# Dark Matter

## ASTR 333/433
## Spring 2026
## TR 11:30am-12:45pm
## Sears 552

http://astroweb.case.edu/ssm/ASTR333/


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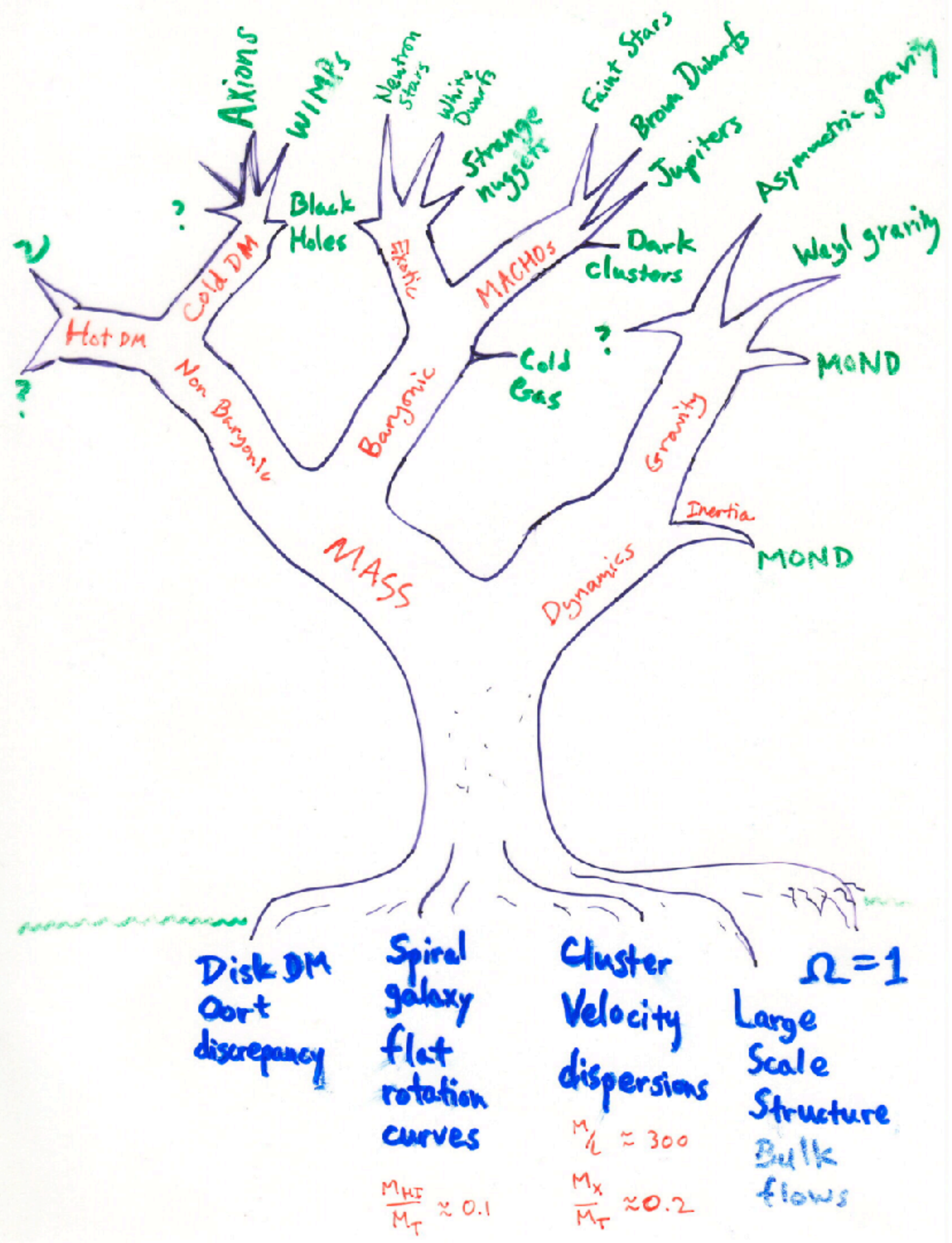
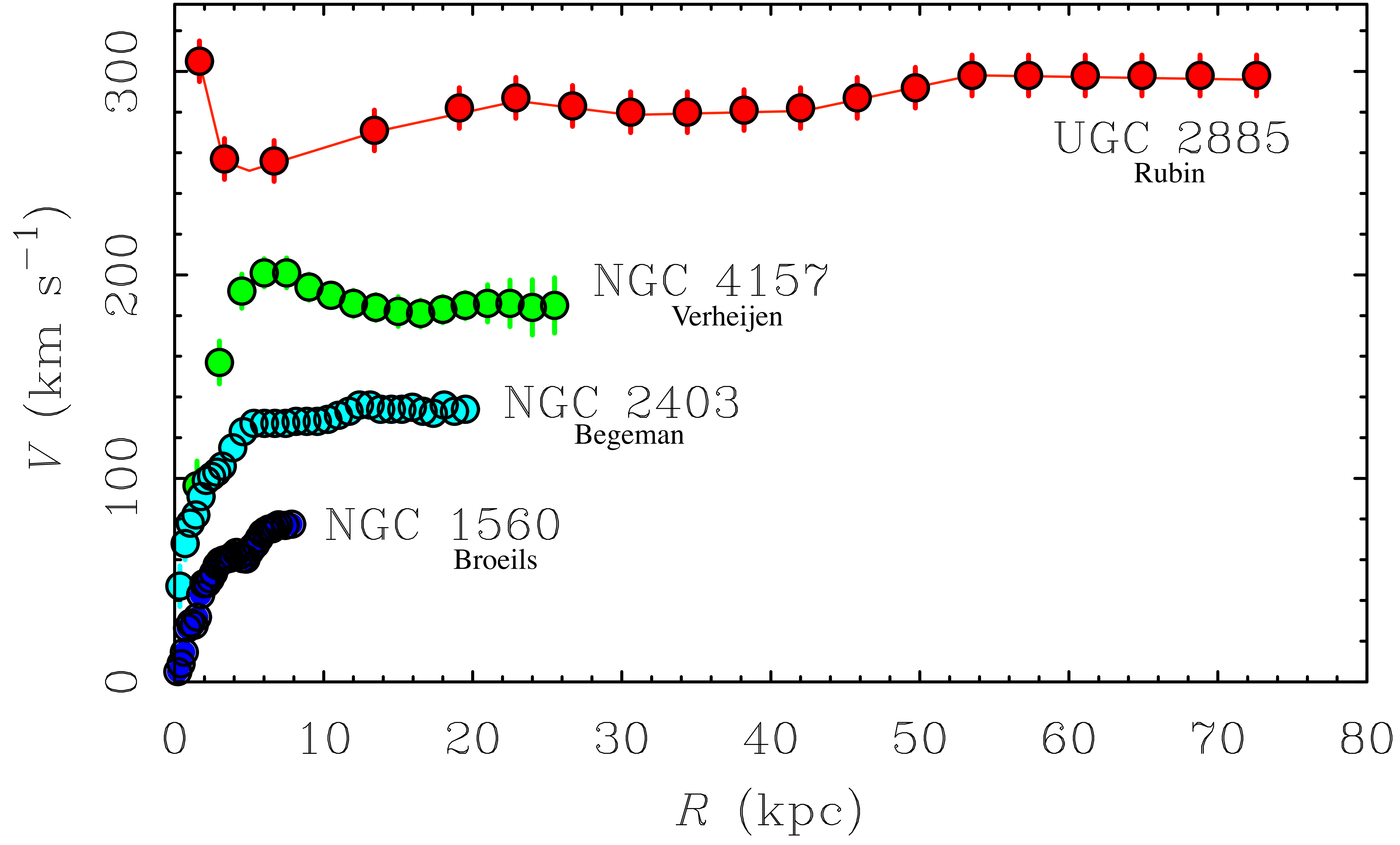
Case Western Reserve University EST. 1826

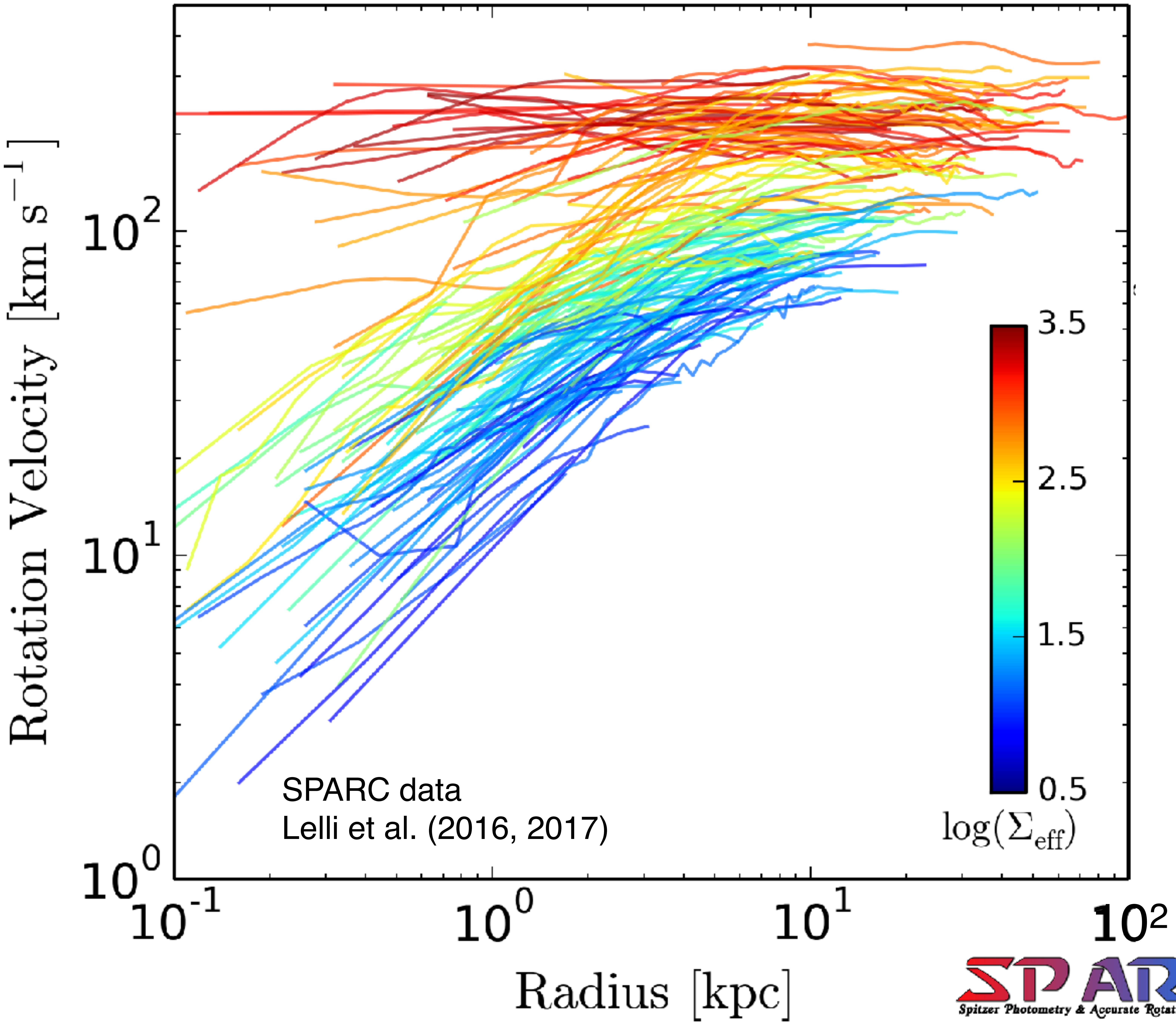


Rotation curve amplitude depends on the mass of stars and gas (BTFR)

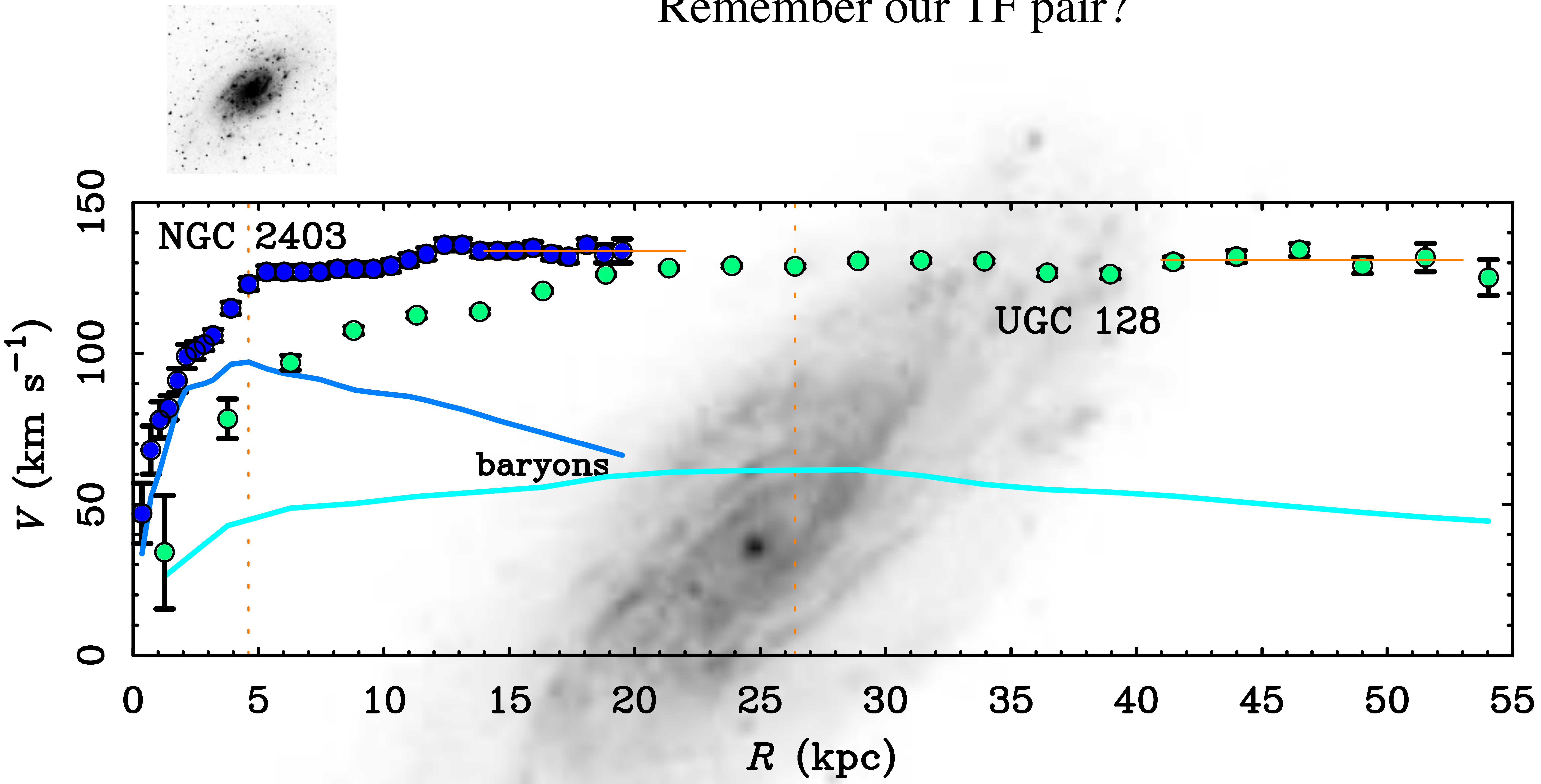
Rotation curve shape depends on the distribution of stars and gas



Rotation curve shape correlates with baryonic surface density

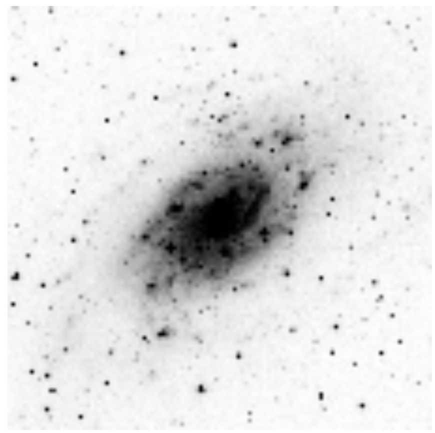


# Remember our TF pair?

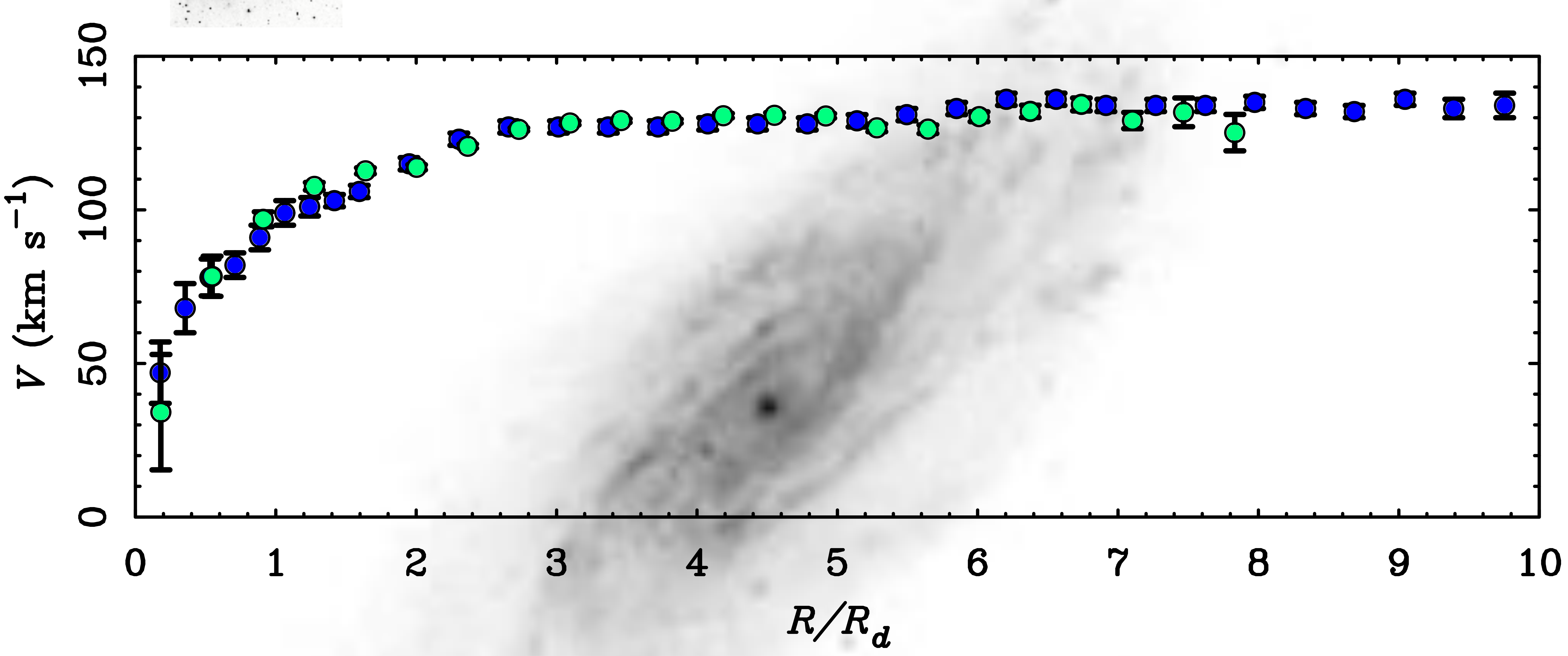



Radius in physical units (kpc)

Persic & Salucci 1996
de Blok & McGaugh 1996
Tully & Verheijen (1998)
Nordermeer & Verheijen (2007)  [URC nor quite right formulation]
Swaters et al. (2009)



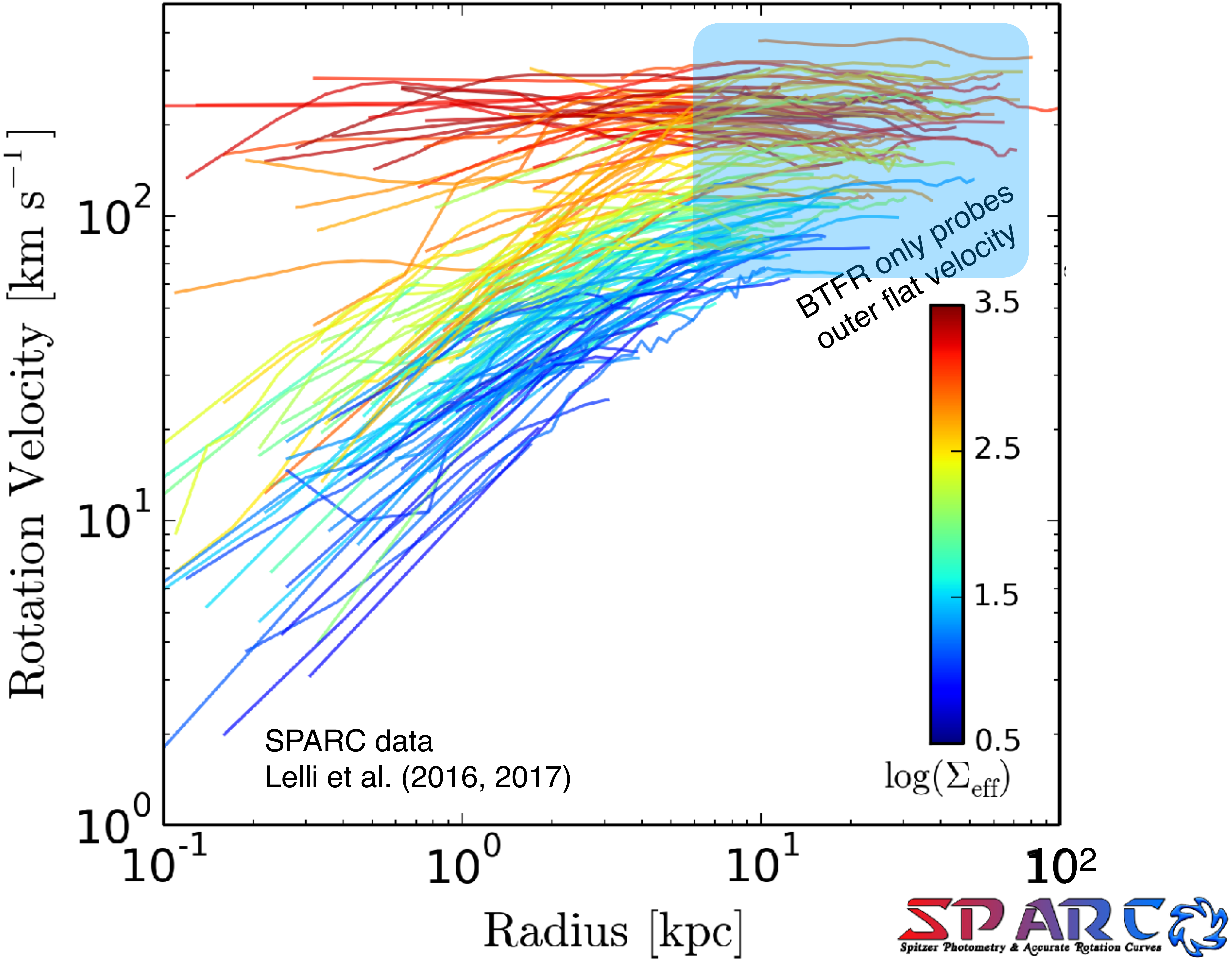
# The dynamics knows about the distribution of baryons, not just their total mass



