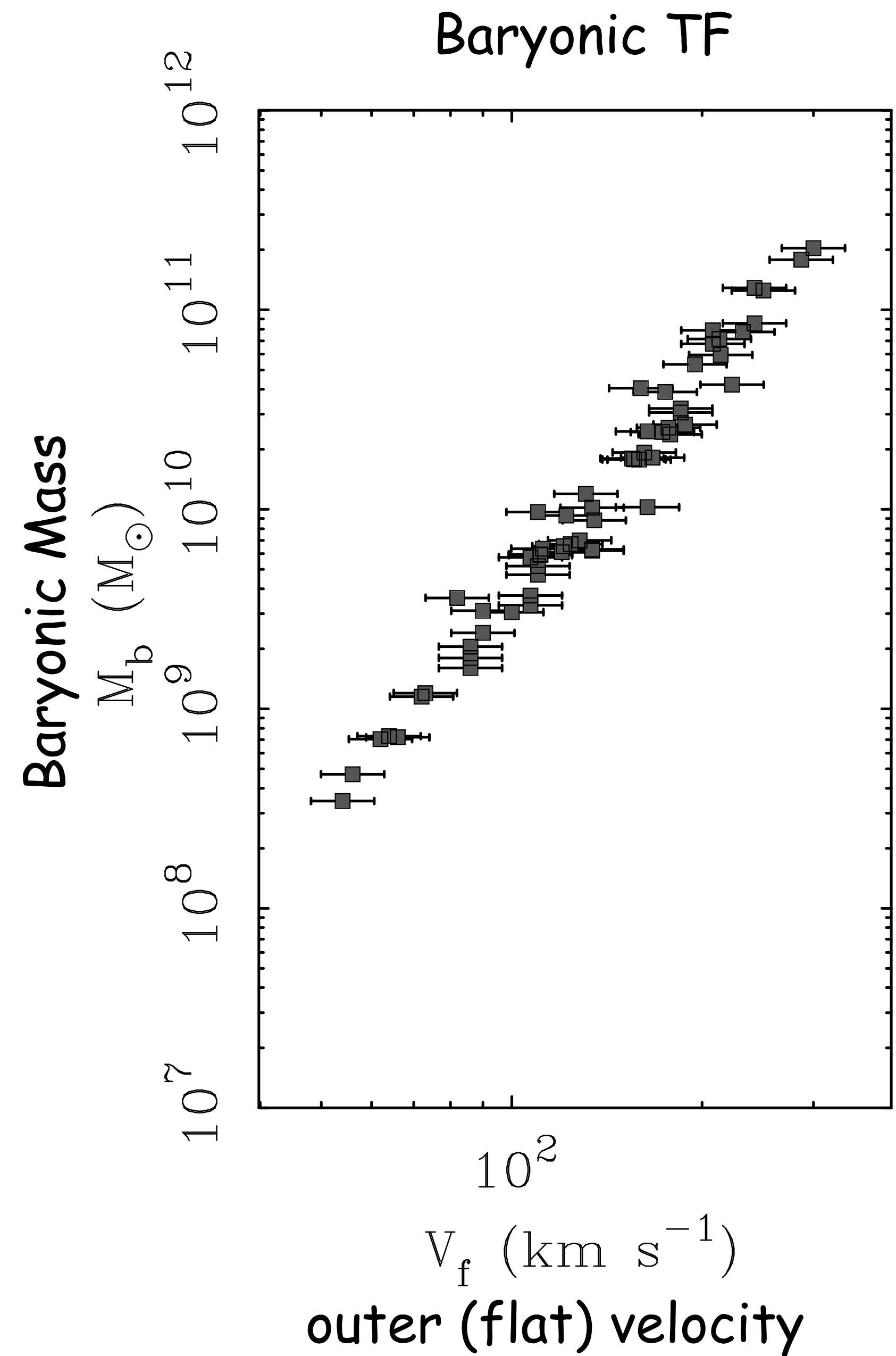
Scatter increases as we diverge from the nominal M^*/L .



Half Nominal M^*/L

Now instead of a translation, the slope pivots as we vary M^*/L .

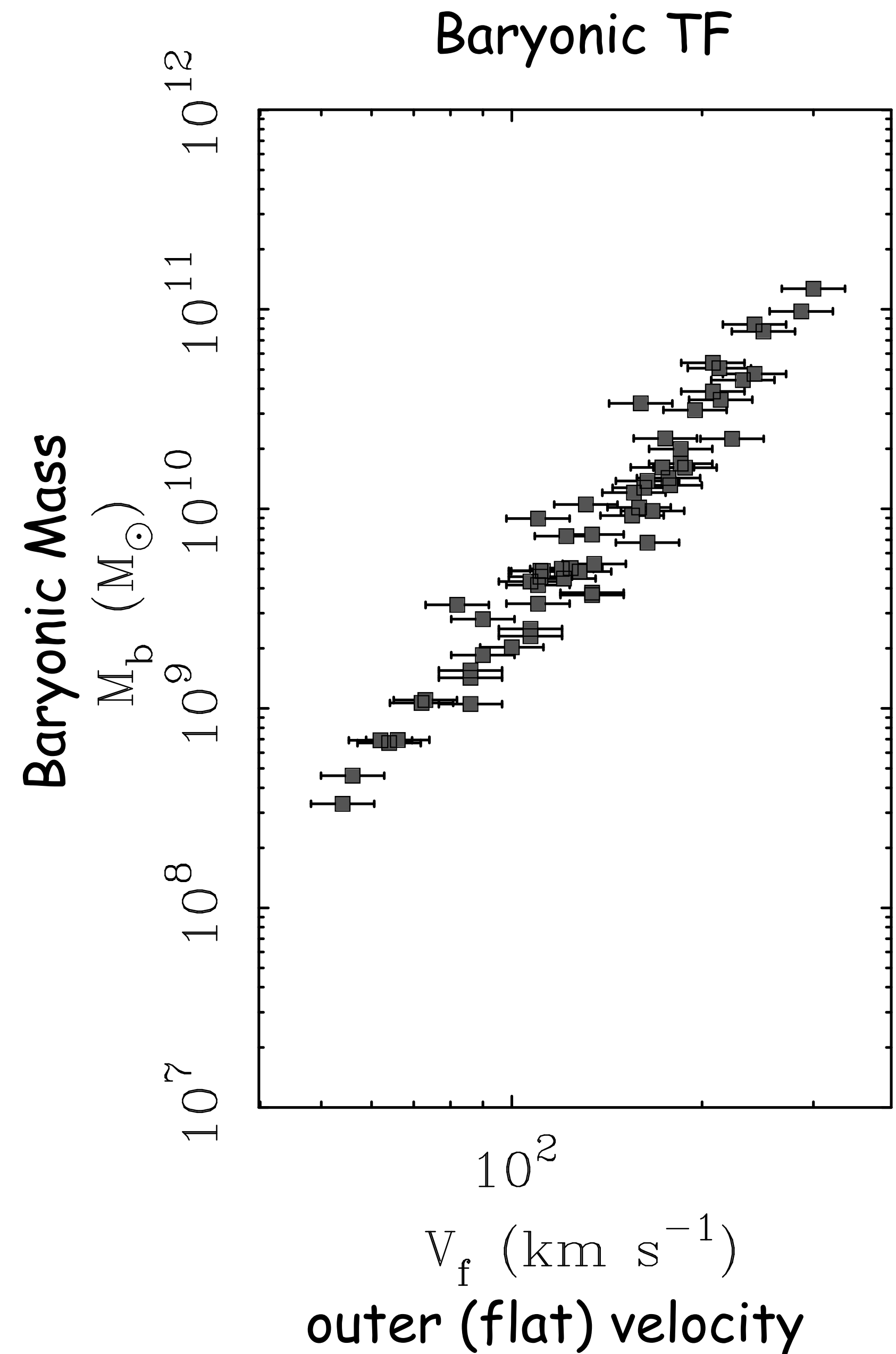
Scatter increases as we diverge from the nominal M^*/L .



Quarter Nominal M^*/L

Now instead of a translation, the slope pivots as we vary M^*/L .

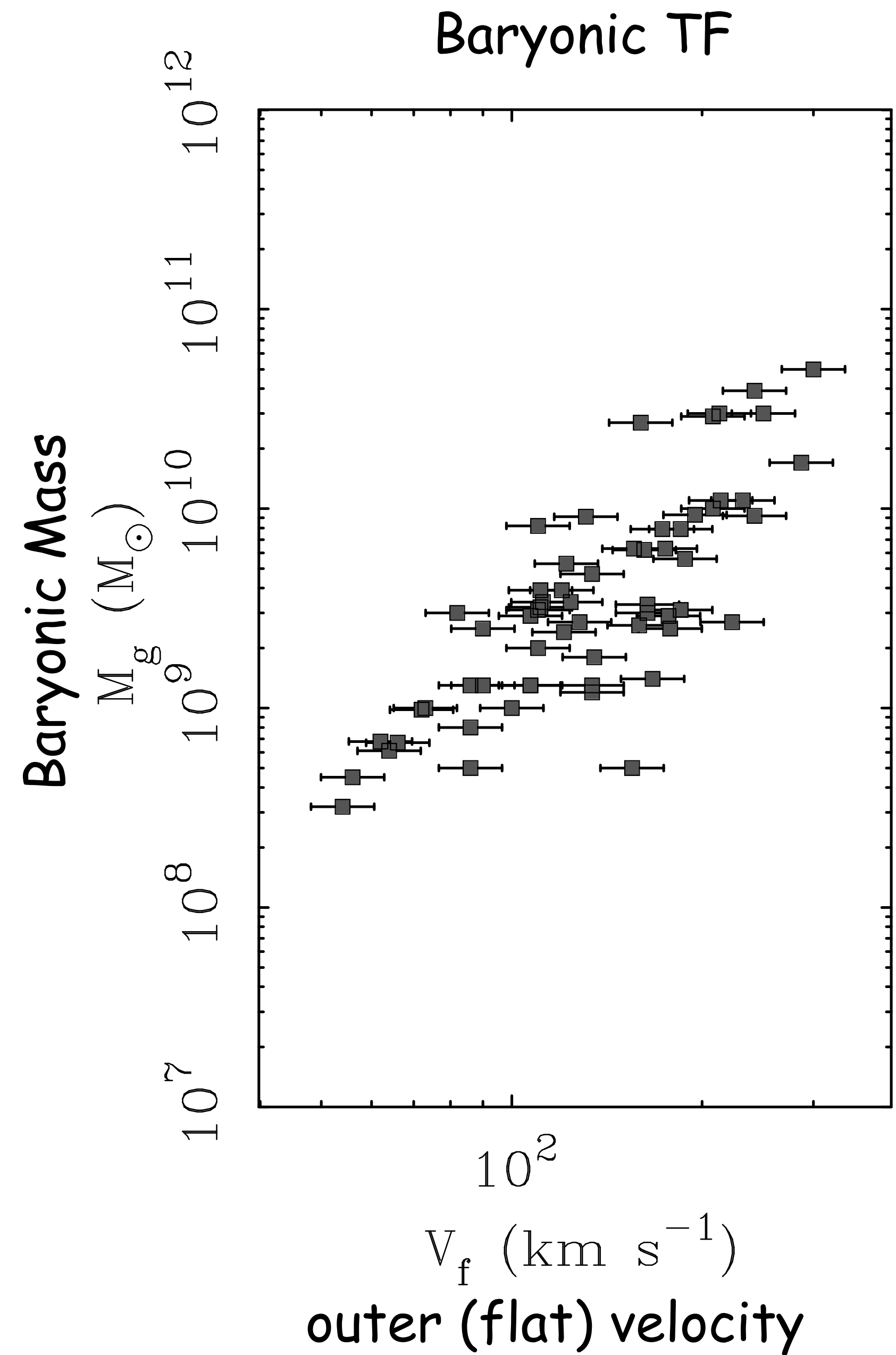
Scatter increases as we diverge from the nominal M^*/L .



Zero M^*/L

Now instead of a translation, the slope pivots as we vary M^*/L .

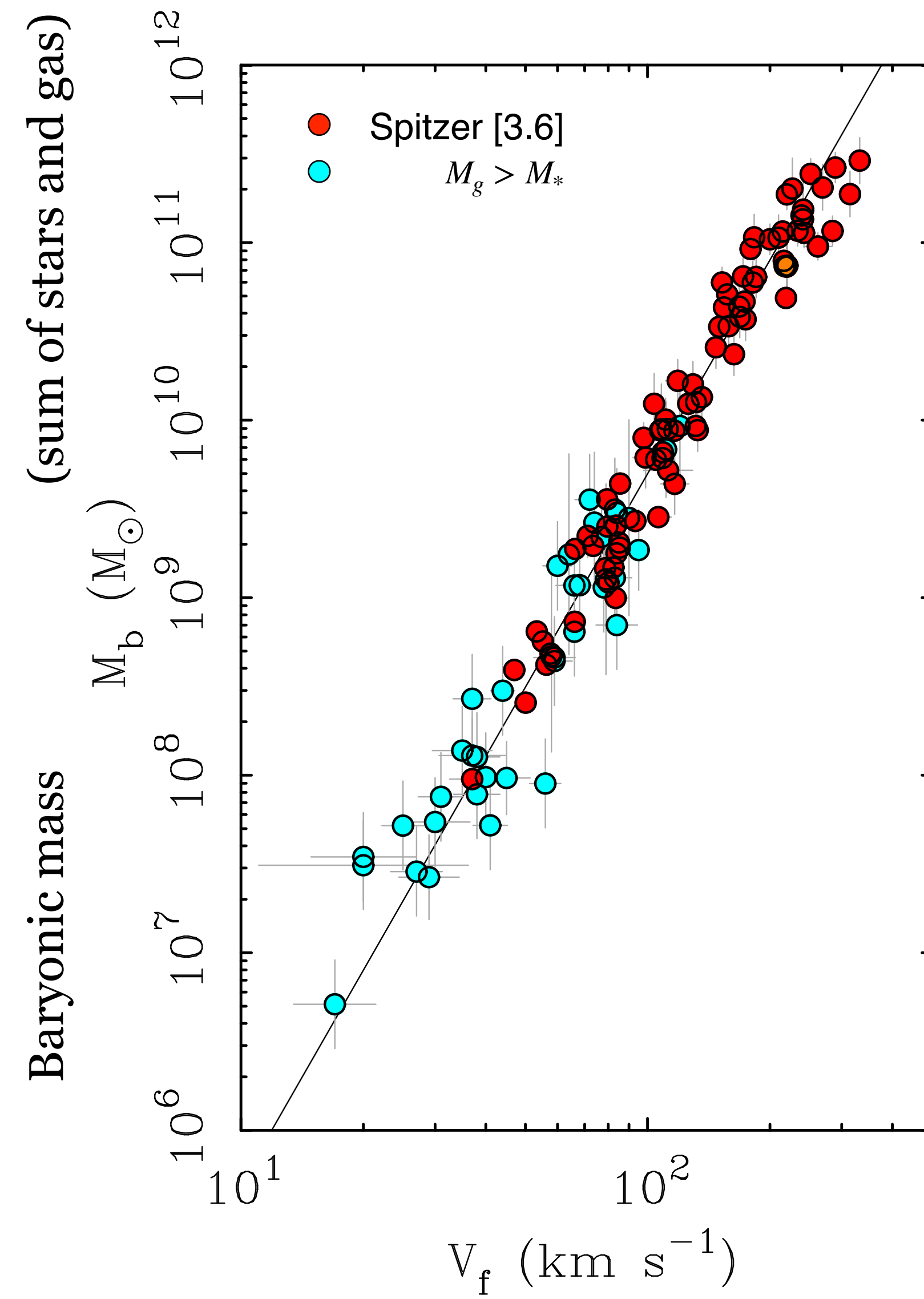
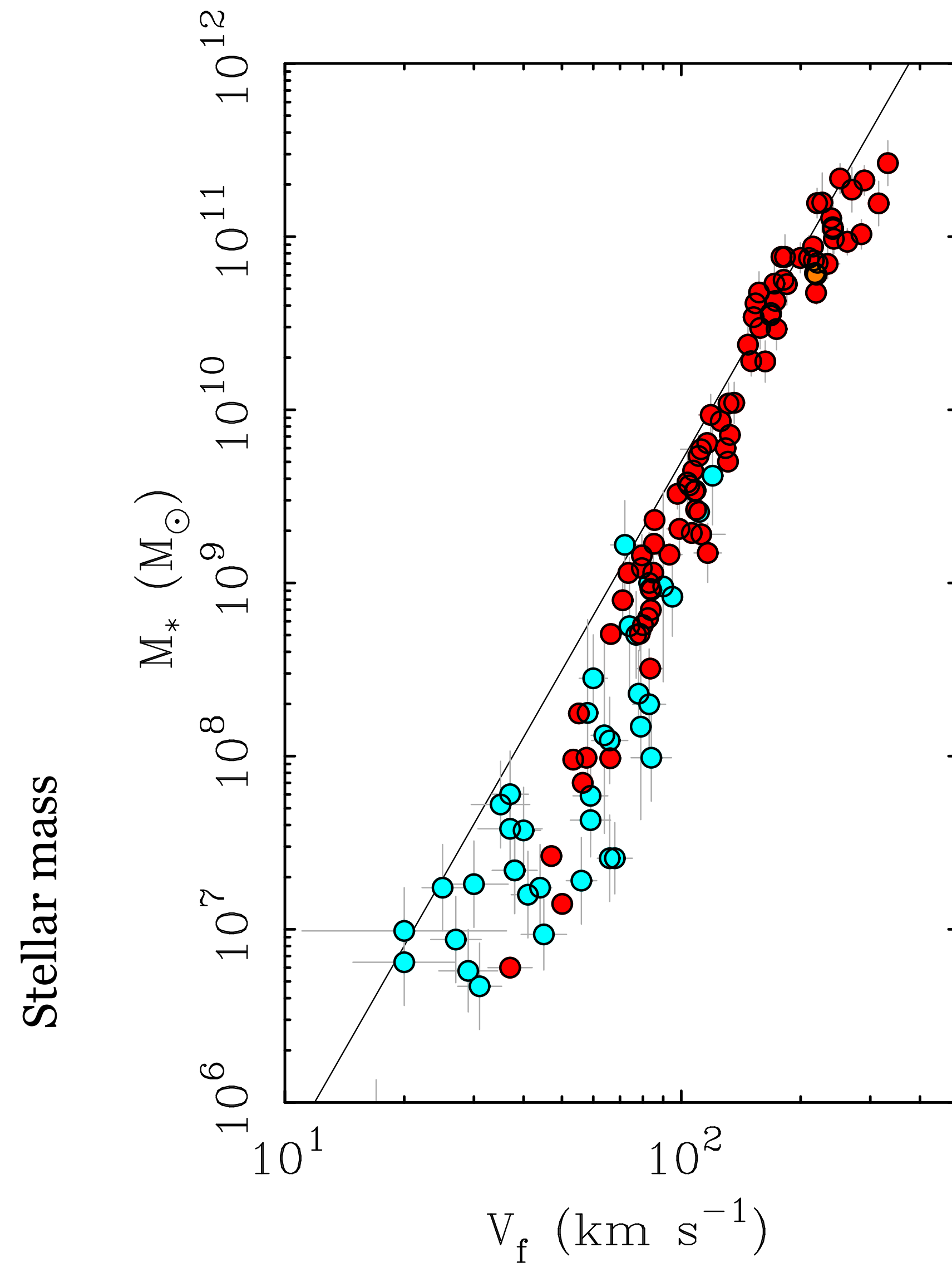
Scatter increases as we diverge from the nominal M^*/L .