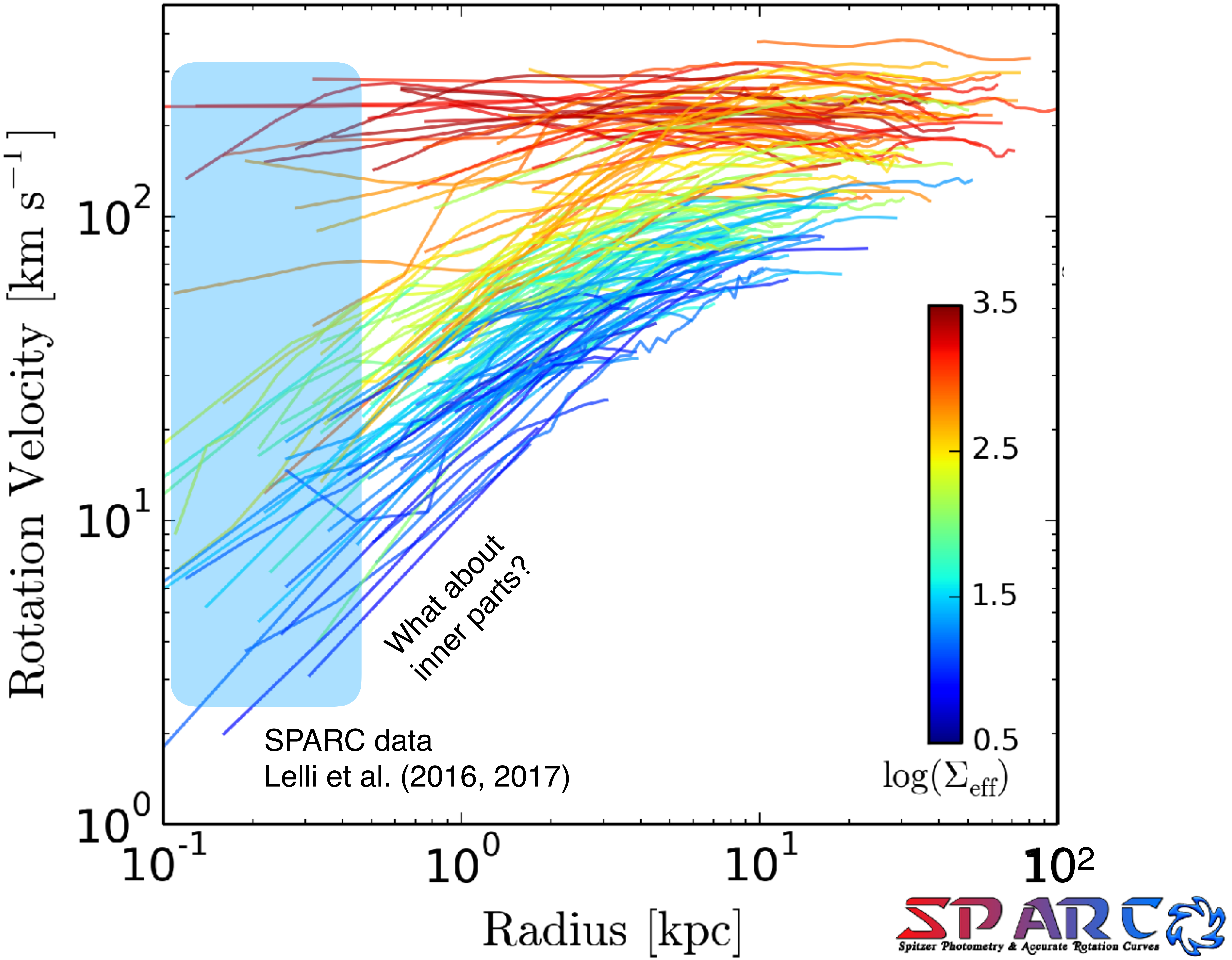

Radius normalized by size of disk.

Persic & Salucci 1996
de Blok & McGaugh 1996
Tully & Verheijen (1998)
Nordermeer & Verheijen (2007)  [URC nor quite right formulation]
Swaters et al. (2009)

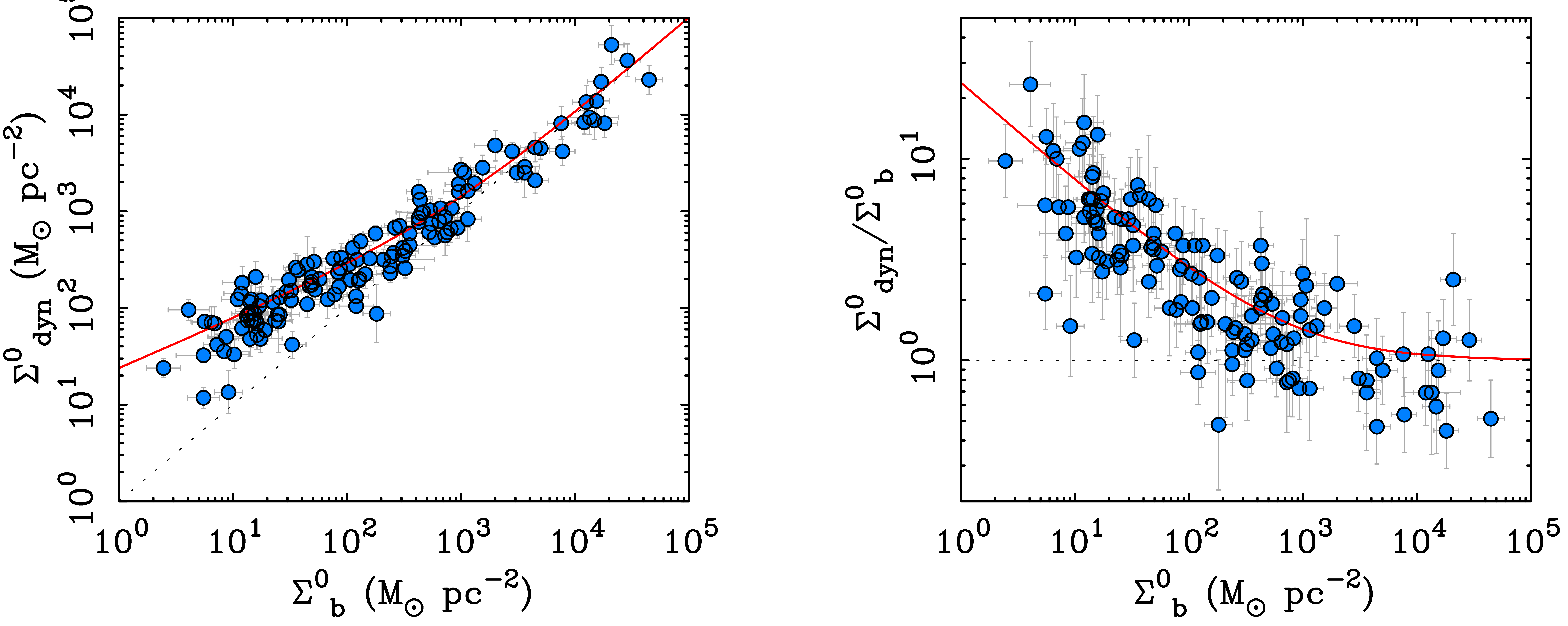
# Rotation curve shape correlates with baryonic surface density



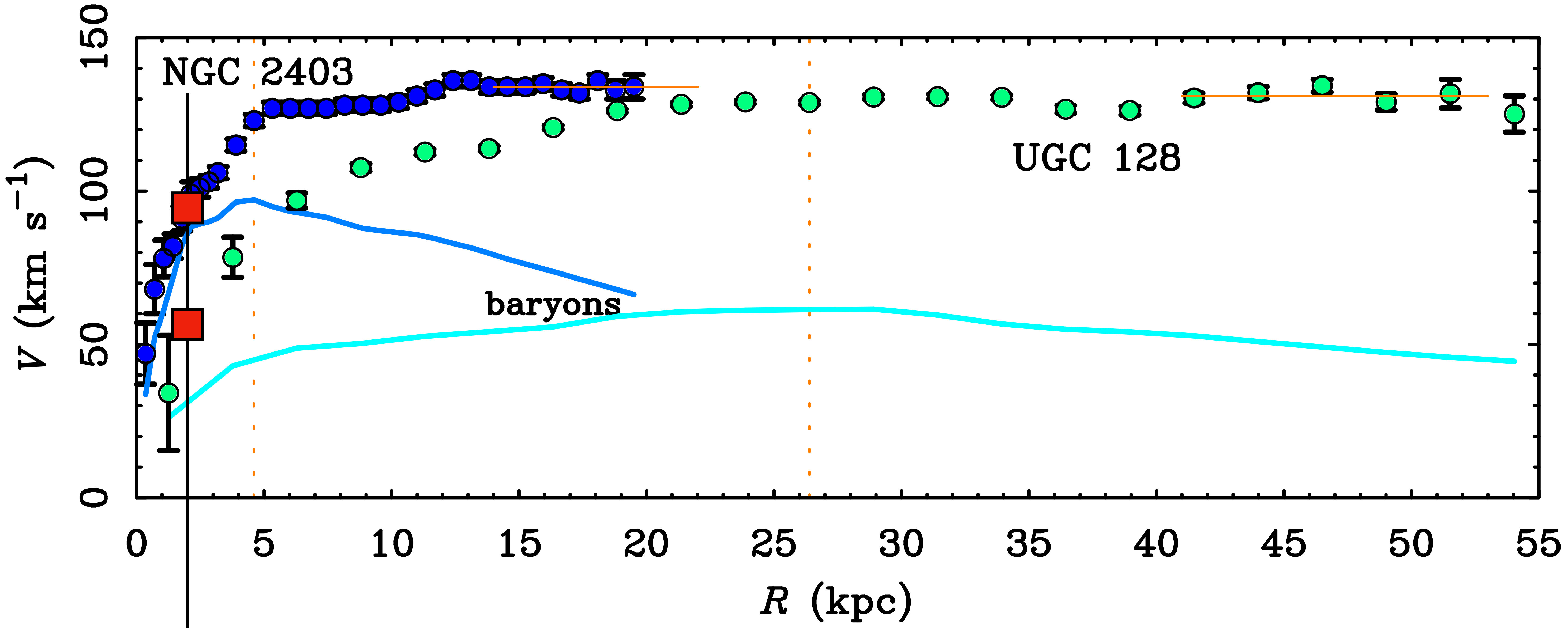
# Rotation curve shape correlates with baryonic surface density



$$\Sigma_{dyn}(0) = \frac{1}{2\pi G} \int_0^\infty \frac{V^2(R)}{r^2} dR$$

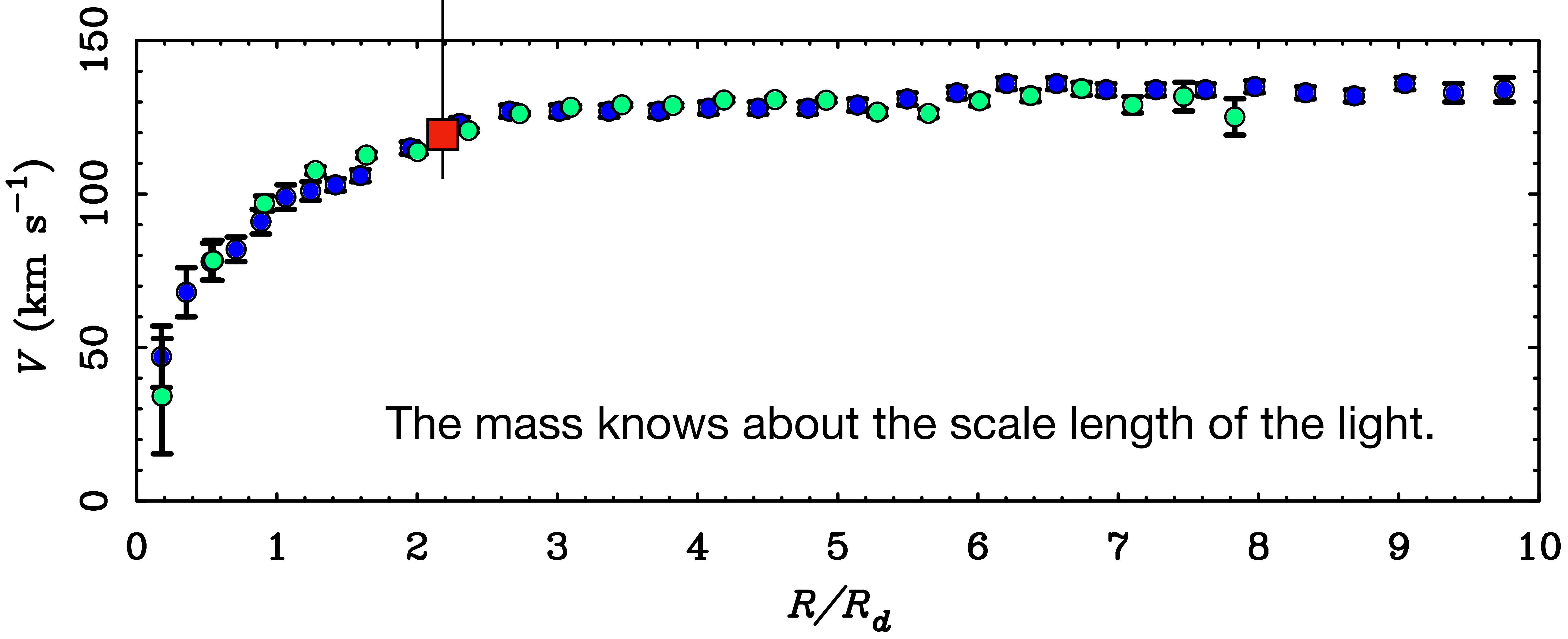


What you get depends on how you look at it: what you assume & what you choose to measure:

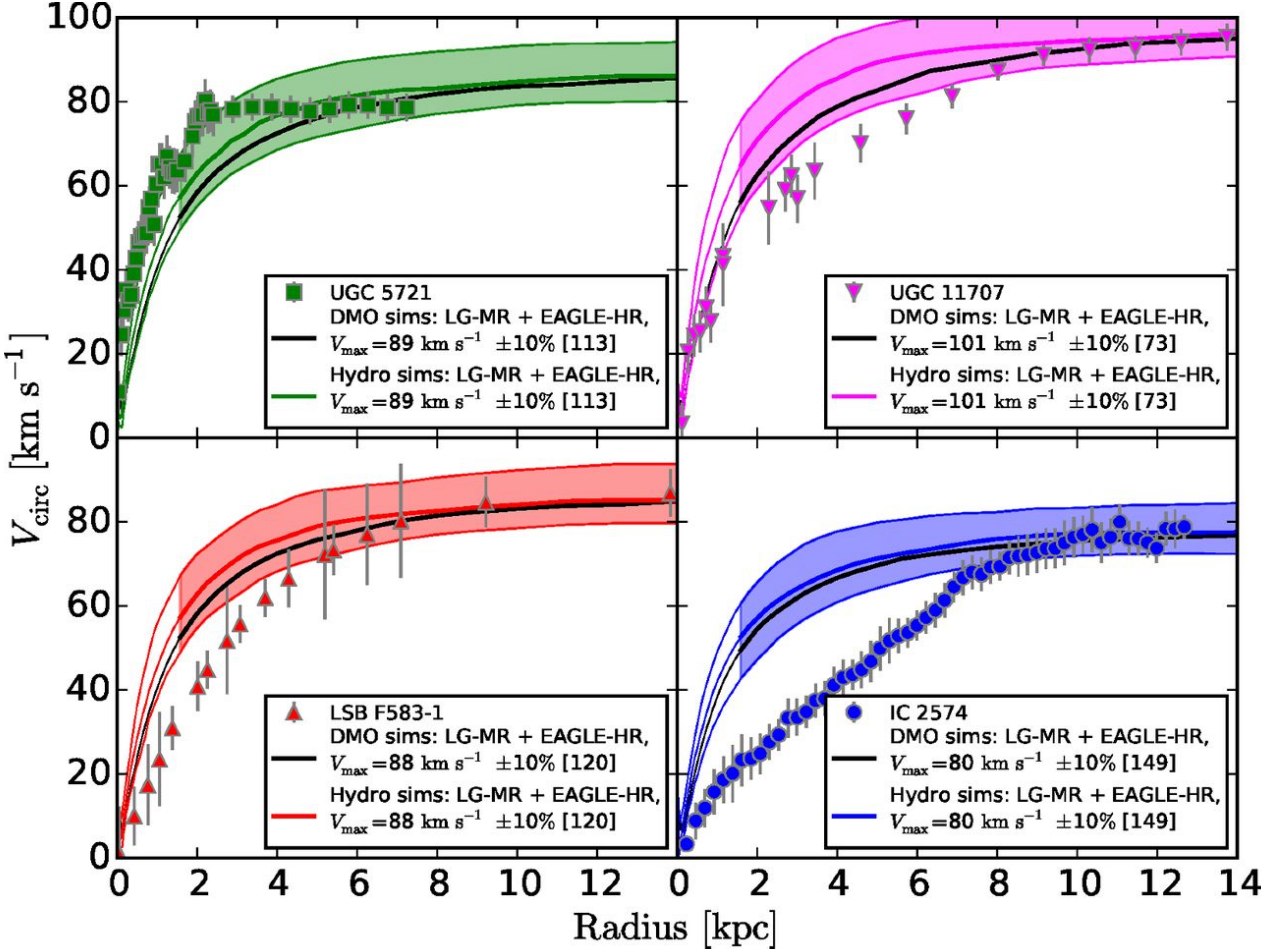



If you measure V(R in kpc) you infer **diversity**.

If you measure V(R in scale lengths) you infer **uniformity**.

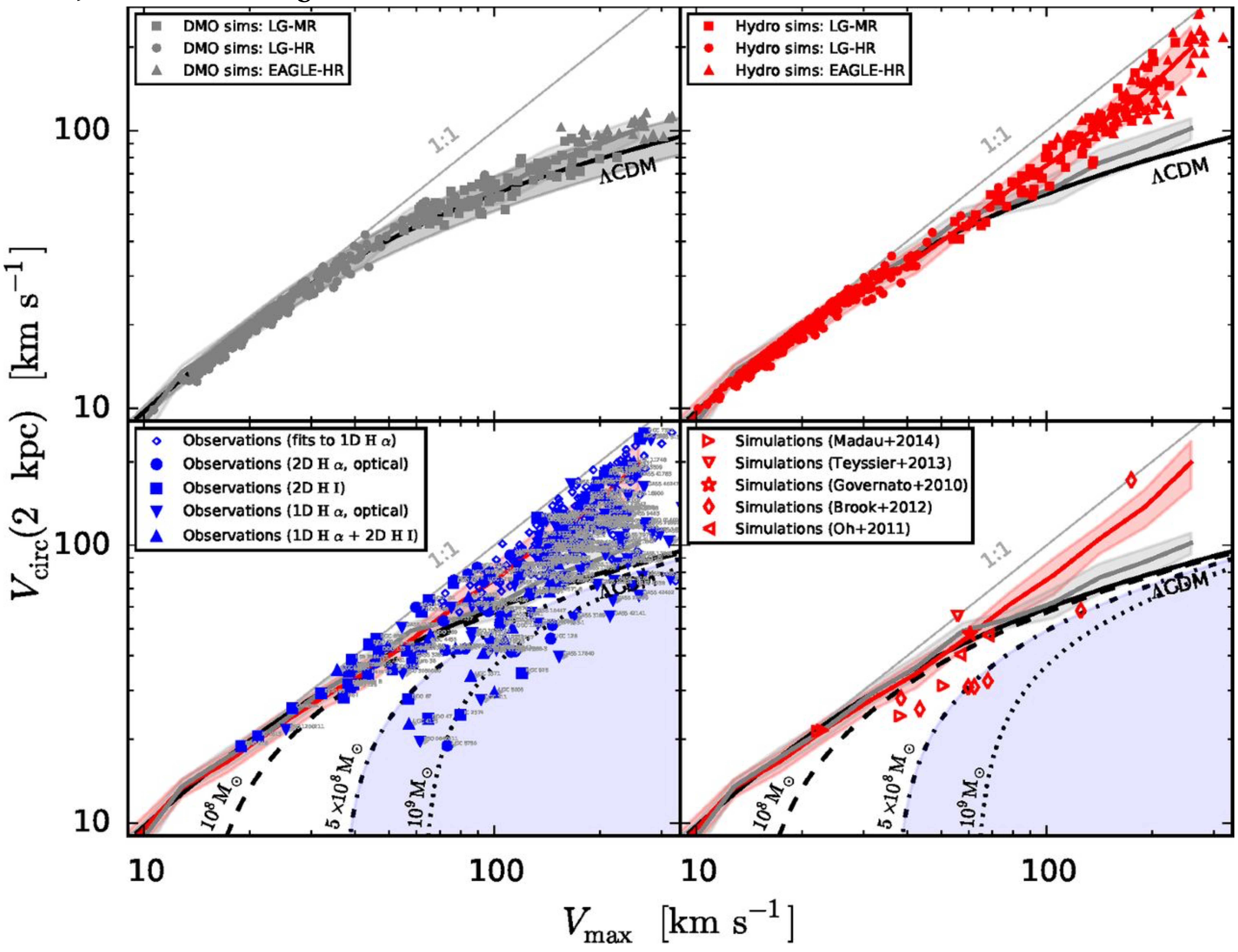


# Rotation curve "diversity" (Oman+ 2015)



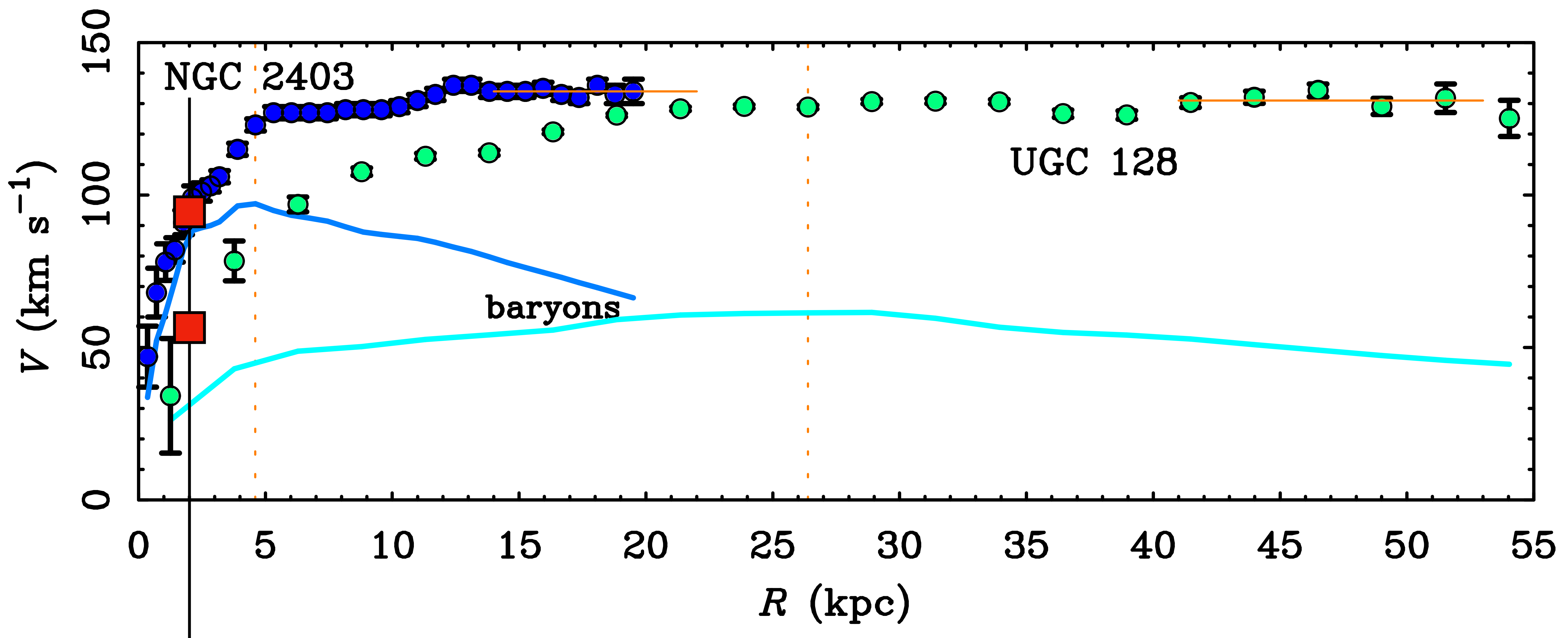

Rotation curves rise less rapidly than predicted by LCDM (see also Lelli 2014)