Tully-Fisher relations

2019

amplitude of flat rotation correlates with mass



Tully-Fisher relations

amplitude of flat rotation correlates with mass

2019

The fundamental relation
is between
baryonic mass
and the amplitude of the
flat rotation speed

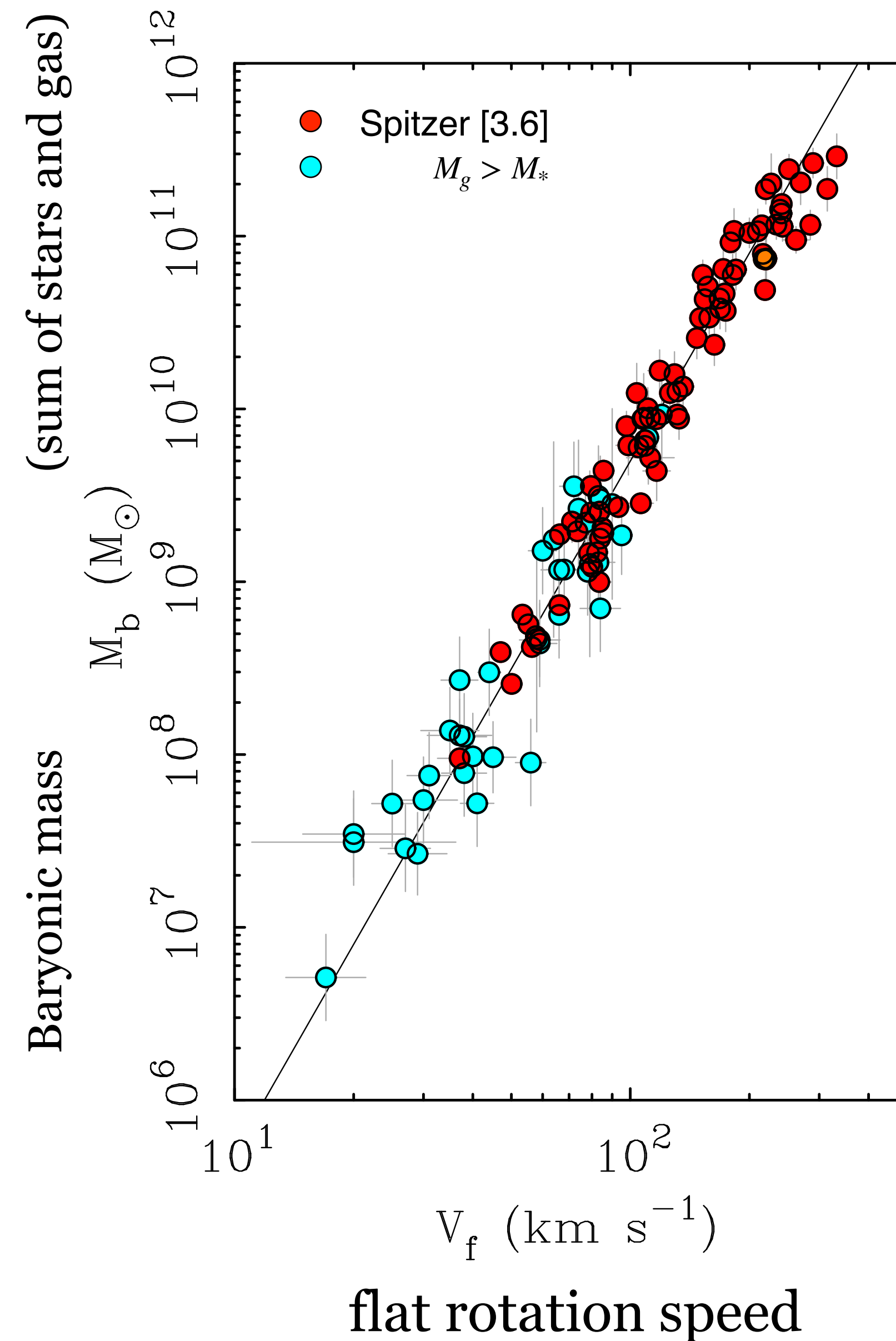
$$M_b = M_* + M_g = AV_f^4$$

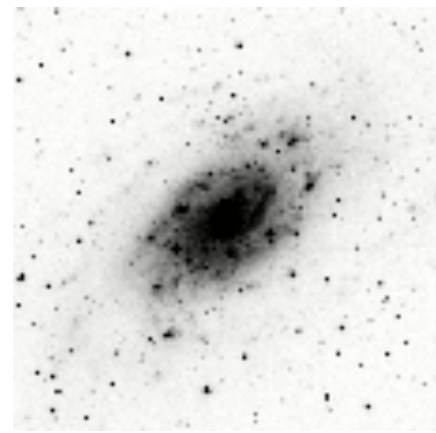
$$A = 50 \pm 3 M_\odot (\text{km s}^{-1})^{-4}$$

there is remarkably little
intrinsic scatter

$$\sigma_M < 0.11 \text{ dex}$$

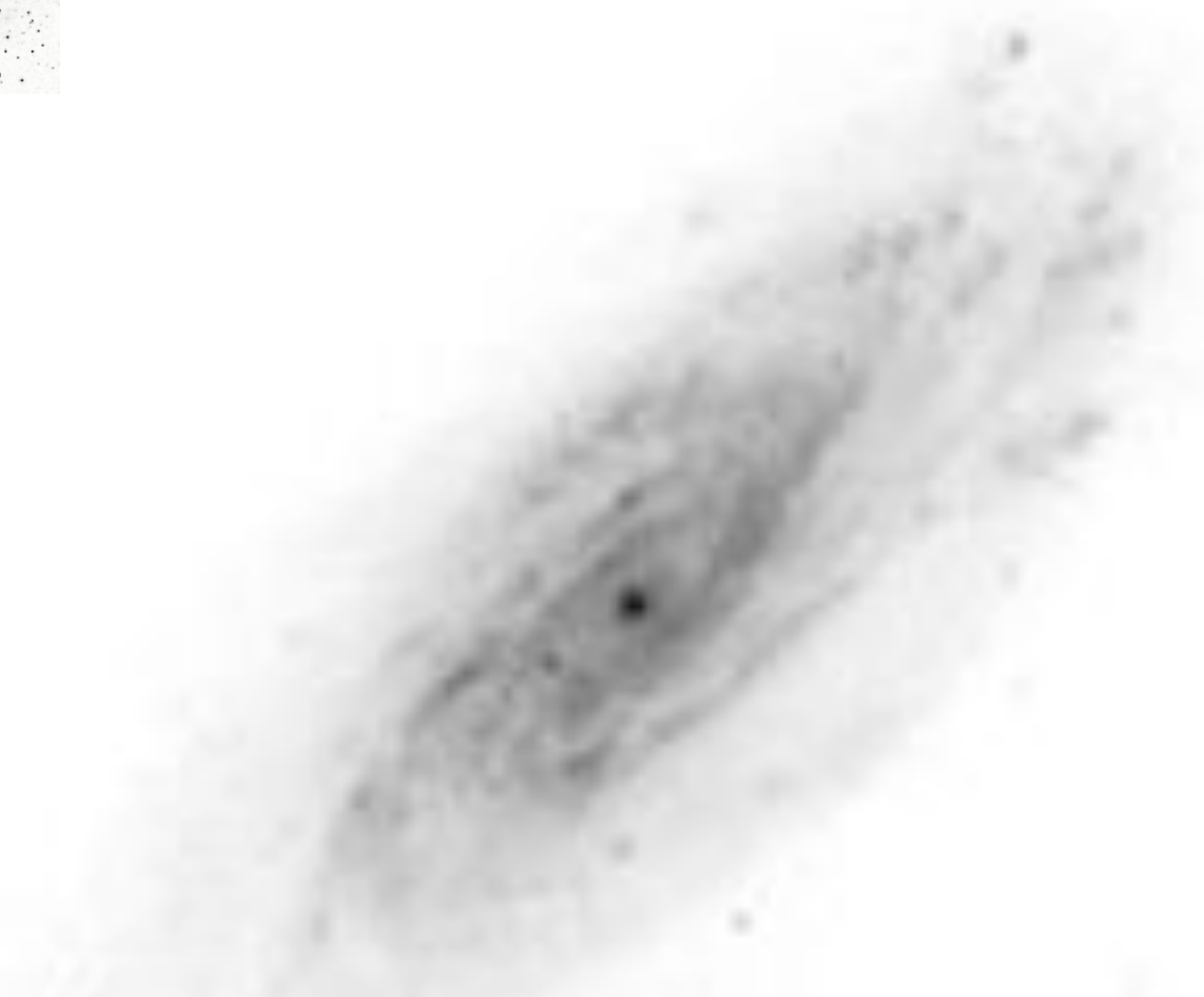
which is about what is
expected for scatter in stellar
population M^*/L