# Rotation curve "diversity" (Oman+ 2015)




Rotation curves indicate less enclosed mass than predicted by LCDM (see also Kuzio de Naray & McGaugh 2014)

What you get depends on how you look at it: what you assume & what you choose to measure:

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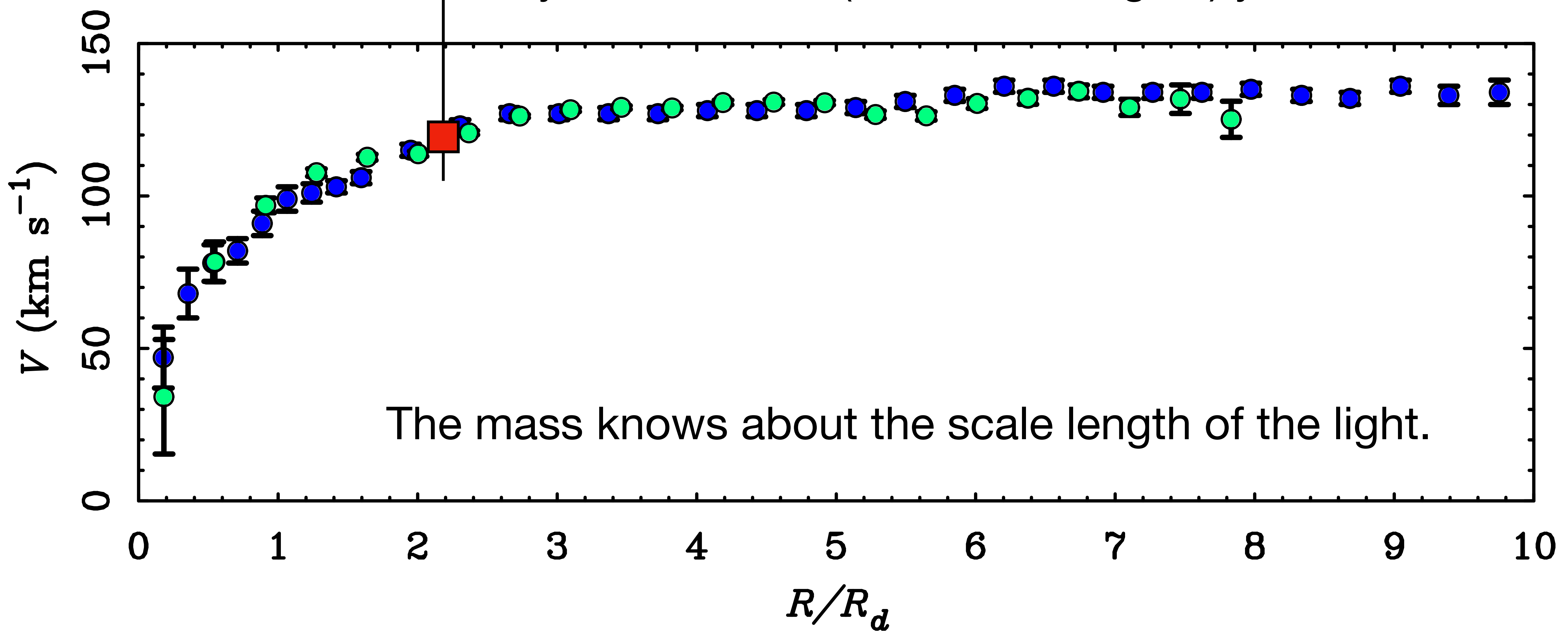



It's not just that rotation curves are **diverse**. It is also that their shapes that cause diversity correlate with surface brightness.

# Central Density Relation

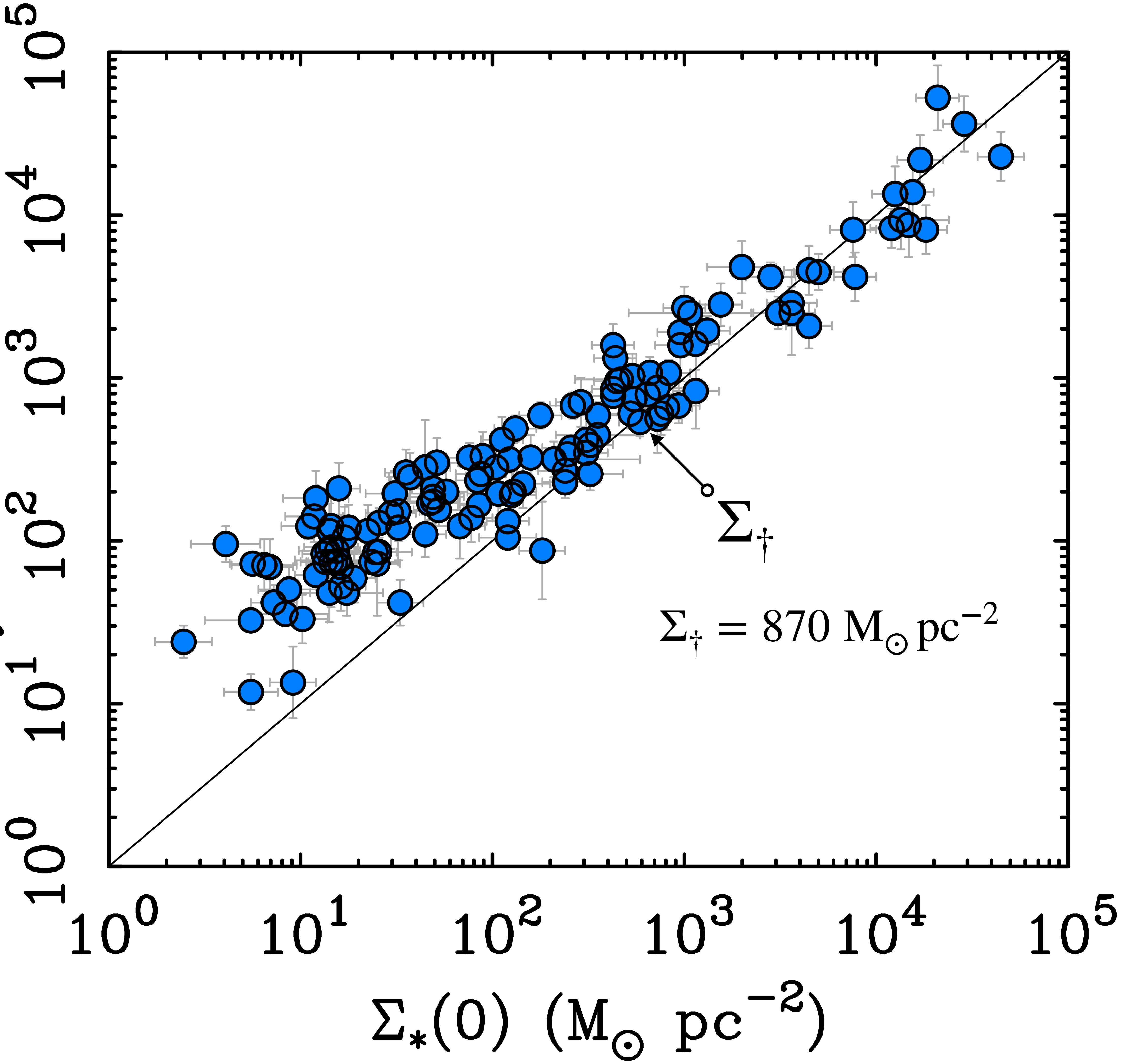
Lelli et al. (2016)

The *dynamical* central mass surface density correlates with the central surface brightness of stars in galaxies.

central dynamical surface density

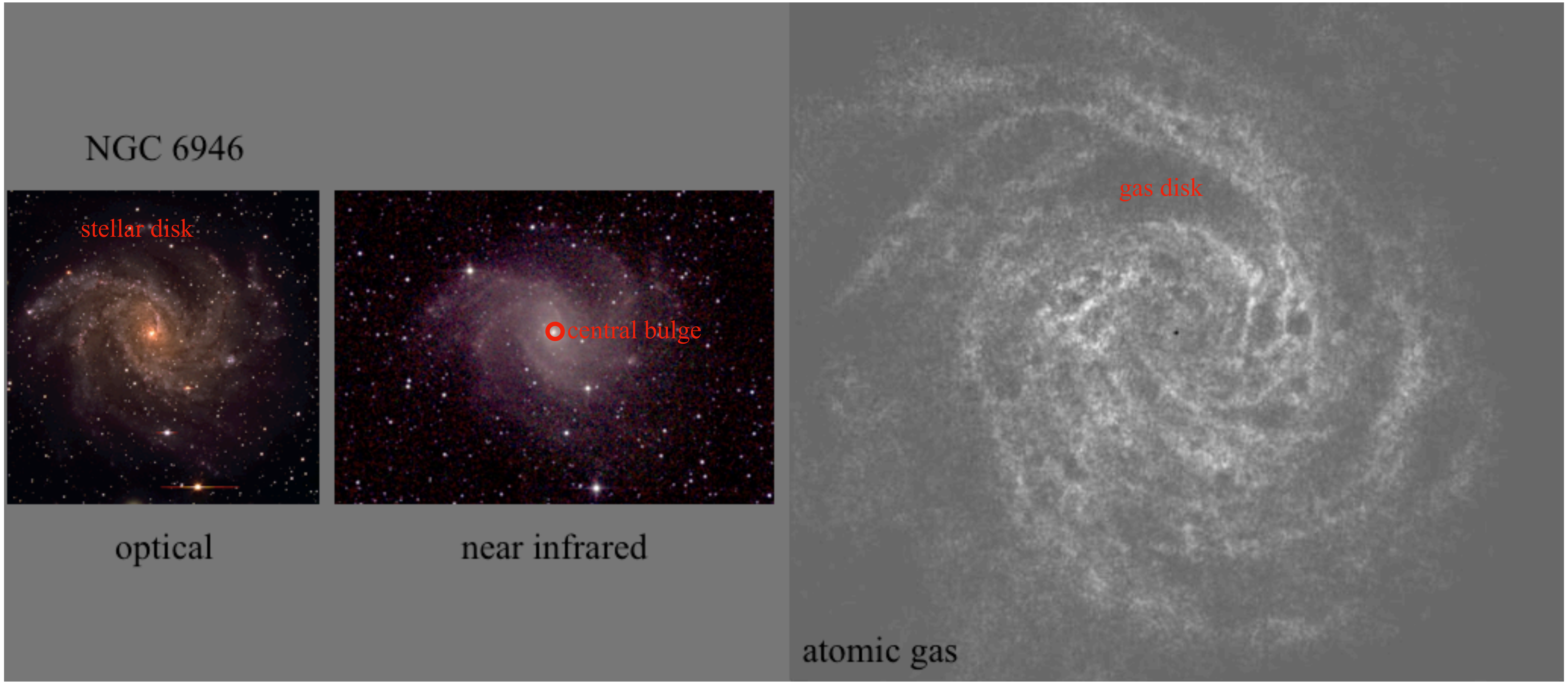
Toomre (1963)

$$\Sigma_{dyn}(0) = \frac{1}{2\pi G}\int_0^\infty \frac{V^2(R)}{r^2} dR$$



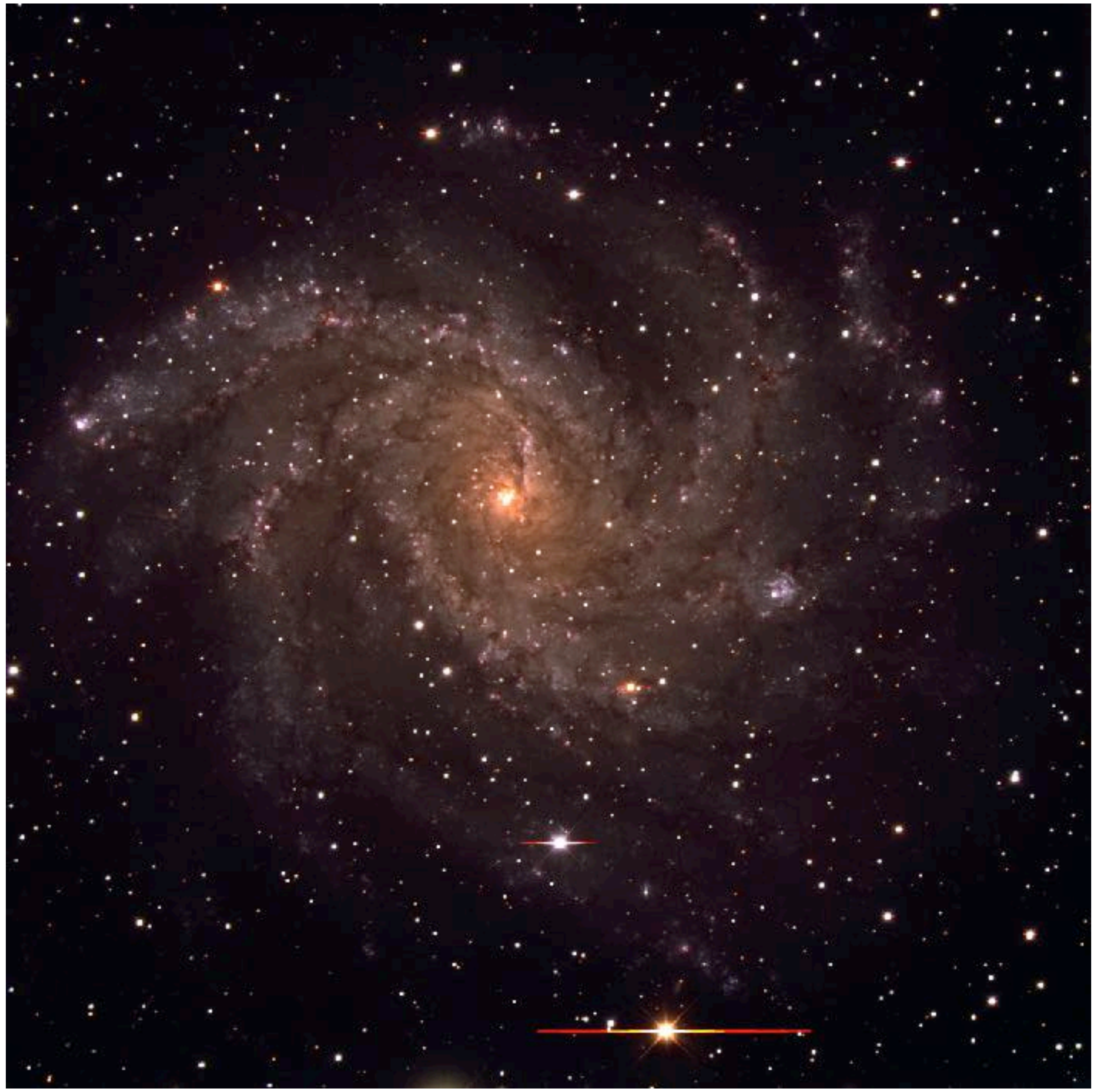

central surface brightness

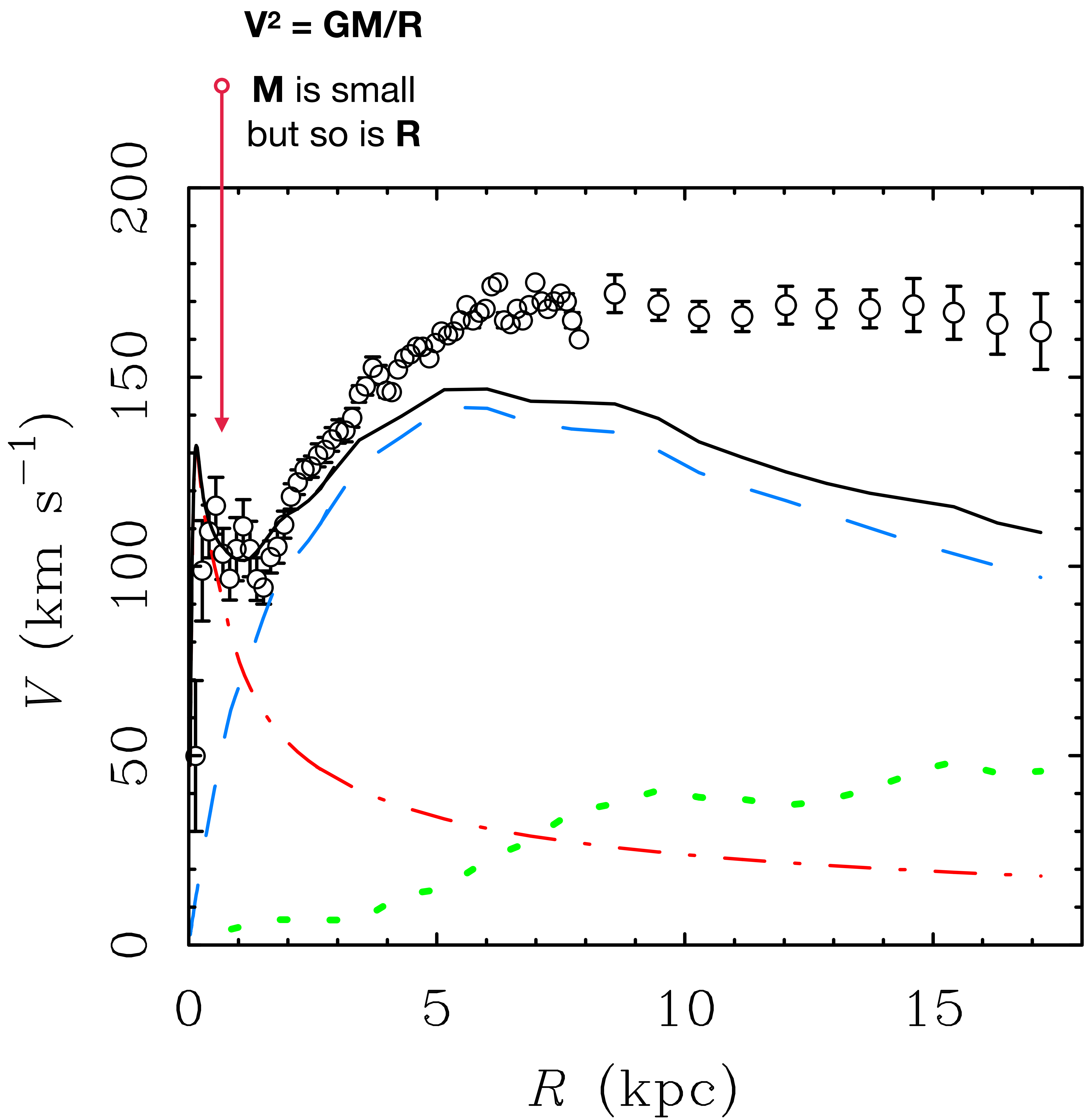
- Renzo's Rule:  (2004 IAU; 1995 private communication) *"When you see a feature in the light, you see a corresponding feature in the rotation curve."*



The central bulge component of NGC 6946 is only 6% of the total light, but it has a perceptible effect on the kinematics.