NGC 2403

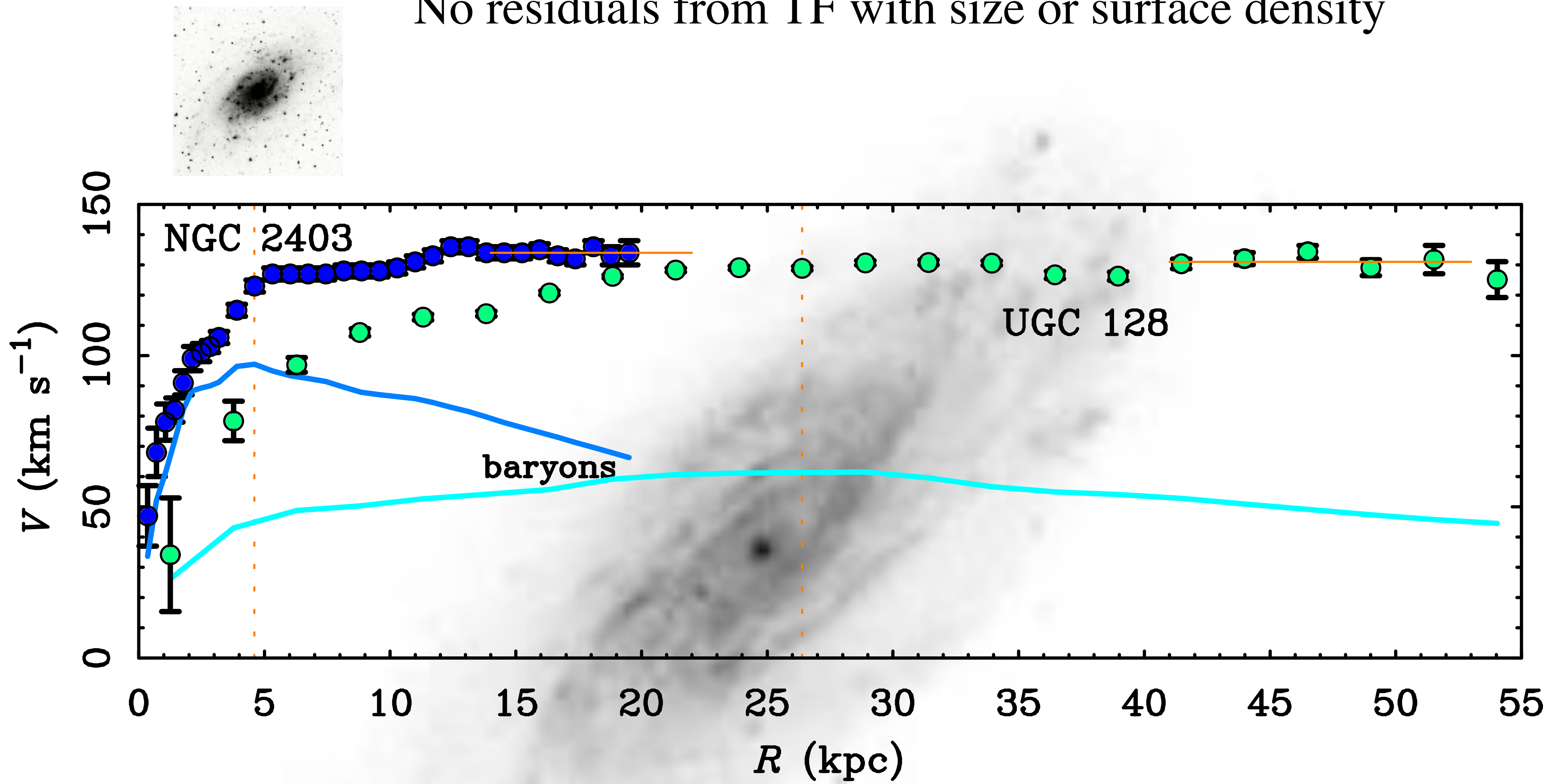
UGC 128



Same (M,V) but very different size and surface density

Size/surface brightness variations from TF

No residuals from TF with size or surface density

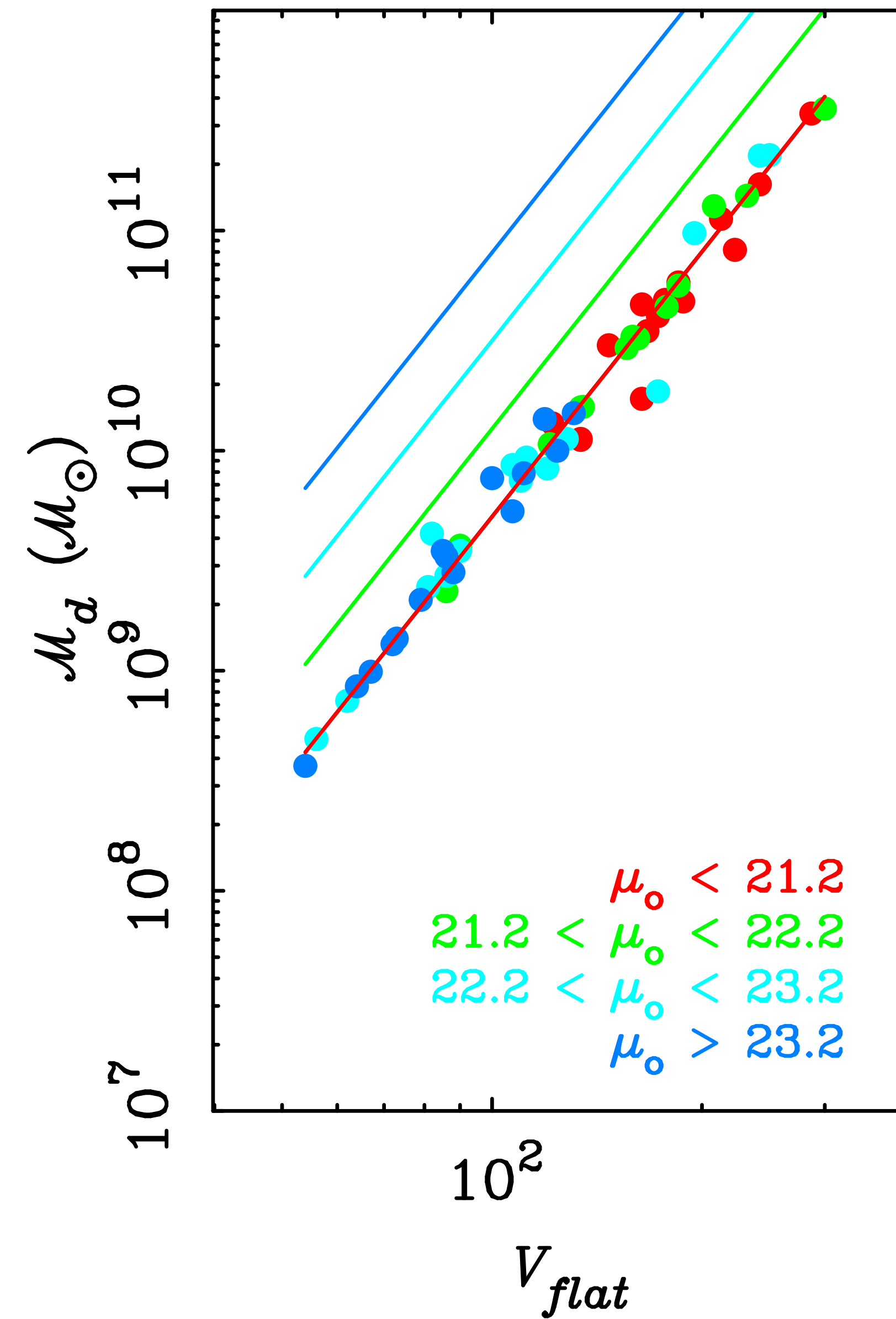


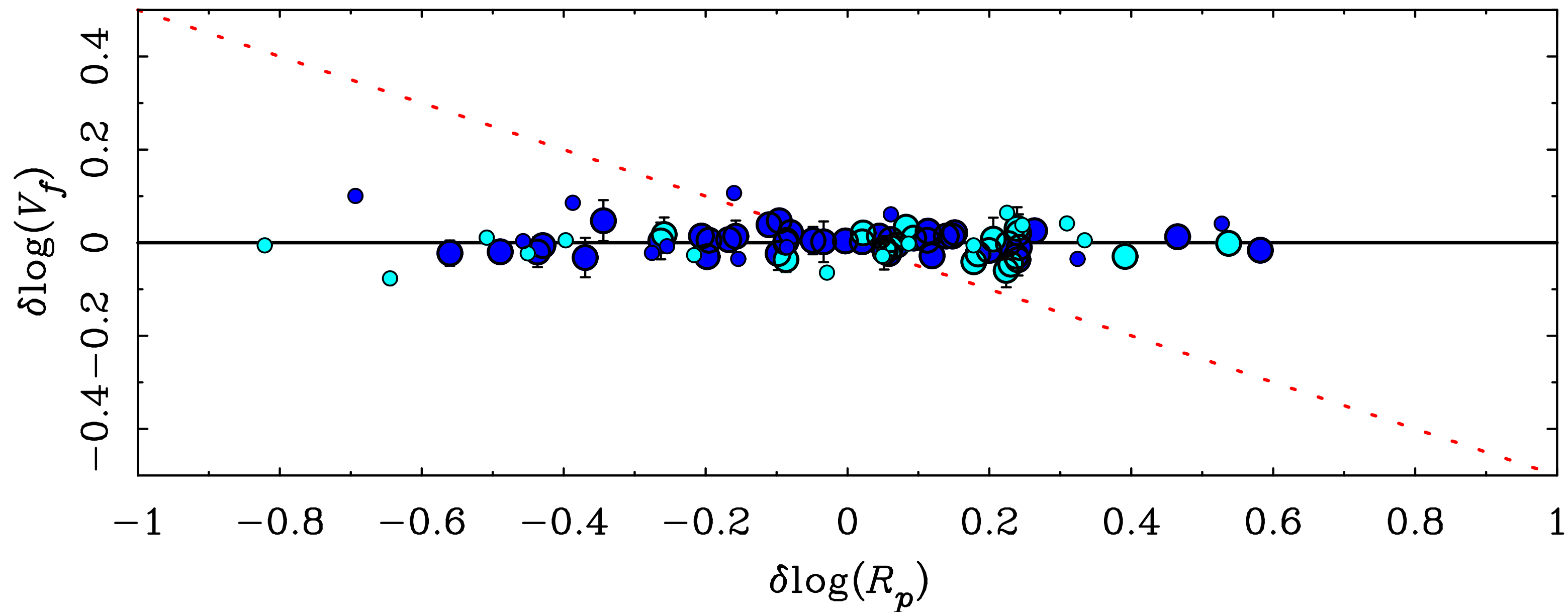
Same (M,V) but very different size and surface density

which is strange, since $V^2 = \frac{GM}{R}$

No residuals from TF with
size or surface brightness

(Zwaan et al 1995;
Sprayberry et al 1995;
McGaugh & de Blok 1998)

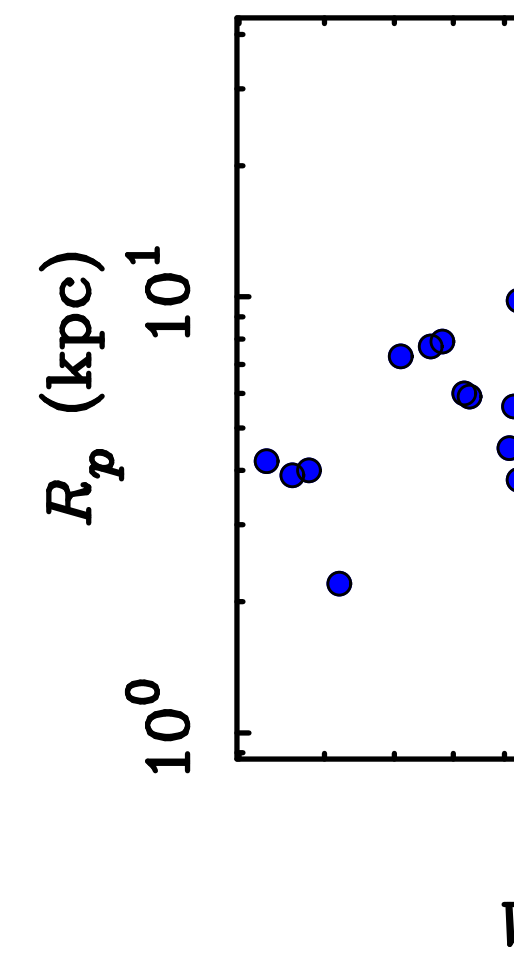
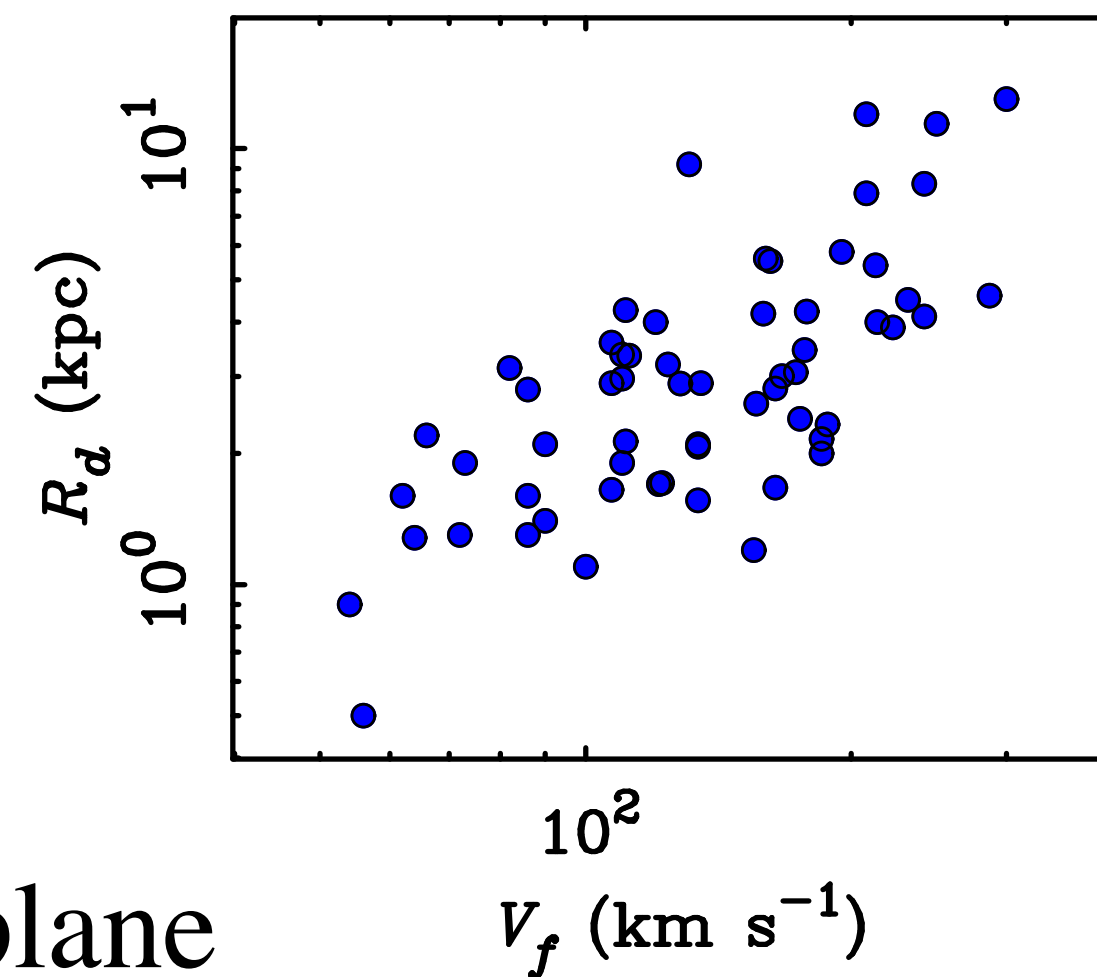




No residuals from TF with size or surface density for disks

$$V^2 = \frac{GM}{R} \rightarrow \frac{\delta \log(V)}{\delta \log(R)} = -\frac{1}{2} \quad \text{expected slope (dotted line)}$$

Note: large range in size at a given mass or velocity



TF already edge-on projection of disk fundamental plane

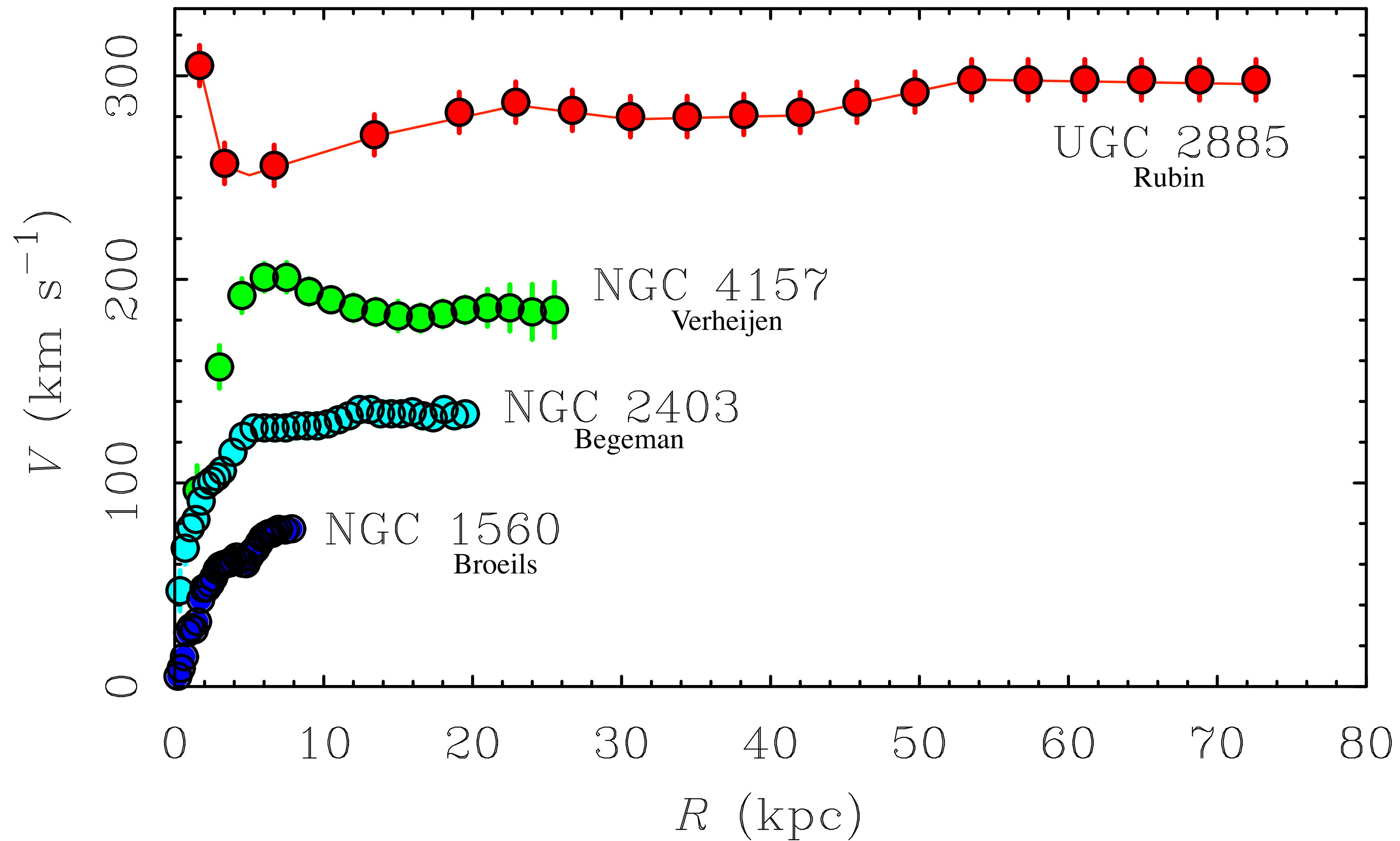
Baryonic TF Relation

- Fundamentally a relation between the baryonic mass of a galaxy and its rotation velocity
 - $M_b = M_* + M_g = 47 V_f^4$ (McGaugh 2012)
 - doesn't matter if it is stars or gas
- Intrinsic scatter negligibly small
 - Can mostly be accounted for by the expected variation in stellar M^*/L
- Physical basis of the relation remains unclear

Relation has real physical units if slope has integer value -
Slope appears to be 4 if V_{flat} is used.

Rotation curve amplitude depends on the mass of stars and gas (BTFR)

Rotation curve shape depends on the distribution of stars and gas



only near the limits of the optical image ($\kappa \approx 1$); velocities of highest-luminosity Sc's reach 100 km s^{-1} in less than 1% of the optical radius ($\kappa < 0.01$). The plot of $\log \kappa$ versus $\log V_{\text{max}}$ (Fig. 3) emphasizes that κ is a reliable estimator of V_{max} and, hence, of absolute magnitude. Ignoring resolution effects for the most distant galaxies, this diagram is distance independent. Furthermore, a comparison of Figures 1 and 2 shows that the $(\log \kappa, M_B)$ relation is similar in form and scatter to the $(\log V_{\text{max}}, M_B)$ relation, i.e., the conventional TF relation. The choice of 100 km s^{-1} as a fiducial mark in measuring κ is not particularly critical to the success of the $(\log \kappa, M_B)$ relation, although it should be located beyond local nuclear effects so as to relate to the overall rotation curve. It should not be affected by nonaxisymmetric barlike motions and local velocity perturbations often observed at small nuclear distances.

From our published rotation curves, it is clear that an adopted measure could be chosen from a fairly wide range of velocities up to and including, of course, the velocity peak. Any one of such measures would serve as a luminosity discriminant. Thus the relationships exhibited in Figures 1 and 2 are representative of a family of dynamical-luminosity relationships. The family of such measures will be explored in detail in a future paper.