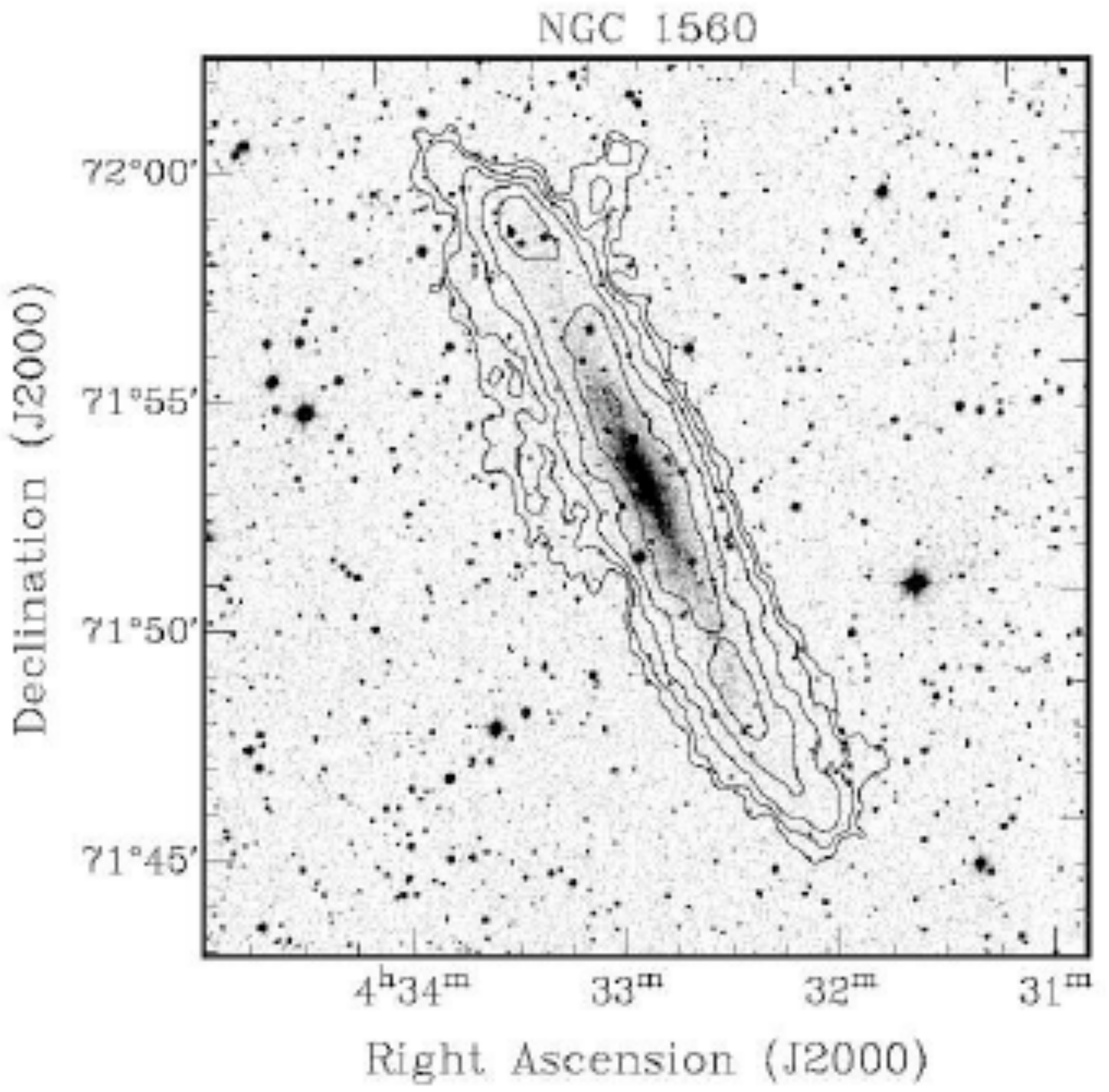
Note the up-down-up morphology - this requires a maximal bulge; can't explain that with a dark matter halo.



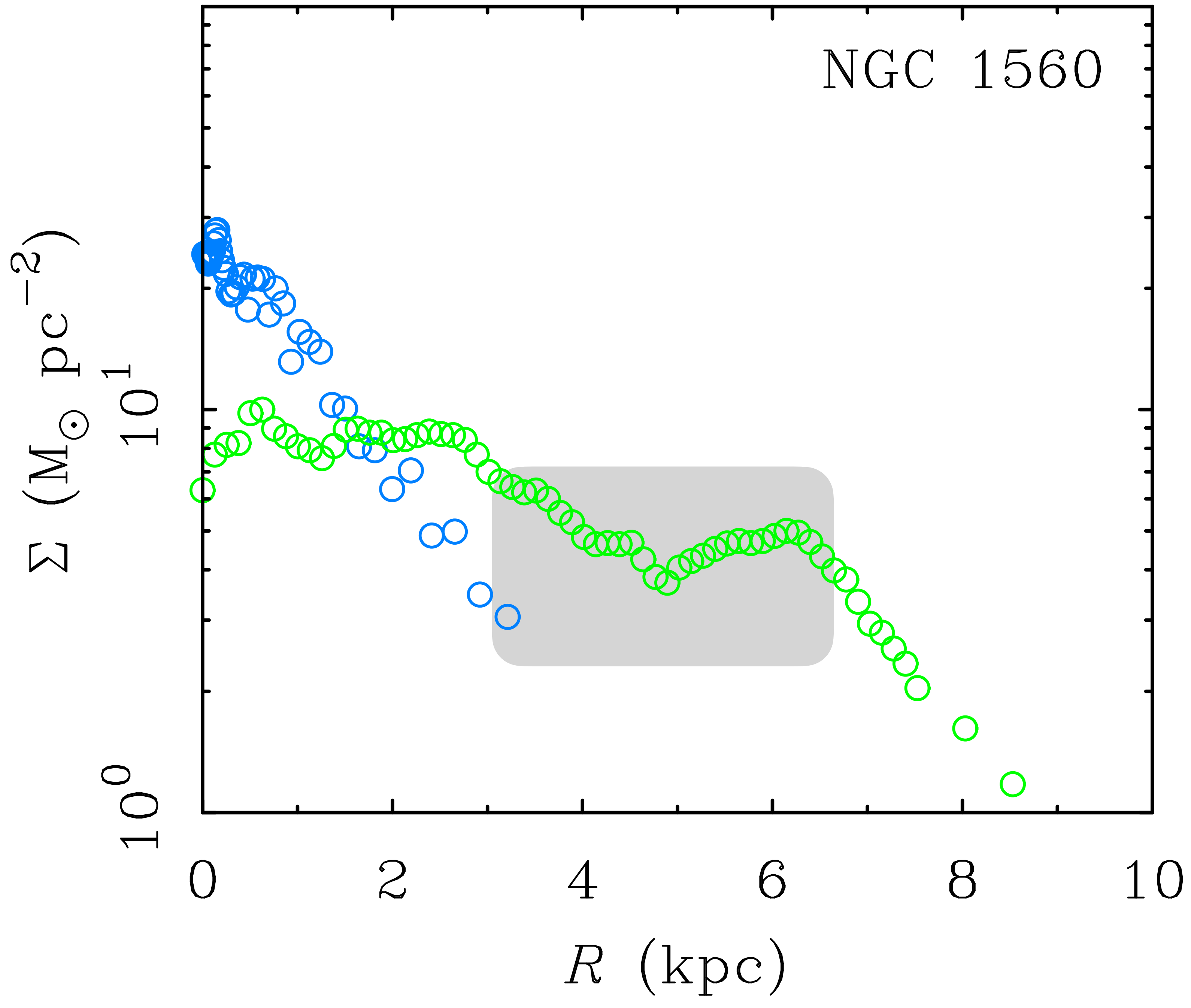


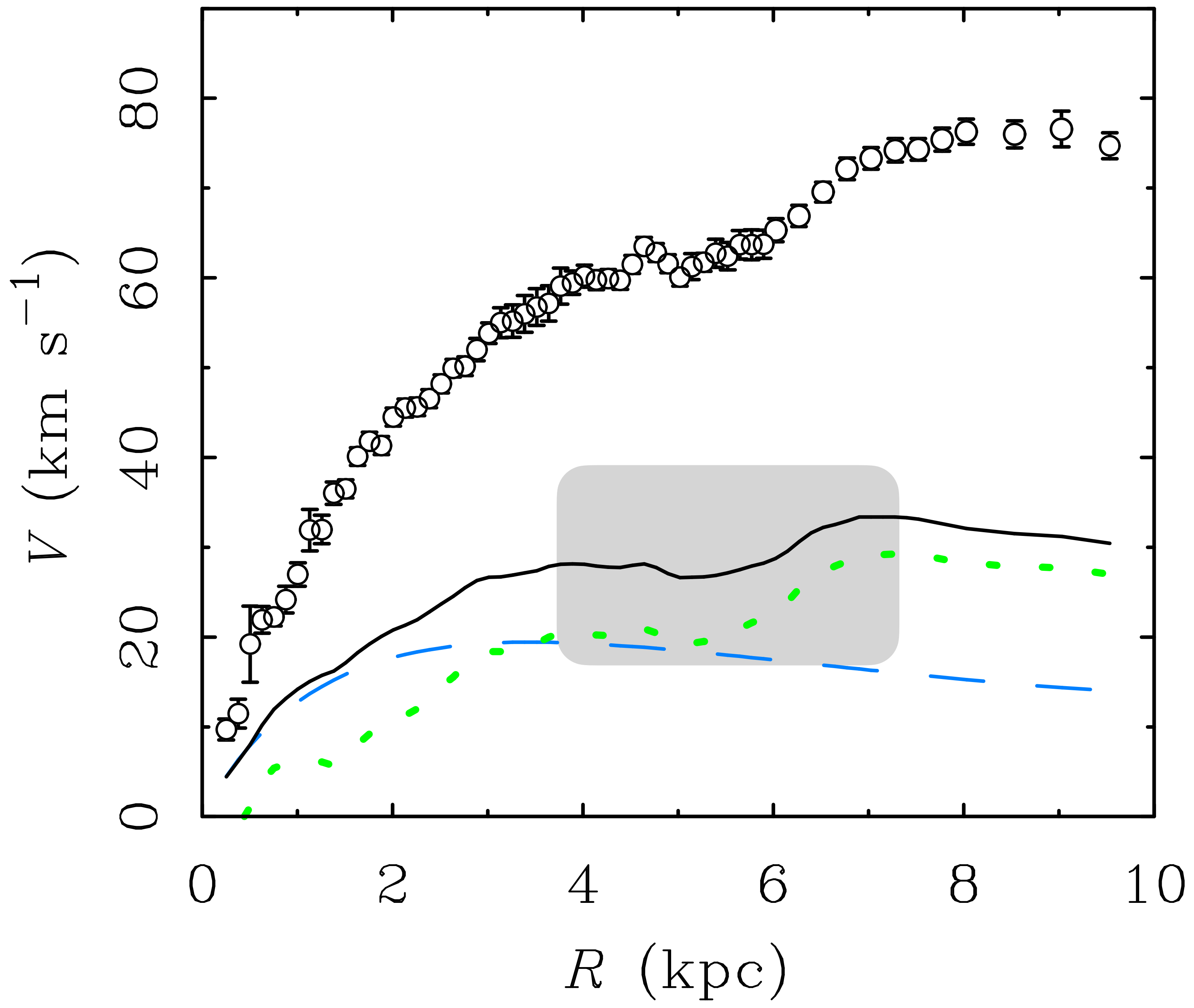
# Renzo's Rule:

*"When you see a feature in the light, you see a corresponding feature in the rotation curve."*




Gentile et al. (2010)




In NGC 1560, a marked feature in the gas is reflected in the kinematics, even though it accounts for little of the dynamical mass.

# Mass models for baryonic components

$$V_b^2(r) = V_{bulge}^2(r) + V_{disk}^2(r) + V_{gas}^2(r)$$

depends on M*/L

$$g_{\rm bar} = \frac{V_b^2}{R}$$

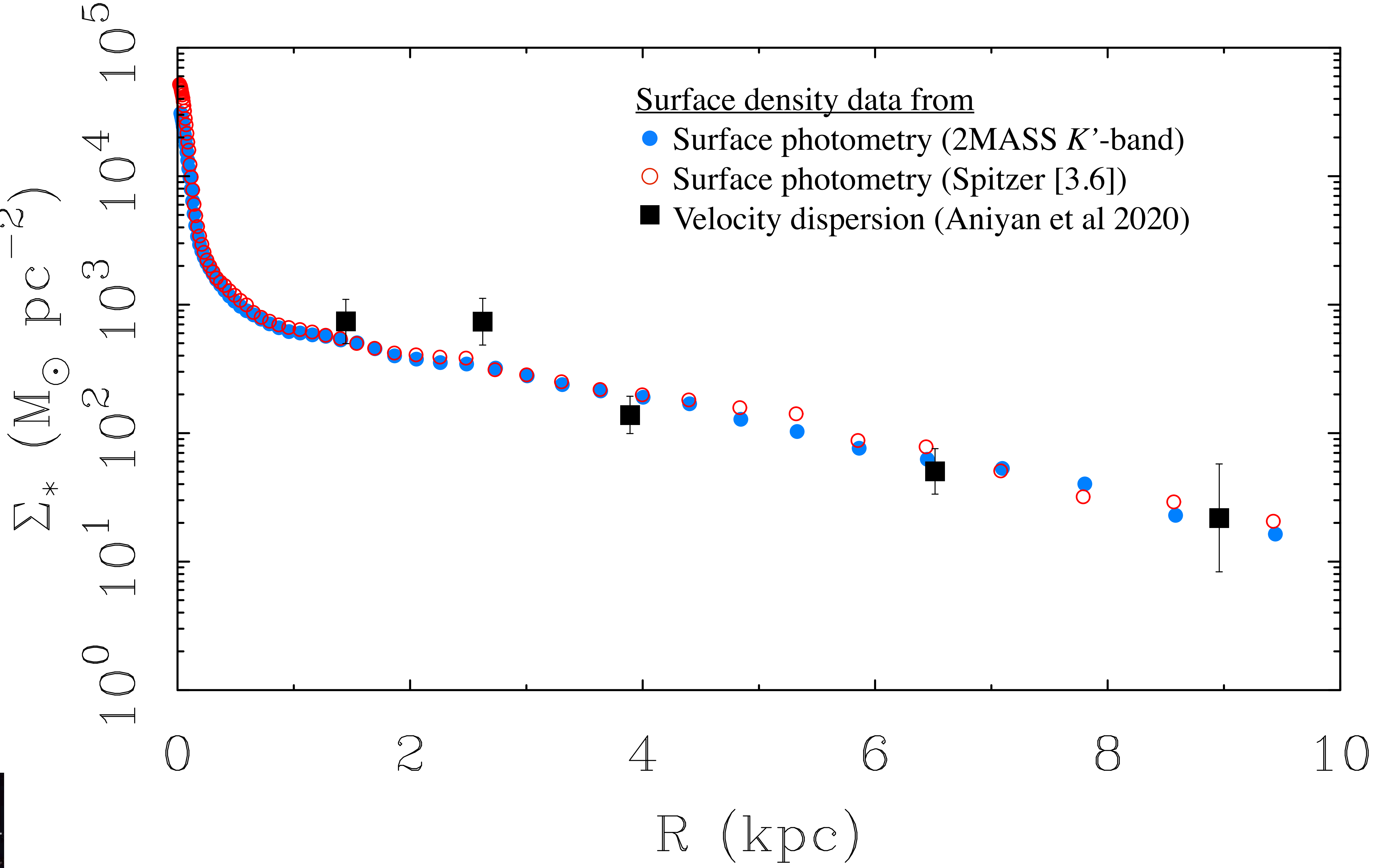
- Bulge
  - not always spherical; sometimes more bar-like
- Stellar Disk
  - exponential a crude approximation
  - in practice, solve numerically for the observed surface brightness profile with DISKFIT or ROTMOD (in GIPSY)
- Gas disk
  - usually just HI; CO tracks stars