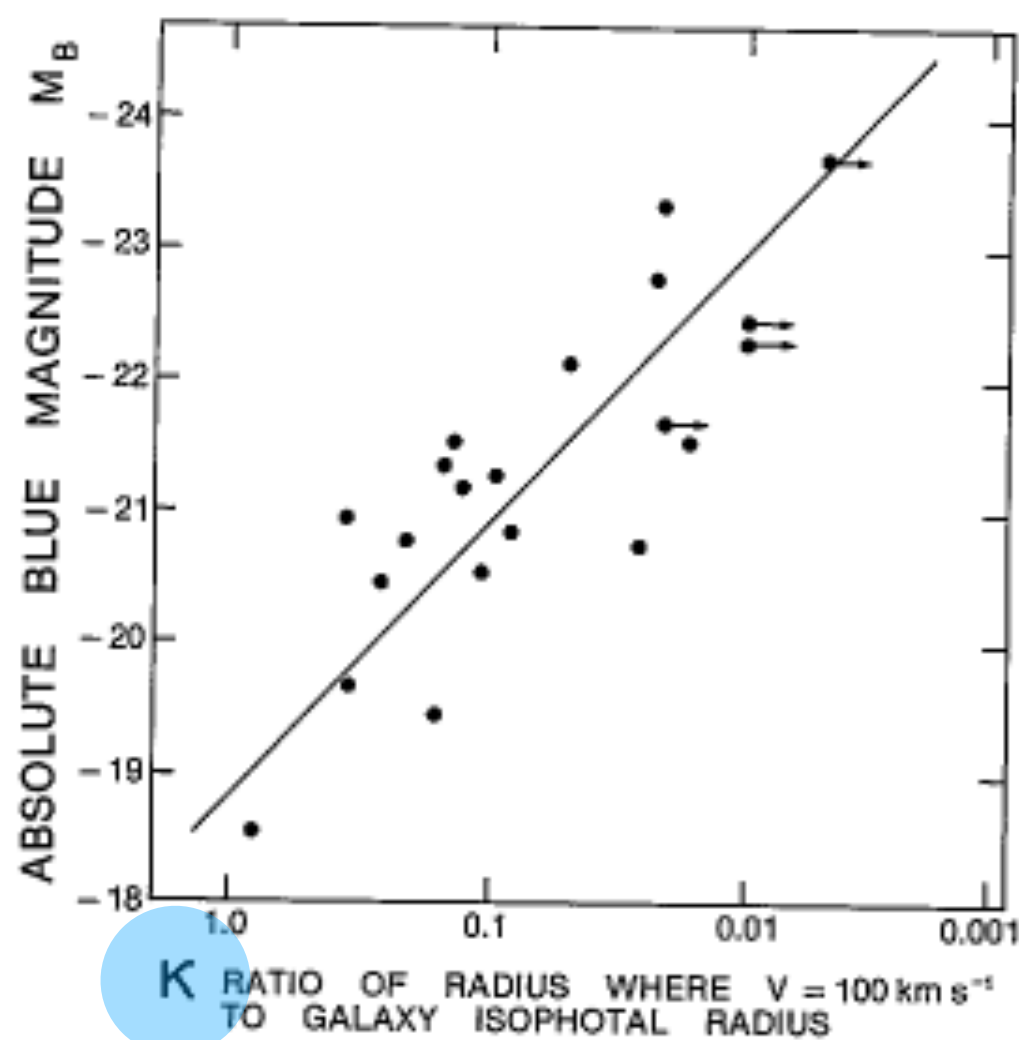


FIG. 2.—The correlation of M_B with $\log \kappa$, the radial distance in the galaxy where the rotational velocity equals 100 km s^{-1} , in units of the isophotal radius. For lowest-luminosity Sc galaxies, the rotational velocity reaches 100 km s^{-1} only near the limits of the optical image ($\kappa \approx 1$), while for high-luminosity Sc's, the rotational velocity reaches 100 km s^{-1} in less than 1% of the optical radius ($\kappa < 0.01$).

linear scale and in Figure 4b scaled to the size of the galaxy. At every radial distance r in the constant-

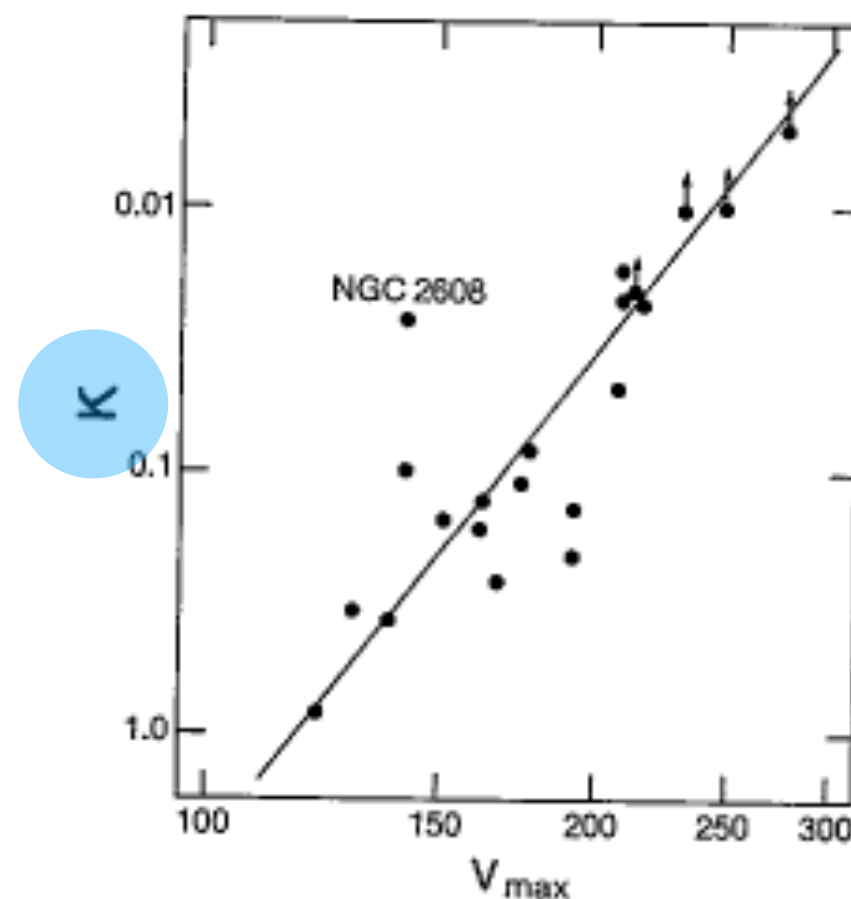


FIG. 3.—The correlation of $\log \kappa$, the radius where the rotational velocity equals 100 km s^{-1} in units of the isophotal radius, vs. $\log V_{\text{max}}$. The line is the mean of the two regressions and has a slope equal to 6.3 ± 0.5 . NGC 2608, the only strongly barred galaxy in the sample, was excluded from the solution.