# Now have

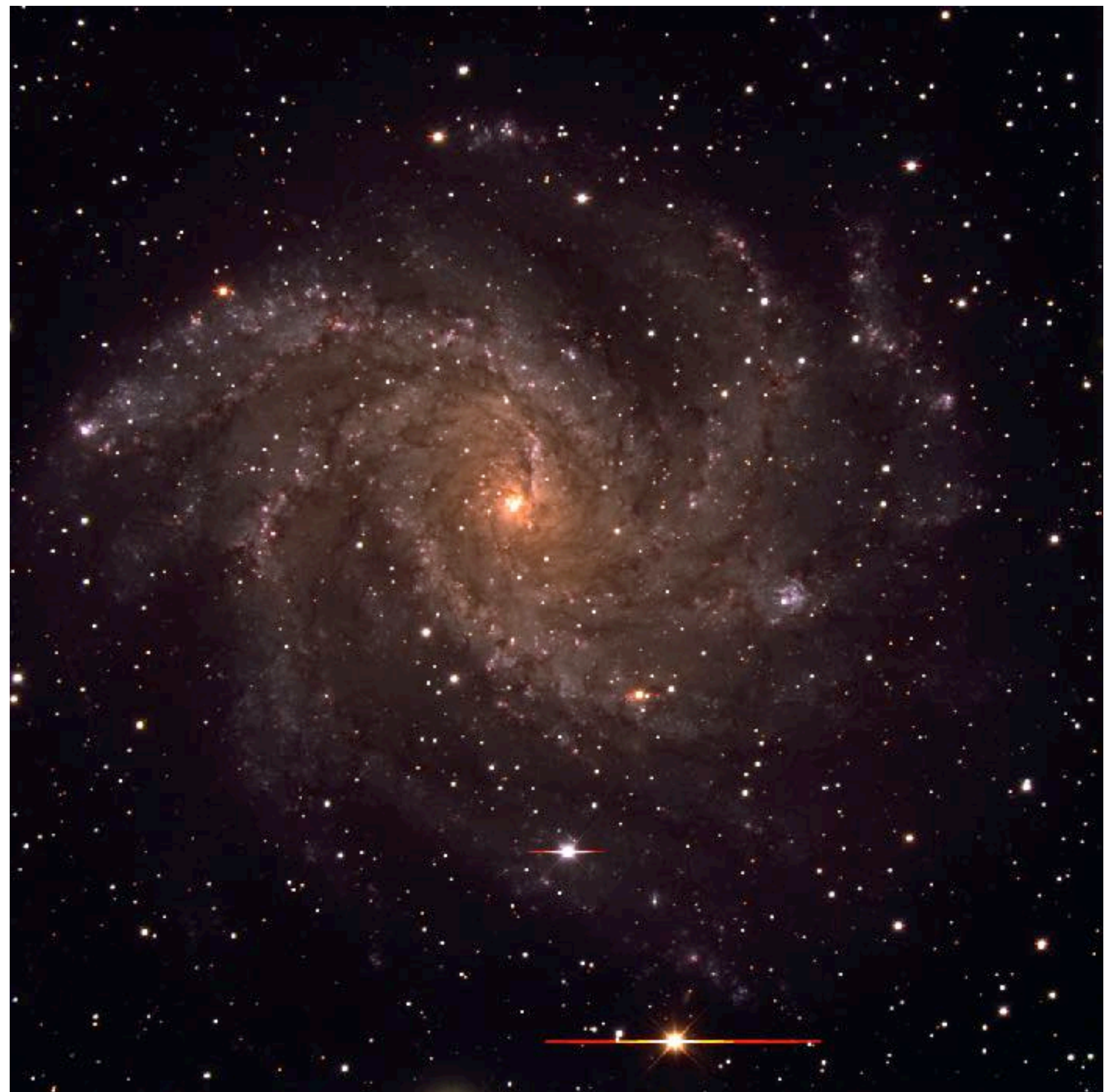
Surface density for
- stars
- gas

and corresponding rotation curves for each component

Observed rotation curve

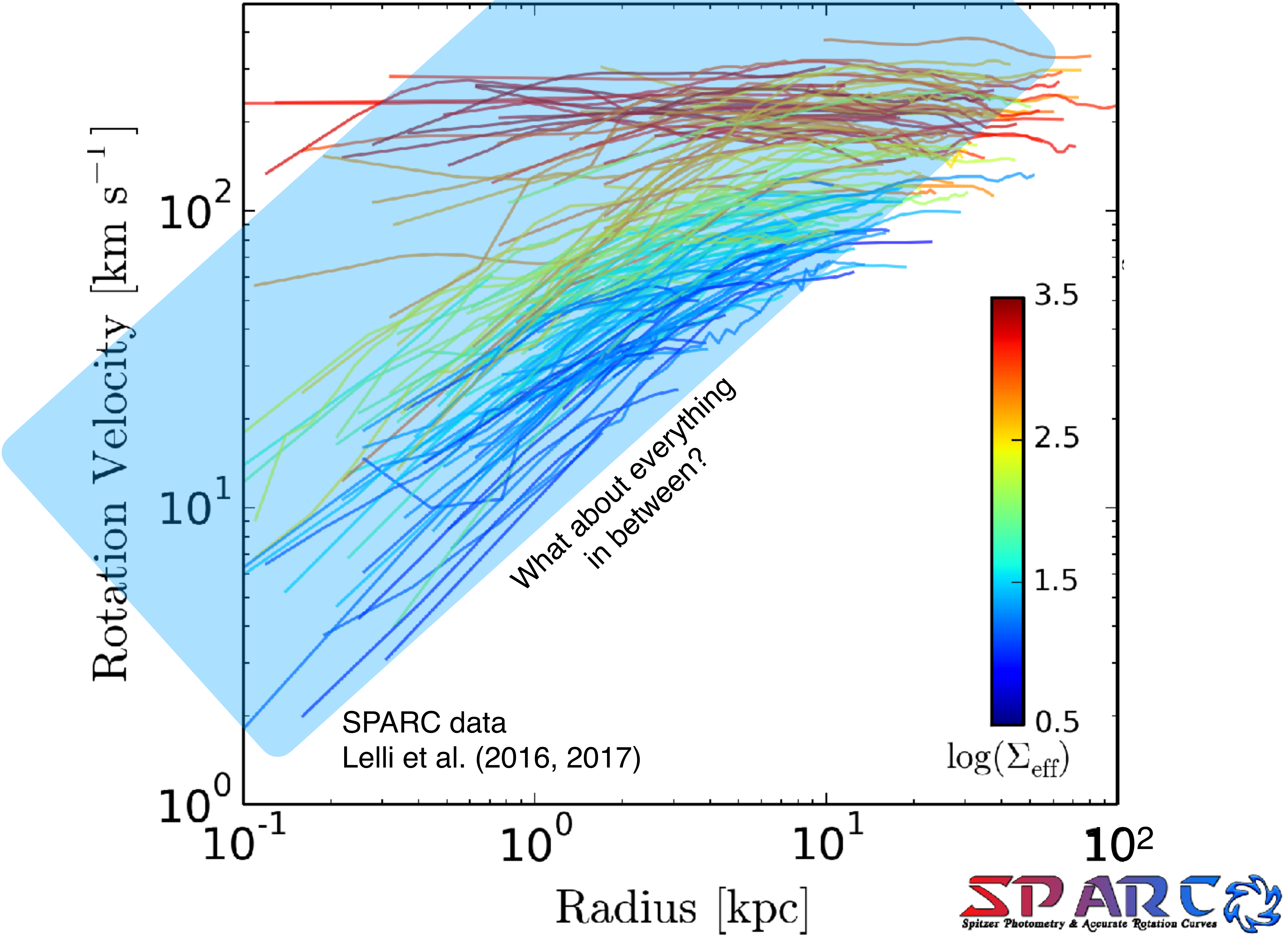



## NGC 6946

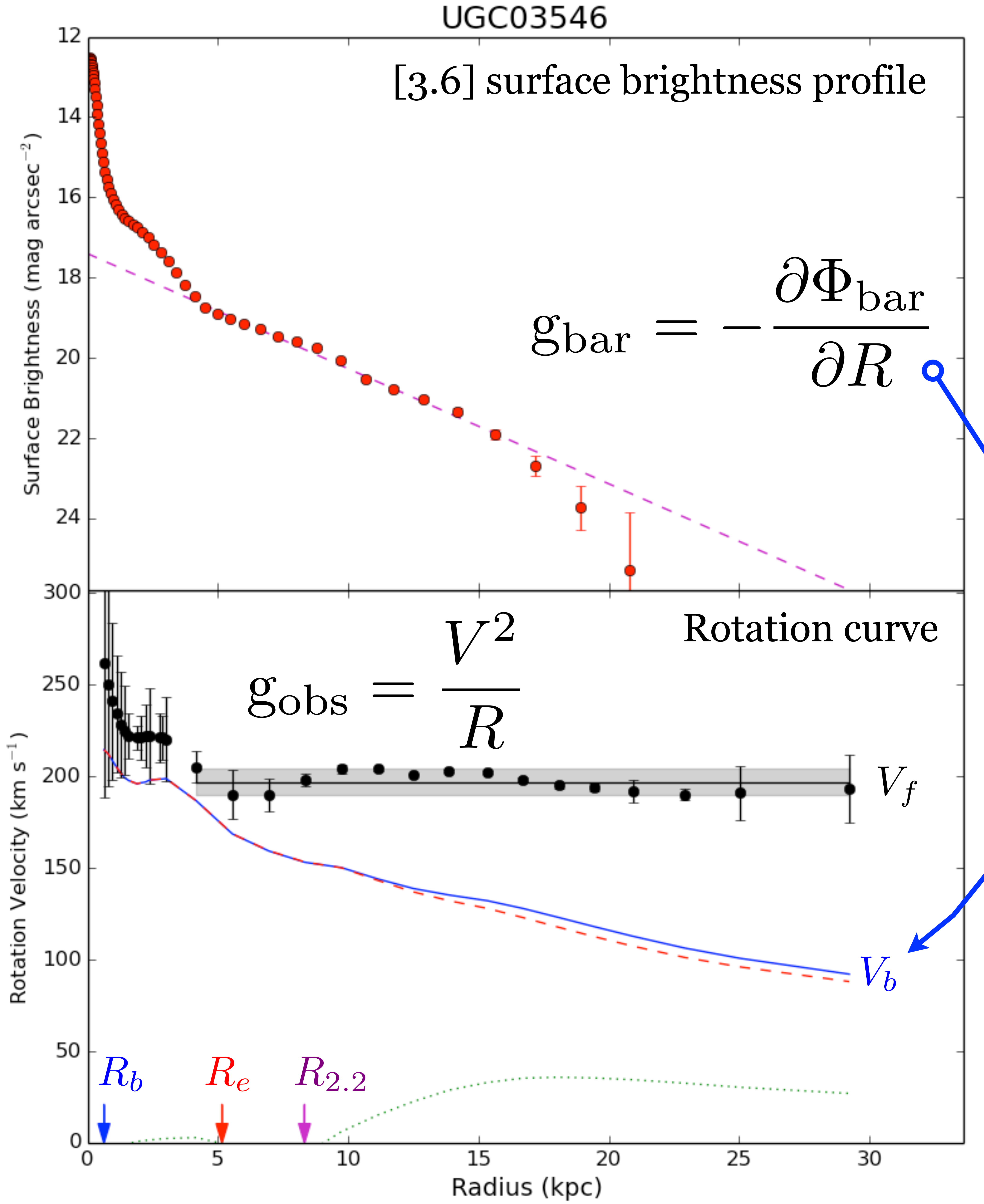


$$\sigma_z^2 \approx 2\pi G \Sigma h_z$$

# Rotation curve shape correlates with baryonic surface density

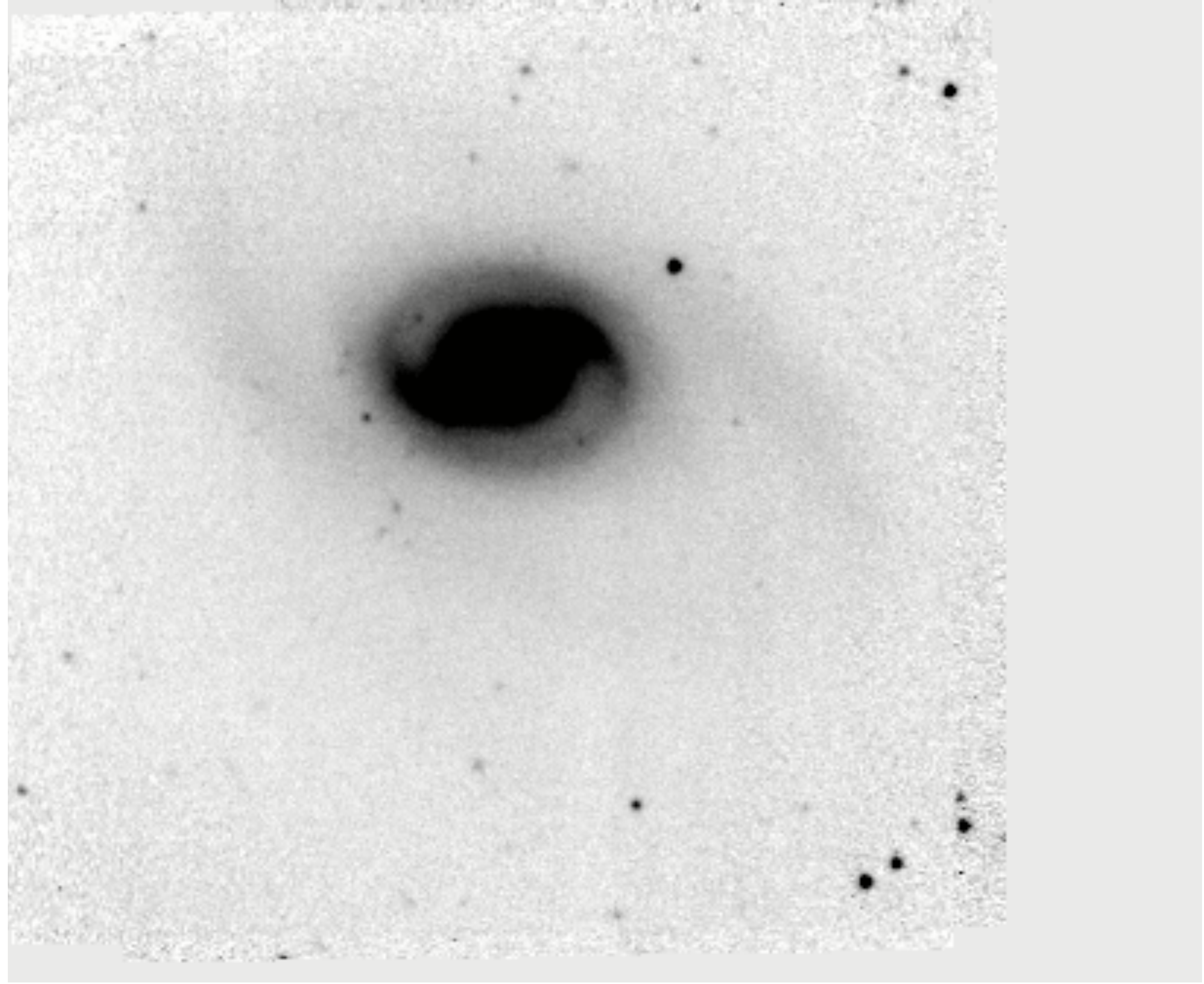


# What about everything in between?

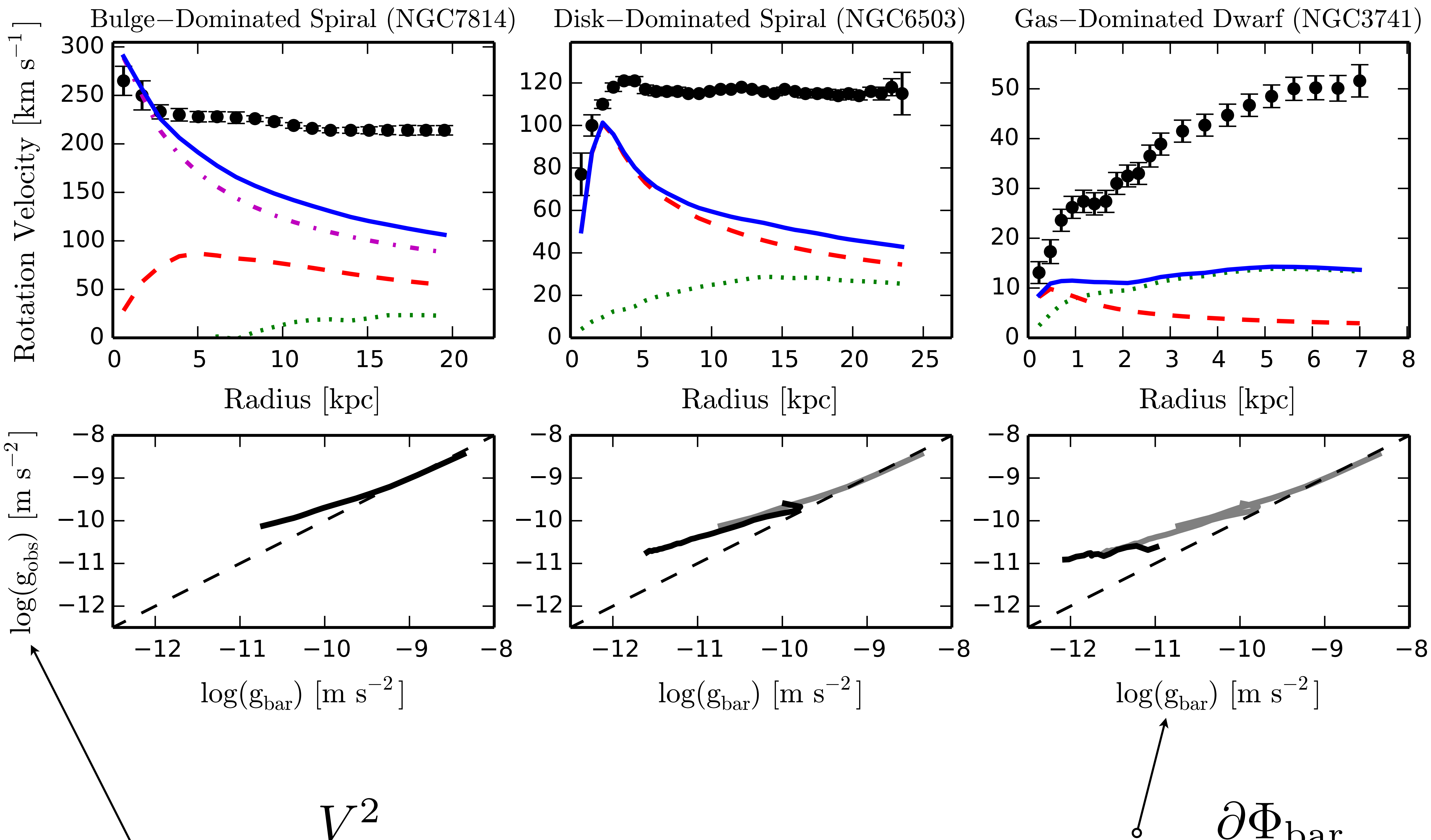



$$g_{\rm bar} = -\frac{\partial \Phi_{\rm bar}}{\partial R}$$

$$g_{\rm obs} = \frac{V^2}{R}$$



The observed centripetal acceleration is linked to that predicted by the observed distribution of baryons.




$$g_{\rm obs} = \frac{V^2}{R}$$

independent quantities

$$g_{\rm bar} = -\frac{\partial \Phi_{\rm bar}}{\partial R}$$

determined from rotation curve

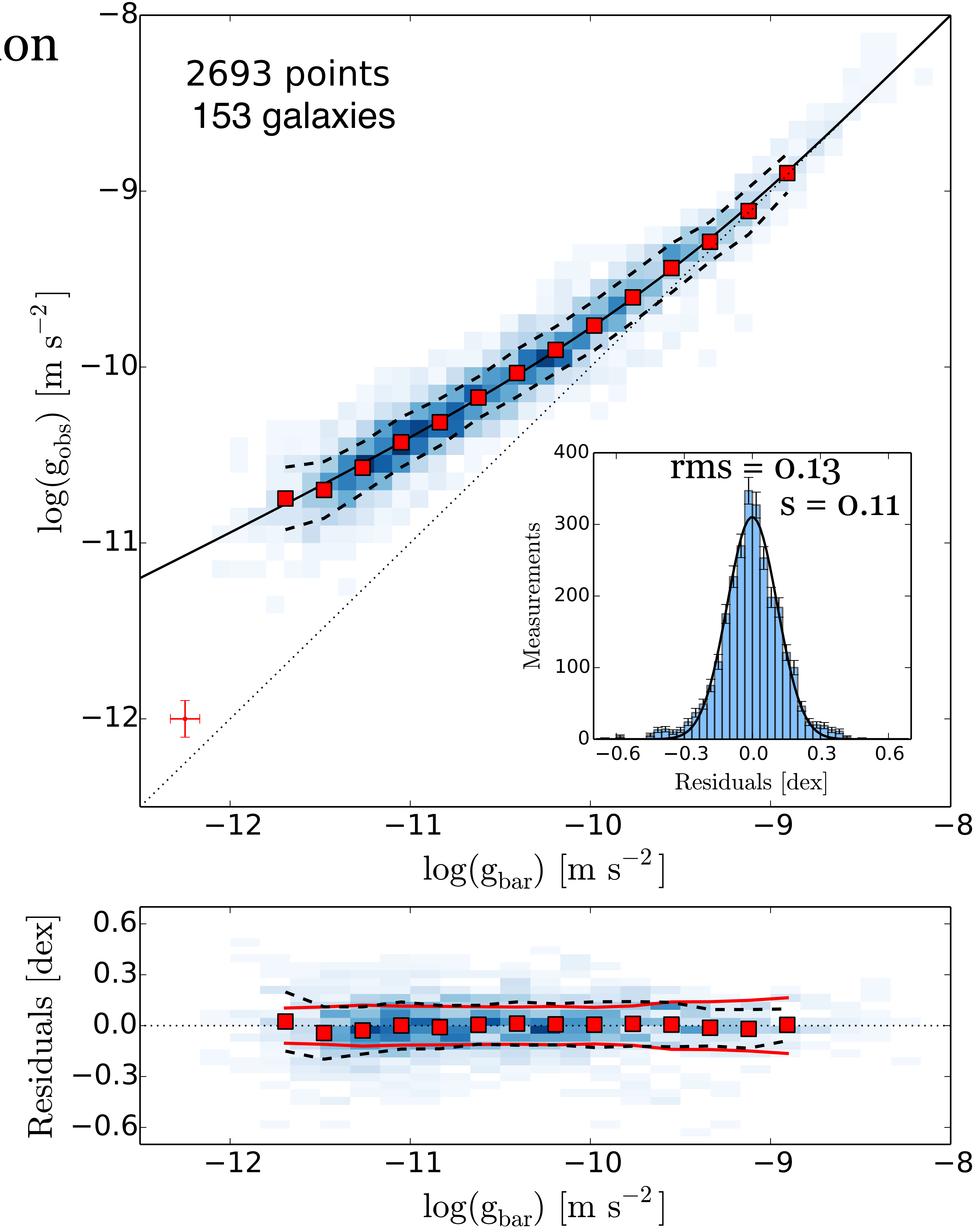
determined from baryon distribution

# Radial Acceleration Relation

# (RAR)

Constructed from 153 galaxies with 21cm rotation curves and near-IR surface photometry from the *Spitzer* space telescope.

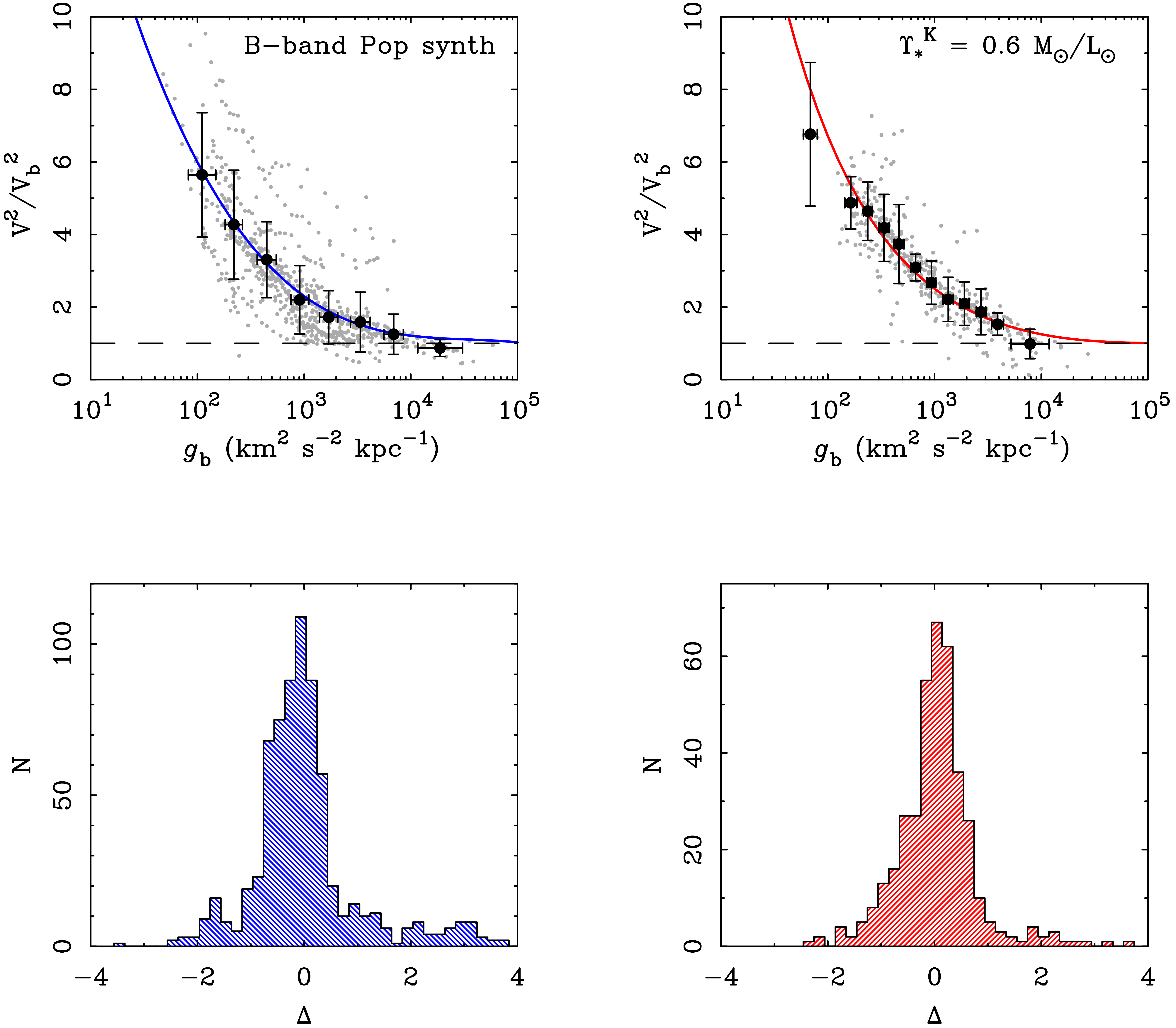
Apparently the mass-to-light ratio in the near-IR is close to constant: individual galaxies do not stand out in this relation.



**MDAR**

$$\mathcal{D} = \frac{g_{\rm obs}}{g_{\rm bar}} = \frac{V^2}{V_b^2}$$

The Radial Acceleration Relation is equivalent to the Mass Discrepancy-acceleration relation, just with independent x & y axes.



# Radial Acceleration Relation

The observed acceleration correlates with that predicted by the baryons

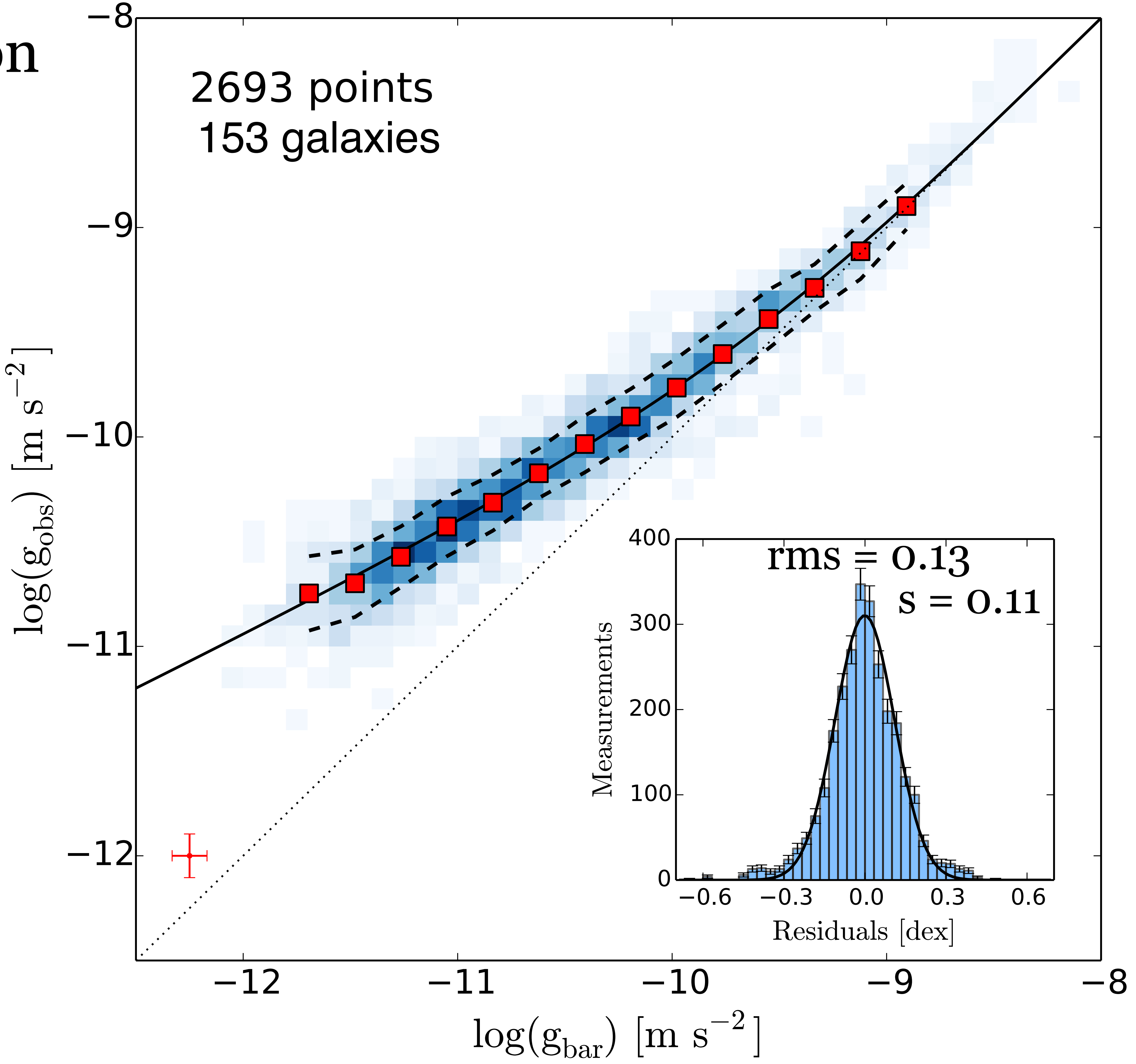
The data are well fit by

$$g_{obs} = \frac{g_{bar}}{1 - e^{-\sqrt{g_{bar}/g_{\dagger}}}}$$

$$g_{\dagger} = 1.20 \times 10^{-10} \ \mathrm{m\,s^{-2}}$$

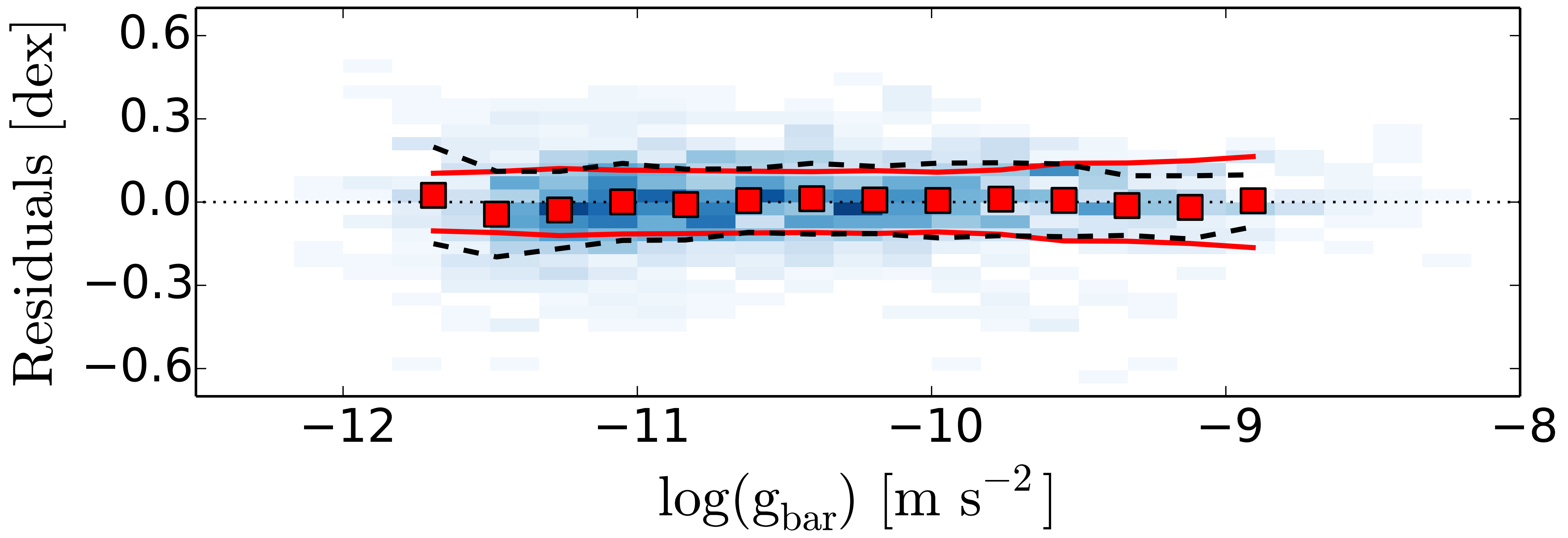
$\pm 0.02$ (random) $\pm\, 0.24$ (systematic)

Lelli et al. (2017)
McGaugh et al. (2016)




observed rms scatter

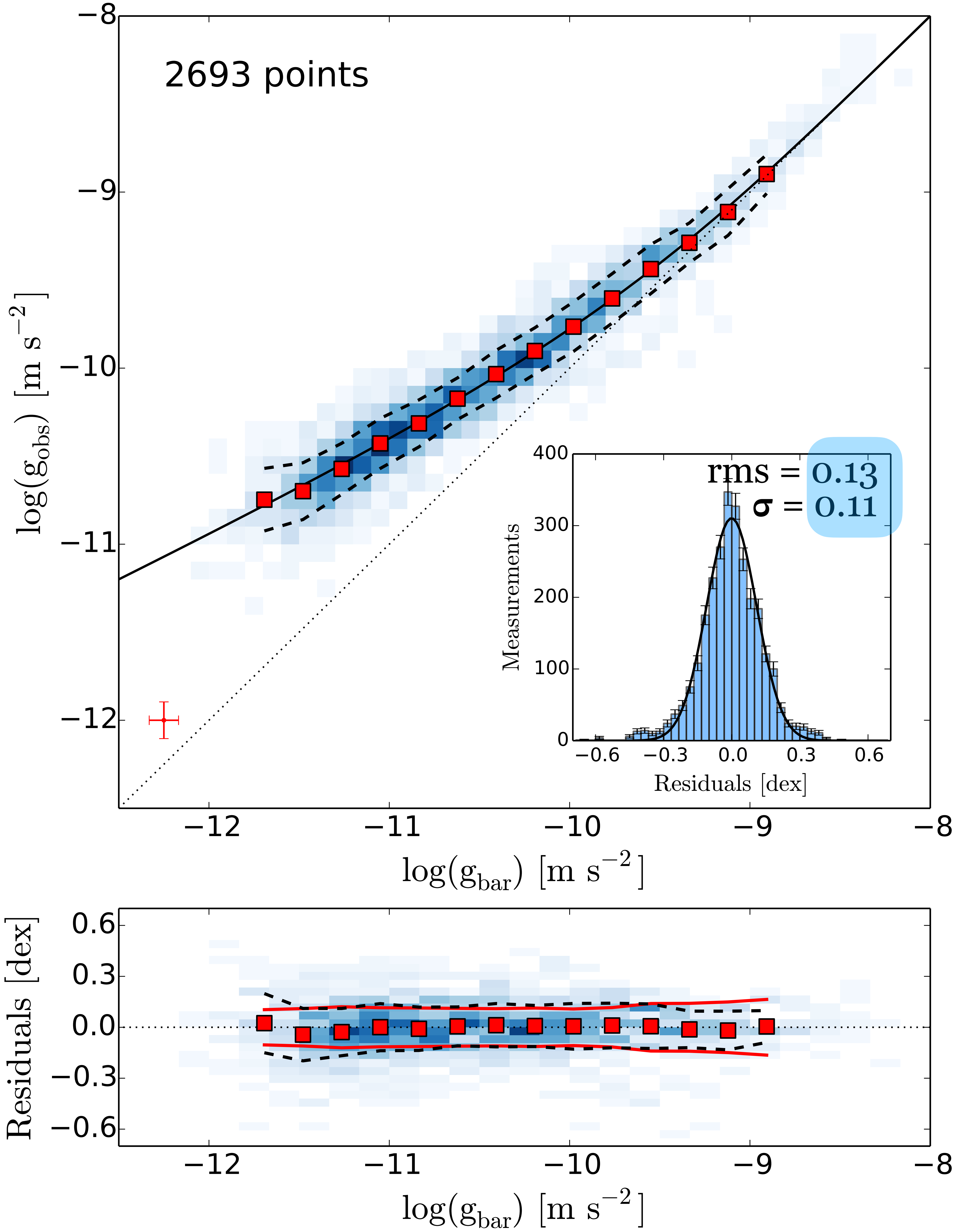
scatter expected from observational errors




The data are consistent with zero intrinsic scatter

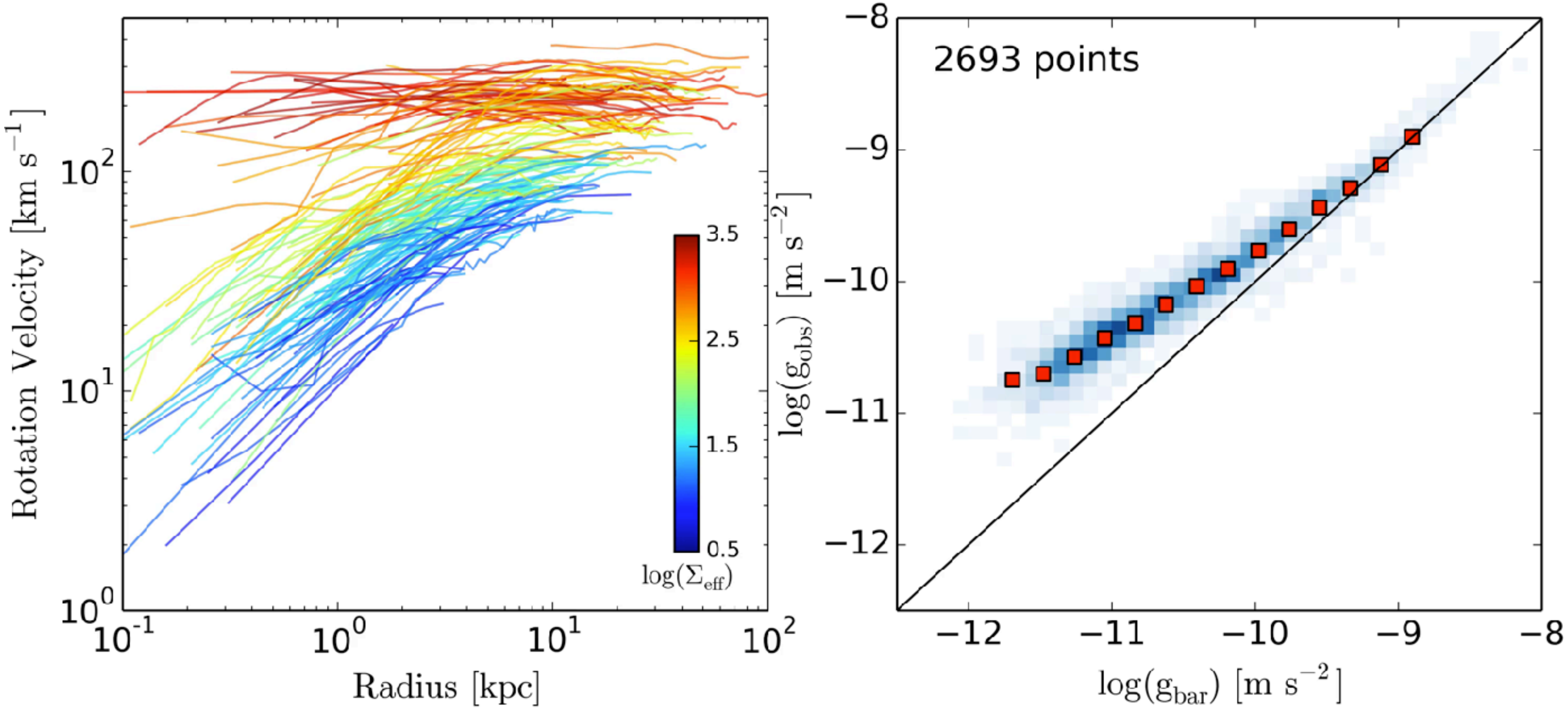
TABLE I. Scatter Budget for Acceleration Residuals

| Source | Residual |
| --- | --- |
| Rotation velocity errors | 0.03 dex |
| Disk inclination errors | 0.05 dex |
| Galaxy distance errors | 0.08 dex |
| Variation in mass-to-light ratios | 0.06 dex |
| HI flux calibration errors | 0.01 dex |
| Total | 0.12 dex |



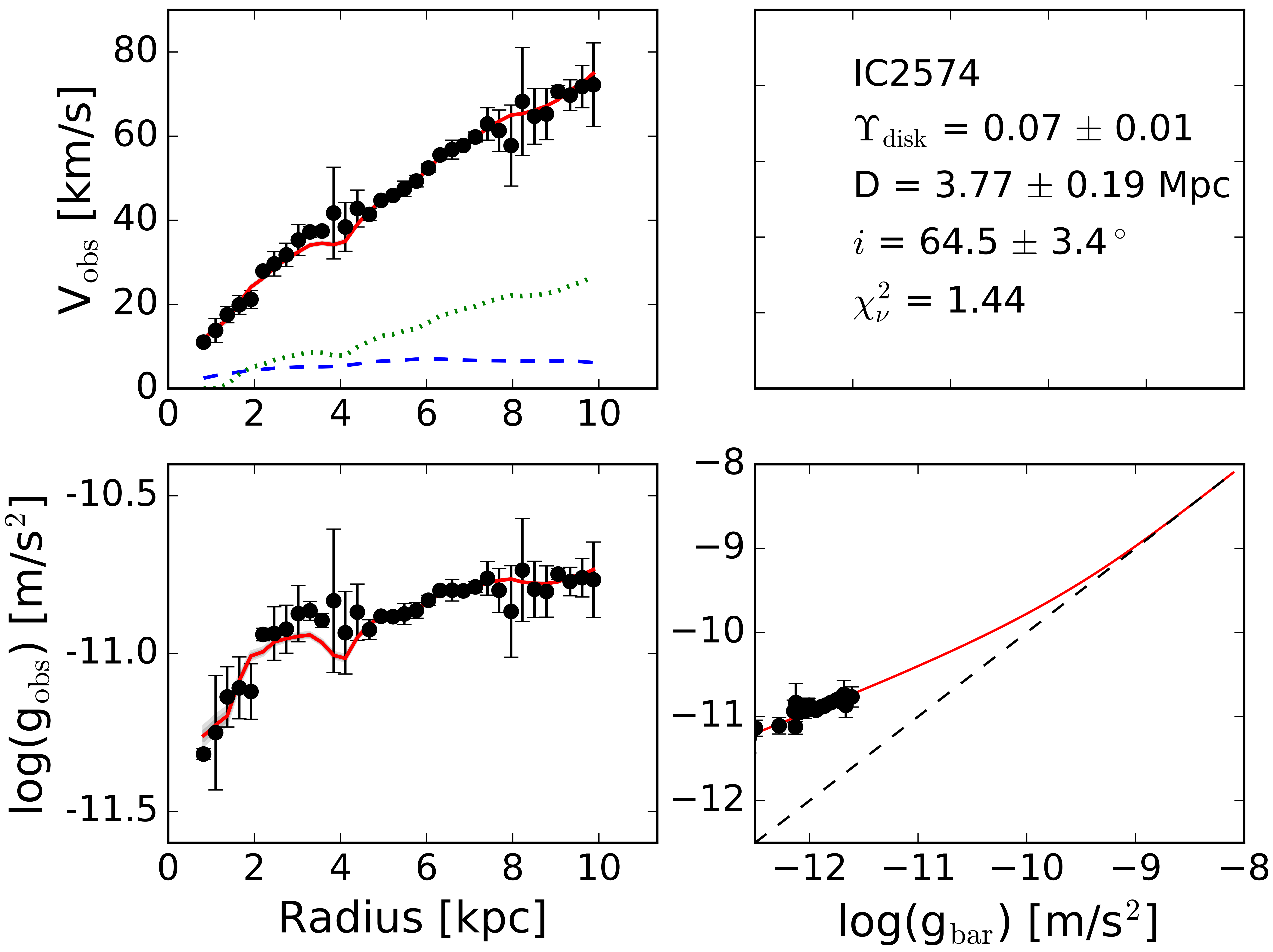

The observed scatter is consistent with that expected from known uncertainties: the radial acceleration relation is consistent has negligible intrinsic scatter.

Lelli et al. 2017, ApJ, 836, 152 (arXiv:1610.08981)
McGaugh et al. 2016, PRL, 117, 201101 (arXiv:1609.05917)

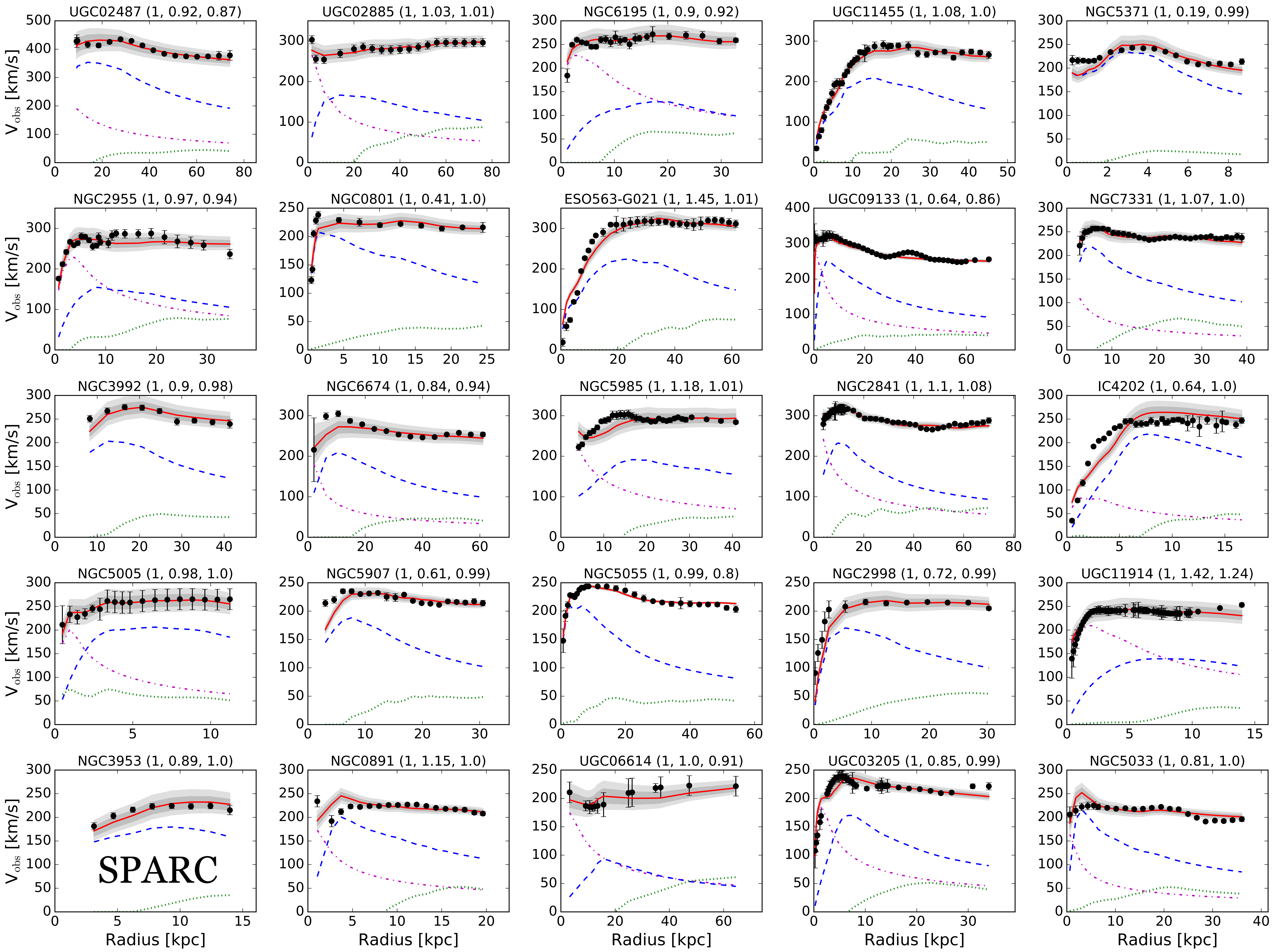


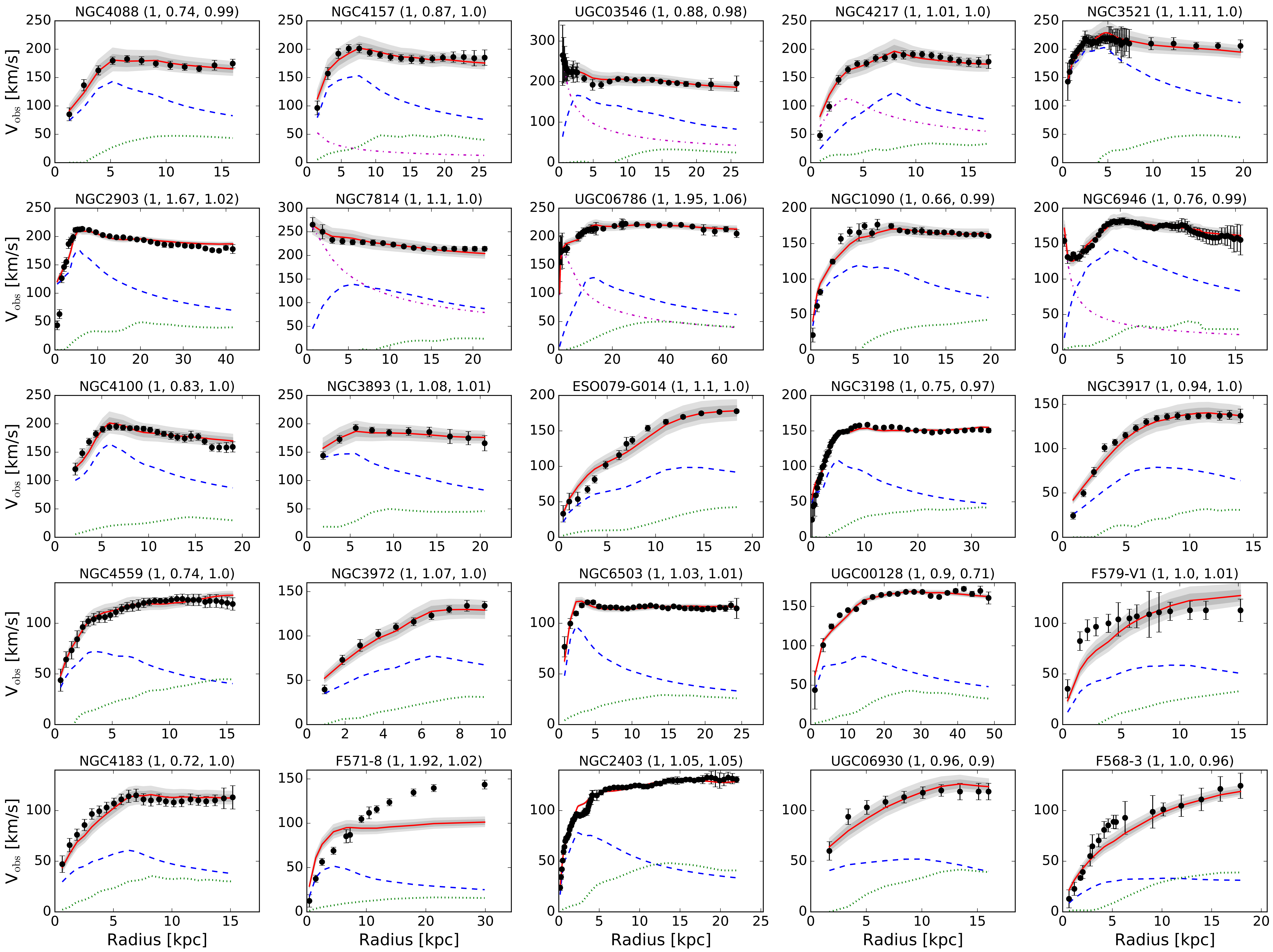

**http://astroweb.case.edu/SPARC/RARmovie.mp4**

That just assumed constant M*/L. We can fit to the mean RAR, marginalizing over distance and inclination as nuisance parameters (Li et al. 2018)