known for a long time

Rotation curve shapes correlate with galaxy properties

Rubin et al. 1980, *ApJ*, 242, L149

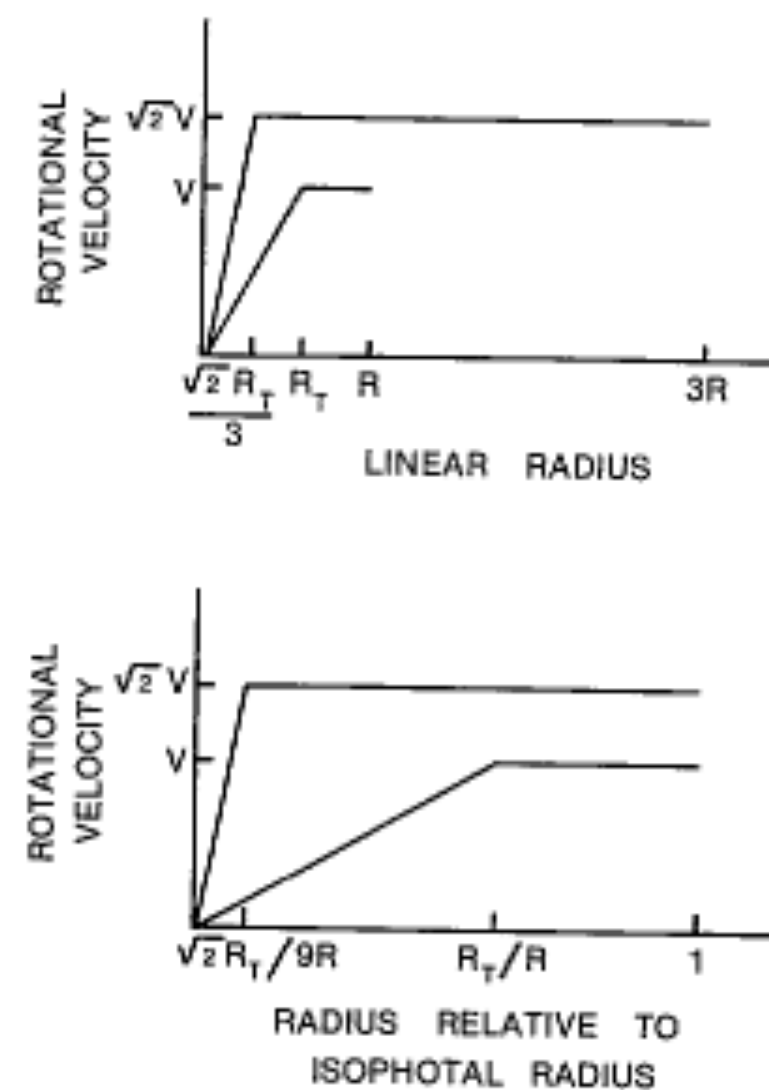
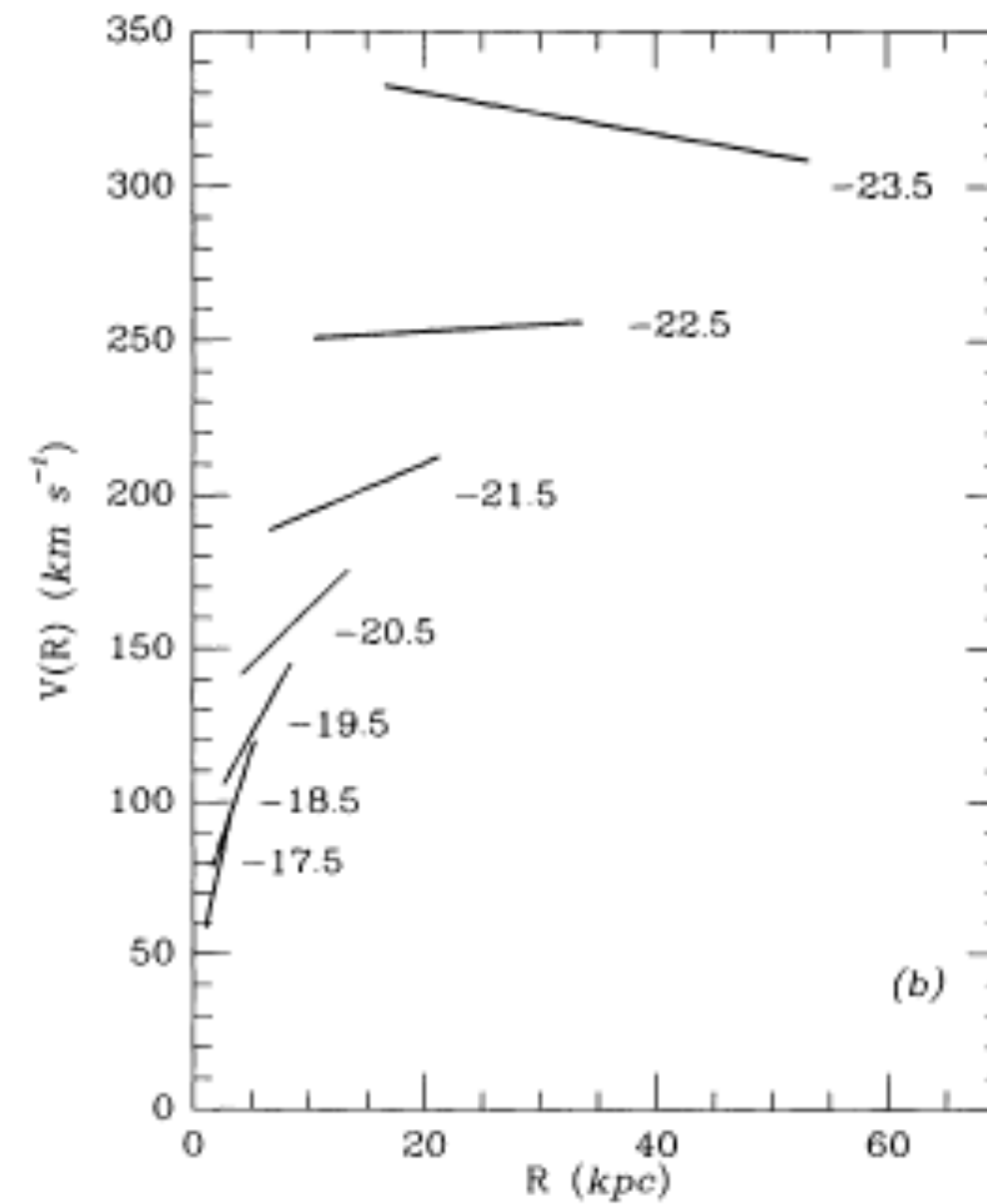


FIG. 4.—Schematic rotation curves for two Sc galaxies on a linear (*upper*) and relative (*lower*) radius scale. The higher-luminosity galaxy is chosen to have its velocity in the flat portion $\sqrt{2}$ times that of the lower-luminosity galaxy. Then the nuclear velocity gradient, turnover radius, and radial extent are fixed by the observations as shown (see text and Table 2).

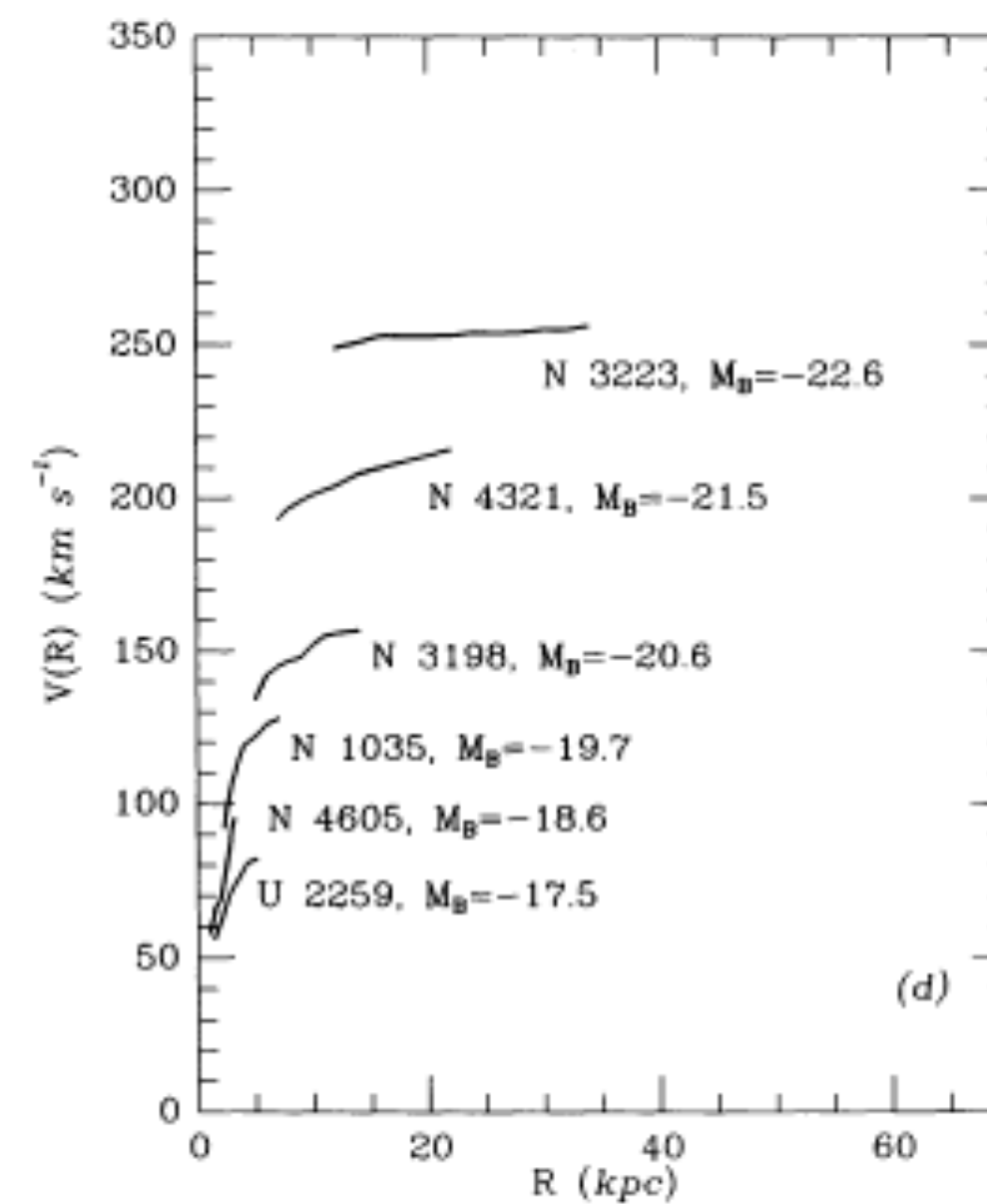
κ here is the radius where $V(R/R_{\text{isophotal}}) = 100 \text{ km/s}$. This is small in bright galaxies, which have steeply rising rotation curves, and large in dim galaxies, which have slowly rising rotation curves.

known for a long time

Rotation curve *shapes* correlate with galaxy properties



Persic & Salucci 1991
“Universal Rotation Curve”



Depends on both the luminosity
and the scale length... turns out
that surface brightness is the key
quantity.

Rotation curve shape correlates with baryonic surface density