Li et al. 2018, A&A, 615, A3 (arXiv:1803.00022)


UGC02487 (1, 0.92, 0.87)
UGC02885 (1, 1.03, 1.01)
NGC6195 (1, 0.9, 0.92)
UGC11455 (1, 1.08, 1.0)
NGC5371 (1, 0.19, 0.99)
NGC2955 (1, 0.97, 0.94)
NGC0801 (1, 0.41, 1.0)
ESO563-G021 (1, 1.45, 1.01)
UGC09133 (1, 0.64, 0.86)
NGC7331 (1, 1.07, 1.0)
NGC3992 (1, 0.9, 0.98)
NGC6674 (1, 0.84, 0.94)
NGC5985 (1, 1.18, 1.01)
NGC2841 (1, 1.1, 1.08)
IC4202 (1, 0.64, 1.0)
NGC5005 (1, 0.98, 1.0)
NGC5907 (1, 0.61, 0.99)
NGC5055 (1, 0.99, 0.8)
NGC2998 (1, 0.72, 0.99)
UGC11914 (1, 1.42, 1.24)
NGC3953 (1, 0.89, 1.0)
NGC0891 (1, 1.15, 1.0)
UGC06614 (1, 1.0, 0.91)
UGC03205 (1, 0.85, 0.99)
NGC5033 (1, 0.81, 1.0)
$V_{obs}$ [km/s]
Radius [kpc]
SPARC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NGC4217 (1, 1.01, 1.0)
NGC3521 (1, 1.11, 1.0)
NGC2903 (1, 1.67, 1.02)
NGC7814 (1, 1.1, 1.0)
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NGC1090 (1, 0.66, 0.99)
NGC6946 (1, 0.76, 0.99)
NGC4100 (1, 0.83, 1.0)
NGC3893 (1, 1.08, 1.01)
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NGC3198 (1, 0.75, 0.97)
NGC3917 (1, 0.94, 1.0)
NGC4559 (1, 0.74, 1.0)
NGC3972 (1, 1.07, 1.0)
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NGC4183 (1, 0.72, 1.0)
F571-8 (1, 1.92, 1.02)
NGC2403 (1, 1.05, 1.05)
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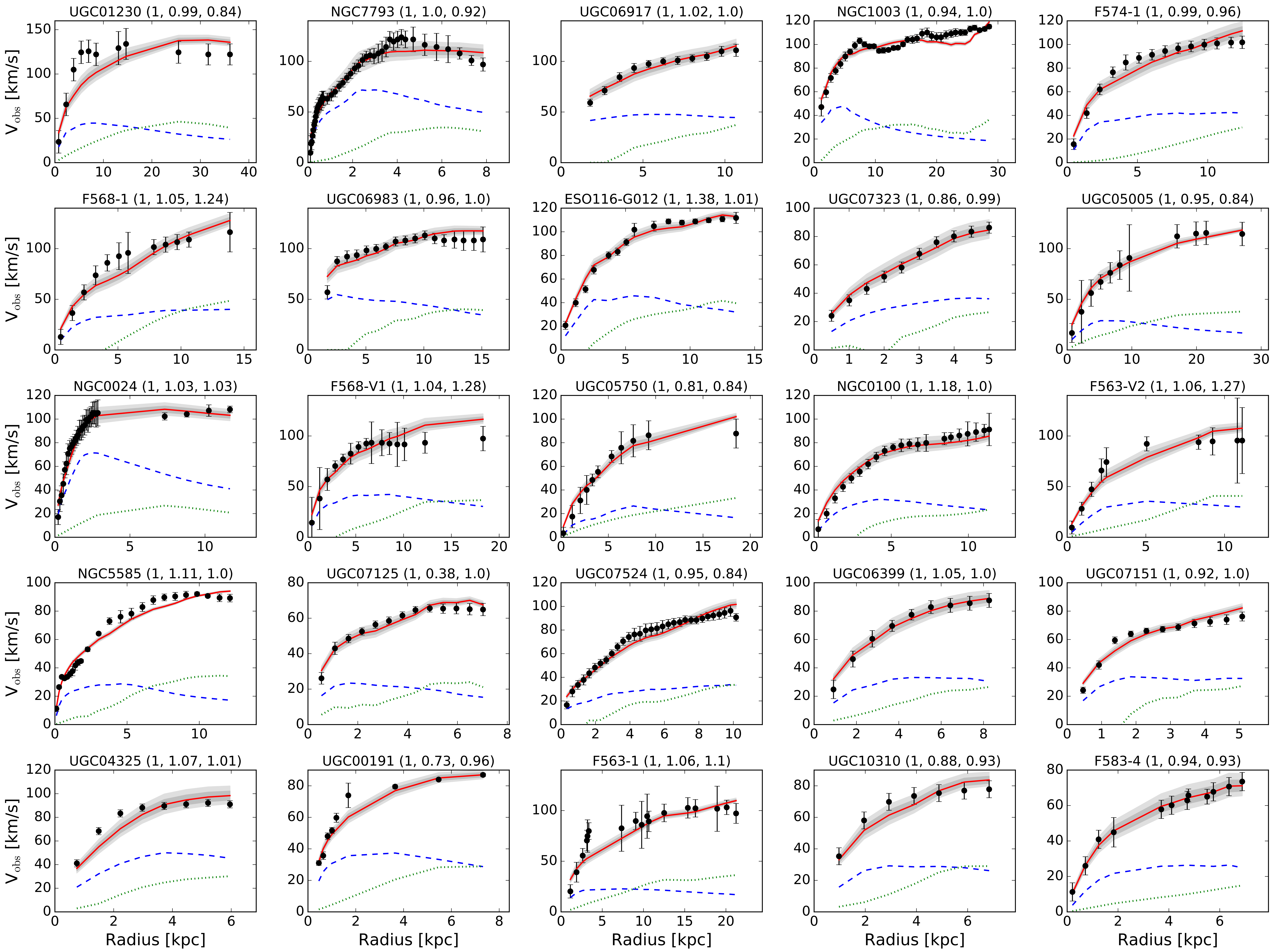
F568-3 (1, 1.0, 0.96)
$V_{obs}$ [km/s]
Radius [kpc]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UGC06917 (1, 1.02, 1.0)
NGC1003 (1, 0.94, 1.0)
F574-1 (1, 0.99, 0.96)
F568-1 (1, 1.05, 1.24)
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UGC05750 (1, 0.81, 0.84)
NGC0100 (1, 1.18, 1.0)
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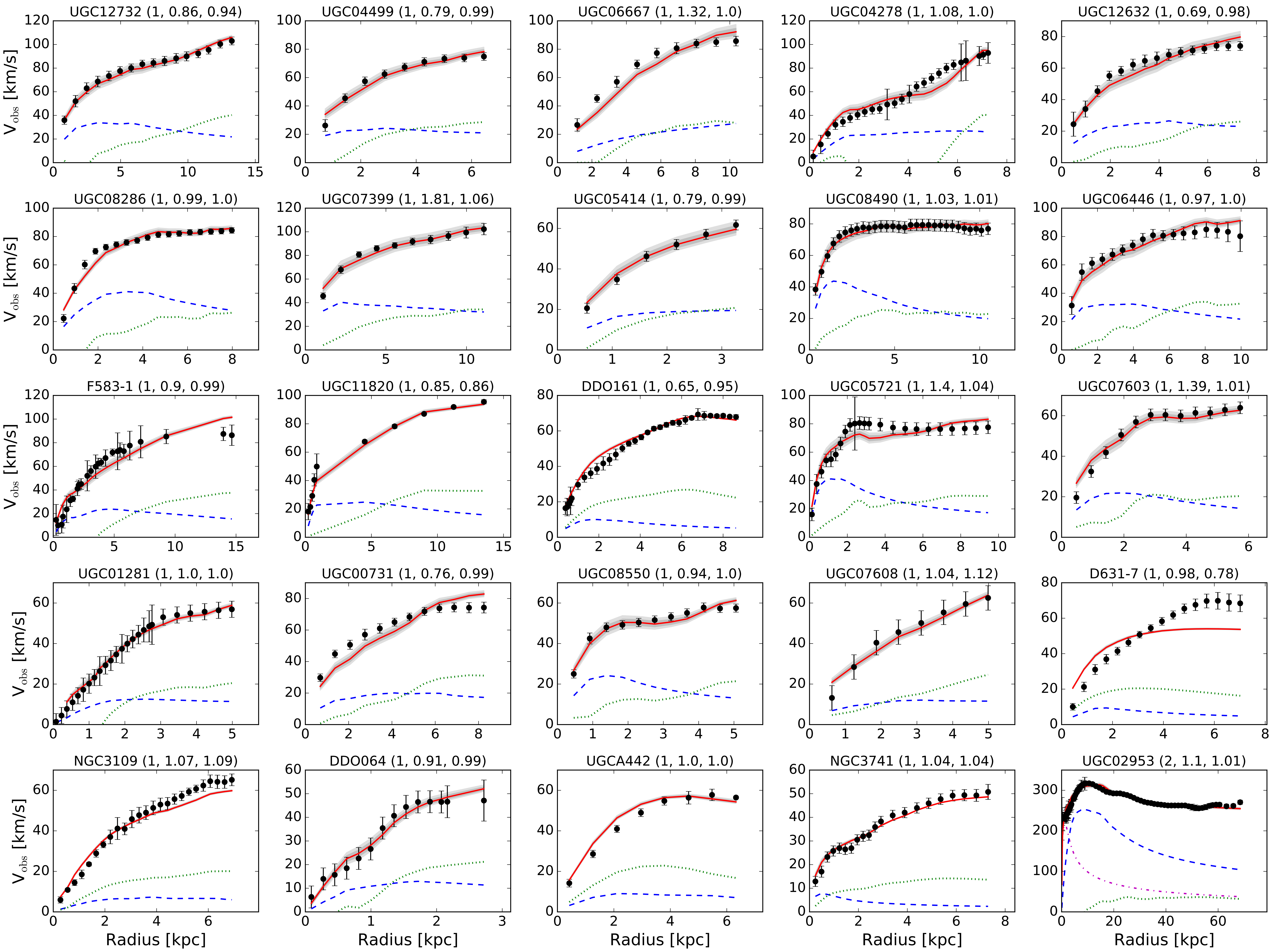
$V_{obs}$ [km/s]
Radius [kpc]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UGC05414 (1, 0.79, 0.99)
UGC08490 (1, 1.03, 1.01)
UGC06446 (1, 0.97, 1.0)
F583-1 (1, 0.9, 0.99)
UGC11820 (1, 0.85, 0.86)
DDO161 (1, 0.65, 0.95)
UGC05721 (1, 1.4, 1.04)
UGC07603 (1, 1.39, 1.01)
UGC01281 (1, 1.0, 1.0)
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UGC07608 (1, 1.04, 1.12)
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NGC3109 (1, 1.07, 1.09)
DDO064 (1, 0.91, 0.99)
UGCA442 (1, 1.0, 1.0)
NGC3741 (1, 1.04, 1.04)
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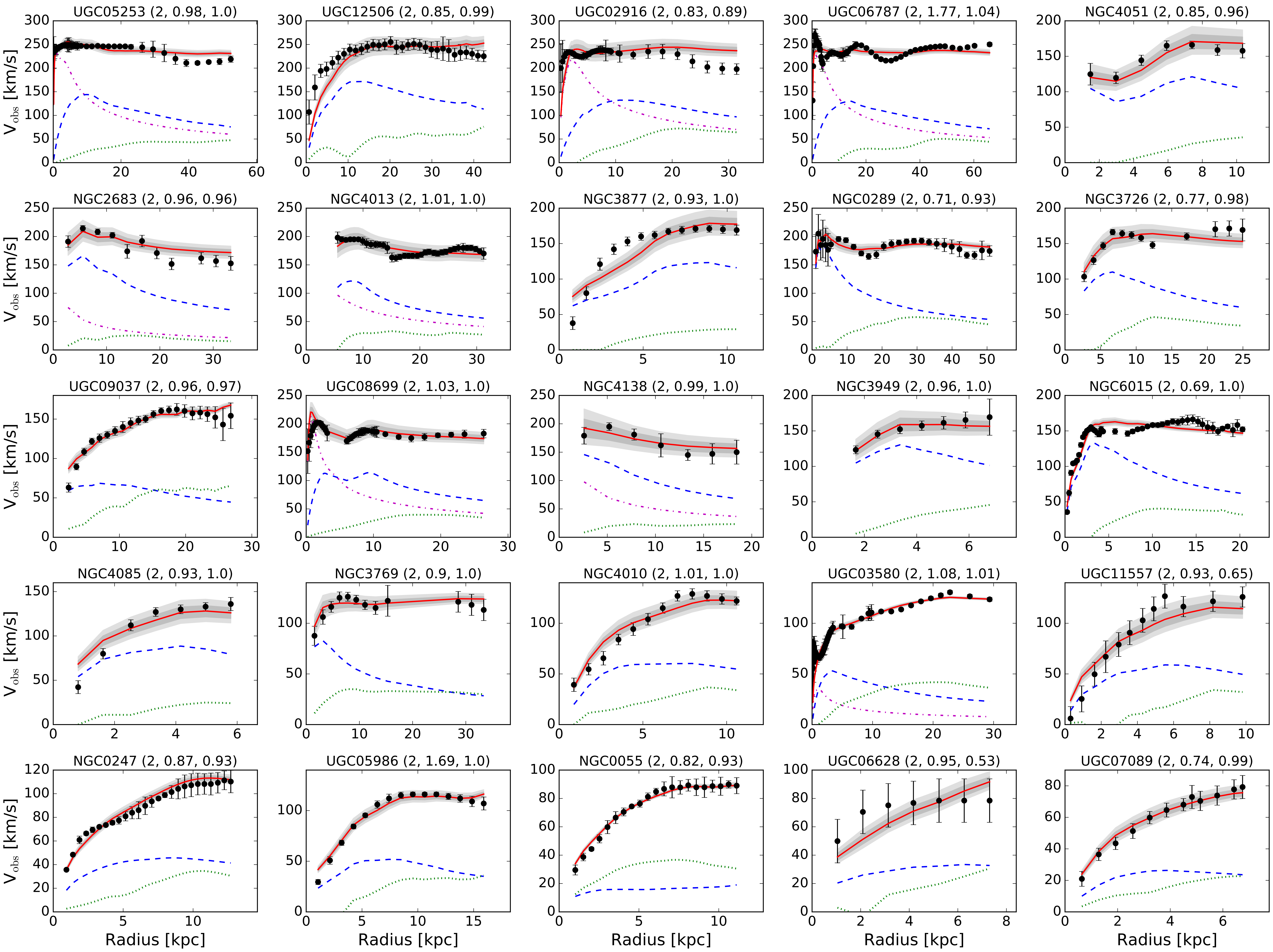
$V_{obs}$ [km/s]
Radius [kpc]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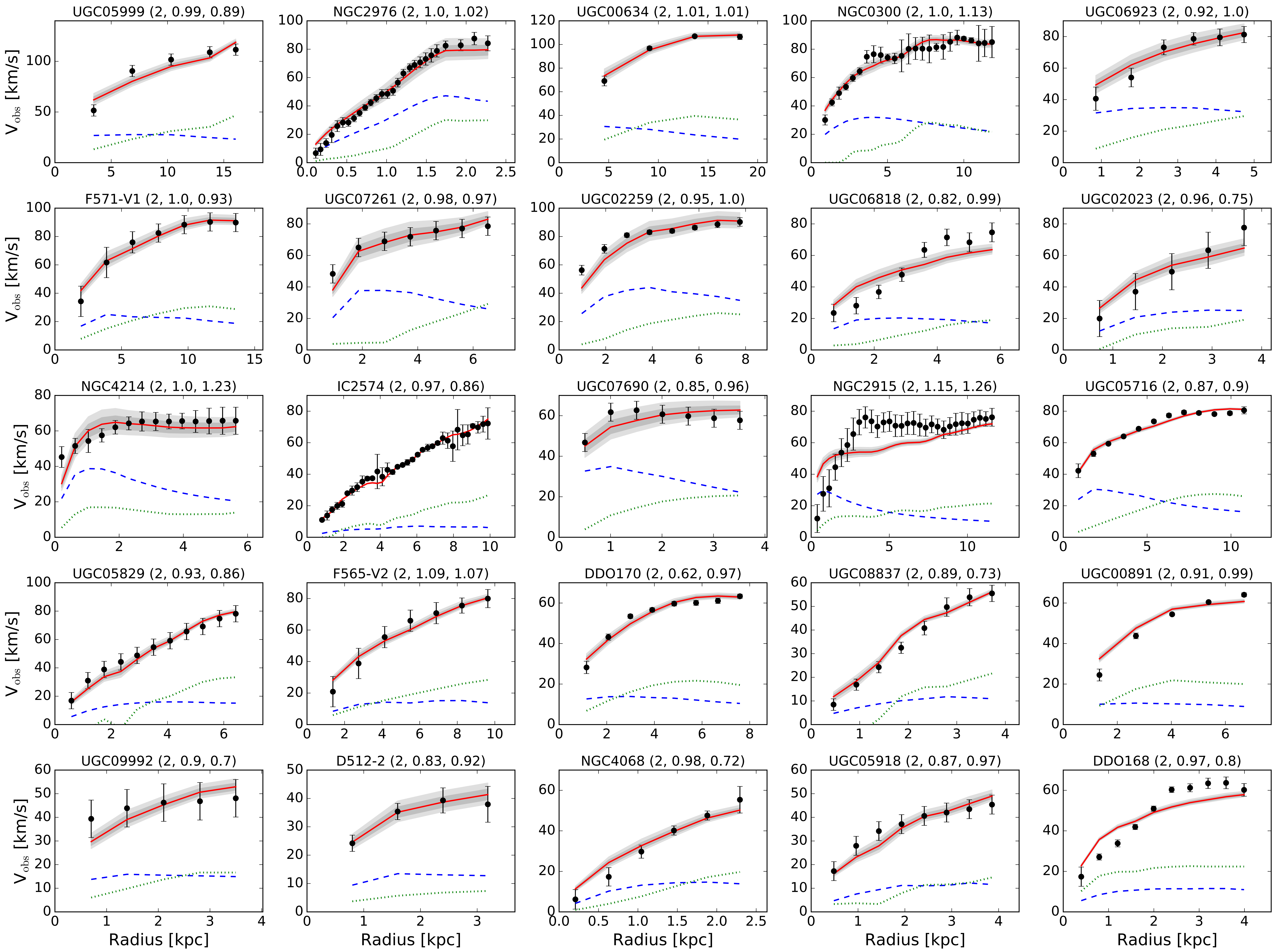
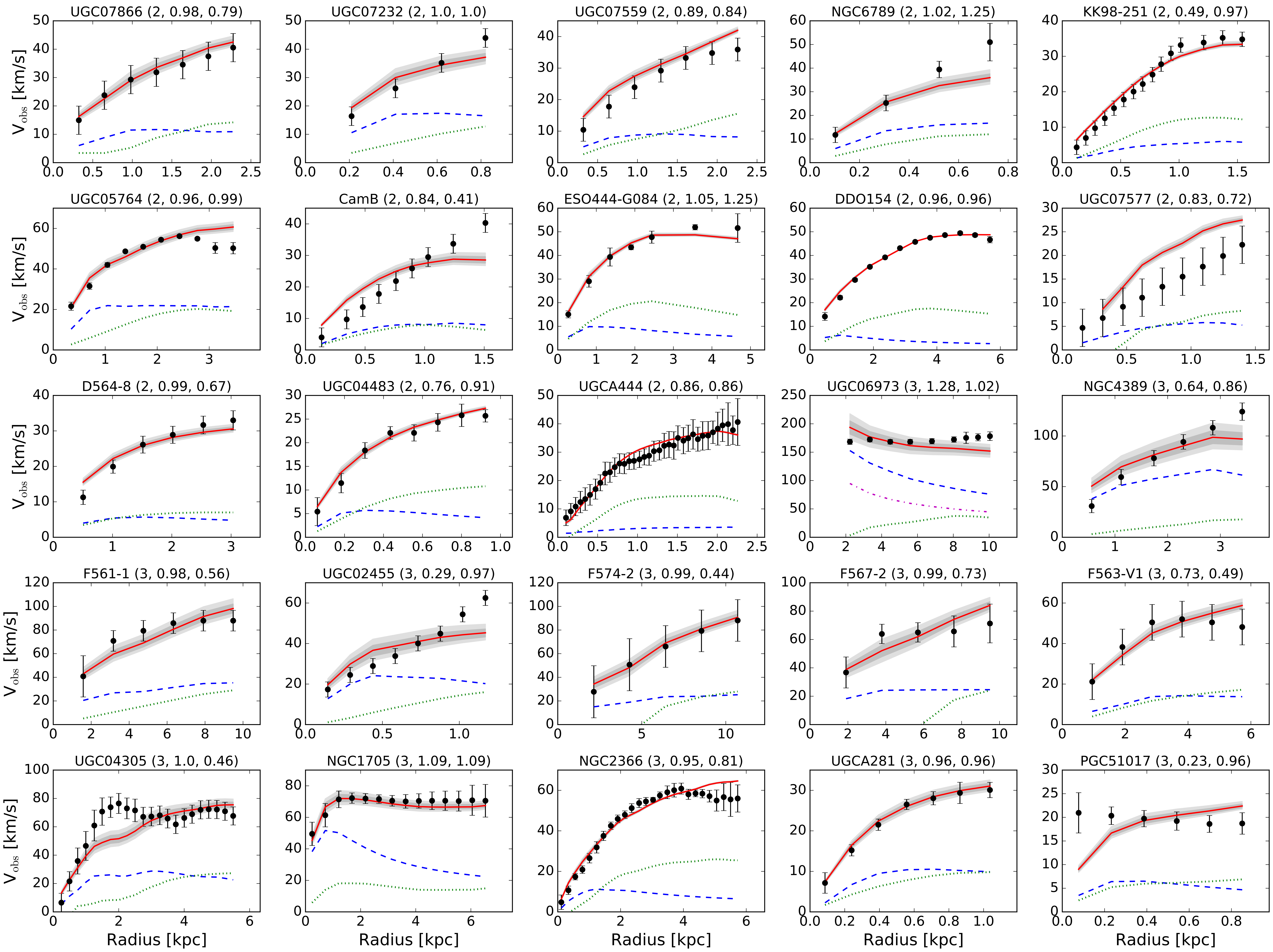
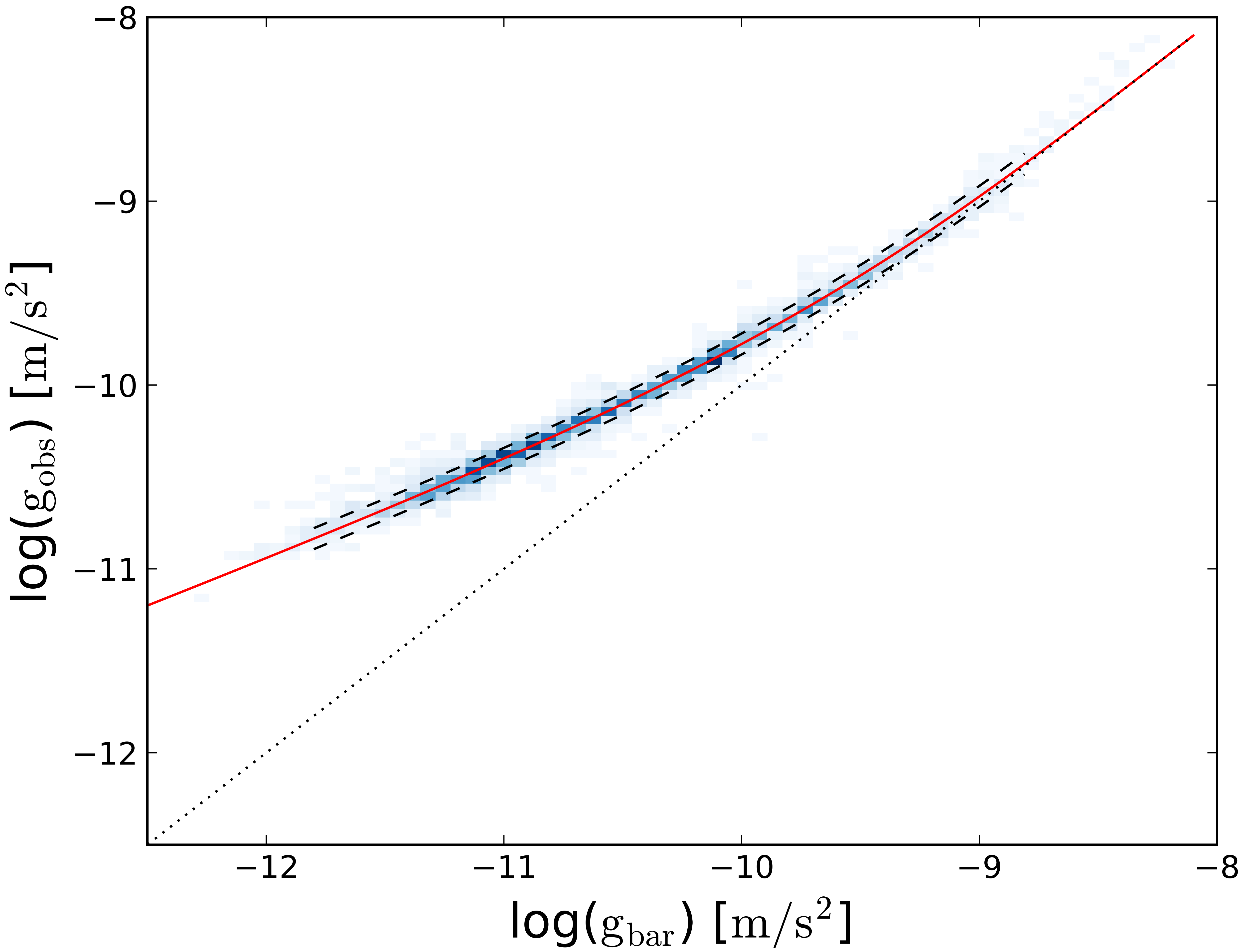
UGC02916 (2, 0.83, 0.89)
UGC06787 (2, 1.77, 1.04)
NGC4051 (2, 0.85, 0.96)
NGC2683 (2, 0.96, 0.96)
NGC4013 (2, 1.01, 1.0)
NGC3877 (2, 0.93, 1.0)
NGC0289 (2, 0.71, 0.93)
NGC3726 (2, 0.77, 0.98)
UGC09037 (2, 0.96, 0.97)
UGC08699 (2, 1.03, 1.0)
NGC4138 (2, 0.99, 1.0)
NGC3949 (2, 0.96, 1.0)
NGC6015 (2, 0.69, 1.0)
NGC4085 (2, 0.93, 1.0)
NGC3769 (2, 0.9, 1.0)
NGC4010 (2, 1.01, 1.0)
UGC03580 (2, 1.08, 1.01)
UGC11557 (2, 0.93, 0.65)
NGC0247 (2, 0.87, 0.93)
UGC05986 (2, 1.69, 1.0)
NGC0055 (2, 0.82, 0.93)
UGC06628 (2, 0.95, 0.53)
UGC07089 (2, 0.74, 0.99)
$V_{obs}$ [km/s]
Radius [kpc]


UGC05999 (2, 0.99, 0.89)
NGC2976 (2, 1.0, 1.02)
UGC00634 (2, 1.01, 1.01)
NGC0300 (2, 1.0, 1.13)
UGC06923 (2, 0.92, 1.0)
F571-V1 (2, 1.0, 0.93)
UGC07261 (2, 0.98, 0.97)
UGC02259 (2, 0.95, 1.0)
UGC06818 (2, 0.82, 0.99)
UGC02023 (2, 0.96, 0.75)
NGC4214 (2, 1.0, 1.23)
IC2574 (2, 0.97, 0.86)
UGC07690 (2, 0.85, 0.96)
NGC2915 (2, 1.15, 1.26)
UGC05716 (2, 0.87, 0.9)
UGC05829 (2, 0.93, 0.86)
F565-V2 (2, 1.09, 1.07)
DDO170 (2, 0.62, 0.97)
UGC08837 (2, 0.89, 0.73)
UGC00891 (2, 0.91, 0.99)
UGC09992 (2, 0.9, 0.7)
D512-2 (2, 0.83, 0.92)
NGC4068 (2, 0.98, 0.72)
UGC05918 (2, 0.87, 0.97)
DDO168 (2, 0.97, 0.8)
V_obs [km/s]
Radius [kpc]

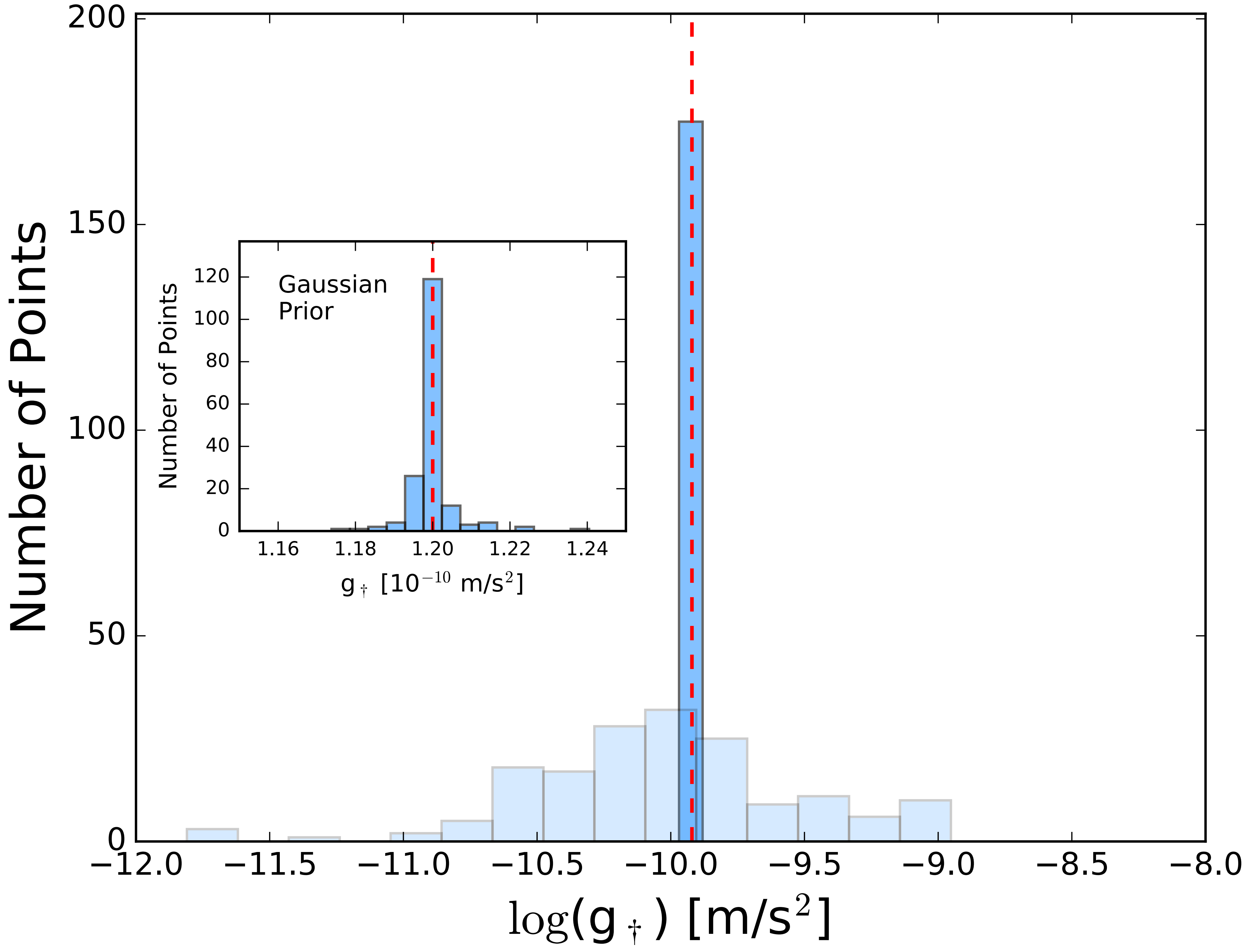


UGC07866 (2, 0.98, 0.79)
UGC07232 (2, 1.0, 1.0)
UGC07559 (2, 0.89, 0.84)
NGC6789 (2, 1.02, 1.25)
KK98-251 (2, 0.49, 0.97)
UGC05764 (2, 0.96, 0.99)
CamB (2, 0.84, 0.41)
ESO444-G084 (2, 1.05, 1.25)
DDO154 (2, 0.96, 0.96)
UGC07577 (2, 0.83, 0.72)
D564-8 (2, 0.99, 0.67)
UGC04483 (2, 0.76, 0.91)
UGCA444 (2, 0.86, 0.86)
UGC06973 (3, 1.28, 1.02)
NGC4389 (3, 0.64, 0.86)
F561-1 (3, 0.98, 0.56)
UGC02455 (3, 0.29, 0.97)
F574-2 (3, 0.99, 0.44)
F567-2 (3, 0.99, 0.73)
F563-V1 (3, 0.73, 0.49)
UGC04305 (3, 1.0, 0.46)
NGC1705 (3, 1.09, 1.09)
NGC2366 (3, 0.95, 0.81)
UGCA281 (3, 0.96, 0.96)
PGC51017 (3, 0.23, 0.96)
$V_{obs}$ [km/s]
Radius [kpc]

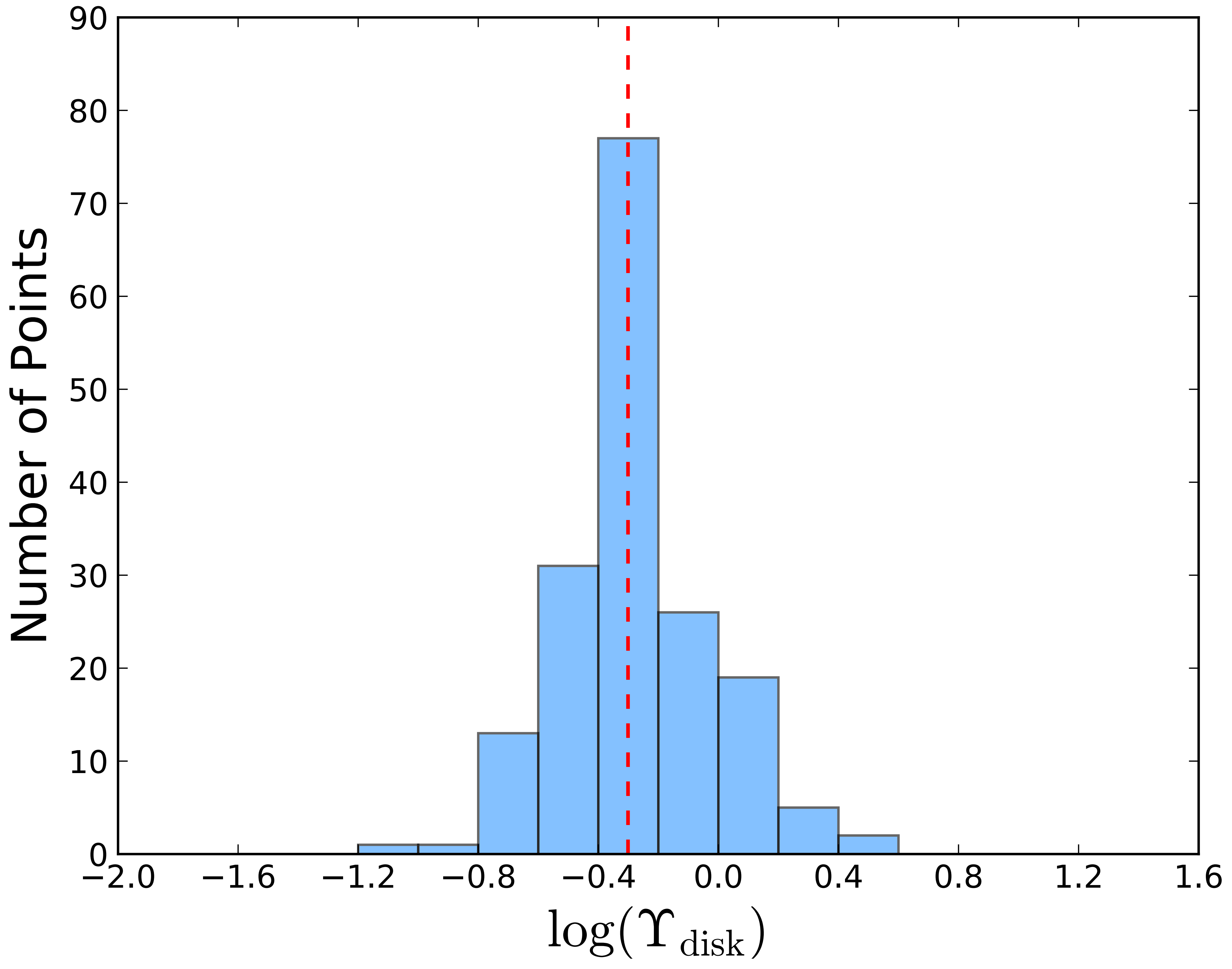
# Residuals from SPARC data
(Li et al. 2018)



No need to vary g+, which covaries with M*/L
The data constrain one or the other; not both
(Li et al. 2018)




The distribution of fitted M*/
L is reasonable



# There are striking regularities in galaxy dynamics

- Flat Rotation Curves
- Baryonic Tully-Fisher Relation
- Central Density Relation
- Renzo’s Rule
- Radial Acceleration Relation

## All the systematic properties involve a critical acceleration scale.

- Baryonic Tully-Fisher Relation

$$g_\dagger^{\mathrm{BTFR}} = 1.24 \pm 0.14 \times 10^{-10}\ \mathrm{m\,s^{-2}}$$

(McGaugh 2011)

- Central Density Relation

$$g_\dagger^{\mathrm{CDR}} = G\Sigma_\dagger = 1.27 \pm 0.05 \times 10^{-10}\ \mathrm{m\,s^{-2}}$$

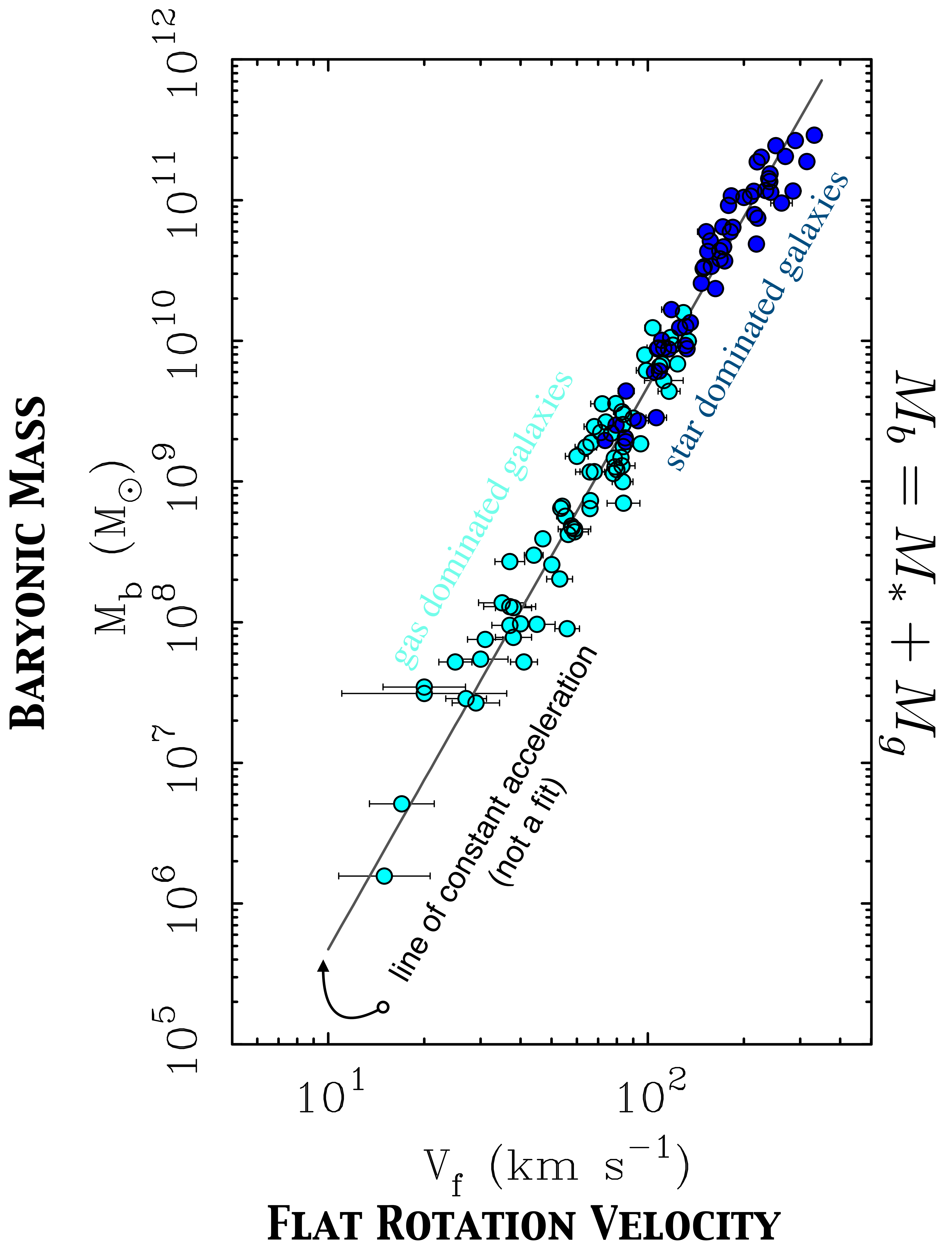
(Lelli et al. 2016)

- Radial Acceleration Relation

$$g_\dagger^{\mathrm{RAR}} = 1.20 \pm 0.02 \times 10^{-10}\ \mathrm{m\,s^{-2}}$$

(McGaugh et al. 2016)

# Baryonic Tully-Fisher Relation




Can construct a characteristic acceleration for each galaxy

$$g_{\dagger} = \frac{\zeta V_f^4}{G M_b}$$

Galaxies closely follow a single, universal acceleration.

$\zeta$ is a factor of order unity that accounts for the geometry of disk galaxies, which are not spherical cows. We adopt $\zeta = 0.8$ (McGaugh 2005).

Over 25 decades in acceleration,
galaxies only exist around 1 Å/s/s

$g_\dagger$ is a special value